Nonlinear frequency chirping of toroidal Alfvén eigenmodes in fusion plasmas with anisotropic energetic particles

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The up-down symmetry of the frequency chirping of a toroidal Alfvén eigenmode is broken due to an anisotropic pitch angle distribution of energetic particles. The evolution of the perturbed distribution function in the P_ϕ phase space only exhibits a hole structure for the cases with $\Lambda \leq 0.5$ while the clump structures are absent, where P_ϕ and $\Lambda = \mu B_0/E$ are the canonical toroidal angular momentum and the pitch angle variable respectively. The downward chirping of the mode frequency is associated with the negative drift of the phase island in the KAM surfaces or the resonant δf structures in the P_ϕ direction. On the other hand, the energetic particle distribution with larger pitch angles leads to upward chirping of the TAE frequency, which is due to the drifting of the resonant structure towards the small Λ region.

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The toroidicity-induced shear Alfvén eigenmode (TAE) can exist with its frequency inside the toroidicity-induced continuum gaps in a nonuniform toroidal magnetized plasma[1, 2]. In burning plasmas, energetic particles generated by nuclear fusion or injected by auxiliary heating method can destabilize TAE modes[3]. With the increase of mode amplitudes driven by wave-particle interactions, the phenomena of the TAE mode frequency chirping are often observed in experiments[4–8]. From magnetically confined plasma experiments, it is suggested that there are three different types of the frequency chirping of the TAE mode: the up-down chirping; mainly downward chirping; and mainly upward chirping[8]. The up-down symmetric frequency chirping was nicely explained by an analytical model based on one-dimensional (1D) bump-on-tail problem with finite collision. It was suggested that the up-down symmetric frequency chirping is a result of spontaneous hole-clump formation and drifting in the energetic particle distribution under a marginally unstable regime[9]. Up-down symmetric TAE mode frequency bifurcations were also reproduced in the reduced kinetic simulations for the marginally unstable regime[6, 10] and a kinetic-MHD hybrid simulation with sources and sinks[11]. However the physical mechanism is still not well understood on why only the mainly downward or upward chirping is observed in some experiments.

In previous theoretical and numerical studies, isotropic distribution or anisotropic distribution with $\Lambda = 0$ were used to simulate fast frequency chirping for simplicity, but for modelling of experiments with auxiliary heatings, e.g, neutral beam injection (NBI), ion cyclotron range of frequencies (ICRF) heating, etc. more realistic energetic particle distributions should be used with anisotropic distribution in pitch angle. In the current work, we will carefully study the nonlinear interaction between energetic particles with a narrow pitch angle distribution and a toroidal Alfvén eigenmode in a toroidal geometry without collision, source and sink. In particular, our investigation focuses on the n=2 TAE mode frequency chirping property associated with anisotropic energetic particle distribution with $\Lambda = 0.50$ and $\Lambda = 0.80$, where n is toroidal mode number.

We use EAC (Energetic particle and toroidal Alfvén wave interaction Code) code to investigate the interaction between energetic particles and Alfvén waves. The detailed description of code can be found in Refs. 10 and 12. In the simulation, the equilibrium fields and TAE mode structures are imported from the NOVA code[13] and are based on a zero beta shifted-circle tokamak equilibrium with major radius R_0=3m (at the magnetic axis) and minor radius a=1m. The safety factor profile is chosen to be $q = 1.1 + \Psi$, where Ψ is the normalized poloidal flux ranging from 0 at the axis to 1 at the edge. The background thermal plasma density is assumed to be uniform. The equilibrium distribution of energetic particles is chosen to be a slowing down in the particle energy, a Gaussian in the pitch angle and an exponential in the real space:

$$f_0 = \frac{1}{v^3 + v_c^3}\left[1 + \mathrm{erf}\left(\frac{v_0 - v}{0.2 v_A}\right)\right] \exp\left(-\frac{P_\phi}{c_1 e \Delta\psi}\right) \times \exp\left[-\frac{(\Lambda - \Lambda_0)^2}{(\Delta\Lambda)^2}\right], \tag{1}$$

where v_A is the Alfvén velocity at the magnetic axis, $v_0 = 1.7 v_A$, $v_c = 0.58 v_0$, P_ϕ is the canonical toroidal angular momentum, $c_1 = 0.37$, and $\Delta\psi = \psi_{\max} - \psi_{\min}$ is the range of poloidal flux ψ. The energetic ion gyroradius

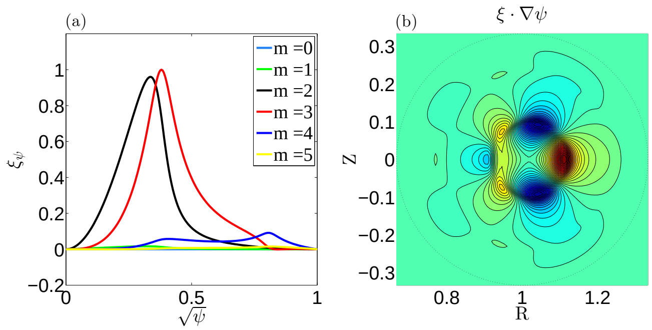

FIG. 1. (a)The n=2 TAE eigenfunction ξ_ψ and (b)Contour plot of the mode structure $\boldsymbol{\xi} \cdot \nabla\psi$.

is chosen to be $\rho_h = 0.085a$ and the finite width in the peaked Gaussian distribution $\Delta\Lambda = 0.1$.

The eigenmode structure of the n=2 TAE obtained from the NOVA code is shown in Fig. 1(a) for poloidal harmonics and in Fig. 1(b) for contours of the radial displacement $\boldsymbol{\xi}\cdot\nabla\psi$. The parameters for the equilibrium and energetic particles are chosen to be the same as those in the single mode simulation study[10].

Fig. 2 shows the amplitude and frequency evolutions of the n=2 TAE mode driven by energetic particles for the marginally unstable case with energetic particle beta $\beta_h = 0.012$, damping rate $\gamma_d = 1.4\%$, and $\Lambda_0 = 0.50$. It is clearly found that after initial saturation, the mode evolves into a quasi-steady state with irregular oscillations and the mode frequency chirps down with a single branch only. This single branch downward-chirping phenomena is quite different from the up-down symmetric chirping picture of the Berk-Breizman model. Their 1D model with fixed mode structure always leads to the symmetric up-down feature of the frequency chirping. It should be pointed out that in the previous isotropic simulations[10, 11] and the $\Lambda = 0$ anisotropic simulations[6, 10], the resulted up-down symmetric frequency chirping is consistent with the hole-clump theory of the Berk-Breizman model. Hence, it can be inferred from our simulation results that the single branch downward chirping of the TAE mode frequency observed in tokamak experiments could be induced by a narrow pitch angle distribution of energetic particles.

Since the TAE mode is mainly driven by the spatial gradient of the energetic particle density $\partial f_0/\partial P_\phi$ while the energy gradient $\partial f_0/\partial E$ plays a damping role associated with the Landau damping, we carefully examine the time evolution of the energetic particle distribution in the real space to illustrate the physical mechanism of the single branch downward chirping. Figure 3 shows the time evolution of the energetic particle distribution function versus the canonical toroidal angular momentum P_ϕ for $E = 0.49v_A^2$. Here, $P_\phi = mv_\parallel RB_\phi/B + e\psi$ can be approximately treated as the radial variable. As shown in Figure 3, near the resonance region the energetic particle distribution in the configuration space is gradually flattened during the early nonlinear phases. When the system gets into the fully nonlinear phase, the depression (hole) structure appears and drifts in the negative P_ϕ direction or gradually moves toward the center of the plasma. This depression (hole) structure is associated with the downward chirping branch. The clump structure is not observed during the whole simulation time.

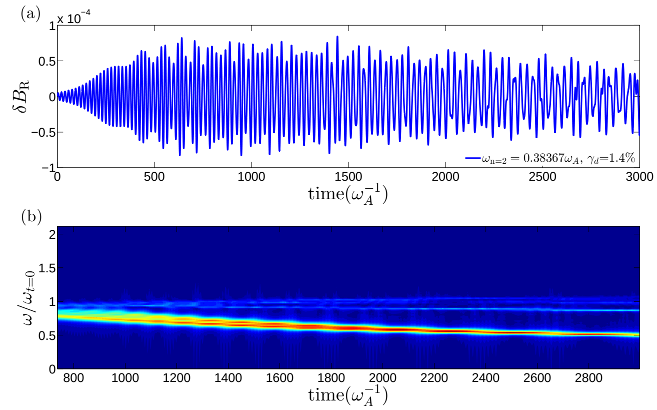

FIG. 2. (a)Time evolutions of the radial perturbed magnetic field and frequency spectrum for the n=2 TAE mode from the single pitch angle($\Lambda = 0.50$) simulation.

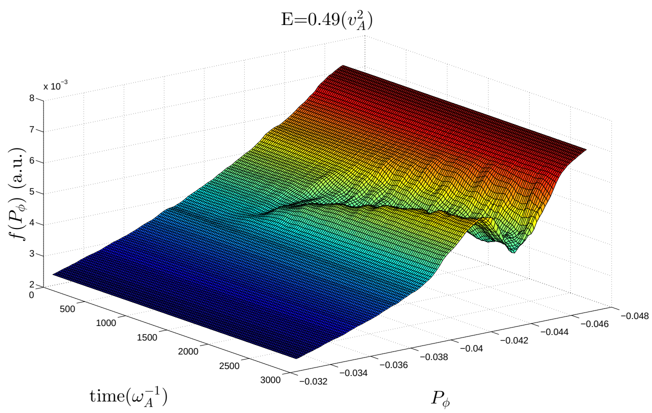

FIG. 3. Time evolution of the distribution function from single pitch angle ($\Lambda_0 = 0.50$) simulation.

The time evolution of the KAM surfaces is also plotted to investigate the phase space structure dynamics associated with the mode frequency chirping. It can be seen clearly in Fig. 4 that the main phase island or the resonant region initially appears in the center region around $P_\phi = 0.046$, then the phase island gradually drifts downward to a smaller P_ϕ region (i.e., the phase island moving towards the plasma core). As P_ϕ decreases, the toroidal orbit frequency ω_ϕ and the poloidal orbit frequency ω_θ increase. In order to satisfy the resonant condition $\dot{\Theta} = \mathrm{n}\omega_\phi + \mathrm{p}\omega_\theta + \omega = 0$, the mode frequency ω has to be reduced as P_ϕ decreases, which gives the downward chirping of the mode frequency.

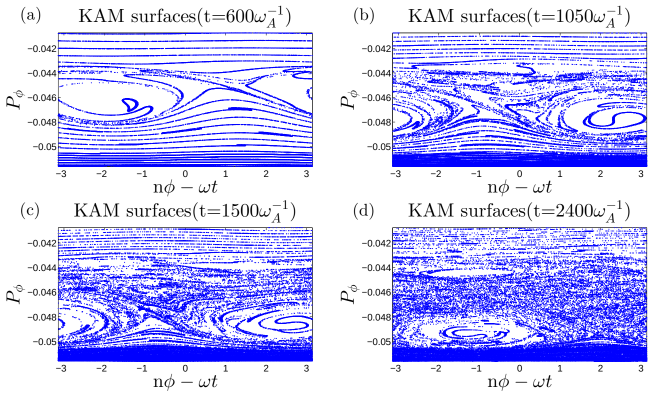

FIG. 4. Time evolution of KAM surfaces in the phase space

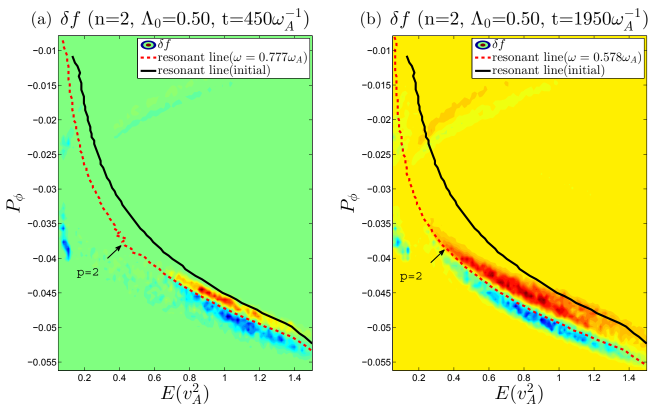

FIG. 5. Color contour plots of theδf structures at two different times. The dashed lines are numerically calculated from the resonant condition: $\dot{\Theta} = \mathrm{n}\omega_\phi + \mathrm{p}\omega_\theta + \omega = 0$, where ω_ϕ and ω_θ are including the nonlinear terms associated with the perturbed field $\delta\Phi$ and $\delta A_\parallel$.

Figure 5 shows the δf structure in the $E - P_\phi$ phase space at two different snapshots. It is evident that the regions with the enhancement and the reduction of the energetic particle density not only are expanding with time and also exhibit a downward drift in the negative P_ϕ direction. The p=2 fitting line based on the resonant condition shown in Fig. 5 is located slightly below the initial resonant line at the early time $t = 450\omega_A^{-1}$ and then appears further away from the original resonant location at the late stage $t = 1950\omega_A^{-1}$. This downward shift property of the 2D global resonant structure shift in the phase space is consistent with that found in the phase island evolution in the KAM surfaces.

When we further increase the pitch angle to $\Lambda_0 = 0.80$ which means that the distribution of energetic particles has a significant fraction of trapped particle component, it is found that the frequency chirping exhibits a upward feature. It is shown in Fig. 6(a) that the δf structure in the $E - P_\phi$ phase space is located just inside the simulation boundary (associated with the minus radius equal to zero). The hole structure associated with the

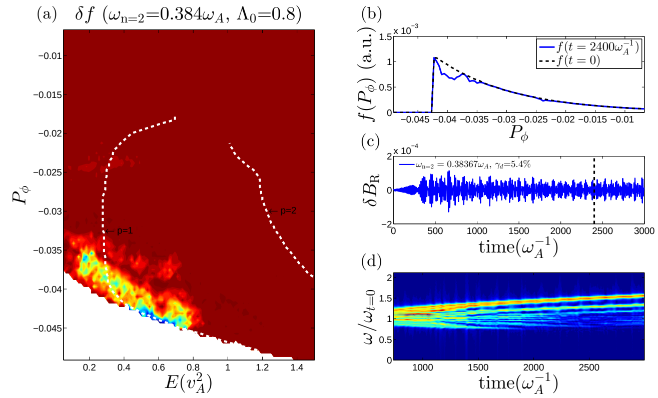

FIG. 6. (a)The δf structure in the $E - P_\phi$ phase space, (b) the distribution function in P_ϕ, (c) the time evolution the mode amplitude, and (d) the time evolution of the frequency spectrum from the anisotropic simulation with $\Lambda_0 = 0.80$.

downward branch of the frequency chirping is strongly affected by the low boundary of P_ϕ. In the early stage, the mode frequency shows both upward and downward frequency chirping as shown in Fig. 6(d), but the downward chirping branches of the mode frequency are gradually dispersed later, which is a result of the hole in the particle distribution function drifts downward and then finally moves out of the region with $\Lambda = 0.80$ across the lower boundary in the $E - P_\phi$ phase space. As shown in Fig. 6(b) the particle distribution with $\Lambda = 0.80$, it seems that the total number of energetic particles in the configuration space is not conserved. Since the simulation code is designed to guarantee the conservation of the total number of energetic particles, the missing particles in the region with $\Lambda = 0.80$ must go into other regions with different Λ.

In order to explain the particle conservation and the redistribution of energetic particles in Λ, the δf structures in the $\Lambda - E$ phase space with fixing $P_\phi = -0.045$ are given in Fig. 7. From these results, it is clear that the main perturbed regions in the δf distribution is located the lower boundary in E with a larger extension in the Λ direction. As we know, a trapped particle performs the motion with a banana orbit. When a trapped particle drifts toward the central region of the plasma, the trapped particle finally reaches the boundary of its forbidding zone associated with its lower boundary in P_ϕ. The larger pitch angle of the trapped particle is reduced and some trapped particles become passing particles, which explains why the resonant structure diffuses into lower pitch angle region as shown in Fig. 7.

It is interesting to note that at the earlier time the perturbed δf structure is mainly extending in the Λ direction at the lower boundary of E while at the later time the perturbed structure drifts away from the lower boundary of E. The number of particles with smaller pitch angles increases evidently. It should be noted that

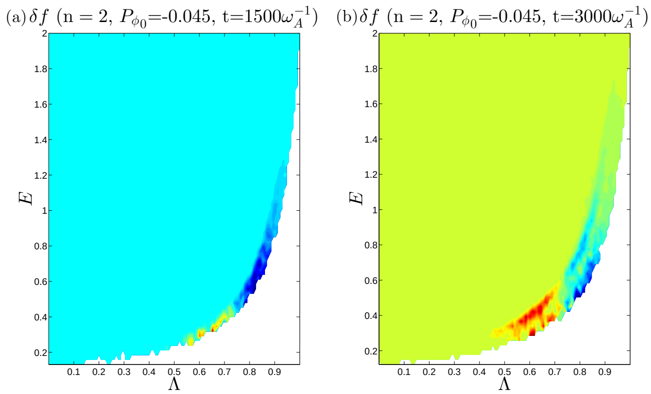

FIG. 7. Comparison of δf structure in the $\Lambda - E$ phase space at (a) t=1500ω_A^{-1} and (b) t= 3000ω_A^{-1} for the $\Lambda_0 = 0.80$ case.

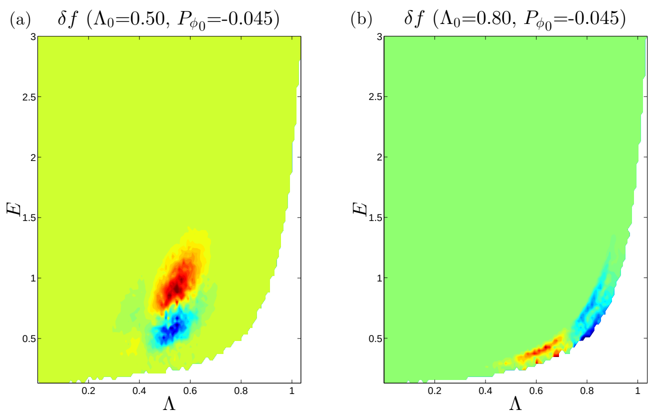

FIG. 8. Comparison of δf structure in $\Lambda - E$ phase space at $t = 2700\omega_A^{-1}$ for different anisotropic case (a)$\Lambda_0 = 0.50$ (b)$\Lambda_0 = 0.80$.

for passing particles, the resonant condition can be expressed as $\omega \propto (\mathrm{n}q - \mathrm{p})\omega_b$ because the precessional frequency is negligible, where poloidal transit frequency can be approximately given by $\omega_b = \sqrt{(1-\Lambda)E}/(2\pi q R_0)$. With fixing E and P_ϕ, a smaller pitch angle Λ is corresponding to a larger transit frequency which leads to the upward chirping of the mode frequency.

Fig. 8 shows the perturbed structure of δf in the $\Lambda - E$ phase space for the cases with $\Lambda_0 = 0.50$ and 0.80. It can be clearly seen that for the $\Lambda_0 = 0.50$ case, the mainly perturbed region δf is still localized around the initial pitch angle Λ_0 while the δf structure for the $\Lambda_0 = 0.80$ case has a larger extension along the Λ direction due to strong anisotropic effect $d\Lambda/dt$.

Under near-marginal instability regime ($\gamma_L - \gamma_d \ll \gamma_L$, $\gamma_L \approx \gamma_d$), Fig. 9 shows the linear growth rate and the chirping properties under different pitch angles of energetic particles. It is found that the linear growth rate is very sensitive to the pitch angle of energetic particles. The linear growth rate driven by deep trapped particles is almost twice that of well passing particles. For the TAE mode excited by barely passing or trapped particles, the linear growth rate is the lowest and is almost one order smaller than that associated with well passing particles. For the small pitch angle ($\Lambda_0 = 0.1 \sim 0.3$), multiple frequency bifurcations with dominant downward chirping is observed, which is similar to that in the previous $\Lambda_0 = 0$ simulation result[10]. A frequency downward chirping with a single branch is found in the parameter regime $\Lambda_0 = 0.4 \sim 0.5$. For the case of $\Lambda_0 = 0.60$, it can be treated as a transition point between passing and trapped particles because of its small linear driving force and weak frequency chirping property. When the distribution of energetic particles has a significant fraction of trapped particle component or the pitch angle is the regime $\Lambda_0 = 0.6 \sim 0.8$, the frequency chirping direction becomes mainly upward. It can be inferred that frequency chirping direction and symmetry is sensitive to the pitch angle distribution.

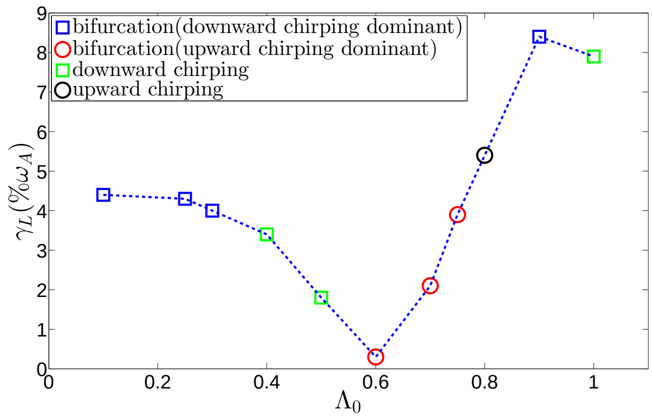

FIG. 9. Pitch angle Λ_0 versus linear growth rate γ_L. The frequency chirping properties are indicated by the different symbols.

In summary, we investigate the chirping properties of the TAE mode frequency driven by anisotropic energetic particle distribution using EAC (Energetic particle and toroidal Alfvén wave interaction Code) code. It is found that only a single upward or downward branch of the mode frequency chirping is observed. This result is quite different from the well-accepted up-down symmetric frequency chirping picture obtained from the 1D Berk-Breizman model. The evolution of the perturbed distribution function in the P_ϕ phase space shows that only a hole structure for the cases $\Lambda \leq 0.5$ is observed while the clump structures are absent. The downward chirping of the mode frequency associated with the hole is caused by the negative drift of the phase island in the KAM surfaces or the resonant δf structures in the P_ϕ direction. On the other hand, the energetic particles distribution with larger pitch angles leads to upward chirping, which is due to the drifting of the resonant structure towards the small Λ region.

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