Hall Effect on Tearing Mode Instabilities in Tokamak

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Abstract. Tearing mode instability is one of the most important dynamic processes in space and laboratory plasmas. Hall effects, resulted from the decoupling of electron and ion motions, could cause the fast development and perturbation structure rotation of the tearing mode and become non-negligible. A high accuracy nonlinear MHD code (CLT) is developed to study Hall effects on the dynamic evolution of tearing modes with Tokamak geometries. It is found that the diamagnetic rotation of the mode structure is self-consistently contained in the Hall MHD model. The self-consistently generated rotation largely alters the dynamic behaviors of the double tearing mode.

I. INTRODUCTION

It is widely believed that many eruptive phenomena in both space[1, 2] and laboratory[3, 4] are closely related to a tearing mode instability[5] that leads to not only magnetic energy converting into kinetic energy and heat[6, 7] but also the barrier breaking down between two different plasma regions. It is regarded as the primary cause for the degradation of plasma performance in magnetic confined fusion device such as Tokamak[8-10]. The tearing mode instability was firstly studied analytically by Furth et al[11] in the framework of resistive magnetohydrodynamics (MHD). It is found that the linear growth rate of the resistive tearing mode instability is $\gamma \sim S^{-3/5}$, where S is Lundquist number.

The Hall-MHD model which describes two-fluid plasma with massless electron is often used to study magnetic reconnection[12-14]. It is believed that Hall effects can largely accelerate dynamic process of magnetic reconnection. The reconnection rate in Hall MHD well agrees with that in full particle simulation. Most studies of magnetic reconnection are carried out in slab geometry where there is no diamagnetic drift effect which may play an important role in dynamics of tearing mode instability in a toroidal geometry such as magnetic confined fusion device Tokamak.

Effects of diamagnetic drift on linear tearing mode instability were firstly studied by Ara et al[15] in the framework of the two-fluid MHD. In previous simulations of the m/n=1/1 resistive kink instability and sawtooth, a diamagnetic drift is included as an initial velocity.[16, 17] In the present study, we carry out a Hall-MHD simulation with the zero initial velocity. It is found that the tearing mode structure is rotated diamagnetically.

One of an advanced mode in Tokamak is operated with a reversed shear q profile,[18, 19]where q is the safety factor. The system may be subject to the double tearing mode (DTM) instability which is excited in neighboring rational surfaces. DTM could have quite different dynamic process from a single tearing mode due to mutual interaction between two tearing mode instabilities in rational surfaces. The tearing mode instabilities in two rational surfaces could excite each other if their perturbations in two rational surfaces are anti-phase, i.e., the expansion of the island in one rational surface compresses the current sheet of another rational surface. The tearing mode instabilities in two rational surfaces could be suppressed each other if their perturbations in two rational surfaces are in-phase, i.e., the expansion of the island in one rational surface collides with the expansion of the island in another rational surface. Since the tearing mode structures in Hall MHD are rotated due to diamagnetic drift, it is worthwhile to investigate dynamic process of the double tearing mode instability when the pressure gradients are different in the two rational surfaces.

To the best of our knowledge, a toroidal tokamak simulation in the framework of Hall-MHD has not been carried out. In this paper, Hall MHD simulations are performed to investigate the dynamic processes of the m/n=2/1 tearing mode and the m/n=3/1 double tearing mode by the CLT code [20, 21] developed at Zhejiang University.

Our article is organized as follows. In Section II, without the initial diamagnetic drift velocity, the diamagnetic rotation frequency associated with the tearing mode instability in the linear stage is derived based on Hall MHD. In Section III, the Hall MHD equations used in CLT are presented. In Section IV.A, we present simulation results from toroidal Hall-MHD code (CLT) and compare these results with theoretical predictions. In Section IV.B, double tearing mode instability is simulated with the different pressure gradients in two rational surfaces. Finally, we summarize our work in Section VI.

II. DIAMAGNETIC RTOTATION FOR TEARING MODE INSTABILITY DUE TO HALL EFFECTS

The diamagnetic rotation frequency is firstly derived by Ara et al [15]. They used two-fluid equations and analyzed the effects of diamagnetic rotation on the tearing mode instability. In this section, we use the incompressible Hall MHD equations to derive the diamagnetic rotation frequency and we will show that the theoretical prediction is in good agreement with our simulation results in Section IV.A.

For the sake of the simplicity, incompressible assumption $\nabla \bullet \mathbf{u} = 0$ is used. In cylindrical geometry, the linearized equations for tearing mode instability [5] can be written as

$$\gamma\psi_{h1} - \psi_{h0}^{'}\gamma\xi = \frac{\eta}{\mu_0}[\frac{1}{r}\frac{d}{dr}(r\frac{d\psi_{h1}}{dr}) - (\frac{m^2}{r^2} + \frac{n^2}{R^2})\psi_{h1}] \quad (1)$$

$$\gamma^2\rho\frac{1}{m}[\frac{1}{r}\frac{d}{dr}(r\frac{d(r\xi)}{dr})-\frac{m^2}{r^2}(r\xi)] \\ =-\frac{m}{\mu_0 r}\psi_{h0}^{'}[\frac{1}{r}\frac{d}{dr}(r\frac{d\psi_{h1}}{dr})-\frac{m^2}{r^2}\psi_{h1}]+\frac{m}{r}\psi_{h1}\frac{dJ_{z0}}{dr} \tag{2}$$

where ψ_{h0} and ψ_{h1} are initial helical flux and perturbed helical flux, respectively. ξ is a radical displacement. In the exterior or ideal region, the resistive diffusion and inertia terms can be neglected. We have

$$\gamma\psi_{h1}-\psi_{h0}^{'}\gamma\xi=0 \tag{3}$$

$$[\frac{1}{r}\frac{d}{dr}(r\frac{d\psi_{h1}}{dr})-\frac{m^2}{r^2}\psi_{h1}]-\frac{\mu_0 dJ_{z0}}{\psi_{h0}^{'}dr}\psi_{h1}=0 \tag{4}$$

In the interior region, since $\psi_{h1}^{''}\gg\frac{1}{r}\frac{d\psi_{h1}}{dr}\gg\frac{m^2}{r^2}\psi_{h1}$, it yields

$$\psi_{h1}-F'x\xi=\frac{r_s^2}{\gamma\tau_R}\psi_{h1}^{''} \tag{5}$$

$$\frac{\gamma^2\tau_A^2}{m^2}\xi^{''}=-F^{'}x\psi_h^{''} \tag{6}$$

where $F^{'}=-q_s^{'}/q_s$, $x=r-r_s$, $\tau_A=r_s\sqrt{\mu_0\rho}/B_{\theta 0}(r_s)$ and $\tau_R=\mu_0 r_s^2/\eta$.

Through the matching condition of the interior and exterior solutions, we obtain the linear growth rate

$$\gamma=(\frac{\Gamma(1/4)}{\pi\Gamma(3/4)}\Delta^{'})^{4/5}(\frac{\eta}{\mu_0})^{3/5}(\frac{mq_s^{'}}{r_s q_s}\frac{B_{\theta 0}(r_s)}{\sqrt{\mu_0\rho}})^{2/5}. \tag{7}$$

With inclusion of the Hall contribution in the Om's law, the linearized equations for tearing mode instability is modified to be

$$\gamma\psi_{h1}-\psi_{h0}^{'}(\gamma+i\omega)\xi=\frac{\eta}{\mu_0}[\frac{1}{r}\frac{d}{dr}(r\frac{d\psi_{h1}}{dr})-(\frac{m^2}{r^2}+\frac{n^2}{R^2})\psi_{h1}]+\frac{im}{ner}J_{0\perp}\psi_{h1}, \tag{8}$$

$$\gamma(\gamma+i\omega)\rho\frac{1}{m}[\frac{1}{r}\frac{d}{dr}(r\frac{d(r\xi)}{dr})-\frac{m^2}{r^2}(r\xi)] \\ =-\frac{m}{\mu_0 r}\psi_{h0}^{'}[\frac{1}{r}\frac{d}{dr}(r\frac{d\psi_{h1}}{dr})-\frac{m^2}{r^2}\psi_{h1}]+\frac{m}{r}\psi_{h1}\frac{dJ_{z0}}{dr}. \tag{9}$$

The last term in Eq. (8) is associated with the Hall effects. In the exterior or ideal region, Eq. (8)

and (9) becomes

$$\gamma\psi_{h1} - \psi_{h0}^{'}(\gamma + i\omega)\xi = \frac{im}{ner} J_{0\perp}\psi_{h1}, \tag{10}$$

$$[\frac{1}{r}\frac{d}{dr}(r\frac{d\psi_{h1}}{dr}) - \frac{m^2}{r^2}\psi_{h1}] - \frac{\mu_0 dJ_{z0}}{\psi_{h0}^{'} dr}\psi_{h1} = 0, \tag{11}$$

respectively. From the real and imaginary parts of Eq. (10), it yields

$$\gamma\psi_{h1} - \psi_{h0}^{'}\gamma\xi = 0, \tag{12}$$

$$-\psi_{h0}^{'}\omega\xi = \frac{m}{ner} J_{0\perp}\psi_{h1}. \tag{13}$$

From Eq. (12) and (13), it easily gets

$$\omega = -\frac{m}{ner} J_{0\perp} = -\frac{m}{nerB_{h0}}\frac{dp}{dr} = m\omega_* \tag{14}$$

where $\omega_* = -\frac{1}{neBr}\frac{dp}{dr}$ is the diamagnetic rotation frequency. It is suggested that the mode structure in the ideal region is rotated with the diamagnetic frequency due to inclusion of Hall effects.

For the case $\omega_* \ll \gamma$, Eq. (5) and (6) for the interior region remain unchanged. Thus, we have the same linear growth rate as in resistive MHD, which means that the diamagnetic rotation of the mode structure in the outer region for Hall MHD will not affect the mode growth rate if $\omega_* \ll \gamma$.

III. HALL-MHD EQUATIONS IN CLT

The full set of the Hall-MHD equations is given as follows:

$$\frac{\partial\rho}{\partial t} = -\nabla\cdot(\rho\mathbf{v}) + \nabla\cdot[D\nabla(\rho - \rho_0)] \tag{16}$$

$$\frac{\partial p}{\partial t} = -\mathbf{v}\cdot\nabla p - \Gamma p\nabla\cdot\mathbf{v} + \nabla\cdot[\boldsymbol{\kappa}\nabla(p - p_0)] \tag{17}$$

$$\frac{\partial\mathbf{v}}{\partial t} = -\mathbf{v}\cdot\nabla\mathbf{v} + (\mathbf{J}\times\mathbf{B} - \nabla p)/\rho + \nabla\cdot[\upsilon\nabla(\mathbf{v} - \mathbf{v}_0)] \tag{18}$$

$$\frac{\partial\mathbf{B}}{\partial t} = -\nabla\times\mathbf{E} \tag{19}$$

$$\mathbf{E} = -\mathbf{v}\times\mathbf{B} + \eta(\mathbf{J}-\mathbf{J}_0) + \frac{d_i}{\rho}(\mathbf{J}\times\mathbf{B} - \nabla P_e) \tag{20}$$

$$\mathbf{J} = \nabla\times\mathbf{B} \tag{21}$$

Where ρ, p, v, B, E, and J denote the plasma density, the thermal pressure, the plasma velocity, the magnetic field, the electric field, and the current density, respectively. $d_i = c/\omega_{pi}$ is the ion skin length. The subscript "0" denotes the equilibrium quantities. $\Gamma(=5/3)$ is the ratio of specific heat of plasma. p_e is the electron pressure. The variables are normalized as follows: $\mathbf{B}/B_0 \to \mathbf{B}$, $\mathbf{x}/a \to \mathbf{x}$, $\rho/\rho_0 \to \rho$, $\mathbf{v}/v_A \to \mathbf{v}$, $t/t_A \to t$, $p/(B_0^2/\mu_0) \to p$, $\mathbf{J}/(B_0/\mu_0 a) \to \mathbf{J}$, $\mathbf{E}/(v_A B_0) \to \mathbf{E}$, $d_i/a \to d_i$ and $\eta/(\mu_0 a^2/t_A) \to \eta$, where a is the minor radius, $v_A = B/\sqrt{\mu_0\rho}$ is the Alfven speed, and $t_A = a/v_A$ is the Alfven time. B_0 and ρ_0 are the magnetic field and the plasma density at the magnetic axis, respectively. For the typical parameters in Tokamak, $n \sim 10^{20}$, $a \sim 1$, $\omega_{pi} \sim 10^{10}$ and $c = 3\times10^8$ so $d_i \sim 0.03$.

In the previous literatures[16, 17], the diamagnetic flow as an initial background flow is added into the momentum equation to study diamagnetic rotation effects on the tearing mode instability, i.e.,

$$\frac{\partial\mathbf{v}}{\partial t} = -\mathbf{v}\cdot\nabla\mathbf{v} - \mathbf{v}_*\bullet\nabla\mathbf{v} + (\mathbf{J}\times\mathbf{B} - \nabla p)/\rho \tag{22}$$

In this equation the contribution of $\mathbf{v}\bullet\nabla v_*$ is neglected. However, these terms are not guaranteed to be zero. In our simulations, the diamagnetic rotation effects on the tearing mode instability are investigated self-consistently by using the Hall-MHD equations.

IV. SIMULATION RESULTS

A. m/n=2/1 tearing mode

The parameters with a toroidal tokamak configuration are chosen as follows: the major radius $R_0 = 4$ and the minor radius $a = 1$. The initial equilibrium profiles of the safety factor q and the plasma pressure p are shown in Figure 1. The equilibrium magnetic field B_0 and the current density J_0 are obtained from the NOVA[22] code. Other diffusion parameters used in our simulations are the resistivity $\eta = 1\times10^{-5}$, the viscosity $\upsilon = 1\times10^{-6}$, the plasma diffusivity $D = 1\times10^{-6}$, and the conductivity $\kappa = 5\times10^{-5}$. The larger conductivity is used to suppress ballooning modes. With the given profile, the most unstable tearing mode is the m/n=2/1 mode occurs at the *q*=2 rational surface.

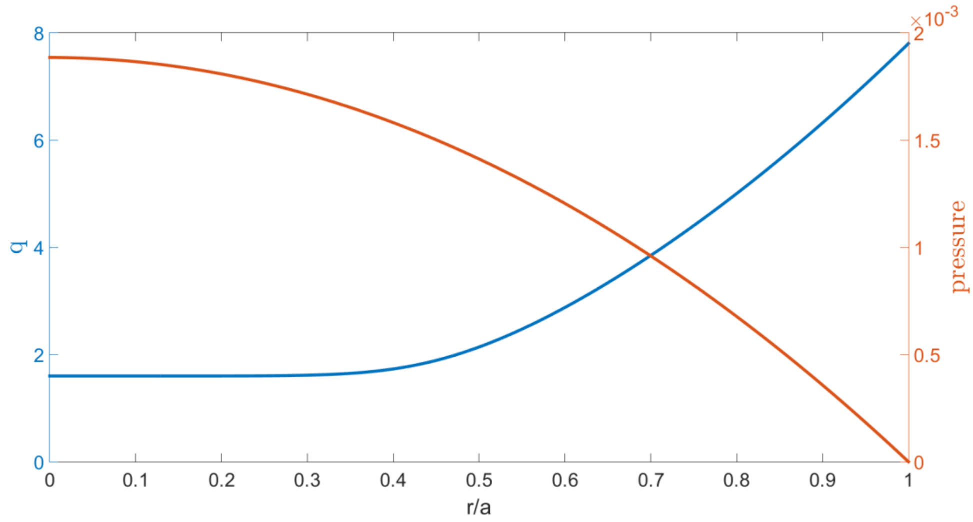

Figure 1 .Initial profiles of the safety factor *q* and the plasma pressure *p*.

Figure 2 gives the mode structures (δE_ϕ) in the $\phi = 0$ cross-section at (a), (c) t=620 t_A and (b), (d) t=1683t_A for cases with/without Hall effects. It is clearly indicated that there is a slow rotation of the mode structure for Hall-MHD. The mode structure at t=1683t_A has been rotated about 30 degree, which gives the period to be about $T_s \sim 2\times10^4 t_A$. After inserting the parameters used in the simulation into Equation (14), we have $T_t \sim 1.7\times10^4 t_A$ which is in a good agreement with that from the simulation. From the pressure profile, the pressure inside the q=2 resonant surface becomes gradually flattening. The low pressure gradient leads to a slower rotation speed. The mode structure will slow down overall mode rotation due to the dragging effect of the slower rotation of the core region. This is why the rotation period in the simulation is slightly longer than that from the theoretical calculation.

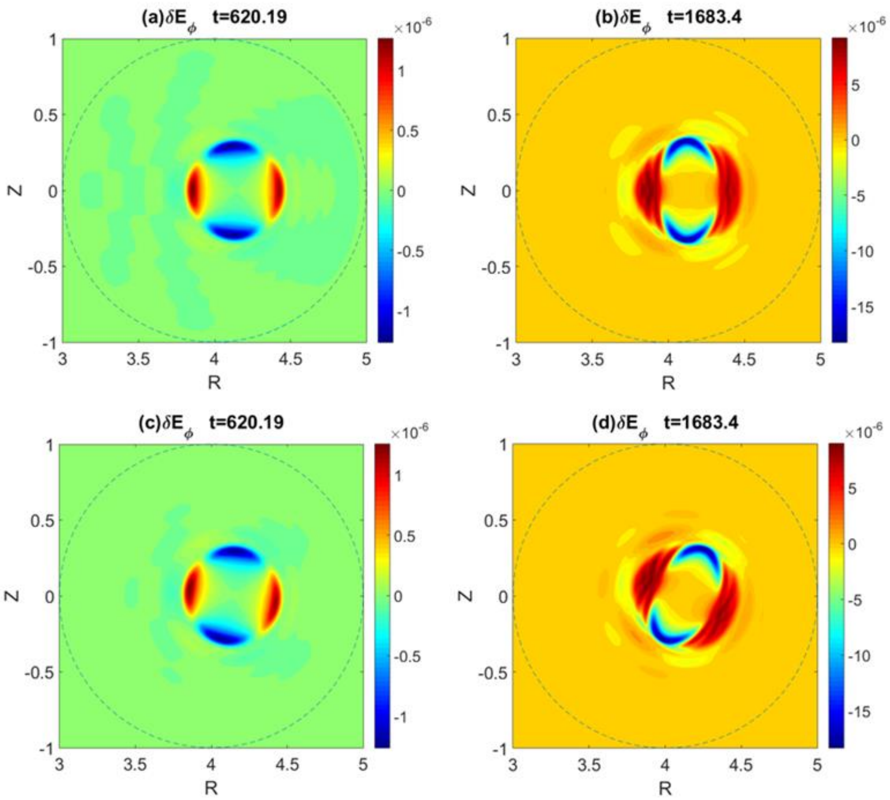

Figure 2 .The mode structure (δE_{ϕ}) from resistive MHD and Hall MHD at t=620 t_A and t=1683 t_A .

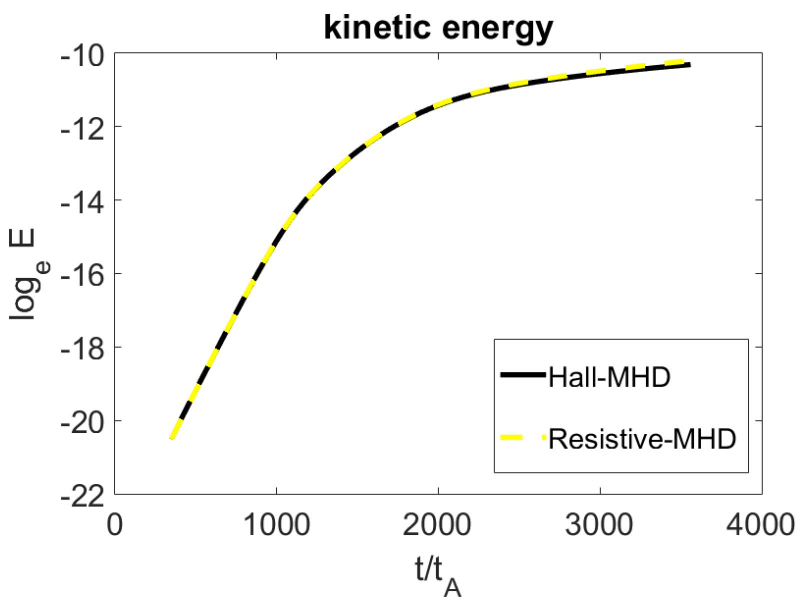

Figure 3 .The time evolution of the kinetic energy with/without Hall effects for the m/n=2/1 tearing mode.

The time evolution of the kinetic energy with/without Hall effects for the m/n=2/1 tearing mode is given in Figure 3. It is found that the linear growth rate is $\gamma = 0.0084$ that is much larger than the rotation frequency of the mode structure in Hall MHD. As we expected, there are the same linear growth rates of the m/n=2/1 mode for cases with/without Hall effects because the current sheet thickness is still larger than the ion inertial length in the linear phase. In the nonlinear phase, the reconnection rate in the slab geometry will largely increase in Hall MHD. In the present simulation with Tokamak geometry, reconnection rates are no change with/without Hall effects, which may be associated with the diamagnetic rotation of the mode structure in Hall MHD. The mode rotation can prevent the current sheet thinning.

B. Double tearing mode

Previous studies[23] suggests that for a reserved q profile, tearing mode instabilities in two resonant surfaces will excite each other and grow much faster than a single tearing mode because the growing mode in one resonant surface becomes an external driven source that accelerates the mode development in another resonant surface.

In the previous subsection for the single tearing mode, it has been shown that the mode structure rotates at the diamagnetic frequency due to inclusion of Hall effects and the rotation speed of the mode structure depends on the local pressure gradient. It can be expected that different rotation frequencies of mode structures, due to different pressure gradients, will largely affect the dynamic evolution of a double tearing mode. We artificially construct an equilibrium with reverse-sheared q profile and different gradients of the plasma pressure to study double tearing mode instability. The initial q and pressure profiles are showed in Figure 4. With the range of q from 2.6 to 3.8, the most unstable modes should be the m/n=3/1 mode which takes place at the two resonant surfaces with q=3.

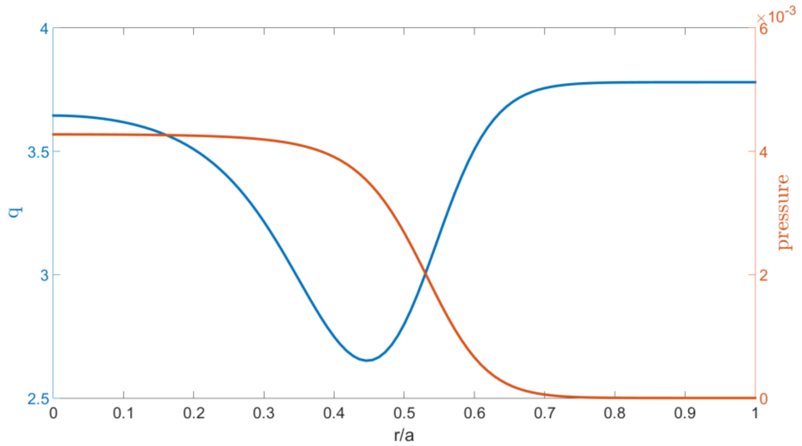

Figure 4. The initial q and pressure profile for m/n=3/1 double tearing mode instability.

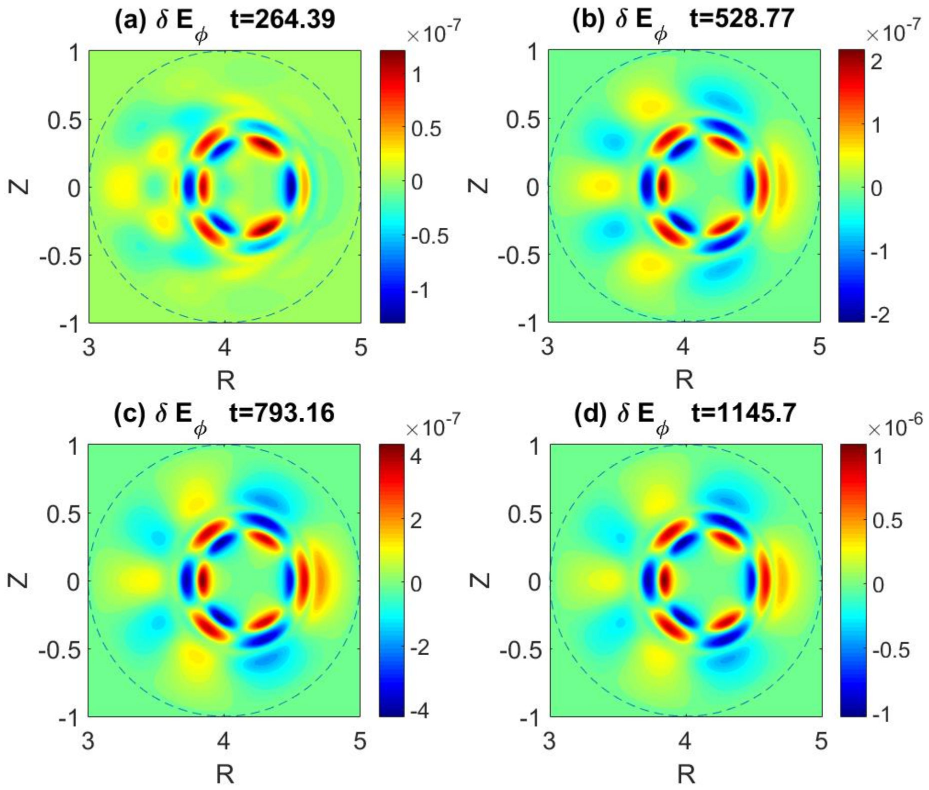

Figure 5. The snapshots of mode structures (δE_{ϕ}) from resistive-MHD.

The snapshots of mode structures (δE_{ϕ}) from resistive MHD are shown in Figure 5. It is evident that the perturbations at the two resonant surfaces have opposite polarities and this property of the opposite polarities remains in the whole simulation period. The opposite polarities of the mode perturbations becomes mutual driven sources that accelerate the development of double tearing mode instability persistently. Therefore, the growth rate of the double tearing mode is much faster than that of a single tearing mode in resistive MHD.

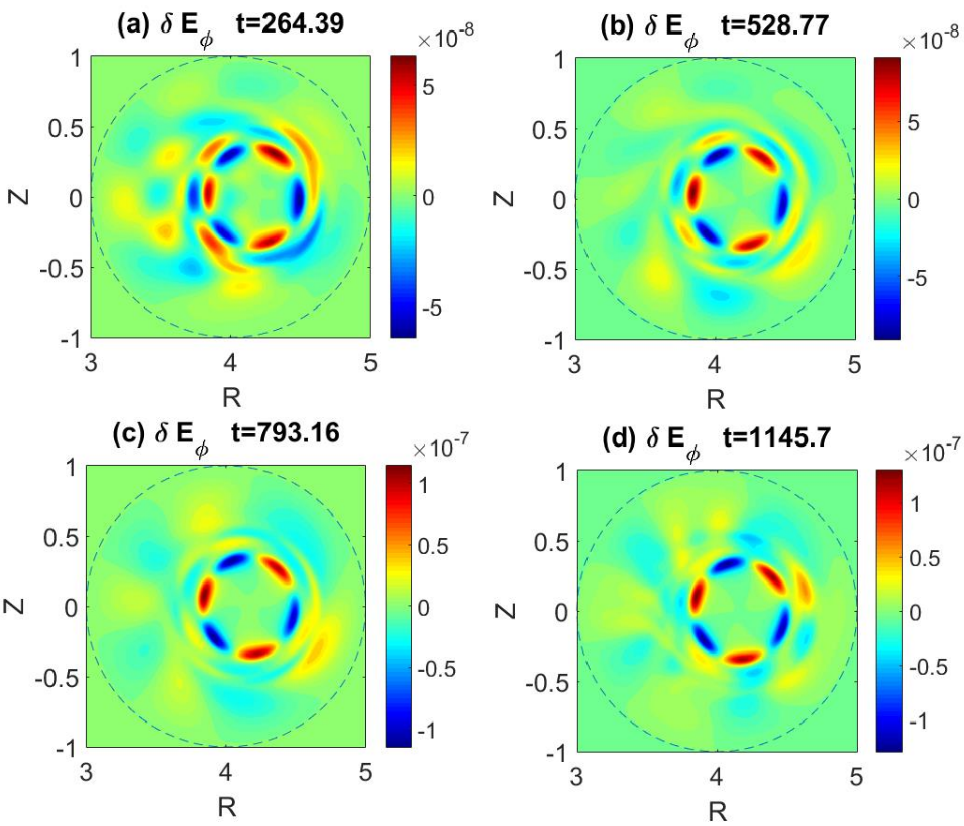

Figure 6. The snapshots of the mode structures (δE_{ϕ}) from Hall MHD.

With inclusion of Hall effect, the simulation and theoretical results of the single tearing mode suggests that the mode structure at the outer resonant surface rotates with the frequency $\omega = 3\omega_{*}$ while the mode structure at the inner resonant surface is stationary due to the flatten pressure profile, which is quite evident from simulation results as given in Figure 6. In the simulation, the mode structure in the outer resonant surface rotates clockwise while the inner mode structure almost remains at rest. The perturbations of two tearing modes gradually switch from anti-phase to in-phase. In other words, the perturbations for the two modes gradually evolve from mutual acceleration to mutual suppression, which is clearly indicated in Figure 6 that the growth rates of the amplitudes of the mode structure reduces with time and the mode finally saturates.

The time evolutions of the kinetic energy for double tearing modes from resistive-MHD and Hall MHD are shown in Figure 7. It is clearly indicated that the growth rate of the double tearing mode in resistive MHD is much larger than in Hall MHD. The kinetic energy in resistive MHD continuously increases and there is no saturation observed in the simulation period. But the kinetic energy in Hall MHD has a low growth rate and quickly saturates at a very low level.

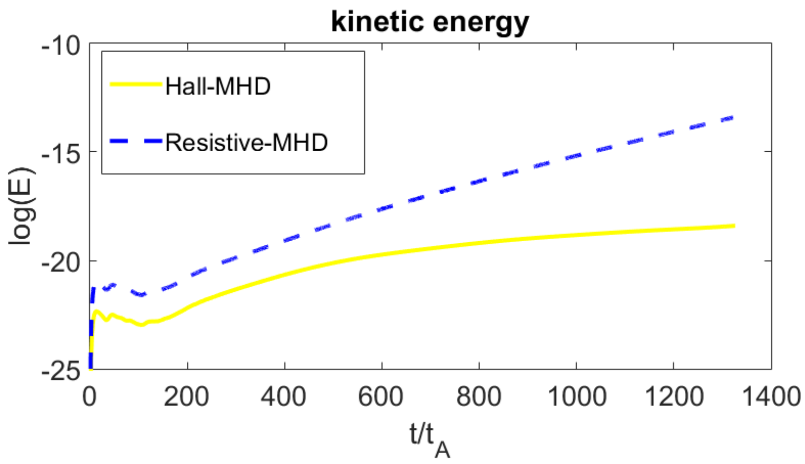

Figure 7. The time evolution of the kinetic energy for double tearing modes from resistive-MHD and Hall MHD.

V. SUMMARY AND DISCUSTION

In magnetic confined fusion device such as Tokamak, the thermal plasma pressure decreases away from the central core region. The pressure gradient leads to a diamagnetic rotation of plasma. The diamagnetic rotations resulted from ion or electron pressure usually play different role on plasma dynamics due to the large mass ratio. The diamagnetic rotation associated with ion pressure causes a plasma flow while it related to the electron pressure leads to the rotation of the magnetic field perturbation due to the frozen-in condition. In Hall MHD, the pressure is only from the electrons because the cold ions are assumed. Thus, we should only observe the diamagnetic rotation of the mode structure without the plasma rotation flow.

Tearing mode instability is one of the most important dynamic processes in space and laboratory plasmas. It is suggested that Hall effects in the slab geometry could cause the fast development and perturbation structure rotation of the tearing mode and become non-negligible. In this paper, we use the new developed high accuracy nonlinear MHD code (CLT) to study Hall effects on the dynamic evolution of tearing modes with Tokamak geometries. It is found that the rotation speed of the mode structure from the simulation is in a good agreement with that from the analytical theory in a single tearing mode. The phenomenon of fast growth of tearing mode instability is not observed, which may be associated with the rotation of the reconnection region.

The Hall effects on dynamic evolution of double tearing mode is also conducted out. With an artificial constructed pressure to amplify the different diamagnetic rotation speeds at two resonant surfaces, it is found that the perturbations of two tearing modes in Hall MHD gradually switch from anti-phase to in-phase while the perturbations in resistive MHD is persistently anti-phase. In Hall MHD, the mutual driven perturbations with opposite polarities gradually evolve to the mutual suppressed perturbations with the same polarities which cause the lower growth rate of the double tearing mode and the mode amplitude saturates at a very low level.

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