Energetic particle physics and optimization methods for stellarators

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Energetic particle (EP) transport in stellarators

• Energetic particle confinement: remains important question for existing and future 3D systems
  – Heating efficiency, ignition margin, PFC damage
  – Very difficult to achieve good alpha particle confinement for stellarator reactors – not at tokamak levels yet

• EP confinement improvement examples
  – LHD inward shifted discharges
  – J-optimization, QO systems
  – Large aspect ratio IPP designs
  – NCSX design and ARIES-CS study

• Should consider both classical and turbulent losses

EP loss channels

Classical orbit loss

Instability-induced loss (Alfvén modes)
Outline

- Introduction
  - Stellarator optimization
  - Confinement optimization targets

- Energetic particle confinement

- Energetic particle stability
Synthesis of 3D configurations

- Coils ➔ outer magnetic surface shape ➔ physics properties
- 3D shapes open up very large design space: effectively ~ 40 independent parameters (A. Boozer, L. P Ku, 2010) based on SVD analysis
- Axisymmetric tokamak shape parameters: \( R_0/a, \kappa, \delta \)
- Thought experiment: quantize shape parameters into 10 levels
  - \( 10^3 \) 2D configurations vs. \( 10^{40} \) 3D configurations => “combinatorial explosion”

**STELLOPT:**

1. Adjust plasma boundary or coil geometry shape
2. Solve 3D equilibrium
3. Levenberg-Marquardt or other method used to minimize \( \chi^2 \)
4. Calculate \( \chi^2 \) (physics + engr. targets)
Orbit confinement has been a dominant factor in stellarator optimization

- **Closed helical contours**
- **Closed toroidal contours**
- **Closed poloidal contours**

**Examples:**
- **HSX**
- **NCSX**
- **W7-X**
- **QPS**

**Quasi-symmetry** $B = B(\psi, M\Theta - N\zeta)$ Nührenberg (1988)
- Ideal (but unachieved) goal
- Quasi-helical ($M, N$ integers), quasi-toroidal ($N=0$), quasi-poloidal ($M=0$)

**Quasi-omnigeneity** can further improve closed contour configurations
- $J = \oint v_\parallel dl = J(\psi)$
- Constant $|B|$ contour spacing
  Cary, Shasharina (1997)

- $B_{\text{min}}$ and $B_{\text{max}}$ contours
  - Min/max along field line make const. on flux surface
  - Deeply trapped: $B_{\text{min}}(\psi)$, Transitional: $B_{\text{max}}(\psi)$

**Additional mathematical expressions**
\[
\frac{(B \times \nabla \psi) \cdot \nabla B}{B \cdot \nabla B} = F(\psi) \quad \text{Helander (2014)}
\]
3 methods for creating rotational transform
(L. Spitzer, 1951; C. Mercier, 1964)

• Plasma current (3D tokamak)
  ![Field Lines and Magnetic Axis]
  Configuration scaling: \( i / N_{fp} = \text{const.}, \langle R \rangle / \langle a \rangle / N_{fp} = \text{const.}, \) \( N_{fp} = \) field periods

• Planar axis, rotating cross-section (LHD, ATF)

• Helical magnetic axis (TJ-II, HSX, W7-X)
• **Stellarator/tokamak hybrids**
  
  – NCSX
  
  – QPS
  
  – Tilted coils
  
  transform amplifier

  ![Proto-CIRCUS - Columbia Univ. A. Clark, F. Volpe, et al. (2014)](image)

• **Reverse field pinches**
  
  – single helicity states
  
  – sustainment, transport barriers
Outline

• Introduction

• **Energetic particle confinement**
  – Orbit equations and types
  – Adiabatic invariants/reduced models
  – Optimization targets
  – Configuration categorization based on closed $|B|$ contours

• **Energetic particle stability**
Particle Orbits in 3D fields

• Guiding-center
  – Canonical coordinates (A. Boozer, R. White, 1981)
    • Only involves $|B|$ and currents in straight field line coordinates
      \[
      \frac{d\psi}{dt} = \frac{1}{D} \left( \mu \frac{mv^2}{B} - g \frac{\partial B}{\partial \theta} \right) \frac{\partial}{\partial \theta} 
      \frac{\partial B}{\partial \psi}, \quad \frac{dp_{\parallel}}{dt} = \frac{t - \rho_{\parallel} g'_{\parallel}}{D} \frac{\partial}{\partial \theta} \frac{\partial B}{\partial \psi}, \quad \frac{d\theta}{dt} = \left( \mu + \frac{mv^2}{B} + e \frac{\partial \Phi}{\partial \psi} + e B_{\parallel} \frac{\partial \rho_{\parallel}}{\partial \theta} \right) \frac{\partial}{\partial \psi} 
      \frac{\partial B}{\partial \psi} + e B_{\parallel} \frac{\partial \rho_{\parallel}}{\partial \psi} 
      \frac{d\zeta}{dt} = \left( \mu + \frac{mv^2}{B} \right) \frac{\partial B}{\partial \psi} + e \frac{\partial \Phi}{\partial \psi} \frac{\partial}{\partial \psi} + e B_{\parallel} \frac{\partial \rho_{\parallel}}{\partial \psi} 
      \]
  – Non-canonical coordinates (Littlejohn, Cary 1979-83)
    • Lie-transform perturbation methods, variational action integral
    • Coordinate-free
      \[
      \frac{d\tilde{R}}{dt} = \frac{1}{B_0} \left( v_{\parallel} \tilde{B}^* + \tilde{E}^* \times \hat{b} + \frac{\mu \hat{b} \times \nabla B}{Ze} \right), \quad \frac{dv_{\parallel}}{dt} = Z e (\tilde{E}^* + \tilde{R} \times \tilde{B}^*) \mu \nabla B 
      \]
      \[
      \tilde{B}^* = \tilde{B} + \frac{mv_{\parallel}}{Ze} \nabla \times \hat{b}, \quad \tilde{E}^* = \tilde{E} - \frac{mv_{\parallel}}{Ze} \frac{\partial \hat{b}}{\partial t} 
      \]
    – Adiabatic invariants: $\mu = \frac{mv^2}{2B}$, $J = \Phi mv_{\parallel}$, $\nabla_d = \nabla J \times \hat{b} / Ze$

• Lorentz equation: $\frac{d\tilde{v}}{dt} = \frac{Ze}{m} (\tilde{E} + \tilde{v} \times \tilde{B})$; g.c.

Issues for simulations: energy conservation, Liouville’s theorem (conservation of phase space volume carried with particle), intersections of fast ion with walls, PFCs

\[
\frac{1}{\sqrt{g}} \frac{\partial}{\partial \tau} \left( \sqrt{g} \frac{\partial z_i}{\partial \tau} \right) = 0
\]
Particle trajectories in 3D configurations: many new classes of orbits

Stellarator ($E_r$ added for confinement of trapped and transitional orbits)

Tokamak with ripple and TBM

Banana-tip jitter (Goldston, White, Boozer, 1981)

- locally trapped
- passing
- trapped-passing transitional
- ripple trapped
J optimization – simplest to apply to ≥ 5 field period QO systems, but should be extendable to other configurations (QH, QA)

\[ J = \Phi \frac{dl}{v_\parallel} \]

D. Spong, et al., POP (1998)
J optimization (contd.)

Large aspect ratio QO configuration

Finite $\beta$ effects on J contour centering [A. Ware, S. Hirshman, et al., PRL 2002]

Also, significant improvements in J seen for W7-X at $\beta \sim 5\%$
**EP confinement improvements**

- Bounce/velocity-averaged radial drift parameter (analogous to NEO $\varepsilon_{\text{eff}}$ for low $v$ transport)

\[
\Gamma_w = \frac{\pi R_0^2}{\sqrt{2}} \lim_{L_s \to \infty} \left( \int_0^{L_s} \frac{ds}{B} \right)^{-1} \frac{1}{\langle |\nabla \psi| \rangle^2} \times \int_{B_{\text{min}}/B_0}^{B_{\text{max}}/B_0} db' \sum_{j=1}^{j_{\text{max}}} \left( \frac{\partial g_j}{\partial b'} \right)^2 \left( \frac{\partial f_j}{\partial b'} \right)^{-1} \]

For $B \approx B_0 [1 + \varepsilon_h \cos(m\theta - n\varphi)]$

\[
\Gamma_w^{(\text{conv})} = \sqrt{\varepsilon_h}
\]

V. Nemov, S. Kasilov, W. Kernbichler, G. Leithold, POP 2005

- **Monte Carlo**
  - Mynick, Boozer, Ku (POP, 2006) find NCSX fast ion confinement improved by adding mirror term: local well near magnetic peak vs. in magnetic slope region
  - Reactor size NCSX $\alpha$ losses reduced from 27% to $< 10$
  - Particle noise may favor genetic or differential evolution optimization methods
Outline

• Introduction

• Energetic particle confinement

• Energetic particle stability
  – EP turbulence impacts confinement
  – Categories of EP instability
  – Alfvén instability basics: gap structure, eigenmodes
  – Gyrokinetic models – full/reduced
  – Gyrofluid models – application to LHD and TJ-II
  – Methods for controlling EP instabilities
Fast ion instabilities and confinement

- Tokamaks show 40-60% of fast ion energy can be lost when EP instabilities are present [DIII-D, W. Heidbrink, Nuclear Fusion 48 (2008)]

- Stellarator options for mitigation
  - High density operation
  - ECH suppression (as in tokamaks, M. Van Zeeland, et al.)
  - ECCD => modify iota (Heliotron-J)
  - Direct optimization of Alfvén gap structures

Loss scaling in LHD – K. Ogawa
Stellarator reactors at high density sample a different EP regime than tokamaks

- At steady-state
  \[ \frac{n_\alpha}{\tau_{s\alpha}} = \frac{n_e^2}{4} \left\langle \sigma_f v \right\rangle \]

- Since \( \tau_{s\alpha} \propto T_e^{3/2}/n_e \), \( \frac{n_\alpha}{n_e} \) depends only on \( T_e \) (for \( T_e = T_i \))
Stellarator reactors at high density sample a different EP regime than tokamaks

- AE stability limits also scale with $T_e$, (with a weak density dependence), e.g., Fu-Van Dam (1989)

$$\beta_\alpha \left( \frac{\omega * \alpha}{\omega} - \frac{1}{2} \right) F > \beta_e \frac{V_A}{V_e} \quad \text{where} \quad F = x(1 + 2x^2 + 2x^4) e^{-x^2} \quad x = \frac{V_A}{V_\alpha}$$

- N. Ohyabu, et al., PRL 2006 consider low temperature, high density reactor with $T_{e,i} = 6$ to 7 keV
  - But thermal stability feedback loop: rising $T \Rightarrow$ more alpha heating
Stellarator reactors at high density sample a different EP regime than tokamaks

High density/low T reactor may be able to stay on stable side of AE’s

High density increases energy scattering range/minimizes lossy pitch angle scattering range
Categories of EP-driven instability

• Thermal plasma-driven MHD instabilities that are augmented or suppressed by energetic particles
  – EIC mode, fishbones, sawteeth, infernal modes
• Stable MHD waves that are destabilized by EP
  – ★AE modes (★ = T, G, H, M, C, RS, B, BA, …)
  – Can be classified as gap (mode coupled) or extremal modes
• Some possible good effects
  – Channeling: direct transfer of EP energy to core ions
    • Beta-induced Alfvén and acoustic Alfvén modes (BAE, BAAE)
    – Alpha ash removal
• But, since highest energy EP component can be rapidly transported, deleterious for PFC’s and power balance
A growing collection of models/codes are available for AE’s in 3D systems

- Alfvén continua and gaps
  - CAS3D, STELLGAP*

- Stable Alfvén modes
  - CAS3D, AE3D*

- Perturbative stability models
  - CAS3D-K, AE3D-K, CKA-EUTERPE

- AE resonance mapping

- Hybrid fluid particle models
  - MEGA, M3D-K

- Gyrofluid: FAR3D

- Gyrokinetic: GTC, GENE, GS2, EUTERPE

*Available on GITHUB
Open/closed Alfvén gaps: show which modes are viable for destabilization

Effective inertia along magnetic field line $\sim n_{ion} m_{ion} / B^2$

Bending energy $\sim k_{\parallel}^2 = \frac{1}{qR} (nq - m)^2 \quad n, m = \text{integers}$

Cylindrical $n = 3$ Alfvén continua: $\omega^2 = k_{\parallel}^2 v_A^2$

Toroidal $n = 3$ Alfvén continua

Toroidal $n = 3$ Alfvén-acoustic continua

\[ \frac{\partial k_{\parallel} v_A}{\partial r} = 0; \quad k_{\parallel, m} \neq -k_{\parallel, m+1} \]

\[ k_{\parallel, m} = -k_{\parallel, m\pm1} \]

\[ k_{\parallel, m} v_A = -k_{\parallel, m} C_S \]
Toroidal mode number (n) is not a good quantum number for 3D configurations

- Field period symmetry: $N_{fp}$ replicated elements
- Toroidal coupling $\Rightarrow$ mode families, rather than single toroidal modes:
  \[
  n' \pm n = kN_{fp}, \; k = 0, 1, 2, \ldots
  \]
- Finite number of families:
  
  - $1 + N_{fp}/2$ for even $N_{fp}$ and $(N_{fp}-1)/2 + 1$ for odd $N_{fp}$
- Computational difficulty
  
  - High $N_{fp}$ easiest
  - Low $N_{fp}$ hardest

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**Diagram:**

- $n = \pm 1$ mode family couplings
- $N_{fp} = \infty$
- $N_{fp} = 10$
- $N_{fp} = 5$
- $N_{fp} = 4$
- $N_{fp} = 3$
- $N_{fp} = 2$
- $N_{fp} = 1$

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**Graph:**

- 6 families
- 3 families
- 3 families
- 2 families
- 2 families
- 1 family

**Lab Experiments:**

- LHD
- W7-AS, -X
- HSX, TJ-II
- NCSX
- QPS
- ITER with TBM's /DIII-D
Stellarators include both tokamak-like Alfvén gaps as well as ones unique to 3D (helical and mirror Alfvén gaps).
Gyrokinetic models for 3D configurations

• **Reduced vs. complete models**
  – Perturbative linear wave-particle energy transfer model
  – Global $\delta f$ PIC or continuum models
    • GEM: flux tube approach generalized to flux surface – each field line different in 3D
    • Global: GTC, EUTERPE

• **Kinetic ions, fast ions: full GC orbits followed**
  – charges, currents allocated over local gyro radius template for field solve
  – Maxwellian, slowing-down distribution options

• **Several options for electron and field models**
  – Adiabatic, Fluid/hybrid, Fully kinetic
  – Electrostatic (ITG), electromagnetic (Alfvén instability)
Linear gyrokinetic wave-particle energy transfer method:

\[ \gamma \propto \int \overrightarrow{\delta j}_{\text{particles}} \cdot \vec{E} \text{ Wave energy} \]

- Used in AE3D-K, EUTERPE, and VENUS codes
- LHD TAE mode
  \( B = 3.1T, n_{\text{fast}}(0) = 1.2 \times 10^{18} \text{ m}^{-3} \)
- Stabilized AE mode structure

\[ n = 1, -9 \text{ continuum structure} \]


Time-average growth rates
Distribution function sensitivity
Gyrokinetic modeling of 3D systems

3D equilibrium preparation

- VMEC fixed or free boundary 3D equilibrium

Transform to Boozer coordinates

Collapse Boozer double Fourier series to single series over toroidal mode number

GTC/gyrokinetic time evolution

Scatter:
-charges/currents interpolated onto grid

Field Solve:
- Calculate $\phi$ and $A_\parallel$
- Apply mode filter

Gather:
-add up forces on each particle

Push:
- advance particle positions/velocities

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GTC modeling of AE instabilities in LHD

• Equilibrium and profiles

• Typical Alfvén instability evolution
GTC finds linear EP-driven modes with frequencies aligned with the primary TAE continuum gaps.
The GTC model has been applied linearly and nonlinearly for several different 3D configurations (2015 ISHW)

LHD $n = 3$ BAE instability

W7-X $n = 6$ TAE instability

2015 JIFT exchange collaboration with Y. Todo – comparison with MEGA

Nonlinear $n = 1$ evolution in LHD
Efficient reduced dimensionality method: gyrofluid

- TAEFL (tokamak only) => FAR3D (tokamak & stellarator)
- MHD + gyrokinetic moments/Hammett-Perkins closure
- ~1000 times faster than gyrokinetic models
  - Good for parameter scans, Potential for optimization target

- Solution methods
  - Initial value – fastest growing instability
  - Eigenvalue hunting => can find multiple/sub-dominant modes

2018 RSAE Benchmark study

Tokamak:
D. Spong, Nuclear Fusion 53 (2013)


Gyrofluid Equations couple reduced MHD with Hammet-Perkins closure relations

**Reduced MHD equations** (assumes small $\beta$, $\varepsilon$, $k_{||}/k_{\perp}$)

Ohm's + Faraday's law: $E_{||} = -\frac{\partial A_{||}}{\partial t} - \hat{b} \cdot \vec{\nabla} \phi = \eta J_{||}$

Toroidal component of vorticity:

$$\hat{\varepsilon} \cdot \vec{\nabla} \times \sqrt{g} \left\{ \rho_m \left( \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \vec{\nabla} \right) = -\vec{\nabla} \left( p_{th} + n_{fast} T_{fast} \right) + \vec{J} \times \vec{B} \right\}$$

Pressure evolution:

$$\frac{\partial p_{th}}{\partial t} + \vec{v} \cdot \vec{\nabla} p_{th} = -\Gamma p_{th} \vec{\nabla} \cdot \vec{v}$$

**Basic fast ion Landau closure equations:**

$$\frac{\partial n_f}{\partial t} = -\Omega_d (n_f) - n_{f0} \nabla_{||} (v_{||f}) - n_{f0} \Omega_d \left( \frac{q_f \phi}{kT_{f0}} \right) + n_{f0} \Omega_\star \left( \frac{q_f \phi}{kT_{f0}} \right)$$

$$\frac{\partial v_{||f}}{\partial t} = -\Omega_d (v_{||f}) - \left( \frac{\pi}{2} \right)^{1/2} \sqrt{\frac{kT_{f0}}{M_f}} \nabla_{||} \left( v_{||f} \right) - \frac{kT_{f0}}{M_fn_{f0}} \nabla_{||} (n_f) + \frac{q_f}{M_f} \Omega_\star \left( \frac{\psi}{R} \right)$$

where $\Omega_d(\ ) = \frac{T_{f0}}{q_f B_0} \frac{\vec{B}_0 \times \vec{\nabla} B_0}{B_0^2} \cdot \vec{\nabla} (\ )$ and $\Omega_\star(\ ) = \frac{T_{f0}}{q_f B_0 n_{f0}} \vec{\nabla} n_{f0} \cdot \frac{\vec{B}_0}{B_0} \times \vec{\nabla} (\ )$
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$$\frac{\partial v_{||f}}{\partial t} = - \Omega_d \left( v_{||f} \right) - \left( \frac{\pi}{2} \right)^{1/2} \sqrt{\frac{kT_{f0}}{M_f}} \nabla_{||} \left( v_{||f} \right) - \frac{kT_{f0}}{M_f n_{f0}} \nabla_{||} \left( n_f \right) + \frac{q_f}{M_f} \Omega_* \left( \frac{\psi}{R} \right)$$

where $\Omega_d \left( \cdot \right) = \frac{T_{f0}}{q_f B_0} \frac{\vec{B}_0 \times \vec{\nabla} \vec{B}_0}{B_0^2} \cdot \vec{\nabla} \left( \cdot \right)$ and $\Omega_* \left( \cdot \right) = \frac{T_{f0}}{q_f B_0 n_{f0}} \vec{\nabla} n_{f0} \cdot \frac{\vec{B}_0}{B_0} \times \vec{\nabla} \left( \cdot \right)$

Recent upgrades (collaboration with Jacobo Varela, Luis Garcia):

- Thermal ion and EP FLR effects added
- Multiple fast ion species
- Two fluid (ion/electron) thermal plasma
- 3 and 4 field EP model
- Nonlinear terms
Application to LHD

- J. Varela, D. Spong, L. Garcia, Nuclear Fusion (2017)
- TAE modes + MHD interchange/ballooning, parameter scans, eigenmode structures

Growth rate contours vs. $S$ and $\beta_{\text{fast}}$
Application to TJ-II

TJ-II shows many examples of dynamical frequency sweeping as the iota profile is changed:

Gyrofluid model reproduces TJ-II frequency sweeping effects as iota profiles are displaced

- Iota variations cause frequency sweeping, showing inverse correlations between growth rates and frequency.

- As iota changes and the frequency sweeps, different helicities become dominant
NSTX Alfvén mode suppression correlated with 3D coils

- ELMs triggered 2 ms after MP starts
- Two dominant TAEs observed
  - $n=2$, 65 kHz; $n=3$, 75 kHz
- 3D Alfvén continuum (STELLGAP model) shows toroidal coupling effects near edge
  => increased damping for TAE
Control of EP-driven modes in stellarators

- **AE control in existing devices:**
  - Rotational transform control, ECH deposition
  - High density operation (shorter $\tau_{\text{slow-down}}$, smaller $\beta_{\text{EP}}$)

- **3D shaping optimization for future designs**
  - AE gap width and locations determined by variation in $g^{\rho \rho}$, $|B|$ and $\beta$
  - Suppress or enhance?
  - Could try to make $g^{\rho \rho}/B^2$ uniform on surface to reduce gap width
  - Target more uniform density of eigenstates vs. frequency => close off open gaps
Conclusions/Summary

- Energetic particle (EP) confinement remains an important stellarator optimization goal
  - Reactor-grade confinement: stellarators still have some “catching up” to get to tokamak levels
  - Fusion power balance probably okay, PFC loading not

- EP confinement different, depending on optimization approach
  - Helically, toroidally, poloidally closed contours of $|B|$, QO-improvements
    - Long vs. short trapped particle bounce orbits
    - Focused vs. diffuse loss patterns
  - Innovations needed: better optimization, or ways to deal with EP losses (divertors for EP, liquid metal walls, etc.)

- A variety of EP instability models have been developed and provide understanding of EP observations
  - Stable Alfvén gap, mode structure and frequency spectra
  - Gyrokinetic, gyrofluid, particle-mode energy transfer

- EP stability optimization tools (ECH, ECCD, iota control, direct optimization) available => more tests needed