Evolutions of safety factor profiles in sawteeth

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Abstract: The helicity of the magnetic field should remain unchanged unless magnetic reconnection takes place. However, in previous literature for sawteeth, the safety factor in the region near the magnetic axis always gives $q_0 = 1.0$ without magnetic reconnection, which is contradictory to Poincare plots of the magnetic field lines. In order to obtain a precise the safety factor during sawteeth, we redefine the poloidal angle, which is used in the calculation of the safety factor $q = \frac{\Delta\varphi}{\Delta\theta}$ ($\Delta\varphi$ and $\Delta\theta$ are the changes of the toroidal and poloidal angles), to relate to the new twisted magnetic axis during sawteeth instead of the initial magnetic axis that used in previous studies. With the new definition of the poloidal angle, the safety factor profiles become consistent with Poincare plots of the magnetic field. We also find that the time evolutions and the spatial distributions of the safety factor are significantly different from previous studies: during normal sawteeth, the safety factor at the magnetic axis q_0 remains unchanged in almost the entire period of a sawtooth and jump up to 1.0 near the end; in the non-axisymmetric equilibrium, q_0 is still far below 1.0; during the incomplete reconnection, q_0 keeps the same with its initial value.

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I. Introduction

Sawteeth are common phenomena for magnetically confined fusion device, whose central safety factor falls below one. [1] During sawteeth, the central plasma pressure periodically crashes after a slow rise. Sawteeth can not only flatten center plasma temperature but also trigger neo-classical tearing modes in nearby resonant surfaces[2], which results in a significant reduction of energy confinement.

Since the sawteeth are deleterious for Tokamak operations, many efforts were taken to understand the mechanism of sawteeth.[3-5] However, after more than 40 years, two fundamental points of sawteeth is still unknown, i.e., the mechanism of the fast pressure crash and whether the magnetic reconnection is complete or incomplete during the crash. For the first problem, there are several candidates, i. e. the Hall effect,[6] the stochasticity of the magnetic field[7], and the pressure-driven instabilities.[8] For the second problem, we are even unable to determine whether the incomplete reconnection has actually occurred, since reported experimental results are totally different. In TFTR[9] and ASDEX-U[10], it was suggested that the safety factor of the magnetic axis q_0 remains almost unchanged. However, in other experiments, q_0 goes above one after the core crash. Therefore, the precise calculation of q_0 is of great importance to understand the insight physical mechanism of the sawteeth, especially for incomplete reconnection.

The safety factor is defined as $q = \frac{\Delta\varphi}{\Delta\theta}$ to reflect the helicity of a magnetic field line, where $\Delta\varphi$ and $\Delta\theta$ are the changes of the toroidal and poloidal angles along the magnetic field line based on the initial untwisting magnetic axis. However, if the magnetic axis is twisted due to kink instabilities, the magnetic field lines not only twist with its helicity but also have to wing around the twisted magnetic axis. If we still use such a definition, the q profile will totally be misleading and deceptive. For example, if the 1/1 kink instability is well developed, the magnetic axis will be twisted and its helicity is m/n=1/1. Assuming the twisting magnetic axis at a toroidal plane locates at $(r_A, \theta_{A0} - \varphi, \varphi)$, where r_A is the distance between the locations of the new and initial magnetic axis, θ_{A0} is the poloidal angle of the new twisted magnetic axis at φ=0. Then, the twisting magnetic axis locates at

$$R_A = R_{A0} + r_A \cos(\theta_{A0} - \varphi), \tag{1}$$

$$Z_A = Z_{A0} + r_A \sin(\theta_{A0} - \varphi), \tag{2}$$

where (R_{A0}, Z_{A0}=0) is the position of the untwisting magnetic axis at φ=0. Magnetic field lines in the region $\sqrt{(R-R_A)^2 + (Z-Z_A)} < r_A$ now wind around the twisted magnetic axis while magnetic field lines in other regions are not affected by the kink instability. Assuming a magnetic field line starts from $(R_A + \tilde{r}, 0, 0)$ and $\tilde{r} < r_A$, and its helicity to the twisting magnetic axis is q_F, the location of the magnetic field line will be

$$R(\varphi) = R_{A0} + r_A \cos(\theta_{A0} - \varphi) + \tilde{r} \cos(-\varphi / q_F) \tag{3}$$

$$Z(\varphi) = r_A \sin(\theta_{A0} - \varphi) + \tilde{r} \sin(-\varphi / q_F) \tag{4}$$

If the poloidal angle θ used in the safety factor is still defined based on the initial magnetic axis, we name it as the old safety factor,

$$q_{old} = \frac{\Delta\varphi}{\Delta\theta} = \frac{\Delta\varphi/(2\pi)}{N_{z=0}/2} = \frac{\Delta\varphi}{\pi N_{z=0}}, \tag{5}$$

where $N_{z=0}$ is the number of times for a magnetic field line crossing the $Z=0$ plane. From $Z(\varphi)=0$, we get

$$r_A \sin(\theta_{A0} - \varphi) + \tilde{r} \sin(-\varphi / q_F) = 0 \tag{6}$$

Since $\tilde{r} < r_A$, then

$$\Delta\varphi = 2[\frac{N_{z=0}}{2}]\pi + \theta_{A0} + \arcsin[\frac{\tilde{r}}{r_A} \sin(-\Delta\varphi / q_F)] \tag{7}$$

or

$$\Delta\varphi = (2[\frac{N_{z=0}}{2}]+1)\pi + \theta_{A0} - \arcsin[\frac{\tilde{r}}{r_A}\sin(-\Delta\varphi / q_F)] \tag{8}$$

To get an accurate q, $N_{z=0} >> 1$ (typically $N_{z=0} \sim 1000$), then

$$\Delta\varphi \approx \pi N_{z=0} \tag{9}$$

And the old safety factor will be

$$q_{old} = \frac{\Delta\varphi}{\pi N_{z=0}} \equiv 1 \tag{10}$$

Equation (10) indicates that the old definition of the safety factor has not considered that the magnetic axis has been twisted by the kink instability, and all magnetic field lines in the region must wind around the twisted magnetic axis. As a result, no matter what the helicity of the magnetic field it is, it always ‘proves’ that the profile of the old safety factor around the magnetic axis ($\tilde{r} < r_A$) is flattened and its value is equal to unit. It should be noted that the helicity or topology of a magnetic field line will remain unchanged unless this field line is reconnected. If the magnetic field lines in the region ($\tilde{r} < r_A$) have not been reconnected, the safety factor should remain unchanged. Therefore, the safety factor from Equation (1.10) fails to reflect the helicity of the magnetic field because in this region because it has not taken into that the magnetic axis is twisted by the kink instability into account.

In the region $\sqrt{(R-R_A)^2+(Z-Z_A)} > r_A$, the magnetic field line is not affected by the twisted magnetic axis, and the old safety factor give an accurate value, i.e.,

$$q_{old} \equiv q_F \tag{11}$$

From the above discussion, the old safety factor defined with the poloidal angle change that is based on the initial magnetic axis will always give a wrong value in the region near the twisted magnetic axis. Note that, since magnetic field lines in the m/n=1/1 island do not wind around the new magnetic axis, the safety factor in the island is not affected and equal to unit.

As discussed above, the old safety factor fails to represent the helicity of the magnetic field and falsely gives that the safety factor profile around the magnetic axis is flattened and its value is equal to unit when a kink instability occurs. Also, the old q profile will be inconsistent with the Poincare plots of the magnetic field in such a situation. However, the inconsistence between the Poincare plots of the magnetic field and the safety factor profile has not be noticed in previous studies[11-13] (including our recent paper[14]). Therefore, the motivation of the present paper is to propose a precise way to calculate the safety factor during sawteeth.

II. New method for the safety factor calculation

Since the main problem is resulted from incorrectly using the poloidal angle in the q definition, we introduce a new poloidal angle $\theta'=\theta\text{-}\theta_{\mathrm{A}}$, where θ and θ_A are the poloidal angles of a magnetic field line and the twisted magnetic axis. Thus, the new poloidal angle $\theta'=\theta\text{-}\theta_{\mathrm{A}}$ of the magnetic field line is relative to the magnetic axis twisted by the kink instability. In the region $\sqrt{(R-R_A)^2+(Z-Z_A)}<r_A$, the relative position of the magnetic field line to the twisting magnetic axis is

$$R_{new}(\varphi)=\tilde{r}\cos(-\varphi/q_F) \tag{12}$$

$$Z_{new}(\varphi)=\tilde{r}\sin(-\varphi/q_F) \tag{13}$$

Similarly,

$$q_{new}=\frac{\Delta\varphi}{\Delta\theta'}=\frac{\Delta\varphi/(2\pi)}{N_{z_{new}=0}/2}=\frac{\Delta\varphi}{\pi N_{z_{new}=0}}, \tag{14}$$

where $N_{z_{new}=0}$ is the number of times for a magnetic field line crossing the $Z_{new}=0$ plane. From $Z_{new}(\varphi)=0$,

$$\tilde{r}\sin(-\varphi/q_F)=0, \tag{15}$$

$$\Delta\varphi=q_F\pi N_{z_{new}=0}, \tag{16}$$

then we name the safety factor based on the new poloidal angle as the new safety factor,

$$q_{new}=\frac{\Delta\varphi}{\pi N_{z_{new}=0}}\equiv q_F \tag{17}$$

Equation (1.16) indicates that the new safety factor in the region $\sqrt{(R-R_A)^2+(Z-Z_A)}<r_A$ can successfully solve the problem resulted from the twisting magnetic axis.

Now we calculate the safety factor in the region with $\sqrt{(R-R_A)^2+(Z-Z_A)}>r_A$. Since magnetic field lines are not affected by the twisted magnetic axis,

$$R_{new}(\varphi)=\tilde{r}\cos(-\varphi/q_F)-r_A\cos(\theta_{A0}-\varphi) \tag{18}$$

$$Z_{new}(\varphi)=\tilde{r}\sin(-\varphi/q_F)-r_A\sin(\theta_{A0}-\varphi) \tag{19}$$

From $Z_{new}(\varphi)=0$,

$$\tilde{r}\sin(-\varphi/q_F)-r_A\sin(\theta_{A0}-\varphi)=0 \tag{20}$$

Since $\tilde{r}>r_A$,

$$\Delta\varphi=q_F\{2[\frac{N_{z_{new}=0}}{2}]\pi-\arcsin[\frac{r_A}{\tilde{r}}\sin(\theta_{A0}-\Delta\varphi)]\}, \tag{21}$$

or

$$\Delta\varphi=q_F\{(2[\frac{N_{z_{new}=0}}{2}]+1)\pi+\arcsin[\frac{r_A}{\tilde{r}}\sin(\theta_{A0}-\Delta\varphi)]\} \tag{22}$$

thus

$$q_{new}=\frac{\Delta\varphi}{\pi N_{z_{new}=0}}\equiv q_F. \tag{23}$$

Equations (17) and (23) indicate that, even if the magnetic axis is twisted by the kink instabilities, we can still obtain the precise safety factor by defining the poloidal angle with the relative position between the twisted magnetic axis and the magnetic field line. It is also indicated that the new definition of the poloidal angle can be appliable in the whole region, no matter whether magnetic field lines are affected by the kink instability or not.

III. Simulation results

All the simulations in the present paper are carried out with the CLT code. [15] Since the purpose of the simulations is to verify the accuracy of the new safety factor, we do not repeat the details of the CLT code. Similar simulation results could be found in our previous studies (W. Zhang et al. to be published) or other literature. [11-13, 16]

i. Normal sawteeth

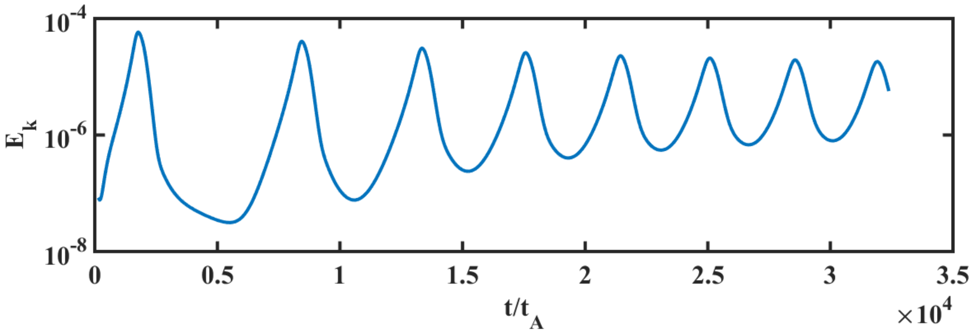

Figure 1. The kinetic energy evolution of normal sawteeth.

The parameters used in this subsection are given as follows: the plasma beta $\beta_0 \sim 2.4\%$, the resistivity $\eta = 2.5\times10^{-6}$, the diffusion coefficient $D = 1.0\times10^{-5}$, the viscosity ν=1.0×10^{-4}, the perpendicular and parallel thermal conductivities $\kappa_\perp = 2.0\times10^{-5}$ and $\kappa_\parallel = 5\times10^{-2}$, respectively. The kinetic energy evolution during the normal sawteeth is shown in Figure 1. The Poincare plots of magnetic field lines and the q profiles at four typical moments in the first cycle ($t = 0t_A$, $t = 1243t_A$, $t = 1597t_A$, and $t = 1775t_A$) are shown in Figure 2. As shown in Figure 2e and 2h, both of the old and new safety factors give the same profiles when the amplitude of the kink mode is small or magnetic reconnection finishes. However, when the m/n=1/1 magnetic island appears, the old and new safety factors at the magnetic axis have a significant difference. The old safety factor is 1.0 at $t = 1243t_A$ and $t = 1597t_A$ as shown in Figure 2f and 2g. However, the new safety factor remains unchanged, i.e., $q = 0.7$ at these two stages.

It should be noted that the helicity of magnetic field lines should remain unchanged until the magnetic field line is reconnected. Therefore, the safety factor at the magnetic axis should remain unchanged and the same with the initial safety factor at $t = 1243t_A$ and $t = 1597t_A$. As a result, the old safety factor that has the flattened profile and becomes equal to unit in the region near the twisting magnetic axis is obviously wrong. As pointed out in the introduction, this mistake is resulted from that the poloidal angle in the old safety factor is still assumed to be relative to the initial magnetic axis. With taking into account the fact that the magnetic axis is twisted by the kink mode, the profile of the new safety factor agrees well with the Poincare plots of the magnetic field. The contour plots of the old and new safety factors at $t = 1597t_A$ are shown in Figure 3. As shown in Figure 2c and 3b, although magnetic field lines become stochastic in the region between the m/n=1/1 magnetic island and the magnetic axis, the new safety factor could still give the accurate q values, which are well consistent with the Poincare plots of the magnetic field.

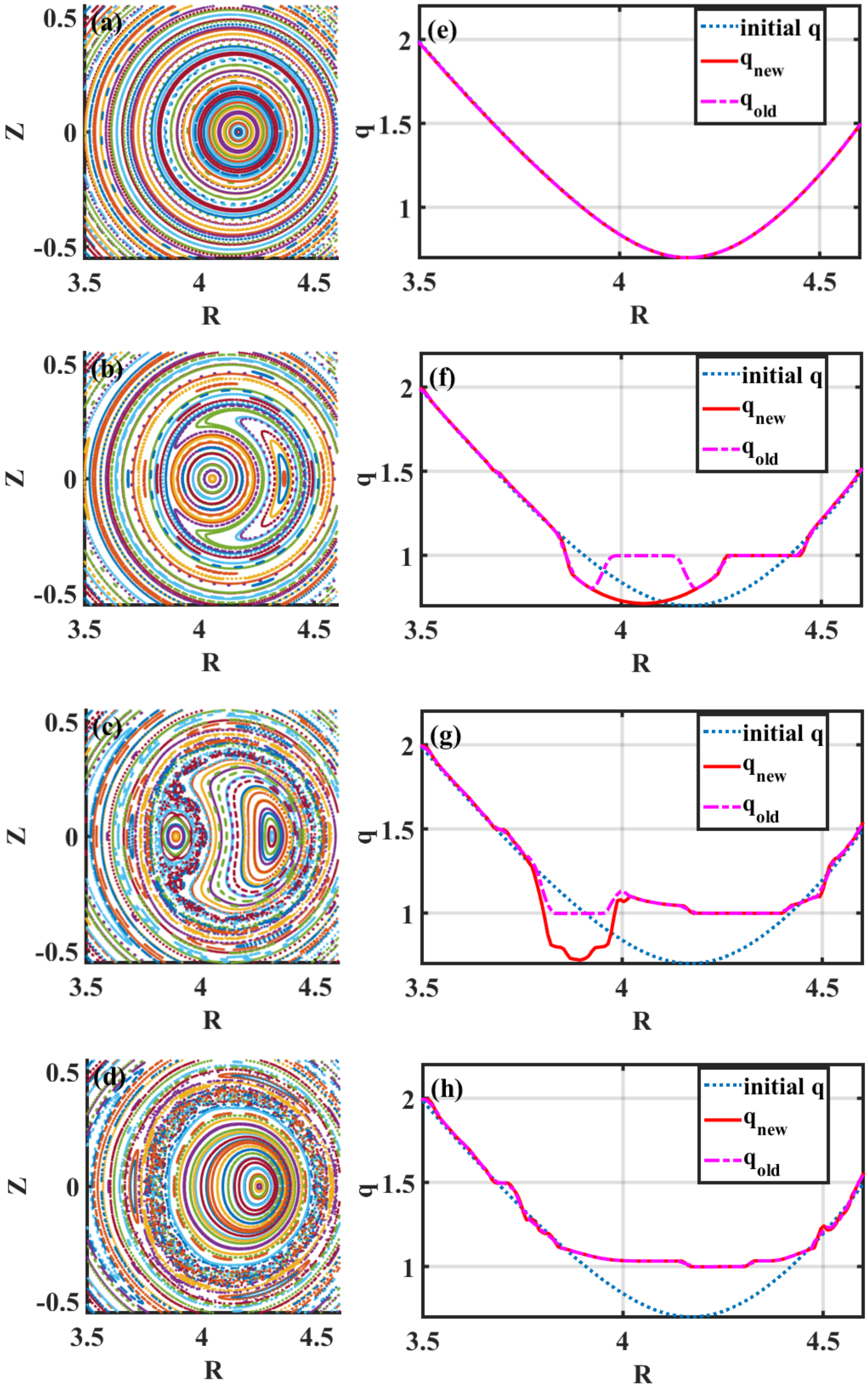

Figure 2 The Poincare plots of the magnetic field and the profiles of the safety factor at $t=0t_A$, $t=1243t_A$, $t=1597t_A$, and $t=1775t_A$. q_{old} is the safety factor with the

old poloidal angle that is still defined based on the untwisting magnetic axis, and q_{new} is the safety factor with the new poloidal angle that is redefined based on the twisting magnetic axis.

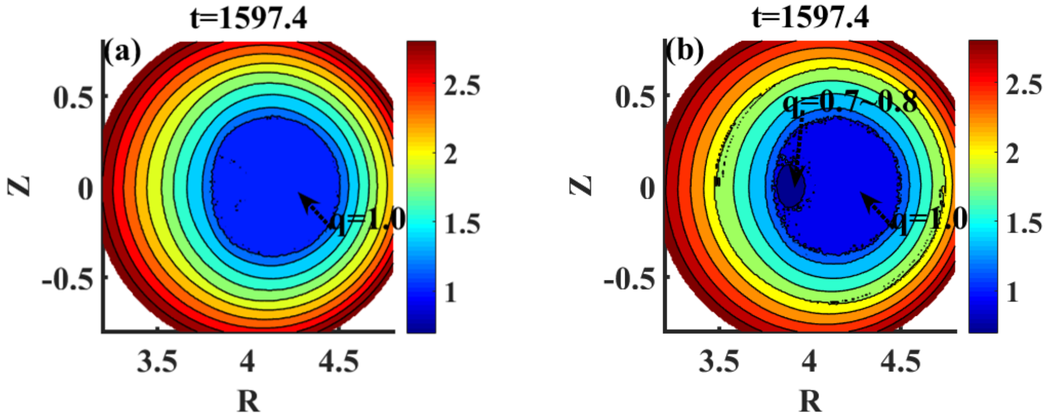

Figure 3 The contour plots of the old (a) and new (b) safety factors at $t = 1597t_A$.

It should be noted that, even at the moment ($t = 1597t_A$), when the original core almost disappears, the safety factor at the magnetic axis still keeps $q_0 \sim 0.7$. It indicates that the safety factor at the magnetic axis almost keeps unchanged during the reconnection (Figure 2g), and suddenly jumps up to 1.0 until the magnetic flux reconnection finishes (Figure 2h), while previous studies always show that the safety factor at the magnetic axis slowly rises to 1.0 during the magnetic reconnection process. From Wesson's theory [5], we know that the interchange instability (or quasi-interchange instability[8]) can only occur when the magnetic shear $s = \frac{rq'}{q}$ becomes much smaller. However, as shown in Figure 2f and 2g, the magnetic shear at the X-point becomes larger instead of smaller, which implies that the quasi-interchange instability should not be responsible for the fast pressure crash during the sawteeth in Tokamaks unless the initial safety factor profiles around the magnetic axis is flattened and close to unit

The evolutions of the safety factor from four different calculations are shown in Figure 4. Firstly, the minimum q_{new}, which is roughly located at the new magnetic axis, keeps almost unchanged for a long time and suddenly jumps up to 1.0 at the end of the reconnection process. Similar to serval previous studies, the minimum q_{old} gradually rises to 1.0 before the reconnection finishes is due to the improper calculation of the safety factor as shown in Figure 2b, 2c, 2f, and 2g. Moreover, if one uses $q_{old}(0)$ that located at the original axis as the safety factor at the magnetic axis, it will always larger or equal to 1.0 during the sawteeth after the first cycle in which the system evolves from the equilibrium. This might be why the safety factor at the magnetic axis remains $q_0 \sim 1.0$ during the sawteeth in some experiments.

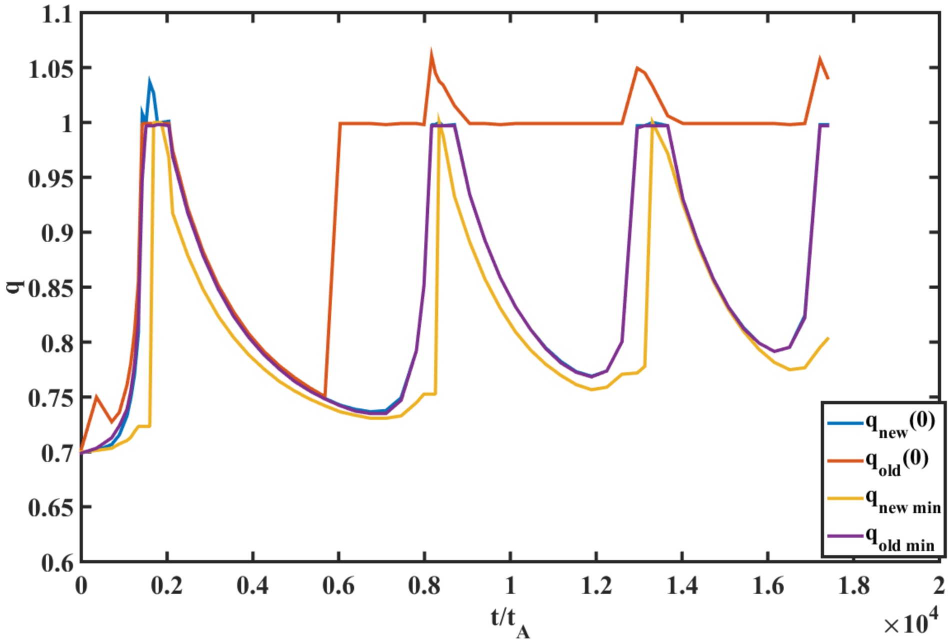

Figure 4 The evolutions of the safety factor at the magnetic axis for four different calculations. $q_{new}(0)$ indicates the new safety factor located at the original magnetic axis. $q_{old}(0)$ indicates the old safety factor located at the original magnetic axis. $q_{new\_\min}$ represents the minimum new safety factor along X=0, which is the new safety factor at the time-varying location of the magnetic axis. $q_{old\_\min}$ represents the minimum old safety factor along X=0.

ii. Stationary state

Recently a non-axisymmetric stationary state that is related to sawteeth has been reported in many experiments[17-19] and has been reproduced by several toroidal codes.[11, 12, 16] As discussed in those papers, both of the magnetic field and the stream function has the helicity of m/n=1/1 at the stationary state. It also has been reported that the safety factor in the core region becomes flattened and is about 1.0. However, we find that the safety factor profile at the stationary state is not fully flattened and is still smaller than 1.0. To illustrate this, the simulations associated with the stationary state are carried out.

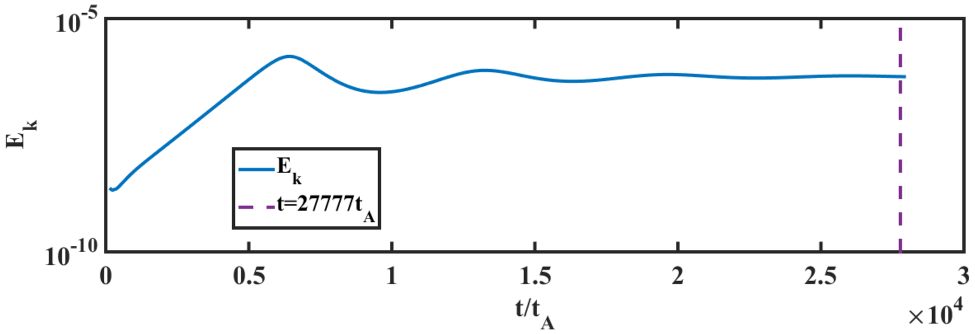

Figure 5 The kinetic energy evolution of the stationary steady of sawteeth with high viscosity.

The parameters used in this subsection are given as follows: the plasma beta $\beta_0 \sim 2.4\%$, the resistivity $\eta = 2.5\times10^{-6}$, the diffusion coefficient $D = 1.0\times10^{-5}$, the perpendicular and parallel thermal conductivities $\kappa_\perp = 2.0\times10^{-5}$ and $\kappa_\parallel = 5\times10^{-2}$, and the viscosity ν=1.0×10^{-3}, respectively. The kinetic energy evolution of the stationary state with the high viscosity is shown in Figure 5. The Poincare plot of magnetic field lines at the stationary state is typically like Figure 6(a). As shown in Figure 6 (b), the old safety factor indicates that the safety factor around the magnetic axis is totally flattened and just above 1.0, while the new safety factor indicates that its safety factor still remains below 1.0, q_{new}=0.86. The contour plots of the new and old safety factors at $t = 27777t_A$ are shown in Figure 7. It is evident that the contour plot of the new safety factor is consistent with the Poincare plot of the magnetic field, while the old safety factor profile is not.

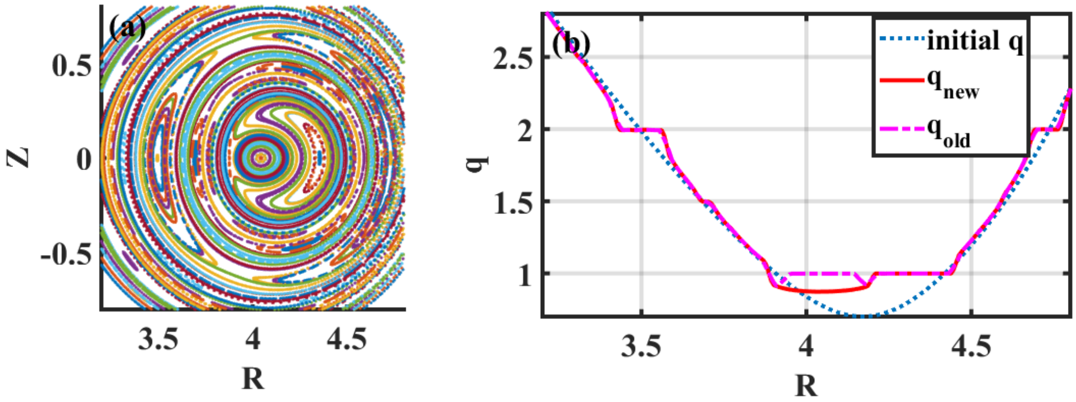

Figure 6 (a) The Poincare plot and (b) the profiles for the old and new safety factors at the stationary state ($t = 27777t_A$).

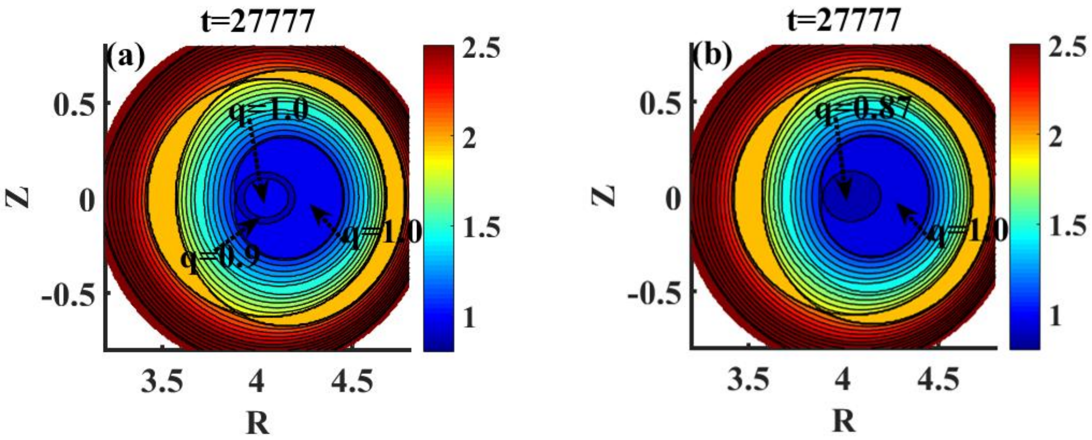

Figure 7 The contour plots of the (a) old and new (b) safety factors at $t = 27777t_A$.

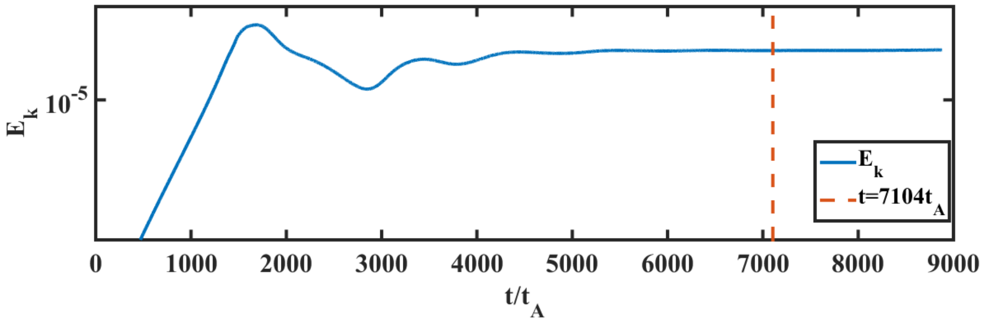

Figure 8 The kinetic energy evolution of the stationary state of sawteeth with low viscosity.

The evolution of kinetic energy with the low viscosity (ν=6.0$\times$10^{-6}) is shown in Figure 8. The system could also achieve the stationary state with a large m/n=1/1 magnetic island in the present case (Figure 9a) rather than with a small m/n=1/1 island with high viscosity. The corresponding profiles of the old and new safety factor are shown in Figure 9b. The real safety factor at the magnetic axis is 0.9387, which is still below 1.0, which could also be seen from the contour plot of the safety factor (Figure 10 (b)).

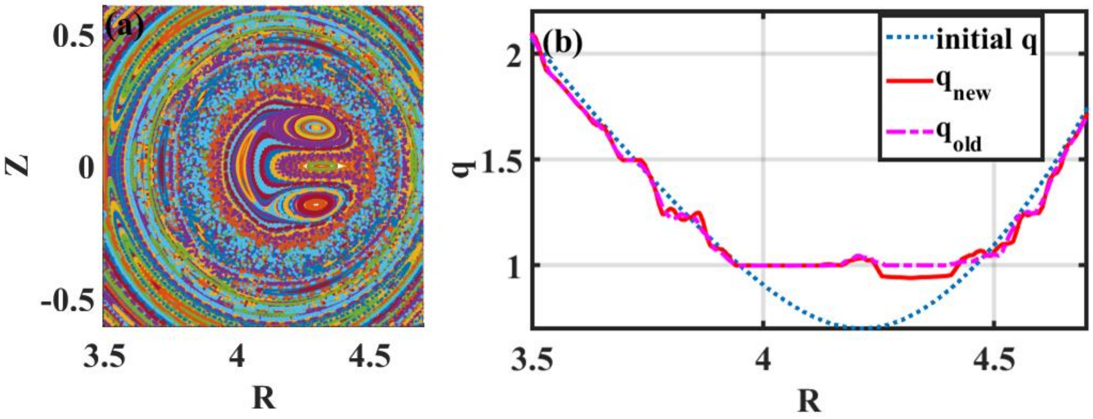

Figure 9 (a) The Poincare plot and (b) the profiles of the old and new safety factors at the stationary state ($t = 7104t_A$).

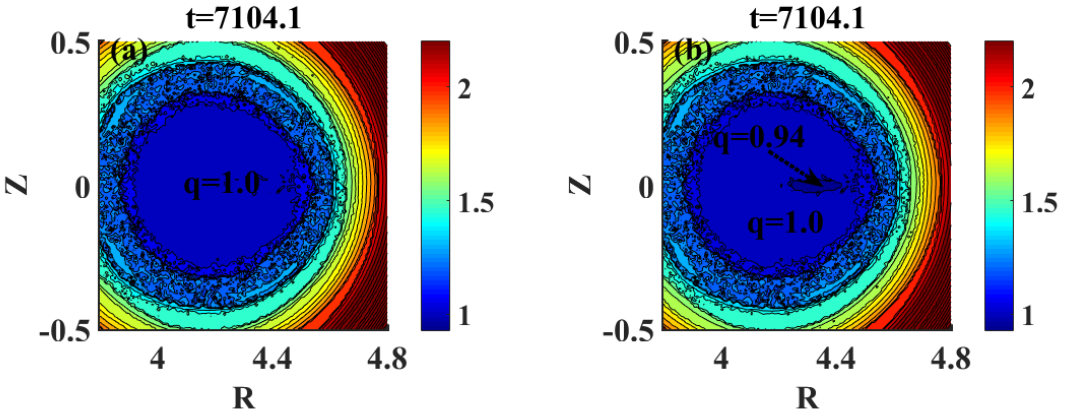

Figure 10 The contour plots for the (a) old and (b) new safety factors at $t = 7104t_A$.

iii. incomplete reconnection

Several studies [20, 21] have reported that magnetic reconnection during sawteeth could be incomplete due to plasmoid instabilities. We believe that it is interesting and important to present the time evolution and the spatial distribution of the safety factor. The parameters used in this subsection are given as follows: the plasma beta $\beta_0 \sim 0$, the resistivity $\eta = 1.0\times10^{-7}$, the diffusion coefficient $D = 1.0\times10^{-4}$, the perpendicular and parallel thermal conductivities $\kappa_\perp = 3.0\times10^{-6}$ and $\kappa_\| = 5\times10^{-2}$, and the viscosity ν=1.0×10^{-8}, respectively. The Poincare plot of magnetic field lines and the corresponding profiles of the old and new safety factors at four typical moments (at $t = 0t_A$, $t = 3423t_A$, $t = 4336t_A$,and $t = 5324t_A$) are shown in Figure 11. The system is unstable for the resistive-kink mode since the initial safety factor at the magnetic axis is 0.9 (Figure 11 e). The m/n=1/1 magnetic island will slowly develop (Figure 11b), and the current sheet near the X-point becomes thinner and thinner. When the current sheet thickness decreases below a critical value, a secondary tearing instability will be triggered and lead to formation of the plasmoid near the original X-point (Figure 11c), which prevents the resistive-kink mode from further growing up and then finally results in an incomplete reconnection (Figure 11d). As shown in Figure 11e~11f, the profiles for the new and old safety factors are the same except the region near the magnetic axis. The old safety factor indicates that the safety factor is flattened and equal to 1.0. However, as shown in the Poincare plot, magnetic reconnection only occurs on the q=1 resonant surface and never occurs around the magnetic axis. Therefore, the safety factor at the magnetic axis should remain unchanged. From the profile of the new safety factor, the safety factor at the magnetic axis indeed remains 0.9 throughout the simulation, and the safety factor profile only become flattened in the two 1/1 magnetic islands. The contour plots of the old and new safety factors at $t = 4336t_A$ are shown in Figure 12 a and b. It is clear that the contour plots of the new safety factor agree well with the Poincare plots, while the results from the old safety factor do not.

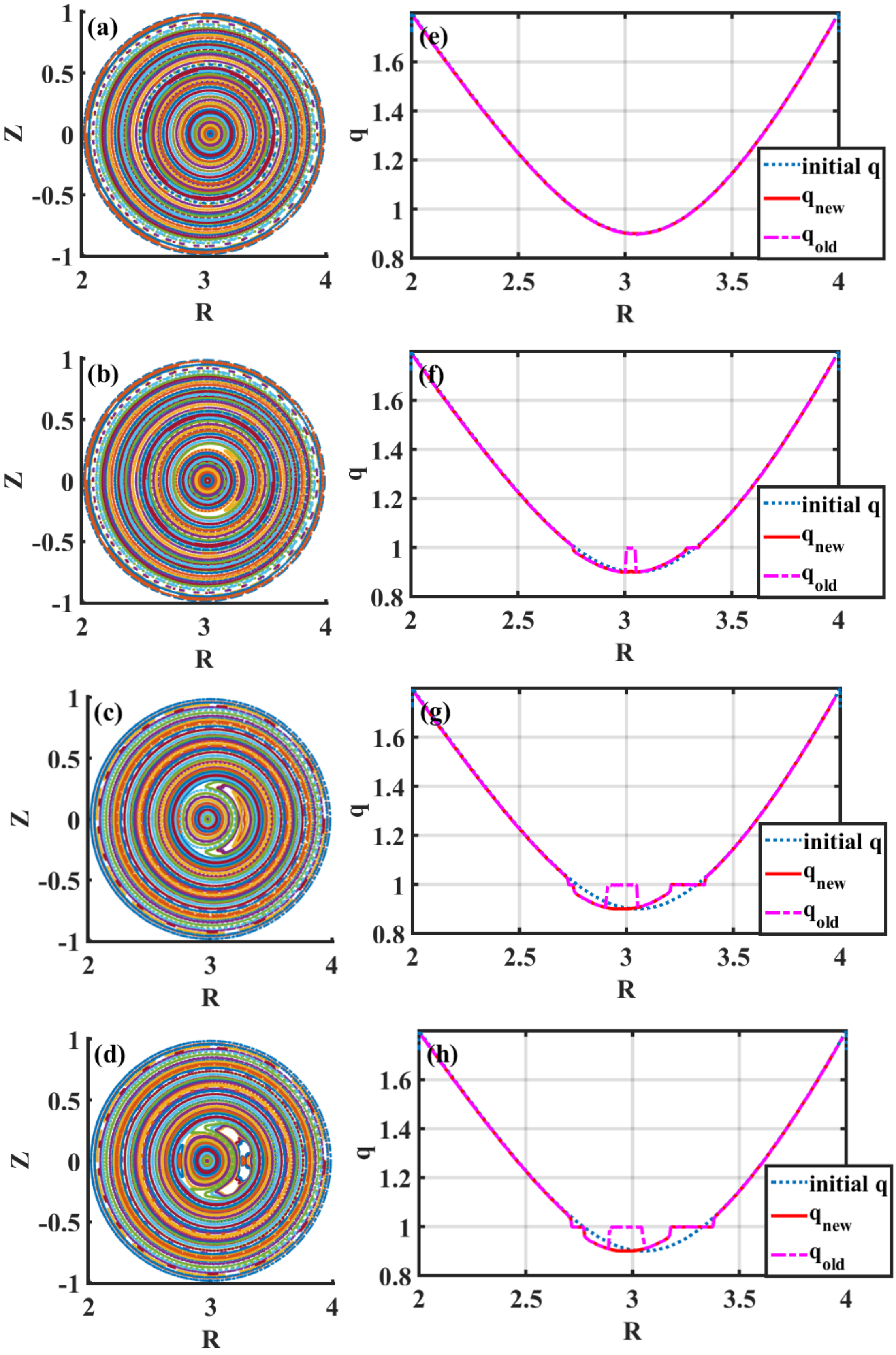

Figure 11 The Poincare plots of the magnetic field and the corresponding profiles of the old and new safety factors at $t = 0t_A$, $t = 3423t_A$, $t = 4336t_A$,and $t = 5324t_A$.

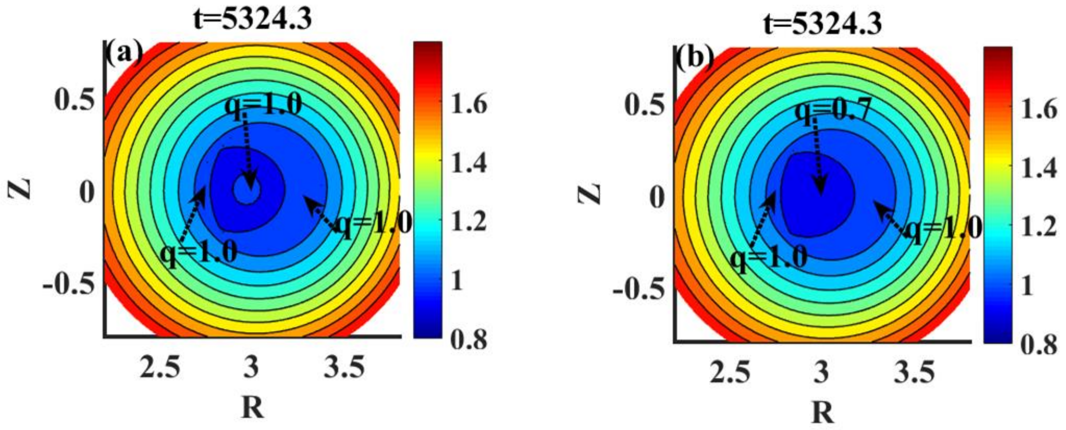

Figure 12 The contour plots of the (a) old and new (b) safety factors at $t = 5324t_A$.

IV. Summary and discussion

The safety factor with two different definitions of the poloidal angle are adopted to investigate the time evolutions and the spatial distribution of the safety factor during the sawteeth with normal oscillations, the stationary state, and the incomplete reconnection. We find that the time evolutions of the old safety factor defined with the poloidal angle based on the initial magnetic axis are inconsistent with the Poincare plots of the magnetic field when the m/n=1/1 magnetic island exists. The old safety factor always indicates that the safety factor around the magnetic axis is flattened and equal to 1.0 with development of the kink instability.

To solve this inconsistence, we define the new safety factor with the poloidal angle based on the new twisted magnetic axis. It is found that the new safety factor agrees well with the Poincare plots of the magnetic field for all kinds of sawteeth. The new safety factor at the magnetic axis remains unchanged for the quite long time of a sawtooth oscillation and then quickly jumps up to 1.0 near the end of magnetic reconnection. For the new safety factor at the magnetic axis, there is no slow ramping phase, Instead, only a sudden transition phase (i.e. q_0 jumps up from the initial safety factor to 1.0) that occurs near the end of magnetic reconnection for the normal sawteeth. It should also be noted that the old safety factor at the location of the original magnetic axis always gives $q_0 = 1.0$ during the entire period of sawteeth.

In many previous studies, it is reported that the safety factor at the magnetic axis is flattened and is about 1.0 when the system achieves the steady-state. However, the Poincare plots of the magnetic field indicates that the safety factor at the magnetic axis should not be 1.0. The inconsistence is resulted from that the poloidal angle in previous studies is improperly related to the initial magnetic axis. From the present study, we find that the safety factor at the stationary state still remains below 1.0, instead of a flatten profile with $q = 1.0$ around the magnetic axis, which is consistent with the Poincare plots of the magnetic field.

For sawteeth with the incomplete reconnection, the disadvantage of the safety factor based on the initial magnetic axis becomes more evident. Since magnetic reconnection is incomplete, the helicity of the magnetic field or the safety factor near the magnetic axis should remain unchanged during sawtooth. But the old safety factor indicates the safety factor at the magnetic axis becomes flattened and equal to 1.0, which is apparent incorrect. From the new and precise calculation of the safety factor, the safety factor indeed remains unchanged during sawteeth with the incomplete reconnection, which is consistent with the experimental observations.[9]

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