

Effective Resistivity in Collisionless Magnetic Reconnection

Z. W. Ma^{1*} and T. Chen¹

¹Institute for Fusion Theory and Simulation, Department of Physics, Zhejiang University, Hangzhou 310027, China.

Abstract: Fast magnetic reconnection (FMR) in collisionless plasma is often attributed to the off-diagonal electron Reynolds stress, which can give rise to a large induction electric field in the reconnection region. However, in MHD simulations of FMR, it is difficult to implement the full Reynolds stress, which is kinetic in nature. In this paper, an effective, or pseudo, resistivity, which only accounts for the kinetic effects relevant to FMR, is introduced through the relation between the electric field and the current density to investigate FMR. Justification of our approach is verified by full particle-in-cell simulations, and the corresponding physics is discussed.

Keywords: Magnetic reconnection, Effective-resistivity, Collisionless plasma, PIC simulation

*) Corresponding author: zwma@zju.edu.cn

27 **1. Introduction**

28 Magnetic reconnection (MR) is an important plasma process that efficiently converts magnetic
29 energy into plasma kinetic and thermal energies [1, 2] and is believed to play crucial roles in the
30 evolution of the solar corona [3-5], geomagnetic tail [6-8], magnetosphere [9, 10], as well as
31 laboratory fusion plasmas [11, 12].

32 In collisionless plasma, a widely accepted physical mechanism for fast MR (FMR) is an
33 increase of the effect of the off-diagonal (with respect to the ambient magnetic field) electron
34 Reynolds stress in the diffusion region, which gives rise to a large reconnection electric field that
35 strongly accelerates the charged particles in the region [13, 14]. However, the Reynolds stress is
36 associated with the electron kinetic effects and can therefore not be easily implemented in fluid
37 descriptions of the plasma. In many MHD models, FMR is attributed to anomalous resistivity
38 arising from current-instability driven turbulence in the diffusion region [15, 16]. However, such
39 an anomalous resistivity often involves artificially given (usually constant) turbulence level or is
40 only current dependent. Speiser [17] introduced an effective conductivity for studying
41 collisionless FMR without invoking turbulence. However, the model does not include the details
42 of the particle motion that give rise to the effective conductivity, so that it is not clear how particles
43 are accelerated.

44 In this paper, we introduce an effective, or pseudo resistivity for considering collisionless
45 FMR. The PR is obtained by replacing the collision mean-free-time in the traditional collisional
46 drag force with the transit time of electrons in the small diffusion region around the X point of the
47 MR. The transit time is obtained by following the motion of test electrons in the region and, as to
48 be expected, is space and time dependent. Validity of our ad hoc model is confirmed by full
49 particle-in-cell (PIC) simulation.

50 The rest of this paper is as follows. Section 2 presents our effective resistivity model and its
51 properties. A theoretical argument justifying the effective resistivity is also given. Section 3
52 presents the corresponding PIC simulation. Section 4 compares the results from the model and the
53 simulation. Section 5 gives a summary of our work.

54

55 **2. Physical Mechanisms and Model Description**

56 Classically, plasma resistivity arises from inter-particle collisions that lead to momentum and
57 energy exchange between the colliding particles. It therefore depends on the collision frequency or

58 the mean free path. In collisionless MR, particles in the small diffusion region around the X point
 59 experience strong electric and magnetic forces. A particle is first decelerated, and then accelerated
 60 as it enters and leaves the diffusion point due to the bent magnetic fields and the induction electric
 61 fields. It thereby exchanges energy with the fields. The interaction can thus lead to a local effective
 62 resistivity around the X point in a region of the order of the electron inertial length. The scenario is
 63 roughly similar to what occurs in a binary collision, namely the interaction takes place in a very
 64 small region around the center of mass or a massive particle, analogous to the X point in MR.

65 We consider the dynamics of a charge particle along an X line (assumed to be in the z direction,
 66 perpendicular to the MR plane) of the diffusion region, where the magnetic field is nearly zero and
 67 the induction electric field E_z is strong. The change in the velocity of the particle can be written as
 68 [17]

$$69 \quad \delta v_z = qE_z \delta t / m, \quad (1)$$

70 where q and m are the particle charge and mass, respectively, and δt is the transit time of the
 71 particle. Accounting for all the particles in the diffusion region, the corresponding change in the
 72 local current density is

$$73 \quad \delta J_z = nq\delta v_z = nq^2 E_z \delta t / m, \quad (2)$$

74 where n is the local particle density. Thus, one can define an effective resistivity

$$75 \quad \eta \equiv E_z / J_z = m / nq^2 \delta t, \quad (3)$$

76 which is valid only near the X point. One must however still determine the particle density and the
 77 transit time.

78 To model the diffusion region in collisionless MR, we consider a two-dimensional (2D) plane
 79 (x, y) with the X line lying in the perpendicular, or z , direction at $(0, 0)$. The vacuum magnetic
 80 and electric fields in this region can be approximated by

$$81 \quad \mathbf{B} = B_0 \left(\frac{y\hat{x}}{L_y} + \frac{xy}{L_x} \right), \quad (4)$$

$$82 \quad \mathbf{E} = -E_0 \hat{z}, \quad (5)$$

83 where B_0 and E_0 are positive constants, L_x and L_y are the local characteristic lengths of B_y
 84 and B_x in the x and y directions, respectively. That is, the induction electric field remains uniform
 85 in this region, and the magnetic field increases with the distance away from the X line (or X point

86 in the (x, y) plane).

87 We first consider a general case of the motion of a test electron in the diffusion region.

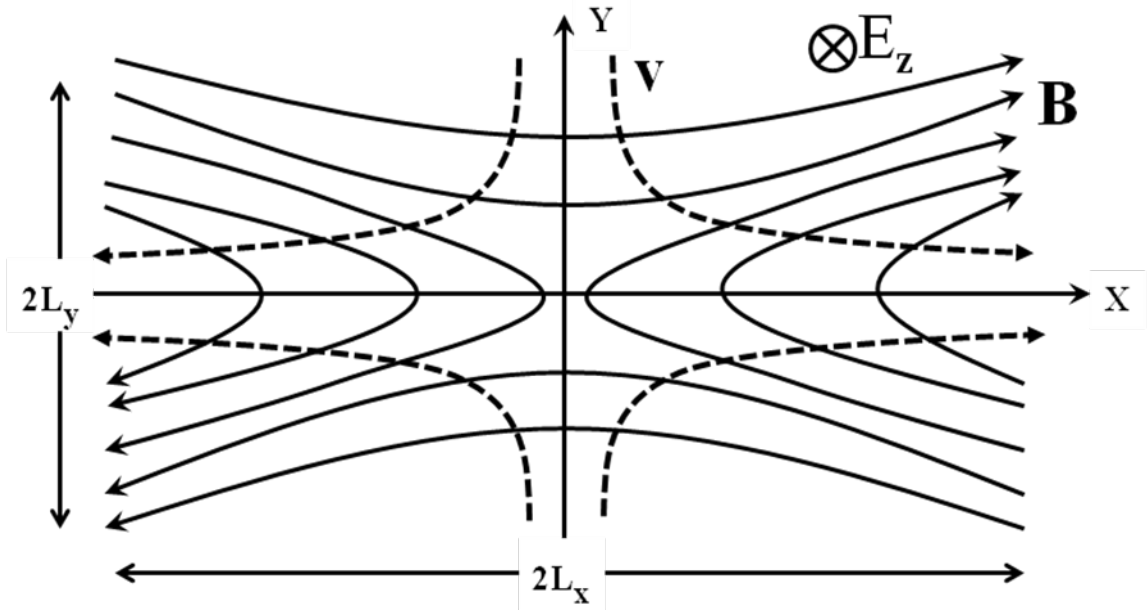
88 Initially, the electron is at (x_0, y_0) and its velocity components are $v_{x0} > 0$, $v_{y0} > 0$, $v_{z0} > 0$.

89 The region considered is $2L \times 2L$. The fields and other parameters are illustrated in Figure 1.

90 The configuration here differs from that of Ref. 17, where the diffusion region is

91 one-dimensional. It is similar to that in Ref. 18, except that here more details are involved, such

92 that the FMR process can be better understood.



93

94 Figure 1. Schematics of magnetic field lines and electron trajectories in the 2D

95 diffusion region. The X line in the z direction is at $(0,0)$.

96

97 An electron inside this box will be driven by the Lorentz and electric forces:

98
$$F_x = -qv_z B_0 x / L_x, \quad (6)$$

99
$$F_y = qv_z B_0 y / L_y, \quad (7)$$

100
$$F_z = q(E_0 + v_x B_0 x / L_x - v_y B_0 y / L_y), \quad (8)$$

101 The trajectory of the electron is then given by

102
$$x(t) = x_0 + \int_0^t [v_{x0} - \int_0^{t'} \frac{q}{mL_x} v_z(t'') B_0 x(t'') dt''] dt' \quad (9)$$

103
$$y(t) = y_0 + \int_0^t [v_{y0} + \int_0^{t'} \frac{q}{mL_y} v_z(t'') B_0 y(t'') dt''] dt' \quad (10)$$

104 where m is now the electron mass. The electron velocity is given by

$$105 \quad v_x(t) = v_{x0} - \int_0^t \frac{q}{mL_x} v_z(t') B_0 x(t') dt', \quad (11)$$

$$106 \quad v_y(t) = v_{y0} + \int_0^t \frac{q}{mL_y} v_z(t') B_0 y(t') dt', \quad (12)$$

$$107 \quad v_z(t) = v_{z0} + \int_0^t \frac{q}{m} [E_0 + v_x(t') B_0 x(t') / L_x - v_y(t') B_0 y(t') / L_y] dt'. \quad (13)$$

108 The initial and boundary conditions are $x(0) = x_0$, $x'(0) = v_{x0}$, $y(0) = y_0$, $y'(0) = v_{y0}$. Since
 109 the transit time of the electron in the small diffusion region is very short [19], we can assume that
 110 during the transient time the change δv_{z0} of v_{z0} satisfies $\delta v_{z0} \ll v_{z0}$ or $v_z(t')$ is constant in
 111 Eqs. (9)-(12). As to be numerically verified in Section 4, the corresponding change in v_z is even
 112 smaller. Eqs. (9)-(13) then yield

$$113 \quad x(t) = x_0 \cosh(t / \tau_d) + v_{x0} \tau_d \sinh(t / \tau_d), \quad (14)$$

$$114 \quad y(t) = y_0 \cos(t / \tau_d) + v_{y0} \tau_d \sin(t / \tau_d), \quad (15)$$

$$115 \quad v_x(t) = x_0 \sinh(t / \tau_d) / \tau_d + v_{x0} \cosh(t / \tau_d), \quad (16)$$

$$116 \quad v_y(t) = -y_0 \sin(t / \tau_d) / \tau_d + v_{y0} \cos(t / \tau_d), \quad (17)$$

$$117 \quad v_z(t) = v_{z0} + \frac{q}{m} E_0 t - \frac{1}{2v_{z0}} [(v_{x0}^2 + x_0^2 / \tau_d^2) \sinh^2(t / \tau_d) + v_{x0} x_0 \sinh(2t / \tau_d) / \tau_d] + \frac{1}{2v_{z0}} [(v_{y0}^2 - y_0^2 / \tau_d^2) \sin^2(t / \tau_d) + v_{y0} y_0 \sin(2t / \tau_d) / \tau_d], \quad (18)$$

118 where $\tau_d = \sqrt{\frac{mL_x}{qv_{z0}B_0}}$ is a characteristic time for electrons in the diffusion region. From Eqs. (14)
 119 and (15), it is indicated that the electron oscillates in the y direction, but it is accelerated in the x
 120 direction. If we assume τ_1 to be the time when the electron leaves the box in the x direction,
 121 from Eq. (14) and $x(\tau_1) = L$ we get

$$122 \quad \tau_1 = \tau_d \ln \left\{ \left[L + \sqrt{L^2 + v_{x0}^2 \tau_d^2 - x_0^2} \right] / (v_{x0} \tau_d + x_0) \right\}. \quad (19)$$

123 In order to see the acceleration process in more detail, we reasonably assume that thermal
 124 effects can be neglected. Thus, the initial in-plane velocity of an electron in the diffusion region is

125 nearly zero. Considering the separation of the electron motion in the x and y direction, we only
 126 need to examine the electron motion in the x direction. With assumption $L = c / 2\omega_{pe}$, the
 127 transit time then becomes

$$128 \quad \tau = \tau_d \ln \frac{c / 2\omega_{pe} + \sqrt{(c / 2\omega_{pe})^2 - x_0^2}}{x_0} . \quad (20)$$

129 Since x_0 can be anywhere inside the box, the averaged transit time is

$$130 \quad \bar{\tau} = \int_0^L \tau dx_0 / L = \frac{\pi}{2} \tau_d , \quad (21)$$

131 so that the effective resistivity is given by

$$132 \quad \eta_e = \frac{2}{\pi} \sqrt{\frac{m v_z B_0}{q^3 n^2 L_x}} . \quad (22)$$

133 If the particle is farther away from the X line, v_{x0} cannot be ignored, and the transit time is

$$134 \quad \bar{\tau} = \frac{1}{L} \int_0^L \tau dx_0 = \tau_d \left[\tan^{-1} \frac{L}{\delta} + \frac{\delta}{L} \ln \frac{(L + \delta)\delta}{L^2 + \delta^2} \right] , \quad (23)$$

135 where $\delta = v_{x0} \tau_d$. The corresponding effective resistivity is

$$136 \quad \eta_{general} = \frac{m}{nq^2 \tau_d} \frac{1}{\tan^{-1} \frac{L}{\delta} + \frac{\delta}{L} \ln \frac{(L + \delta)\delta}{L^2 + \delta^2}} . \quad (24)$$

137 Outside the region, δ is much larger (more precisely, v_{x0} is much larger and v_{z0} is smaller),
 138 making effective resistivity much smaller there. Since v_{z0} is smaller outside the region, the
 139 assumption $\delta v_z \ll v_{z0}$ may breakdown. That is, our interaction model is applicable only in the
 140 small diffusion region around the X line.

141 If we include the ion motion, the current in Eq. (2) can be rewritten as

$$142 \quad \mathbf{J}_{z\delta} = nq^2 \mathbf{E}_z (\boldsymbol{\tau}_e) / m_e \tau_i / m_i . \quad (25)$$

143 where we have assumed that the plasma is quasi-neutral. Eq. (3) then becomes

$$144 \quad \eta_{tot} = \frac{m_e m_i}{nq^2} \frac{1}{m_i \tau_e + m_e \tau_i} . \quad (26)$$

145 Substituting Eq. (22), we get

146
$$\eta_{tot} = \eta_e \left(1 + \sqrt{\frac{m_e v_{ze}}{m_i v_{zi}}} \right)^{-1}, \quad (27)$$

147 where v_{ze} and v_{zi} are the local electron and ion velocities in the z direction.

148

149 3. Simulation Model

150 We have performed 2.5D PIC simulations for electrons on the (x,y) plane by assuming
 151 $\partial/\partial z = 0$. For convenience, we use the charge-conservation scheme (CCS) [20] instead of solving
 152 the Poisson equation, and the finite difference time domain method (FDTD) to solve the other
 153 Maxwell's equations. The particles are driven by the electric and Lorenz forces and the
 154 corresponding equations used in the PIC simulations are

155
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (28)$$

156
$$\nabla \times \mathbf{B} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}, \quad (29)$$

157
$$\frac{d\mathbf{p}_j}{dt} = q_j(\mathbf{E} + \mathbf{v}_j \times \mathbf{B}), \quad (30)$$

158 where c is the light speed, $\mathbf{J} = n_i q_i \mathbf{V}_i + n_e q_e \mathbf{V}_e$, \mathbf{V}_j ($j = i, e$) is the bulk velocity of species j , \mathbf{v}_j
 159 and $\mathbf{p}_j = m_j \mathbf{v}_j$ are the particle velocity and momentum, respectively. The variables are
 160 normalized as follows: $\mathbf{x}/d_{i0} \rightarrow \mathbf{x}$, $(\mathbf{V}_j, \mathbf{v}_j)/v_{Ai0} \rightarrow (\mathbf{V}_j, \mathbf{v}_j)$, $\omega_{ci0} t \rightarrow t$, $\mathbf{B}/B_0 \rightarrow \mathbf{B}$,
 161 $\mathbf{E}/E_0 \rightarrow \mathbf{E}$, $\mathbf{J}/J_0 \rightarrow \mathbf{J}$, $n/n_0 \rightarrow n$, $\mathbf{p}_j/m_e v_{Ai0} \rightarrow \mathbf{p}_j$, where $d_{i0} = c/\omega_{pi0} = c/\sqrt{n_0 q_i^2/\mu_0 m_i}$,
 162 $v_{Ai0} = B_0/\sqrt{\mu_0 n_{i0} m_i}$, $\omega_{ci0} = q_i B_0/m_i$, $E_0 = v_{Ai0} B_0$, and $J_0 = n_0 q_0 v_{Ai0}$.

163 Our 2D simulation domain is $-D_x/2 \leq x \leq D_x/2$, $-D_y/2 \leq y \leq D_y/2$, where $D_x = 12.8d_{i0}$,
 164 $D_y = 6.4d_{i0}$, $dx = dy = 0.01d_{i0}$. Closed boundary condition is adopted in the y direction and
 165 periodic boundary condition is used in the x direction. The time step is $\omega_{ci0} \Delta t = 0.0002$, and the
 166 duration of the simulations is $\omega_{ci0} t = 40$, corresponding to 200,000 time steps. Nearly 82 million
 167 particles for each species are used in this simulation. We also assume $v_{Ai0}/c = 0.05$ and $\beta = 0.2$.
 168 The ion-to-electron mass ratio $M_{ie} = m_i/m_e$ is from 25 to 400, and the ion-to-electron initial

169 temperature ratio $T_{ie} = T_i / T_e = 5$.

170 We shall use the Harris equilibrium as the initial configuration. The initial magnetic field is
171 given by

$$172 \quad B_x = B_0 \tanh(y / b_0), \quad B_y = B_z = 0, \quad (31)$$

173 and the initial density profile is

$$174 \quad n = n_0 / \cosh(y / b_0)^2 + n_b, \quad (32)$$

175 where $B_0 = 1.0$, $b_0 = 0.5$, $n_0 = 1.0$, $n_b = 0.2$, and b_0 is the width of the current sheet with the
176 current intensity given by

$$177 \quad I_z = B_0 / b_0 \cosh(y / b_0)^2. \quad (33)$$

178 In order to shorten the initial stage in the simulation, we impose a small periodic excitation in
179 the initial system, such that Eq. (31) and (33) become

$$180 \quad B_x = B_0 \tanh(y / b_0) + \varepsilon \pi \cos(2\pi x / D_x) \sin(\pi y / D_y) / D_y, \quad (34)$$

$$181 \quad B_y = -2\varepsilon \pi \sin(2\pi x / D_x) \cos(\pi y / D_y) / D_x, \quad B_z = 0, \quad (35)$$

$$182 \quad I_z = B_0 / b_0 \cosh(y / b_0)^2 + \varepsilon \pi^2 \cos(2\pi x / D_x) \cos(\pi y / D_y) (1 / D_y^2 + 4 / D_x^2), \quad (36)$$

183 where $\varepsilon = 0.01$.

184 Pressure balance yields

$$185 \quad P + \frac{B^2}{2} = (1 + \beta) \frac{B_0^2}{2}, \quad (37)$$

186 where P and B are the local thermal pressure and magnetic field, $\beta = P / B^2$, and P and B are
187 normalized by $B_0^2 / 2\mu_0$.

188

189 **4. Numerical Results and Comparison**

190 First, we consider $M_{ie} = 25$, i.e., the same as that for the Geospace Environment Modeling
191 (GEM) MR challenge [13]. Figure 2 shows the evolution of the induction electric field and
192 reconnected magnetic flux at the X line. We can see that MR occurs at $t = 20 - 32$, followed by a
193 nonlinear stage of the process. Figure 3 shows the current J_z and the magnetic field lines at

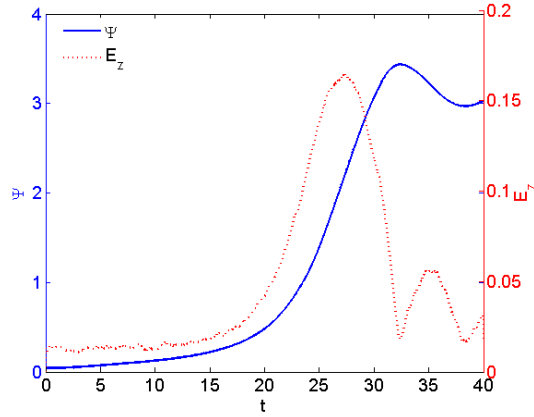
194 different times. During the MR, the current sheet is compressed around the X line, and then
 195 separated into two parts.

196 The electric field in the z direction as from the 2D electron fluid momentum equation is

$$197 \quad E_z = -\frac{m_e}{e} \left[\frac{\partial V_{ez}}{\partial t} + \mathbf{V}_e \cdot \nabla V_{ez} \right] - \frac{1}{n_e e} \left(\frac{\partial \Pi_{exz}}{\partial x} + \frac{\partial \Pi_{eyz}}{\partial y} \right) - (\mathbf{V}_e \times \mathbf{B})_z, \quad (38)$$

198 where the pressure tensor is given by $\Pi_e = m_e \int (\mathbf{v} - \mathbf{V})(\mathbf{v} - \mathbf{V}) f_e(\mathbf{v}) d\mathbf{v}$, where $f_e(\mathbf{v})$ is the
 199 electron velocity distribution function. Figure 4 shows the contribution of each term in Eq. (38) in
 200 the current sheet when MR occurs. We see that the sum of the off-diagonal pressure tensors leads
 201 to 80% of the induction electric field, similar to that found in Ref. 13.

202

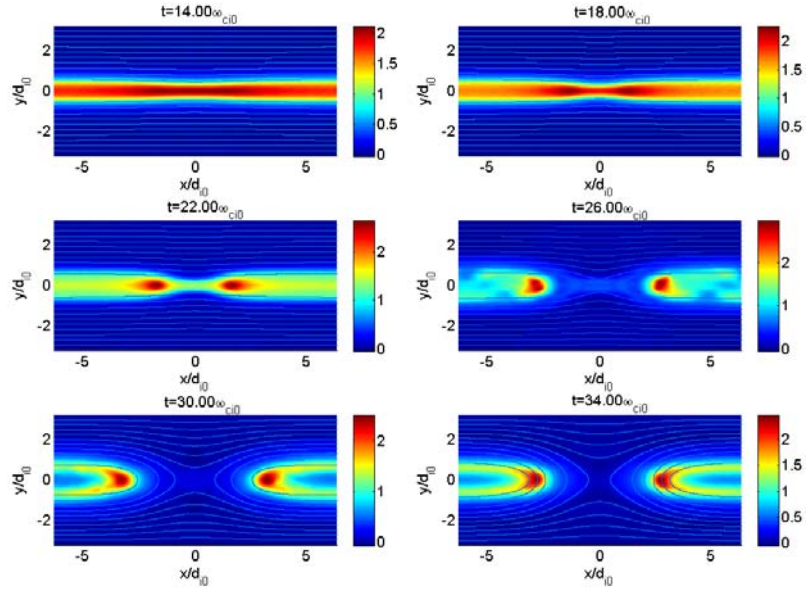


203

204

205

Figure 2 Evolution of reconnecting magnetic flux and the induction electric field on the X line. Here ψ is normalized by $B_0 c / \omega_{pi0}$, and E_z is normalized by $B_0 v_{Ai0}$.



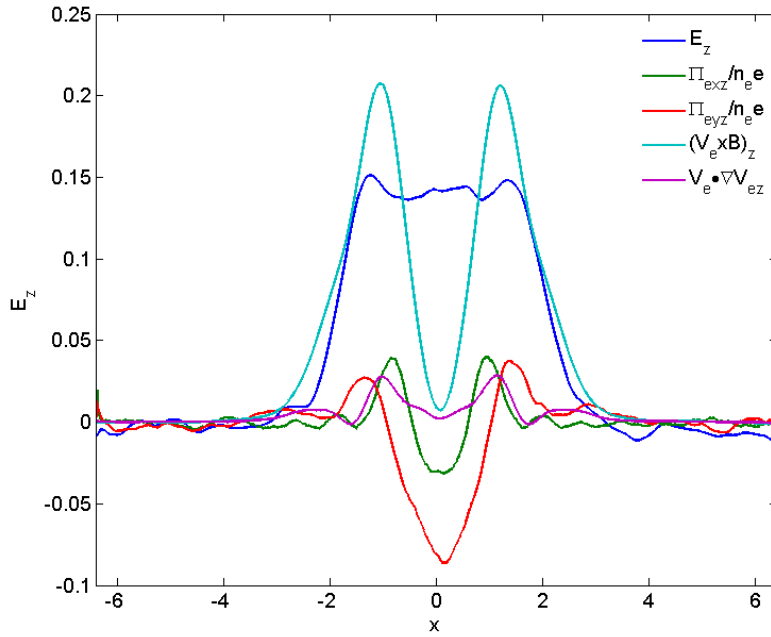
206

207

208

209

Figure 3 The distribution of the current density in the out of plane direction superposed with magnetic field lines at different simulation times.



210

211

212

213

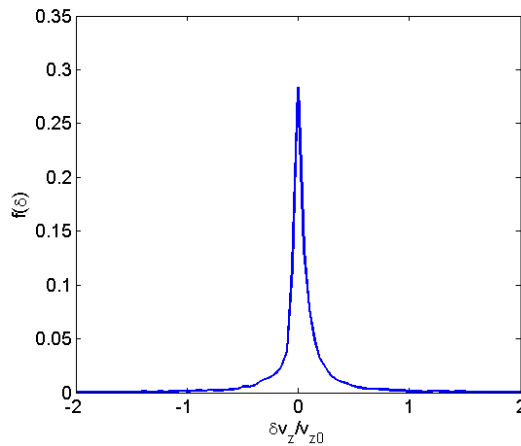
214

215

Figure 4 Contribution of each term from Eq. (38) in the current sheet along the x direction (at $y=0$) at the peak reconnection time $t=26$.

In order to verify the assumption $\Delta v_z \ll v_{z0}$ used in Section 2, we compare the speeds of the

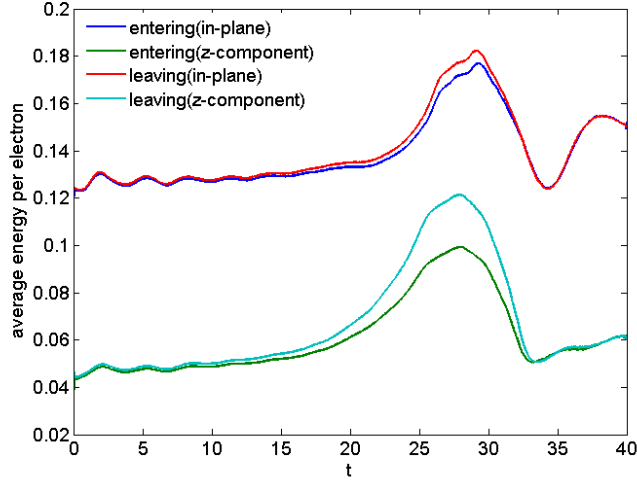
216 particles which are just before entering and after leaving the electron diffusion region. Figure 5
 217 shows the distribution of electron velocity variation $f(\delta v_z)$ during $t=28$ to 29 in the peaked MR
 218 period. Here, $\delta v_z = v_{z1} - v_{z0}$, where v_{z0} and v_{z1} are the electron velocity when it is just before
 219 entering and after leaving the diffusion region, respectively. The mean of this distribution is
 220 0.0470, and the variance is 0.2937. Electrons with $|\delta v_z/v_{z0}| \leq 0.2$ constitute 78.33% of the total
 221 ejected electrons, implying that most of the electrons suffer little change in the z -direction velocity.
 222 In the earlier MR stage, such as from $t = 18$ to 19, the percent of electrons with $|\delta v_z/v_{z0}| \leq 0.2$ is
 223 90.96%. Thus, the velocity changes Δv_z for the majority of electrons are limited when they stay
 224 in the smaller diffusion region, so that our assumption in the derivation of the effective resistivity
 225 is justified. It is clearly also valid for ions, whose velocities are much less.
 226



227
228 Figure 5. Distribution of δv_z during $t=28$ to 29.

229
 230 Figure 6 shows the time evolution of the average energy per electron for different
 231 components. "entering" and "leaving" means for electrons just before entering and after leaving
 232 the electron diffusion region, respectively. We can see that the difference of the average energy
 233 per electron in the in-plane component for the "entering" and "leaving" electrons is relatively
 234 small at all times, which agrees with our assumption that the in-plane electric field is nearly zero
 235 in Eq. (5). The energy gain of electrons in the z component increases with development of MR
 236 during the period in the diffusion region. The energy gain is about 20% when MR reaches its
 237 peak, which means the net change of the velocity in the z direction is about 10%. Therefore, it is
 238 further confirmed that our assumption $\Delta v_z \ll v_{z0}$ is valid. The energy gain of electrons

239 disappears after the fast reconnection stage ends. This behavior can be attributed to the fact that in
 240 the period of FMR, the induced electric field in the z direction is strong around the X line. On the
 241 other hand, the magnetic field is weak in this region and they are not sufficient to alter the
 242 trajectory of the hot electrons, which leads to electrons continuously accelerated in the z direction.
 243



244

245 Figure 6. Time evolution of the average energy per electron for different
 246 components. "entering" and "leaving" means for electrons just before
 247 entering and after leaving the diffusion region, respectively.

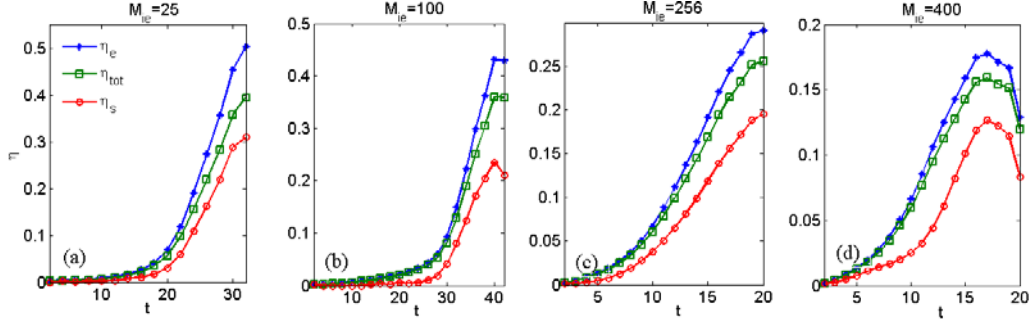
248

249 Figure 7 shows time evolutions of the effective resistivity in the electron diffusion region for
 250 different mass ratios M_{ie} . Since larger M_{ie} corresponds to a longer linear growth phase, we use
 251 larger initial excitation ($\varepsilon = 0.05$) for $M_{ie}=256$ and 400 to shorten simulation time. The other
 252 parameters remain the same. The theoretical η_e and η_{tot} , as well as the simulation result
 253 $\eta_s = E_z / J_z$, are all normalized by $B_0 / (n_0 q_0)$.

254 We can see that as MR enters into the fast reconnection phase, the effective resistivity
 255 exhibits quickly enhancement and the tendencies are almost the same for all the three effective
 256 resistivities. With increasing M_{ie} , not only do the peak values of the effective resistivity decrease,
 257 but also the difference between η_e and η_{tot} decreases, as can also be seen in Eq. (27). Since the
 258 PIC simulation involves larger noise level compared to the MHD simulation, the resistivity η_s
 259 directly from simulation fitting the modeled effective resistivity are reasonably well.

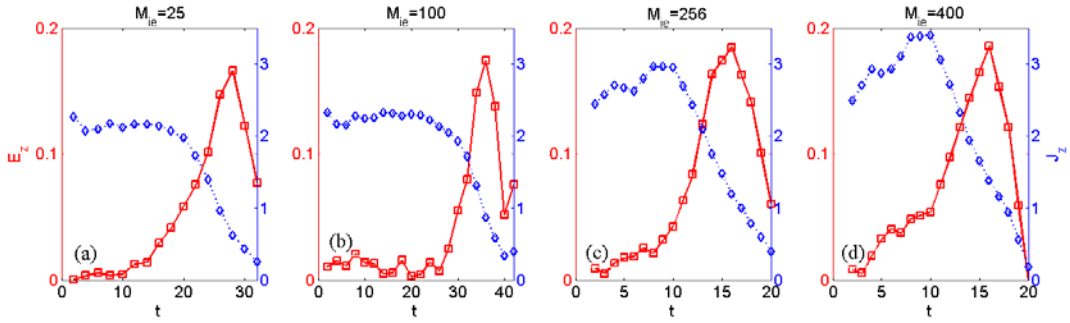
260

261



262
 263 Figure 7 Time evolutions of the effective resistivity from the model and PIC simulation for different M_{ie} 's. The
 264 time interval between the points is $dt = 2$.
 265

266 We also present the electric field E_z and the current density J_z in the z direction directly
 267 from the simulation in Figure 8. It is found that the changes of E_z and J_z are not in phase
 268 with the effective resistivity η_s . The effective resistivity further increases after the reconnection
 269 electric field E_z decreases. This is because the current density J_z always decreases and the
 270 decreasing speed is proportional to the electric field E_z , which is mainly attributed to the
 271 decrease of the electron density in the diffusion region.
 272



273
 274 Figure 8 Time evolutions of the electric field and the current density in the z direction for different M_{ie} 's. The
 275 time interval between the points is $dt = 2$.
 276

277 The inertial conductivity σ_i from Eq. (14) of Ref. [17] is

278

$$\sigma_i = \frac{ne^2}{m} \tau = \frac{ne^2}{m} \frac{L}{v} = \left(\frac{Ln^2 e^3}{2mE} \right)^{1/2}, \quad (39)$$

279 where L is the length of the accelerating region, v is the particle velocity and E is the electric
 280 field [17]. Here m we take as the electron mass. It is evident that this model is not able to

281 implement into MHD because we first have to know the resistivity to calculate the electric field.
282 Another problem is that the estimated resistivity from this model is about 5 times larger than the
283 numerical results from $\eta_s = E_z / J_z$.

284 **Summary**

285 This paper introduces a simple model for energy conversion in FMR. Using the simple
286 equation $E = \eta J$, we define a space-time dependent effective resistivity η that can be obtained
287 from numerical solutions of test electron trajectories in the diffusion region. We find that η rises
288 with development of MR, reaching its maximum when MR reaches its peak. It then falls and
289 finally reaches a low value. The results from the model agree fairly well with that from the PIC
290 simulations. It is also found that with the increase of M_{te} , the peak value of effective resistivity
291 tends to be smaller.

292 We wish this paper can give a new view on anomalous resistivity in MHD simulation, whose
293 idea is derived from a collisionless fast reconnection model, and the physical meaning is
294 reasonable.

295 **Acknowledgements**

296 We thank Prof. M. Y. Yu for many useful advices and polishing on this paper, and also thank H.
297 W. Zhang for many useful discussions. This work is supported by the National Natural Science
298 Foundation of China under Grant No. 41474123, National Magnetic Confinement Fusion
299 Science Program of China under Grant No. 2013GB104004 and 2013GB111004, the Special
300 Project on High-performance Computing under the National Key R&D Program of China No.
301 2016YFB0200603, Fundamental Research Fund for Chinese Central Universities.

302

303 **References**

- 304 [1] J. W. Dungey, Interplanetary magnetic field and the auroral zones, *Physical Review Letters*, 6,
305 47 (1961).
- 306 [2] V. M. Vasyliunas, Theoretical models of magnetic field line merging, *Reviews of Geophysics*,
307 13, 303 (1975).
- 308 [3] R. A. Kopp and G. W. Pneuman, Magnetic reconnection in the corona and the loop prominence

309 phenomenon, *Solar Physics*, 50 (1), 85 (1976).

310 [4] M. Hesse, T. G. Forbes and J. Birn, On the relation between reconnection magnetic flux and
311 parallel electric fields in the solar corona, *The Astrophysical Journal*, 631, 1227 (2005).

312 [5] M. H. Hsieh, C. L. Tsai, Z. W. Ma and L. C. Lee, Formation of fast shocks by magnetic
313 reconnection in the solar corona, *Physics of plasma*, 16 (9), 092901 (2009).

314 [6] J. Birn and M. Hesse, the substorm current wedge and field-aligned currents in MHD
315 simulations of magnetotail reconnection, *Journal of geophysical research*, 96, 1611 (1991).

316 [7] M. Oieroset, T. D. Phan, M. Fujimoto, R. P. Lin and R. P. Lepping, In situ detection of
317 collisionless reconnection in the earth's magnetotail, *Nature*, 412, 6845 (2001).

318 [8] A. Bhattacharjee, Impulsive magnetic reconnection in the earth's magnetotail and the solar
319 corona, *Annual review of astronomy & astrophysics*, 42 (1), 365 (2004).

320 [9] X. H. Deng and H. Matsumoto, Rapid magnetic reconnection in the earth's magnetosphere
321 mediated by whistler waves, *Nature*, 410 (6828), 557 (2001).

322 [10] M. L. Goldstein, W.H. Matthaeus and J. J. Ambrosiano, Acceleration of charged particles in
323 magnetic reconnection: solar flares, the magnetosphere, and solar wind, *Geophysical research
324 letters*, 13 (3), 205 (1986).

325 [11] H. P. Furth, P. H. Rutherford and H. Selberg, Tearing mode in the cylindrical tokamak, *Physics
326 of fluids*, 16, 1054 (1973).

327 [12] S. Wang and Z. W. Ma, Influence of toroidal rotation on resistive tearing modes in tokamaks,
328 *Physics of plasmas*, 22 (12), 2251 (2015).

329 [13] P. L. Pritchett, Geospace environment modeling magnetic reconnection challenge:
330 simulations with a full particle electromagnetic code, *Journal of geophysical research*, 106 (A3),
331 3783 (2001).

332 [14] H. J. Cai and L. C. Lee, The generalized Ohm's law in collisionless magnetic reconnection,
333 *Physics of plasmas*, 4, 509 (1997).

334 [15] M. Ugai, Self-consistent development of fast magnetic reconnection with anomalous plasma
335 resistivity, *Plasma physics and controlled fusion*, 26 (12B), 1549 (1984).

336 [16] L. M. Mal'ushkin, T. Linde and R. M. Kulsrud, magnetic reconnection with anomalous
337 resistivity in two-and-a-half dimensions. I. Quasistationary case, *Plasma of physics*, 12 (10),
338 102902 (2005).

- 339 [17] T. M. Speiser, Conductivity without collisions or noise, *Planetary &Space Science*, 18 (4),
340 613, 1970.
- 341 [18] R. W. Moses, J. M. Finn and K. M. Ling, Plasma heating by collisionless magnetic
342 reconnection: analysis and computation, *Journal of geophysical research*, 98 (A3), 4013 (1993).
- 343 [19] J. S. Wagner, P. C. Gray, J. R. Kan, T. Tajima and S. I. Askasofu, Particle dynamics in
344 reconnection field configurations, *Planetary &Space Science*, 29 (4), 391, 1981.
- 345 [20] J. Villasenor and O. Buneman, Rigorous charge conservation for local electromagnetic field
346 solvers, *Computer Physics Communications*, 69, 306 (1992).
- 347