On Drift Wave Instabilities Excited by Strong Plasma Gradients in Toroidal Plasmas

Hao-Tian Chen[1,*] and Liu Chen[1,2,†]

[1] Institute for Fusion Theory and Simulation and Department of Physics, Zhejiang University, Hangzhou, 310027, People's Republic of China

[2] Department of Physics and Astronomy, University of California, Irvine, California 92697, USA

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Motivated by the recent global gyrokinetic simulations of electrostatic drift-wave instabilities (DWIs) in the strong plasma gradient region of toroidal plasmas [1–3], we have carried out the corresponding analytical and numerical investigations in the case of ion temperature gradient (ITG) driven modes. It is shown that, for sufficiently strong plasma gradients, the eigenmodes are slab-like and predominantly bounded by the plasma non-uniformities. Our results are qualitatively consistent with the simulation observations.

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Recently, gyrokinetic simulations have been carried out to investigate the characteristics of the electrostatic drift-wave instabilities (DWIs) in the strong plasma gradient region of tokamak plasmas [1–3]. It is found that, unlike the weak gradient case, the eigenmodes can peak away from the outboard midplane and have shorter radial correlation lengths [2, 3]. In this work, motivated by the simulation observations, we have carried out corresponding theoretical analyses in order to clarify the nature of DWI eigenmodes in the strong plasma gradient region of toroidal plasmas. Our results indicates that, in the limit of strong plasma gradients, the eigenmodes are radially bounded by the plasma non-uniformities [4] and exhibit little ballooning structures. In other words, in the lowest order, these DWI eigenmodes can be understood in the cylindrical limit with weak couplings to the poloidal sidebands.

Let us consider electrostatic drift-wave instabilities in an axisymmetric, low-β, large aspect-ratio tokamak with concentric circular magnetic surfaces. Here, β is the ratio between plasma and magnetic pressures. The (r, θ, ζ) coordinates are adopted here; corresponding to the minor radius, poloidal and toroidal angles, respectively. The equilibrium magnetic field is given by $\vec{B} = B_0(1 - \epsilon\cos\theta)(\hat{e}_\zeta + \epsilon/q\hat{e}_\theta)$, with $\epsilon = r/R \ll 1$, R the major radius, and q the safety factor. The equilibrium distribution function is taken to be local Maxwellian, $F_{0,j} = N(\pi v_{tj}^2)^{-3/2}e^{-v^2}$. Here, $j = i, e$ denotes the particle species, and the velocity is normalized to the thermal velocity $v_{tj} = \sqrt{2T_j/m_j}$. Other notations are standard. The density and temperature profiles, meanwhile, are modeled as

$$N(r) = N_0 e^{-\arctan(\frac{r-r_0}{\bar{r}_n})}, \quad T_j(r) = T_{0,j}e^{-\arctan(\frac{r-r_0}{\bar{r}_{t,j}})},$$

where r_0 is the reference radius, and $\bar{r}_n, \bar{r}_{t,j}$ are inhomogeneity constants. The plasma non-uniformity function responsible for the instability drive is then given by $r_n^{-1} \equiv -\partial_r N/N = [\bar{r}_n(1 + (r - r_0)^2/\bar{r}_n^2)]^{-1}$. Considering perturbations of a single toroidal mode number, n, the electrostatic potential and non-adiabatic distribution function are expressed as sums of poloidal harmonics:

$$[\delta\phi, \delta G] = e^{-i\omega t - in\zeta}\sum_m e^{im\theta}[\delta\hat{\phi}_m(r), \delta\hat{G}_m(r)].$$

Before we present the detailed analysis, it is instructive to give some qualitative remarks. Let Δ_s denote the distance between mode rational surfaces of adjacent poloidal harmonics, and $\iota = r_n/\Delta_s$ be the normalized density scale length. Note that, in the usual weak non-uniformity limit with $r_n \gg \Delta_s$, the radial envelope is localized by the plasma non-uniformities with a typical width of the order of $\sqrt{\rho_{i0}r_n}$. We can then treat $\delta \equiv \sqrt{\rho_{i0}r_n}/\Delta_s = \sqrt{\epsilon_\rho}\iota \le 1$ as the strong plasma gradient condition, where $\epsilon_\rho = \rho_{i0}/r_n$. That is, if the strong plasma gradient condition is satisfied, plasma non-uniformities will render the meso-scale radial envelope comparable to the microscopic scale, and thus, can be expected to make significantly impacts on the eigenmode structures. Furthermore, due to the breakdown of scale-length separation, one would also expect the usual large-n ballooning representation [6, 7], in general, is not valid here. Using the typical parameters in [3]: $s \simeq 0.5$, $\epsilon_\rho \simeq 8.0 \times 10^{-2}$, $k_\theta\rho_{i0} \simeq 0.3$, we find $\delta \simeq 0.53$, which suggests that the simulation results should fall within the strong plasma non-uniformities regime.

Retaining explicitly the plasma non-uniformities, the ion linear gyrokinetic equation [5] can be readily expressed as

$$\begin{aligned}&[\omega + \omega_{ti}v_\parallel(z-p)]\delta\hat{G}_m + \bar{\omega}_{*i,0}e^{-\arctan(\frac{\bar{\eta}_i z}{\bar{\iota}})}\bar{\epsilon}_n(v_\parallel^2 + \frac{1}{2}v_\perp^2)\\ &\times[(1+s\partial_z)\delta\hat{G}_{m+1} + (1-s\partial_z)\delta\hat{G}_{m-1}]\\ = \;&(\omega + \omega_{*i}^t)\frac{eF_0}{T_i}\langle\delta\hat{\phi}\rangle_m. \end{aligned} \tag{1}$$

Here, $s = r_0q_0'/q_0$, $q_0 = q(r_0)$, $k_\theta = nq_0/r_0$, $\epsilon_n = r_n/R$, $z = k_\theta s(r - r_0)$, $p = m - nq_0$, $\bar{\iota} = k_\theta\rho_{i0}s\bar{\epsilon}_\rho^{-1}$, $\iota = \bar{\iota}/\sigma$, $\sigma = (1 + z^2/\bar{\iota}^2)^{-1}$, $\langle A\rangle$ is the gyrophase average of A, $\omega_{ti} = v_{ti}/(qR)$ and $\omega_{*i} = k_\theta T_i/(eBr_n) = \sigma\bar{\omega}_{*i}$

*Email: haotianchen@zju.edu.cn

†Email (Corresponding author):liuchen@zju.edu.cn

are, respectively, the ion transit and diamagnetic frequencies, and $\omega_{*i}^{t} = \omega_{*i}[1+\eta_i(v^2-3/2)]$, with $\eta_i = \bar{\eta}_i(1+z^2/\bar{\iota}^2)(1+\bar{\eta}_i^2 z^2/\bar{\iota}^2)^{-1}$, $\bar{\eta}_i = \bar{r}_n/\bar{r}_{t,i}$. In terms of the parameters above, the strong plasma gradient condition becomes $\delta = k_\theta \rho_{i0} s \epsilon_\rho^{-1/2} \le 1$. Thus, under the gyrokinetic ordering $\epsilon_\rho \ll 1$, the condition can be more readily satisfied for weak magnetic shear and long wavelength.

To simplify the analysis and, thereby, gain more insights into the main features of DWIs in the strong plasma gradient region, we solve Eq. (1) for circulating ions by adopting the following ordering: $b_i \equiv k_\theta^2 \rho_{i0}^2 \sim \iota^{-2} \sim \bar{\epsilon}_n/|\Omega| \ll 1$, and neglecting the $v_\parallel$ modulation along the orbit. We further assume that the electron response is adiabatic, and, hence, focus on the ion temperature gradient (ITG) driven mode as an example of DWIs. Applying the quasi-neutrality condition then straightforwardly yields the eigenmode equation,

$$\partial_z^2 \delta\hat{\phi}_m + Q\delta\hat{\phi}_m = \hat{\mathcal{C}}(\delta\hat{\phi}_{m+1}, \delta\hat{\phi}_{m-1}). \tag{2}$$

Here, Q is the corresponding potential structure in the slab limit and is given by

$$Q = \frac{2}{b_i s^2} \frac{(g_- - 1)e^{\arctan\frac{\bar{\eta}_i z}{\bar{\iota}}} - \frac{e^{\arctan\frac{\bar{\eta}_e z}{\bar{\iota}}}}{\tau_0}}{g_+} - \frac{1}{s^2}, \tag{3}$$

with $\tau_0 = T_{0,e}/T_{0,i}$, $\bar{\eta}_e = \bar{r}_n/\bar{r}_{t,e}$,

$$g_\pm = \{\Omega + \sigma e^{-\arctan\frac{\bar{\eta}_i z}{\bar{\iota}}}[1+\eta_i(-\partial_\lambda|_{\lambda=1} \pm \frac{1}{2})]\}I_1,$$

$$I_1 = -\alpha Z(\sqrt{\lambda}\alpha\Omega), \quad \alpha(p) = \frac{e^{\frac{1}{2}\arctan\frac{\bar{\eta}_i z}{\bar{\iota}}}}{\Omega_t|z-p|},$$

and the frequencies are normalized by $\bar{\omega}_{*i,0}$, i.e., $\Omega = \omega/\bar{\omega}_{*i,0}$, $\Omega_t = 2\bar{\epsilon}_n/(q_0 k_\theta \rho_{i0})$. Meanwhile, the poloidal coupling term $\hat{\mathcal{C}}$ is

$$\begin{aligned} &\hat{\mathcal{C}}(\delta\hat{\phi}_{m+1}, \delta\hat{\phi}_{m-1}) \\ &= -\frac{2\bar{\epsilon}_n}{b_i s^2 g_+}[(\Omega + \sigma e^{-\arctan\frac{\bar{\eta}_i z}{\bar{\iota}}})(-\frac{1}{2} + \partial_\lambda|_{\lambda=1}) \\ &\quad -\eta_i \sigma e^{-\arctan\frac{\bar{\eta}_i z}{\bar{\iota}}}(\frac{1}{4} + \partial_\lambda^2|_{\lambda=1})] \\ &\quad \times[I_2^+(1+s\partial_z)\delta\hat{\phi}_{m+1} + I_2^-(1-s\partial_z)\delta\hat{\phi}_{m-1}], \end{aligned} \tag{4}$$

where

$$I_2^\pm = \frac{\alpha(p)\alpha(p\pm1)[SZ(\sqrt{\lambda}\alpha(p)\Omega) - Z(\sqrt{\lambda}\alpha(p\pm1)\Omega)]}{[\alpha(p) - S\alpha(p\pm1)]\sqrt{\lambda}\Omega},$$

with $S = \mathrm{sign}[(z-p)(z-(p\pm1))]$ and Z is the plasma dispersion function. It is to be emphasized that Eq. (2) retains the transit resonance and indicates that the eigenmode structures can be strongly affected by plasma non-uniformities.

The radial eigenmode equation, Eq. (2), has been solved numerically with typical simulation parameters [3] and varying $\bar{\delta} \equiv \delta(z=0)$ in order to investigate the effects of plasma non-uniformities on the ITG-DWI eigenmodes. Specifically, the parameters are $\epsilon = 0.21$, $\bar{\epsilon}_\rho \bar{\epsilon}_n = 0.001$, $s = 0.8$, $q_0 = 2.5$, $\tau_0 = 1$, $\bar{\eta}_i = 4$, $\bar{\eta}_e = 1$, $k_\theta \rho_{i0} = 0.3$. Note also that, within the current regime of parameters, we have found that it is sufficient to keep the three harmonics $p = 0, \pm 1$ for the most unstable eigenstates.

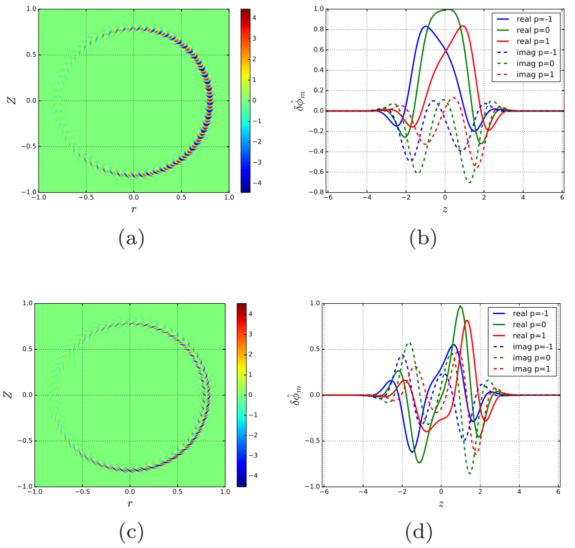

FIG. 1: (Color online) Plots of poloidal mode structures for the toroidicity-induced (a) and slab-like (c) modes. Corresponding radial mode structures of the three poloidal harmonics, $p = 0$ (green), $p = 1$ (red), $p = -1$ (blue), are plotted in (b) and (d). The solid (dashed) lines are for the real (imaginary) components. $\bar{\delta} = 4.15692$. Space coordinates of the poloidal plane are normalized to the minor radius.

We start from the weak plasma non-uniformities case with $\bar{\delta} = 4.15692$. The mode structures are plotted in Fig. 1, from which we can identify two types of unstable eigenmodes corresponding to the slab-like and toroidicity-induced modes [8]. As expected, the poloidal mode structure of the toroidicity-induced mode exhibits the usual ballooning structure, with the intensity peaking outward around $\theta = 0$. The poloidal harmonics, meanwhile, tend to peak about the corresponding mode rational surfaces.

As $\bar{\delta}$ decreases, the poloidal mode structures begin to lose the ballooning characteristics and the intensities peak away from $\theta = 0$; as shown in Fig. 2a, 2c. The poloidal harmonics are plotted in Fig. 2b, 2d. It can be seen that, as the profile becomes sharper, the harmonics peak away from their own mode rational surfaces and become radially localized near the maximum of the diamagnetic frequency ω_{*i}. Essentially such a phenomenon arises because the potential structure Q is modified by the strong plasma non-uniformities. For sufficiently strong plasma gradients with $\bar{\delta} < 1$, the toroidicity-induced

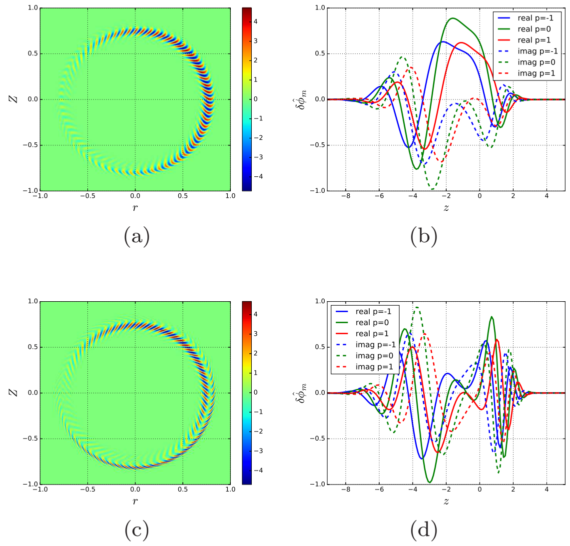

FIG. 2: The same as Fig. 1, except $\bar{\delta} = 1.2$.

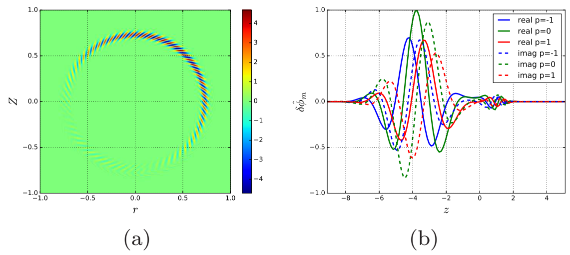

FIG. 3: (Color online) Plots of the poloidal mode structure (a) and radial mode structures of the three poloidal harmonics (b) for the slab-like mode with $\bar{\delta} = 0.734847$. The rest is the same as Fig. 1.

mode disappears. Fig. 3 shows that the slab-like mode is localized within a narrower strong plasma gradient region, along with a shorter radial correlation length. We note that the above features are consistent with the results of gyrokinetic simulations [1–3].

In Fig. 4, we plot the eigenvalues of ITG-DWIs as a function of $\bar{\delta}$. It clearly illustrates that the toroidicity-induced and slab-like modes co-exist for the weak plasma non-uniformities ($\bar{\delta} > 0.96$) with similar growth rates, but very different real frequencies, and the toroidicity-induced mode is more unstable. However, for smaller $\bar{\delta}$, the toroidicity-induced mode disappears, and the slab-like mode becomes the most unstable mode. Thus, the qualitative change in the poloidal mode structure as $\bar{\delta}$ varied is associated with the change of the most unstable eigenmode branch. To further verify that the dominant unstable mode is slab-like in the small $\bar{\delta}$ regime, we have solved Eq. (2) in the slab limit by dropping the poloidal coupling term $\hat{C}$. The corresponding eigenvalues are also

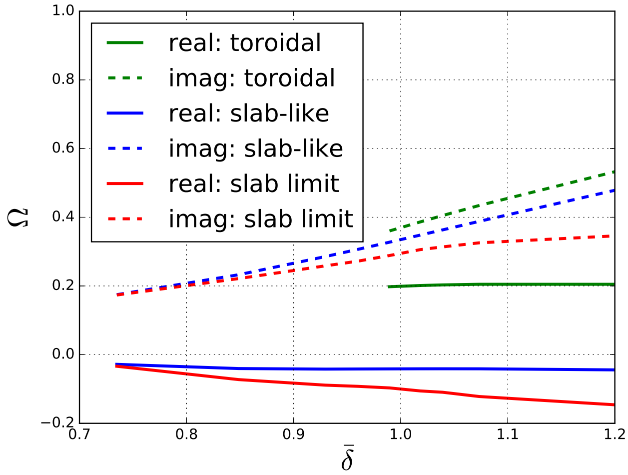

FIG. 4: (Color online) Normalized eigenvalues Ω versus $\bar{\delta}$ for toroidicity-induced (green), slab-like (blue), and slab limit (red) modes. The solid (dashed) lines are for the real frequencies (growth rates).

plotted in Fig. 4, and they coincide well with those of the slab-like mode.

To summarize, we have derived a two-dimensional kinetic eigenmode equation for the ion temperature gradient driven DWIs in the strong plasma gradient region of toroidal plasmas. It is found that the strong plasma gradients can significantly modify the corresponding potential structures and, as a consequence, the slab-like eigenmodes, which are predominantly bounded by the plasma non-uniformities, become the dominant unstable modes. Numerical solutions show that the corresponding poloidal mode structures can peak away from the outboard midplane. Individual poloidal harmonics, meanwhile, localize away from their own mode rational surfaces, and have shorter radial correlation lengths. The obtained results are found to be in qualitative agreement with the recent gyrokinetic simulation observations [1–3].

Finally, as the present results are due to the modification of the potential structures by the strong plasma non-uniformities, we remark that even though the emphasis is placed on the ITG modes in this study, the results are expected to also hold qualitatively for other types of DWIs.

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