

Physics of Kinetic Alfvén Waves: a gyrokinetic theory approach

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Abstract The transverse shear Alfvén wave (SAW) is a fundamental anisotropic electromagnetic oscillation in plasmas with a finite background magnetic field. In realistic plasmas with spatial inhomogeneities, SAW exhibits the interesting spectral feature of a continuous spectrum. That is, the SAW oscillation frequency varies in the non-uniform (radial) direction. This continuum spectral feature then naturally leads to the phase-mixing process; i.e. time asymptotically, the effective radial wave-number increases with time. Any initial perturbation of SAW structures will, thus, evolve eventually into short-wavelength structures; termed as kinetic Alfvén wave (KAW). Obviously, one needs to employ kinetic theory approach to properly describe the dynamics of KAW; including effects such as finite ion-Larmor radius (FILR) and/or wave-particle interactions. When KAW was first discovered and discussed in 1975-1976, it was before the introduction of the linear electromagnetic gyrokinetic theory (1978) and nonlinear electromagnetic gyrokinetic theory (1982). Kinetic treatments then often involved the complicated procedures of taking the low frequency limit of the Vlasov kinetic theory and/or employing the drift-kinetic theory approach; forsaking, thus, the FILR effects. In recent years,

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the powerful nonlinear gyrokinetic theory has been employed to re-examine both the linear and nonlinear physics of KAWs. This brief review will cover results of linear and nonlinear analytical theories, simulations, as well as observational evidences. We emphasize, in particular, that due to the enhanced electron-ion decoupling in the short-wavelength regime, KAWs possess significantly enhanced nonlinear coupling coefficients and, thereby, play important roles in the heating, acceleration and transport processes of charged particles in magnetized plasmas.

Keywords Kinetic Alfvén waves · Nonlinear phenomena and parametric effects · Wave-wave and wave-particle interactions

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1 Introduction

In 1942 [1], Hannes Alfvén discovered that if a perfectly conducting medium (*e.g.*, a fully ionized gas; *i.e.*, a plasma) is immersed in a finite background magnetic field \mathbf{B}_0 ; electromagnetic waves can then propagate within it. The reason is that, while the free electron mobility remains extremely large along \mathbf{B}_0 , its mobility perpendicular to \mathbf{B}_0 is inhibited. That is, a magnetized plasma is an anisotropic conducting medium with an extremely large parallel (to \mathbf{B}_0) and finite perpendicular (to \mathbf{B}_0) conductivities. Thus, correspondingly, these propagating electromagnetic waves, called Alfvén waves, have nearly vanishing parallel (to \mathbf{B}_0) and finite perpendicular (to \mathbf{B}_0) electric fields.

There are two types of Alfvén waves; the compressional (fast) and shear Alfvén waves [35]. The compressional Alfvén wave (CAW) compresses the magnetic field as well as plasma and its group velocity propagates almost isotropically. The shear Alfvén wave (SAW), meanwhile, is nearly incompressible and, thus, more readily excitable by either external perturbations (*e.g.*, solar wind, antenna) or intrinsic collective instabilities [17]. This brief review is focused on the SAW or, more specifically, its kinetic extension; *i.e.*, the kinetic Alfvén wave (KAW).

In a uniform plasma immersed in a uniform background magnetic field, $\mathbf{B}_0 = B_0 \hat{\mathbf{z}}$, and adopting the ideal magnetohydrodynamic (MHD) fluid description, it is well-known that the SAW satisfies the following linear dispersion relation

$$\omega^2 = k_{\parallel}^2 v_A^2. \quad (1)$$

Here, ω and $\mathbf{k} = \mathbf{k}_{\perp} + k_{\parallel} \mathbf{b}_0$ are, respectively, the wave angular frequency and wave vector, $\mathbf{b}_0 = \mathbf{B}_0/B_0$, \mathbf{k}_{\perp} is the perpendicular (to \mathbf{B}_0) component of \mathbf{k} , $k_{\parallel} = \mathbf{k} \cdot \mathbf{b}_0$, $v_A = B_0/(4\pi \varrho_m)^{1/2}$ is the Alfvén speed with $\varrho_m \simeq n_0 m_i$ being the mass density, and $m_i \gg m_e$. The corresponding wave polarization is

$$\left| \frac{c \delta \mathbf{E}_{\perp}}{\delta \mathbf{B}_{\perp}} \right| = v_A, \quad (2)$$

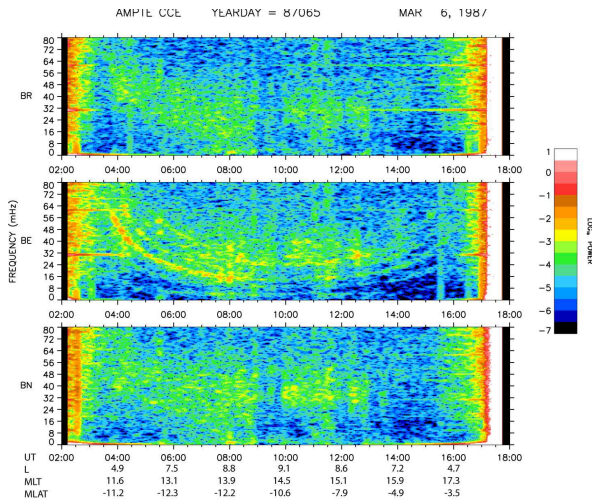


Fig. 1: Three-component dynamic power spectrum of magnetic field data from AMPTE CCE satellite [From original figure in Ref. [22]]. The geomagnetic B_R , radially outward from the center of the Earth; B_E , magnetically Eastward; and B_N , approximately along local magnetic field lines correspond to, respectively, $\delta B_x, \delta B_y, \delta B_z$.

with $\delta \mathbf{E}_\perp$ and $\delta \mathbf{B}_\perp$ denoting, respectively, the fluctuating components of electric and magnetic field perpendicular to \mathbf{B}_0 . Equation (1) indicates that SAW is an anisotropic electromagnetic wave; *i.e.*, while its phase velocity can propagate in any direction, its group velocity, $\mathbf{v}_g = v_A \mathbf{b}_0$, propagates only along \mathbf{B}_0 . This property, of course, has the direct bearing on the feature of Alfvén wave resonant absorption [28, 47, 29, 12, 13].

In a non-uniform plasma, SAW attains the interesting property of a continuous spectrum. To illustrate this feature, let us consider the simplified slab model of a cold plasma with a non-uniform density, $\rho_m = \rho_m(x)$, and a uniform $\mathbf{B}_0 = B_0 \hat{\mathbf{z}}$. Given an initial perturbation, one can readily derive, assuming time asymptotically that $|\partial_x^2| \gg |k_y^2|, |k_z^2|$, to be justified *a posteriori*, the following wave equation

$$[\partial_t^2 + \omega_A^2(x)] \delta B_y(x, t) = 0. \quad (3)$$

Here, $\omega_A^2(x) = k_z^2 v_A^2(x)$ and the solution is

$$\delta B_y(x, t) = \delta \hat{B}_y(x, 0) \exp[-i\omega_A(x)t]. \quad (4)$$

Equation (4) shows that every point in x oscillates at a different frequency, $\omega_A(x)$. With a continuously varying $\omega_A(x)$; the wave frequency, thus, constitutes a continuous spectrum. While the above result is based on a model with a one-dimensional non-uniformity in x , this general feature of SAW continuous spectrum also holds in magnetized plasmas with two- or three-dimensional

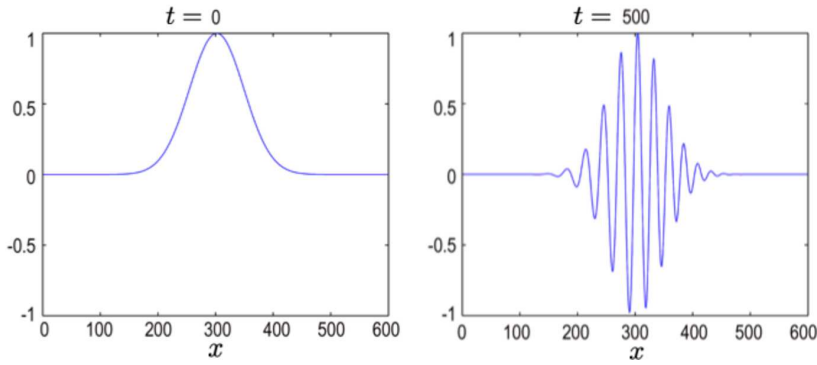


Fig. 2: Snapshots of $\delta B_y(x, t)$ spatial structure vs. x at different times, illustrating the formation of shorter scales at later times [From original figure in Refs. [48, 49]].

non-uniformities [29, 11, 55, 17]. A good example is geomagnetic pulsations in the Earth's magnetosphere. Figure 1 shows the oscillations in the Earth's magnetic field as observed by the satellite AMPTE CCE [22, 23], illustrating the three-component dynamic power spectrum of magnetic field data from for a full orbit from 02 : 30 to 17 : 30 UT March 6, 1987. Apogee is at the center of the figure. As the satellite moved outward from the morning side, ω_A should decrease due to the decreasing $|\mathbf{B}_0|$ and $|k_{\parallel}|$ (increasing field-line length), and this was clearly exhibited in the wave frequency of B_E , the azimuthal (East-West) component of $\delta \mathbf{B}$ (*i.e.*; the effective δB_y). B_E also shows that the wave frequency increases as the satellite moved inward toward the dusk side; consistent, again, with ω_A . Furthermore, the observed wave frequency consisted of several bands, which could be understood as harmonics of standing waves along the field line; *i.e.*, different $|k_{\parallel}|$.

$\delta B_y(x, t)$ given by Eq. (4) also indicates an unique and important property of SAW continuous spectrum; *i.e.*, the spatial structure evolves with time. Specifically, the wave number in the non-uniformity direction is, time asymptotically, given by

$$\begin{aligned} |k_x| &= |\partial_x \ln \delta B_y / \partial x| \\ &\simeq |d\omega_A(x)/dx|t \equiv |\omega'_A|t. \end{aligned} \quad (5)$$

That $|k_x|$ increases with t is significant, since it implies that any initially long-scale perturbations will evolve into short scales. This point is illustrated in Fig. 2; showing the evolution of a smooth δB_y at $t = 0$ to a spatially fast varying δB_y at a later t [48, 49]. Another consequence of $|k_x|$ increasing with t is the temporal decay of δB_x . From $\nabla \cdot \delta \mathbf{B} \simeq \nabla_{\perp} \cdot \delta \mathbf{B}_{\perp} = 0$, we can readily derive that, for $|\omega'_A t| \gg |k_y|$

$$\delta B_x(x, t) \simeq \frac{k_y}{\omega'_A(x)t} \delta \hat{B}_y(x, 0) e^{-i\omega_A(x)t} \left[1 + \mathcal{O} \left(\frac{k_y}{|\omega'_A t|} \right) + \dots \right]. \quad (6)$$

That is, δB_x decays temporally due to the phase-mixing of increasingly more rapidly varying neighboring perturbations. This property also explains why, in Fig. 1, the radial component of $\delta \mathbf{B}$, B_R , is much weaker than B_E .

Noting that, as $t \rightarrow \infty$, $|k_x| \rightarrow \infty$, it, thus, suggests that the perturbation will develop singular structures toward the steady state. Indeed, taking $\partial_t = -i\omega$, the SAW governing wave equation for the cold-plasma becomes [12, 13]

$$\left\{ \frac{d}{dx} [\omega^2 - \omega_A^2(x)] \frac{d}{dx} - k_y^2 [\omega^2 - \omega_A^2(x)] \right\} \delta B_x(x) = 0. \quad (7)$$

δB_x , thus, exhibits a logarithmic singularity at the Alfvén resonant point (layer), x_0 , where $\omega^2 = \omega_A^2(x_0)$ along with a finite resonant wave-energy absorption rate.

That the solution exhibits singularities naturally suggests that the microscopic length-scale physics neglected in the ideal MHD fluid description should be included in the long-time-scale dynamics of SAWs. For low-frequency SAWs, one can readily recognize the relevant perpendicular (to \mathbf{B}_0) microscopic scales are either the ion Larmor radius, $\rho_i = v_{ti}/\Omega_i$, and/or $\rho_s = c_S/\Omega_i$ with $c_S^2 = T_e/m_i$. Including the effects of finite ρ_i and/or ρ_s in the SAW dynamics then led to the discovery of the so-called kinetic Alfvén wave (KAW) [32, 33].

The pioneering discovery of KAW was carried out before the introduction of linear electromagnetic gyrokinetic theory [7, 3] and, later, nonlinear electromagnetic gyrokinetic theory [27]. The analyses employed, therefore, involved taking the low-frequency ($|\omega| \ll |\Omega_i|$) limit of the Vlasov dynamics. This makes theoretical analysis of KAW dynamics in non-uniform plasmas with realistic $\mathbf{B}_0(\mathbf{x})$ intractable; especially when dealing with the nonlinear physics. Indeed, previous nonlinear analyses adopted either the drift-kinetic or the two-fluid description [41, 43, 44, 45, 46, 65]. As our later discussions will show, such approximations not only are inadequate for treating realistic plasma regimes; but also often leave out important physics. The above discussions have, thus, motivated us [15, 16, 68] to re-visit and explore further the KAW physics employing the powerful gyrokinetic theories.

Section 2 presents a brief review of the linear gyrokinetic theory (cf. Sec. 2.1) and its applications to KAW (cf. Secs. 2.2 and 2.3) along with KAW observations by satellites (cf. Sec. 2.4). The nonlinear gyrokinetic theory is then briefly reviewed in Sec. 3.1. It is then applied to examine the physics of the three-wave parametric decay instabilities, the modulational instabilities associated with the spontaneous generation of convective cells, and the quasilinear phase space transport induced by KAW (cf. Secs. 3.2, 3.3 and 3.4). Results from corresponding numerical simulations are also presented. Final conclusions and discussions are given in Sec. 4.

2 Linear KAW physics

Here, we first introduce the foundation of the linear gyrokinetic formalism in Sec. 2.1. Linear KAW properties are then derived in Sec. 2.2 for uniform plasmas. Section 2.3 contains brief discussions of KAW in nonuniform plasmas; including the resonant mode conversion process. Observational evidences of KAWs by satellites are presented in Sec. 2.4.

2.1 Linear gyrokinetic theory

In magnetically confined plasmas, there exists a natural smallness parameter, $\epsilon = \rho/a$ with ρ and a being, respectively, the charged particle's Larmor radius and the macroscopic system scale length. Typically, we have $\epsilon \lesssim \mathcal{O}(10^{-2}) \ll 1$. Since low-frequency but short-wavelength fluctuations are of interest here, one, thus, adopts the following linear gyrokinetic orderings [7, 3, 27, 57, 6, 58]

$$\left| \frac{\omega}{\Omega_i} \right| \sim \mathcal{O}(\epsilon), \quad |k_{\perp} \rho_i| \sim \mathcal{O}(1); \quad (8)$$

and, in order to include Landau resonance,

$$k_{\parallel} v_{\parallel} \sim \omega, \quad \text{or} \quad \left| \frac{k_{\parallel}}{k_{\perp}} \right| \sim \left| \frac{\omega}{\Omega_i} \right| \sim \mathcal{O}(\epsilon). \quad (9)$$

Noting, furthermore, for $|k_{\perp} \rho_i| \sim \mathcal{O}(1)$ and $\beta_i \lesssim \mathcal{O}(1)$, with $\beta_i = 8\pi P_{0i}/B_0^2$ the ratio of plasma ion pressure to the background magnetic field energy density,

$$\left| \frac{\omega}{k_{\perp} v_A} \right| \sim \left| \frac{\omega}{\Omega_i} \right| \left| \frac{1}{k_{\perp} \rho_i} \right| \beta_i^{1/2} \lesssim \mathcal{O}(\epsilon); \quad (10)$$

compressional Alfvén (fast) waves are systematically suppressed in the gyrokinetic orderings.

In the next step, linear gyrokinetic theories perform the following coordinate transformation from the charged particle's phase space (\mathbf{x}, \mathbf{v}) to the corresponding guiding-center phase space (\mathbf{X}, \mathbf{V}) ; where

$$\mathbf{X} = \mathbf{X}_{\perp} + X_{\parallel} \mathbf{b}_0, \quad \mathbf{X}_{\perp} = \mathbf{x}_{\perp} + \boldsymbol{\rho}, \quad \boldsymbol{\rho} = \mathbf{v} \times \mathbf{b}_0 / \Omega_c; \quad (11)$$

$$\mathbf{V} = [\mathcal{E} = v^2/2, \mu = v_{\perp}^2/2B_0, \sigma = \text{sgn}(v_{\parallel})]. \quad (12)$$

Here, $v_{\parallel} = \mathbf{v} \cdot \mathbf{b}_0$, μ is the magnetic moment adiabatic invariant ($\mu = v_{\perp}^2/2B_0$ at the leading order) and, assuming there is no equilibrium electrostatic potential, \mathcal{E} is an equilibrium constant of motion.

In the guiding-center phase space, charged particle dynamics is naturally separated into the fast cyclotron motion and the slow guiding-center motion. One can then apply the gyrokinetic orderings and systematically average out

the fast cyclotron motion (*i.e.*, the gyrophase averaging) and obtain the asymptotically dominant (in terms of the smallness parameter ϵ) perturbed distribution function response. This perturbed distribution function in the guiding-center phase space can then be inversely transformed back to the charged particle phase space and applied toward the field equations (*i.e.*, Maxwell's equations) for a self-consistent kinetic description [7, 3].

For the purpose of the present review, we shall limit our considerations to that of a simple uniform plasma with an isotropic Maxwellian equilibrium distribution function. Readers interested in the detailed analyses and/or broader applications may consult References [3, 14]. Assuming, furthermore, $\beta \ll 1$, such that there is negligible magnetic compression, the particle velocity distribution is then given by

$$f(\mathbf{x}, \mathbf{v}, t) = F_M(\mathcal{E}) + \delta f(\mathbf{x}, \mathbf{v}, t), \quad (13)$$

where

$$\delta f = \frac{q}{T} F_M(\mathcal{E}) \delta \phi + e^{-\rho \cdot \nabla} \delta g, \quad (14)$$

δg satisfies the following linear gyrokinetic equation

$$\left(\frac{\partial}{\partial t} + v_{\parallel} \mathbf{b}_0 \cdot \nabla \right) \delta g = \frac{q}{T} F_M(\mathcal{E}) \frac{\partial}{\partial t} \langle \delta L_g \rangle_{\alpha}, \quad (15)$$

$$\delta L_g = e^{\rho \cdot \nabla} \delta L, \quad (16)$$

$$\delta L = \delta \phi - v_{\parallel} \delta A_{\parallel} / c, \quad (17)$$

and $\langle \dots \rangle_{\alpha}$ denotes averaging over the gyrophase angle, α . Here, the field variables are the scalar and vector potentials, $\delta \phi$ and $\delta \mathbf{A}$, with $\delta A_{\parallel} = \delta \mathbf{A} \cdot \mathbf{b}_0$ and the $\nabla \cdot \delta \mathbf{A} = 0$ Coulomb gauge. The operator $e^{\rho \cdot \nabla}$, meanwhile, represents the transformation between the particle and guiding center positions.

The corresponding field equations are the Poisson's equation and the parallel Ampère's law, $\nabla^2 \delta A_{\parallel} = -4\pi \delta J_{\parallel} / c$. In the low-frequency and $|k \lambda_D|^2 \ll 1$ limit with λ_D being the Debye length, Poisson's equation can be approximated as the quasi-neutrality condition; $\sum_j n_{0j} q_j \langle \delta f_j \rangle_{\mathbf{v}} \simeq 0$. Here, $\langle \dots \rangle_{\mathbf{v}} = \int d^3 \mathbf{v} (\dots)$ is the velocity-space integral, and subscript j runs over the particle species. Meanwhile, substituting the parallel Ampère's law into the $\nabla \cdot \delta \mathbf{J} \simeq 0$ quasi-neutrality condition as given by the parallel velocity moment of Eq. (15) yields a generalized linear gyrokinetic vorticity equation; which is often convenient to use in studying SAW/KAW dynamics [14, 15, 17, 68].

2.2 Linear KAW properties

For plane-wave (ω, \mathbf{k}) perturbations, Eq. (15) gives

$$\delta g_{\mathbf{k}} = -\frac{q}{T} F_M J_0(k_{\perp} \rho) \frac{\omega}{k_{\parallel} v_{\parallel} - \omega} \left(\delta \phi - \frac{v_{\parallel}}{c} \delta A_{\parallel} \right)_{\mathbf{k}}. \quad (18)$$

Note, here, that $J_0(k_\perp \rho)$ corresponds to the gyro-averaging of the coordinate transformation; *i.e.*,

$$\langle \exp(-\boldsymbol{\rho} \cdot \nabla) \rangle_\alpha = J_0(k_\perp \rho). \quad (19)$$

In SAW/KAW analyses, it is sometimes convenient to introduce an effective induced potential defined by $\mathbf{b}_0 \cdot \nabla \delta\psi = -\partial_t \delta A_\parallel / c$ or

$$\delta\psi_{\mathbf{k}} = \omega \delta A_{\parallel \mathbf{k}} / (ck_\parallel). \quad (20)$$

$\delta\psi$, thus, corresponds to the induced parallel electric field; that is, the net parallel electric field is given by

$$\begin{aligned} \delta E_\parallel &= -\mathbf{b}_0 \cdot \nabla (\delta\phi - \delta\psi); \text{ or} \\ \delta E_{\parallel \mathbf{k}} &= -ik_\parallel (\delta\phi - \delta\psi)_{\mathbf{k}}. \end{aligned} \quad (21)$$

The quasi-neutrality condition then becomes

$$\sum_j \left(\frac{n_0 q^2}{T_0} \right)_j \{ \delta\phi_{\mathbf{k}} + \Gamma_{0kj} [\xi_{kj} Z_{kj} \delta\phi_{\mathbf{k}} - (1 + \xi_{kj} Z_{kj}) \delta\psi_{\mathbf{k}}] \} = 0. \quad (22)$$

Here, $\xi_{kj} = \omega / |k_\parallel| v_{tj}$, $Z_{kj} = Z(\xi_{kj})$ with Z the plasma dispersion function, and $\Gamma_{0kj} = I_0(b_{kj}) \exp(-b_{kj})$ with I_0 the modified Bessel function and $b_{kj} = k_\perp^2 \rho_j^2 / 2 = k_\perp^2 (T_j / m_j) / \Omega_j^2$. The linear gyrokinetic vorticity equation, meanwhile, is given by

$$i \frac{c^2}{4\pi\omega} k_\parallel^2 k_\perp^2 \delta\psi_{\mathbf{k}} - i \sum_j \left(\frac{n_0 q^2}{T_0} \right)_j (1 - \Gamma_{0kj}) \omega \delta\phi_{\mathbf{k}} = 0. \quad (23)$$

Noting that, for KAW, $|k_\perp \rho_i| \sim \mathcal{O}(1)$ and $|k_\perp \rho_e| \ll 1$ and, thus, $\Gamma_{0ke} \simeq 1$, Eqs. (22) and (23) then become

$$\begin{aligned} \epsilon_{s\mathbf{k}} \delta\phi_{\mathbf{k}} &= [1 + \xi_{ke} Z_{ke} + \tau (1 + \Gamma_k \xi_{ki} Z_{ki})] \delta\phi_{\mathbf{k}} \\ &= [1 + \xi_{ke} Z_{ke} + \tau \Gamma_k (1 + \xi_{ki} Z_{ki})] \delta\psi_{\mathbf{k}}, \end{aligned} \quad (24)$$

and

$$\omega^2 \delta\phi_{\mathbf{k}} = k_\parallel^2 v_A^2 \frac{b_k}{1 - \Gamma_k} \delta\psi_{\mathbf{k}}. \quad (25)$$

Here, $\tau = T_{0e} / T_{0i}$, $b_k = b_{ki}$, $\Gamma_k = \Gamma_{0ki}$, and $\epsilon_{s\mathbf{k}}$ is the dielectric constant for the slow-sound (ion-acoustic) wave (SSW).

It is also instructive, as done in some literatures, to define the effective parallel potential, $\delta\phi_{\parallel \mathbf{k}} = \delta\phi_{\mathbf{k}} - \delta\psi_{\mathbf{k}}$, and rewrite Eqs. (24) and (25) as

$$\epsilon_{s\mathbf{k}} \delta\phi_{\parallel \mathbf{k}} = -\tau (1 - \Gamma_k) \delta\psi_{\mathbf{k}}, \quad (26)$$

and

$$\left[\omega^2 - k_\parallel^2 v_A^2 \frac{b_k}{1 - \Gamma_k} \right] \delta\psi_{\mathbf{k}} = -\omega^2 \delta\phi_{\parallel \mathbf{k}}. \quad (27)$$

Equations (26) and (27) demonstrate the coupling between SAW and SSW via the finite $|k_\perp \rho_s|$ term. In the $|k_\perp \rho_i| \sim \mathcal{O}(1)$ short-wavelength limit, SAW

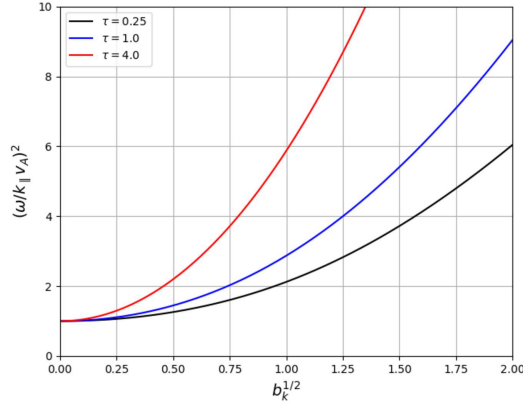


Fig. 3: Dispersion curves illustrating $(\omega_{kr}/k_{\parallel}v_A)^2$ versus $b_k^{1/2}$ for different τ values.

evolves into KAW due to both the finite $|k_{\perp}\rho_i|$ and $|k_{\perp}\rho_s|$ effects. More specifically, the coupled KAW-SSW dispersion relation becomes

$$\omega_k^2 \left[1 - \frac{\tau(1 - \Gamma_k)}{\epsilon_{s\mathbf{k}}} \right] = k_{\parallel}^2 v_A^2 \frac{b_k}{1 - \Gamma_k}. \quad (28)$$

Let us concentrate on the KAW branch and, to further simplify the analysis, assume $1 \gg \beta_i \sim \beta_e \gg m_e/m_i$. With $|\omega| \sim |k_{\parallel}v_A|$, we then have $|\xi_{ki}| = |\omega/k_{\parallel}v_{ti}| \sim \beta_i^{-1/2} \gg 1 \gg |\xi_{ke}| \sim (m_e/m_i\beta_e)^{1/2}$, and, keeping only the lowest-order $\mathcal{O}(1)$ terms,

$$\epsilon_{s\mathbf{k}} \simeq 1 + \tau(1 - \Gamma_k) \equiv \sigma_k. \quad (29)$$

From Eq. (28), we then have

$$\omega_{kr}^2 \simeq k_{\parallel}^2 v_A^2 \frac{\sigma_k b_k}{1 - \Gamma_k}. \quad (30)$$

A sketch of $(\omega_{kr}/k_{\parallel}v_A)^2$ versus $b_k^{1/2}$ for different τ values is given in Fig. (3).

As to wave polarizations, which are useful for wave identification in observations, we can readily derive

$$\left| \frac{c\delta\mathbf{E}_{\perp}}{\delta\mathbf{B}_{\perp}} \right| = v_A \left[\frac{b_k}{\sigma_k(1 - \Gamma_k)} \right]^{1/2}, \quad (31)$$

and

$$\left| \frac{c\delta E_{\parallel}}{\delta\mathbf{B}_{\perp}} \right| = v_A \left| \frac{k_{\parallel}}{k_{\perp}} \right| \tau \left[\frac{b_k(1 - \Gamma_k)}{\sigma_k} \right]^{1/2}. \quad (32)$$

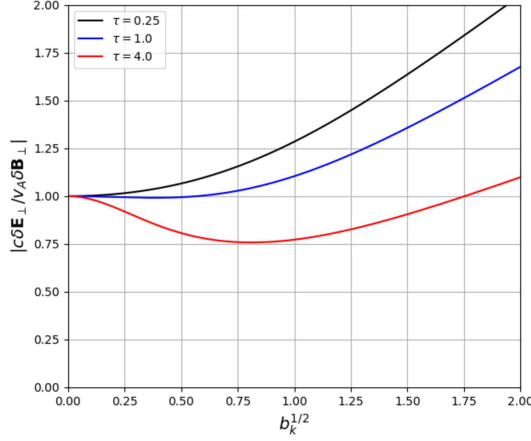


Fig. 4: Polarization curves illustrating $|c\delta\mathbf{E}_\perp/v_A\delta\mathbf{B}_\perp|$ versus $b_k^{1/2}$ for different τ values.

Sketches of $|c\delta\mathbf{E}_\perp/v_A\delta\mathbf{B}_\perp|$ and $|c\delta E_\parallel k_\perp/v_A\delta\mathbf{B}_\perp k_\parallel\tau|$ are given in, respectively, Figs. 4 and 5.

Equation (32) and Fig. 5 show that, for a fixed $|k_\parallel/k_\perp|$, $|\delta E_\parallel/\delta\mathbf{B}_\perp|$ increases with b_k . Since wave-particle energy and momentum exchanges are proportional to $|\delta E_\parallel|$, short-wavelength KAW are, thus, expected to play crucial roles in the heating, acceleration, and transport of charged particles.

In addition to having a significant δE_\parallel , another important property of KAW, in contrast to SAW, is that KAW has a finite perpendicular (to \mathbf{B}_0) group velocity, $\mathbf{v}_{g\perp}$. Assuming $|k_\perp\rho_i|^2 \ll 1$, we have, letting $\omega_A^2 \equiv k_\parallel^2 v_A^2$,

$$\omega_{\mathbf{k}}^2 \simeq \omega_A^2 (1 + k_\perp^2 \hat{\rho}^2), \quad (33)$$

where

$$\hat{\rho}^2 = (3/4 + \tau) \rho_i^2. \quad (34)$$

Thus,

$$\mathbf{v}_{g\perp} \simeq \frac{\omega_A^2}{\omega_{\mathbf{k}}} \hat{\rho}^2 \mathbf{k}_\perp, \quad (35)$$

2.3 Linear mode conversion of KAW

Equation (33) has a significant implication in non-uniform plasmas. Consider, again, a slab plasma with a non-uniform $\omega_A^2(x)$ and $k_\perp^2 = k_x^2(x)$ being the WKB wavenumber in the non-uniformity direction, x . Equation (33) then indicates that KAW is propagating ($k_x^2 > 0$) in the $\omega_{\mathbf{k}}^2 > \omega_A^2(x)$ region, and it is cutoff

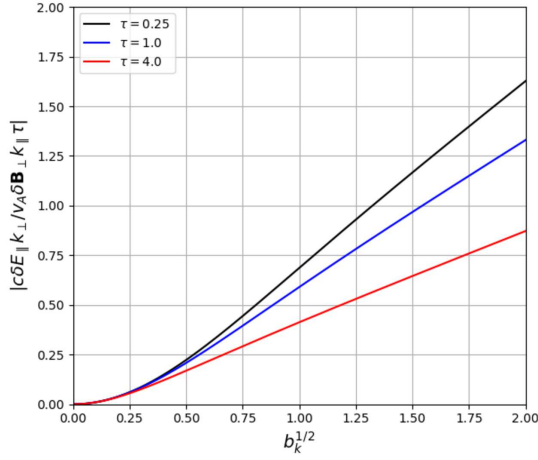


Fig. 5: Polarization curves illustrating $|c\delta E_{\parallel}k_{\perp}/v_A\delta B_{\perp}k_{\parallel}\tau|$ versus $b_k^{1/2}$ for different τ values.

($k_x^2 < 0$) in the $\omega_k^2 < \omega_A^2(x)$ region. That $\mathbf{v}_{g\perp}$ is finite also suggests that, in contrast to SAW, an initial smooth perturbation will not only evolve into short wavelengths but also propagate toward the lower $-\omega_A^2(x)$ region. These features are illustrated in Fig. 6(b); where the spatial-temporal evolution of KAW is solved explicitly according to the following wave equation

$$\left[\hat{\rho}^2 \frac{\partial^2}{\partial x^2} - 1 - \frac{1}{\omega_A^2(x)} \frac{\partial^2}{\partial t^2} \right] \delta B_y(x, t) = 0. \quad (36)$$

The spatial profile of $\omega_A^2(x)/\omega^2 = 1/(1 + x^2/L^2)$ is shown in Fig. 6(a), with L indicating the profile length-scale, so that the KAW wave-packet frequency is assumed to be consistent with the SAW frequency at $x = 0$. Figure 6(b) shows the propagation of the KAW wave packet in the direction of radial non-uniformity, consistent with Eq. (35).

That there exists a finite perpendicular group velocity also implies, in the steady state, the removal of “singular” resonance and linear mode conversion process [33]. More specifically, the corresponding wave equation is given by

$$\left\{ \hat{\rho}^2 \frac{\partial^2}{\partial x^2} + \left[\frac{\omega_0^2}{\omega_A^2(x)} - 1 \right] \right\} \delta \hat{B}_y(x) = \delta \hat{B}_{y0}. \quad (37)$$

Here, ω_0 is the external driving frequency. In the ideal SAW ($\hat{\rho} \rightarrow 0^+$) limit, there is the resonance singularity at x_0 where $\omega_0^2 = \omega_A^2(x_0)$. Noting that, near $x = x_0$, $\omega_A^2(x) \simeq \omega_0^2 + (\omega_A^2)'(x_0)(x - x_0) \equiv \omega_0^2 - (\omega_0^2/L_A)(x - x_0)$, Eq. (37) can be approximated as an inhomogeneous Airy equation and solved

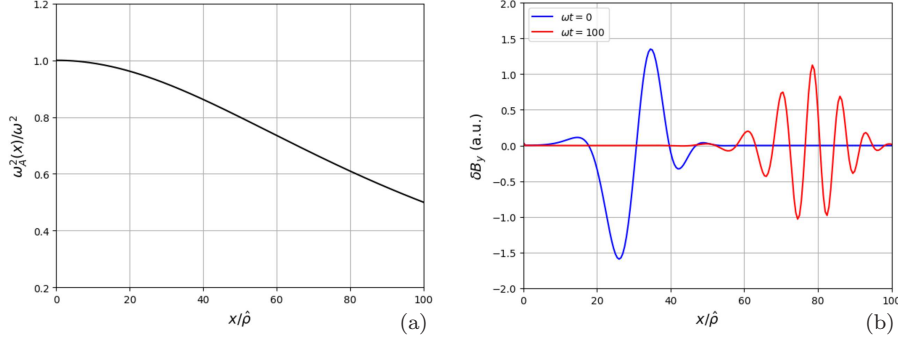


Fig. 6: (a) Spatial dependence of ω_A^2 . (b) Propagation of the KAW wave-packet in the non-uniformity direction.

analytically. Equation (37) can then be solved, with appropriate boundary conditions, by connecting the solutions valid away from the $x = x_0$ resonance layer via the analytic solution of the inhomogeneous Airy equation valid near $x = x_0$ [32, 33]. The solutions away from the singular layer are given by

$$\delta\hat{B}_y(x) = \begin{cases} \frac{\delta\hat{B}_{y0}}{\epsilon_A(x)}, & \text{for } \omega_0^2 < \omega_A^2(x); \\ \frac{\delta\hat{B}_{y0}}{\epsilon_A(x)} - \frac{\sqrt{\pi}\delta\hat{B}_{y0}}{(\hat{\rho}^2/L_A)^{1/2}} \left(\frac{\hat{\rho}^2}{\epsilon_A(x)}\right)^{1/4} \\ \quad \times \exp\left[i\int_{x_0}^x \left(\frac{\epsilon_A(x')}{\hat{\rho}^2}\right)^{1/2} dx' + i\frac{\pi}{4}\right], & \text{for } \omega_0^2 > \omega_A^2(x); \end{cases} \quad (38)$$

where

$$\epsilon_A(x) = \frac{\omega_0^2}{\omega_A^2(x)} - 1. \quad (39)$$

The corresponding numerical solutions are plotted in Fig. 7. Both the analytical results and mode conversion process exhibit two important features. One is, instead of being singular, the amplitude at $x = x_0$ (where $\omega_A(x_0) = \omega_0$) is amplified by the Airy swelling factor; $(L_A/\hat{\rho})^{2/3}$. The other is the singularity at $x = x_0$ is being replaced by the Airy scale length; $\Delta_0 = (\hat{\rho}^2 L_A)^{1/3}$. Recalling, from Eq. (5), $|k_x| \simeq |\omega'_A|t \simeq (\omega_0/L_A)t$, there then exists a KAW formation time scale given by $(\omega_0/L_A)t_0 \simeq 1/\Delta_0$; *i.e.*, $\omega_0 t_0 \simeq (L_A/\hat{\rho})^{2/3}$. Taking, for an example, a typical laboratory plasma, $L_A/\hat{\rho} \simeq \mathcal{O}(10^3)$, we have $\omega_0 t_0 \simeq \mathcal{O}(10^2)$; suggesting that it is reasonable to anticipate, in the presence of SAW continuous spectrum, the appearance of KAW in such plasmas.

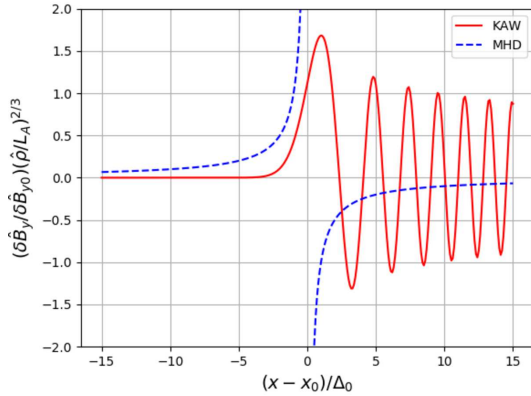


Fig. 7: Illustration of ideal MHD (dashed blue line) and KAW (red line) solutions, which asymptotically match Eq. (38) for $|x - x_0|/\Delta_0 \gg 1$. The Airy swelling factor is evident from the normalization of the ordinate.

2.4 Satellite observations of KAWs

Due to the diagnostics constraints in laboratory plasmas, most of the KAW observations were made by satellites in the Sun-Earth space plasma environments. Shear Alfvénic oscillations in the magnetosphere have been linked to drivers from the upstream solar wind. Due to the collisionless nature of space plasmas, kinetic effects create large-amplitude waves and pressure pulses in the foreshock region upstream from the quasi-parallel bow shock. The foreshock is found to be an important source of magnetospheric waves in the Pc3-4 and Pc5 ranges [25, 24, 19, 21, 61]. The mode conversion process associated with the compressional modes of the foreshock waves has been suggested as a directly-driven mechanism for the generation of the frequently observed discrete harmonic frequencies of shear Alfvénic field line resonances (see Figure 1) [31, 32, 36]. Indeed, near the magnetopause boundary, a sharp transition is frequently found in wave polarization from predominantly compressional waves in the magnetosheath to transverse in the boundary layer [56, 51, 8]. THEMIS observations by Chaston et al. [8] show a direct evidence of a turbulent spectrum of KAWs at the magnetopause with sufficient power to provide massive particle transport. Using coordinated observations in the foreshock and the magnetosphere, Wang et al. [61] found direct evidence of Pc5 field line resonances driven by the foreshock perturbations. As remarked earlier, the main mode identification method for KAWs is based on the measurement of the wave polarization, $|c\delta\mathbf{E}_\perp/v_A\delta\mathbf{B}_\perp|$. Two cases are illustrated here. One is observation by the *Van Allen Probes* in the Earth's inner magnetosphere [9] (cf. Fig. 8); the other is observations by the *Cluster* satellites in the solar wind [54] (cf. Fig. 9). Both observations showed the measured polarizations,

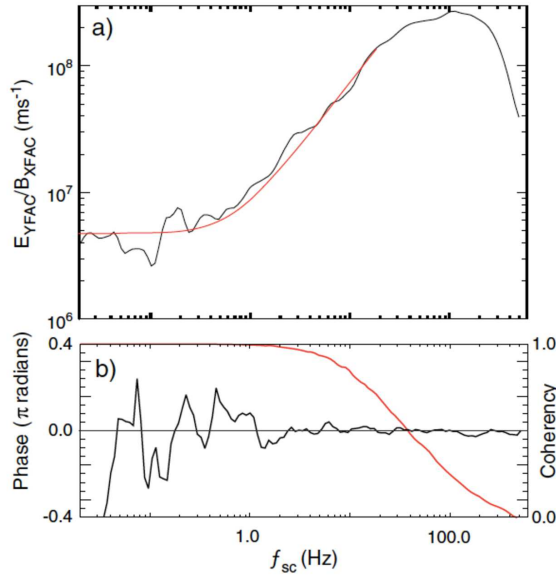


Fig. 8: (a) The time averaged ratio E_{YFAC}/B_{XFAC} in field aligned coordinates (MKS units). Red line shows the fit of the local KAW dispersion relation (cf. Fig. 4). [Reproduced from Ref. [9]]. (b) Relative phase and coherency (red) between E_{YFAC} and B_{XFAC} [Reproduced from Ref. [9]].

$|c\delta\mathbf{E}_\perp/v_A\delta\mathbf{B}_\perp|$, agree qualitatively and/or quantitatively with those theoretically predicted for KAWs.

Finally, we remark KAW physics has also been applied theoretically in laboratory fusion plasmas [32, 33, 16, 17]. For example, in toroidal fusion plasmas, the Kinetic Toroidal Alfvén Eigenmodes (KTAEs) [40] may exist within the SAW continuum and their dynamics are intrinsically related to those of KAWs. Furthermore, laboratory plasma experiments have shown evidence of coupling between SAW eigenmodes and KAWs [63] that may also be externally driven by mode conversion of fast modes [26]. Since KAW carries significant implications to plasma heating and transport, it will be interesting to see more focused investigations on KAW physics in laboratory plasma experiments and/or simulations.

3 Nonlinear KAW physics

In this Section, we first discuss the nonlinear gyrokinetic orderings and present the corresponding equations. We then apply the nonlinear gyrokinetic equations to the fundamental three-wave parametric decay instabilities. Here, we emphasize the qualitative and quantitative differences between the results of

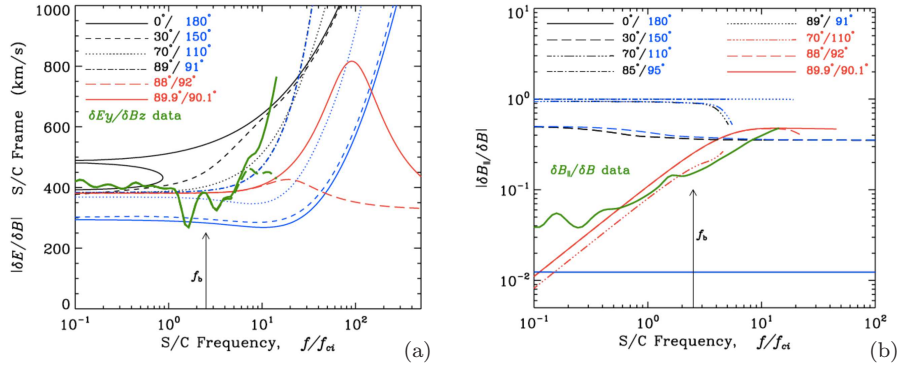


Fig. 9: (a) Prediction of $|\delta\mathbf{E}/\delta\mathbf{B}|_{s/c}$ for kinetic Alfvén waves (red curves) or whistler waves (black and blue curves) with specified angle θ . Cluster measurements of $|\delta E_y/\delta B_z|$ up to 2 Hz, or $12 f_{ci}$, are presented without (green solid) and with (green dashed) the EFW noise floor removed. [Reproduced from Ref. [54]]. (b) Prediction of $|\delta B_{||}/\delta\mathbf{B}|_{s/c}$ for kinetic Alfvén waves (red) or whistler waves (black/blue) with specified angle θ . Cluster FGM measurements up to 2 Hz, or $12 f_{ci}$, are shown in green. [Reproduced from Ref. [54]].

nonlinear gyrokinetic theory and those based on the ideal MHD theory. Corresponding simulations not only support the gyrokinetic theory results, but also suggest the excitation of $k_{||} \simeq 0$ fluctuations; *i.e.*, convective cells. This motivated the studies on the spontaneous excitations of convective cells by KAWs. The results demonstrate the significant effects of finite ion Larmor radius; and, thus, the nonlinear gyrokinetic theory as a powerful theoretical tool. Finally, we present a quasilinear description of plasma transport due to KAWs.

3.1 Nonlinear gyrokinetic theory

In extending the linear gyrokinetic theory to the nonlinear regime, one allows the fluctuations to be of finite amplitudes with, however, the constraint that the corresponding nonlinear frequencies, ω_{nl} , be much less than the cyclotron frequency. In other words, consistent with the linear gyrokinetic orderings:

$$|\omega_{nl}| \sim |\delta\mathbf{u}_{\perp} \cdot \nabla_{\perp}| \sim |\omega| \sim \mathcal{O}(\epsilon) |\Omega_i|. \quad (40)$$

Here, $\delta\mathbf{u}_{\perp}$ represents the fluctuation-induced particle (guiding-center) jiggling velocity. Taking, for example, $\delta\mathbf{u}_{\perp} \simeq v_{||} \delta\mathbf{B}_{\perp}/B_0$ due to magnetic fluctuation, $\delta\mathbf{B}_{\perp}$, $v_{||} \sim v_t$, and $|\nabla_{\perp}| \sim 1/\rho_i$, we then obtain the following nonlinear gyrokinetic orderings [27]:

$$|\delta f/F_0| \sim |\delta\mathbf{B}/B_0| \sim |c\delta\mathbf{E}_{\perp}/(B_0 v_t)| \sim \mathcal{O}(\epsilon). \quad (41)$$

Again, let us consider the case of a uniform plasma in order to simplify the presentation and highlight the important underlying physics. In a uniform

case, the perturbed distribution function, δf as in the linear case, can be decomposed into an adiabatic and a non-adiabatic components; *i.e.*,

$$\delta f = -\frac{q}{T} F_M \delta \phi + \exp(-\boldsymbol{\rho} \cdot \boldsymbol{\nabla}) \delta g. \quad (42)$$

Here, we have taken the background distribution to be Maxwellian, and δg satisfies the following nonlinear gyrokinetic equation [27]

$$\left[\frac{\partial}{\partial t} + v_{\parallel} \mathbf{b}_0 \cdot \boldsymbol{\nabla} + \langle \delta \mathbf{u}_{\perp g} \rangle_{\alpha} \cdot \boldsymbol{\nabla} \right] \delta g = \frac{q}{T} F_M \frac{\partial}{\partial t} \langle \delta L_g \rangle_{\alpha}, \quad (43)$$

δL_g given by Eq. (16) and (17), and

$$\begin{aligned} \langle \delta \mathbf{u}_{\perp g} \rangle_{\alpha} &= (c/B_0) \mathbf{b}_0 \times \boldsymbol{\nabla} \langle \delta L_g \rangle_{\alpha} \\ &= (c/B_0) \langle \delta \mathbf{E}_{\perp g} \rangle_{\alpha} \times \mathbf{b}_0 + v_{\parallel} \langle \delta \mathbf{B}_{\perp g} \rangle_{\alpha} / B_0. \end{aligned} \quad (44)$$

Expanding in terms of plane-wave solutions, Eq. (43) yields

$$\begin{aligned} i(k_{\parallel} v_{\parallel} - \omega_k) \delta g_k &= -i\omega_k \frac{q}{T} J_k \delta L_k F_M \\ &\quad + \frac{c}{B_0} \Lambda_{k'}^{k''} [J_{k'} \delta L_{k'} \delta g_{k''} - J_{k''} \delta L_{k''} \delta g_{k'}]; \end{aligned} \quad (45)$$

where $J_k \equiv J_0(k_{\perp} v_{\perp} / \Omega)$, J_0 is the Bessel function,

$$\Lambda_{k'}^{k''} = \mathbf{b}_0 \cdot (\mathbf{k}'_{\perp} \times \mathbf{k}''_{\perp}), \quad (46)$$

$$\delta L_k = \delta \phi_k - v_{\parallel} \delta A_{\parallel k} / c, \quad (47)$$

and (ω_k, \mathbf{k}) satisfy frequency and wave-vector matching conditions; *i.e.*, $\omega_k = \omega_{k'} + \omega_{k''}$ and $\mathbf{k} = \mathbf{k}' + \mathbf{k}''$.

The field equations remain the same; *i.e.*, the Poisson's equation or the quasi-neutrality condition and the parallel Ampère's Law or the generalized nonlinear gyrokinetic vorticity equation. The quasi-neutrality condition is formally the same as in the linear theory; *i.e.*,

$$(1 + \tau) e \delta \phi_k / T_e = \langle J_k \delta g_{ki} - \delta g_{ke} \rangle_v; \quad (48)$$

with $\tau \equiv T_e / T_i$, consistent with the definition introduced below Eq. (25) and where we dropped the subscript "0" on equilibrium temperature; and $J_k = J_{ki}$ for brevity. The nonlinear gyrokinetic vorticity equation [17, 18, 66, 67], meanwhile, is given by

$$ik_{\parallel} \delta J_{\parallel k} - i \frac{c}{4\pi} \frac{\omega_k}{v_A^2} \frac{k_{\perp}^2}{b_k} (1 - \Gamma_k) \delta \phi_k = (\text{NL})_A + (\text{NL})_{\phi}, \quad (49)$$

where $b_k = k_{\perp}^2 \rho_i^2 / 2$, $\Gamma_k = I_0(b_k) \exp(-b_k)$, consistent with the definitions introduced below Eq. (22),

$$(\text{NL})_A = -\Lambda_{k'}^{k''} (\delta A_{\parallel k'} \delta J_{\parallel k''} - \delta A_{\parallel k''} \delta J_{\parallel k'}) / B_0, \quad (50)$$

and

$$\begin{aligned} (\text{NL})_\phi &= (ec/B_0) A_{k'}^{k''} \langle (J_k J_{k'} - J_{k''}) \delta L_{k'} \delta g_{k''} \\ &\quad - (J_k J_{k''} - J_{k'}) \delta L_{k''} \delta g_{k'} \rangle_{i,v} . \end{aligned} \quad (51)$$

We remark that $(\text{NL})_A$ corresponds to the Maxwell stress term due to the $\delta J_\parallel \mathbf{b}_0 \times \delta \mathbf{B}_\perp$ force with δJ_\parallel mainly carried by electrons due to $m_e \ll m_i$. $(\text{NL})_\phi$, meanwhile, is the gyrokinetic stress tensor; which is dominated by ions and reduces to the well-known fluid expression in the $k_\perp^2 \rho_i^2 \ll 1$ limit [17, 18].

3.2 Parametric decay instabilities

Parametric decay instability (PDI) is a fundamental nonlinear process involving three nonlinear coupled waves/oscillators [34, 39]. One is the pump (“mother”) wave and the other two are the decay (“daughter”) waves. The PDI can be either resonant if both decay waves are marginally stable or weakly damped normal modes, or non-resonant if one of the decay waves is a heavily damped quasi mode. Since the pump wave can be either spontaneously or externally excited, PDI, thus, is an important channel for wave energy transfer along with its associated consequences on plasma heating, acceleration and transports.

Interested readers may refer to the original work [15] for the detailed derivations of the KAW PDI dispersion relations. Here, we will just present the key points and results. Let the three interacting waves be the pump wave $\boldsymbol{\Omega}_0 = (\omega_0, \mathbf{k}_0)$, the low-frequency daughter SSW $\boldsymbol{\Omega}_s = (\omega_s, \mathbf{k}_s)$ and the daughter KAW $\boldsymbol{\Omega}_- = (\omega_-, \mathbf{k}_-)$ with $\omega_- = \omega_s - \omega_0$ and $\mathbf{k}_- = \mathbf{k}_s - \mathbf{k}_0$. Let the small but finite pump wave amplitude be denoted as $\Phi_0 = e\delta\phi_0/T_e$. As $\boldsymbol{\Omega}_s$ could be a quasi mode, we then need to retain $\mathcal{O}(|\Phi_0|^2)$ terms in order to properly account for non-resonant PDI. Carrying on the straightforward algebra [15], we then derive the KAW PDI dispersion relation

$$\epsilon_{sk} \left(\epsilon_{Ak-} + \chi_{A-}^{(2)} \right) = C_k |\Phi_0|^2 . \quad (52)$$

Here,

$$\epsilon_{sk} = 1 + \tau + \tau \Gamma_s \xi_s Z(\xi_s) \quad (53)$$

and

$$\epsilon_{Ak-} = \left[(1 - \Gamma_-) / b_- - \left(k_\parallel^2 v_A^2 / \omega^2 \right)_- \sigma_- \right] \quad (54)$$

are, the linear dielectric constants of, respectively, the $\boldsymbol{\Omega}_s$ -SSW and $\boldsymbol{\Omega}_-$ -KAW decay waves. $\chi_{A-}^{(2)}$, as will be further discussed later, corresponds to nonlinear ion Compton scattering

$$\chi_{A-}^{(2)} = \epsilon_{sk} \left(\lambda^2 / \Gamma_s \right) G |\Phi_0|^2 , \quad (55)$$

$$\lambda^2 = (\Omega_i / \omega_0)^2 \left(\rho_s^2 A_0^s \right)^2 / (\sigma_- b_{s-}) , \quad (56)$$

$$G = \langle J_0^2 J_-^2 F_{0i} \rangle_v / n_0 - F_1^2 / \Gamma_s , \quad (57)$$

and

$$F_1 = \langle J_s J_0 J_- F_{0i} \rangle_v / n_0, \quad (58)$$

where $\rho_s^2 = \tau \rho_i^2$ and $b_{s-} = \tau b_-$. Note that $G \geq 0$ from Schwartz inequality. C_k on the right hand side of Eq. (52) represents the nonlinear coupling coefficient between Ω_s and Ω_- daughter waves via the pump wave $|\Phi_0|$, and

$$C_k = (\lambda H)^2 \quad (59)$$

with

$$H = (\sigma_0 \sigma_- - F_1 \sigma_s / \Gamma_s). \quad (60)$$

Furthermore, in the PDI dispersion relation, Eq. (52), we have dropped the term associated with nonlinear frequency shift to focus on the stability property [15].

Let us first consider the resonant decay, which occurs when both decay daughter waves, Ω_s and Ω_- , are weakly damped normal modes. This generally requires $\tau \equiv T_e/T_i \gtrsim 5$ [32, 33] in order to minimize the ion Landau damping of the Ω_s (SSW) mode. In this case, letting, $\omega_s = \omega_{sr} + i\gamma$ as well as noting $\epsilon_{skr}(\omega_{sr}) = 0$ and $\epsilon_{Ak-r}(\omega_{A-r}) = \epsilon_{Ak-r}(\omega_{sr} - \omega_0) = 0$, Eq. (52) reduces to

$$(\gamma + \gamma_{dA-})(\gamma + \gamma_{ds}) = (\lambda H |\Phi_0|)^2 \left[-\frac{\partial \epsilon_{skr}}{\partial \omega_{sr}} \frac{\partial \epsilon_{Ak-r}}{\partial \omega_{A-r}} \right]^{-1}, \quad (61)$$

where γ_{dA-} and γ_{ds} are, respectively, the linear damping rates of the KAW and SSW daughter waves. We also note that, in order to have a parametric growth ($\gamma > 0$), the round bracket term on the right hand side of Eq. (61) must be positive; *i.e.*, $\omega_{sr}\omega_0 > 0$. Thus, the KAW decay wave has its normal-mode real frequency lower than that of the KAW pump frequency, ω_0 , by the amount of the SSW normal-mode frequency, ω_{sr} . Noting that, for $\beta \ll 1$, we have $|\omega_0| \sim |k_{\parallel 0} v_A| \gg |k_{\parallel 0} c_S|$ and $|\omega_{A-r}| \sim |k_{\parallel A-r} v_A| \gg |k_{\parallel A-r} c_S|$. Thus, in order to satisfy the frequency and wave-number matching conditions for the resonant decay, $|\omega_{sr}| \sim |k_{\parallel s} c_S|$, we must have $k_{\parallel A-} \simeq k_{\parallel 0}$ or $k_{\parallel s} \simeq 2k_{\parallel 0}$. Consequently, we have $(\omega_0/k_{\parallel 0})(\omega_{A-r}/k_{\parallel A-}) < 0$; *i.e.*, the decay KAW daughter wave, Ω_- , has parallel (to \mathbf{B}_0) group velocity opposite to that of the pump wave. In other words, Ω_- can be understood as a KAW due to backscattering of the Ω_0 pump wave by Ω_s fluctuations. Finally, $|\Phi_0|$ must be over a threshold value set by γ_{ds} and γ_{dA-} to achieve $\gamma > 0$.

For $\tau \equiv T_e/T_i \lesssim 5$, the Ω_s SSW mode is, in general, heavily ion Landau damped; *i.e.*, it becomes a quasi mode. The Ω_- KAW mode, meanwhile, remains a weakly damped normal mode. The PDI growth rate, γ , is then determined by the imaginary part of the dispersion relation, Eq. (52);

$$\begin{aligned} (\gamma + \gamma_{dA-}) \left(-\frac{\partial \epsilon_{Ak-r}}{\partial \omega_{A-r}} \right) &= \mathbb{Im} \left[\chi_{A-}^{(2)} - \frac{C_k}{\epsilon_{sk}} |\Phi_0|^2 \right] \\ &= |\lambda \Phi_0|^2 \left[G/\Gamma_s + H^2/|\epsilon_{sk}|^2 \right] \mathbb{Im} \epsilon_{sk}, \end{aligned} \quad (62)$$

where, again, $G \geq 0$,

$$\mathbb{Im} \epsilon_{sk} = \tau \Gamma_s \mathbb{Im} [\xi_s Z(\xi_s)], \quad (63)$$

and $\xi_s = \omega_{sr}/|k_{\parallel s}|v_{ti} = (\omega_0 + \omega_{A-r})/|k_{\parallel 0} + k_{\parallel A-}|v_{ti}$. Since $\Im\epsilon_{sk}$ maximizes around $\xi_s \sim \mathcal{O}(1)$ or $\omega_{sr} \sim |k_{\parallel 0} + k_{\parallel A-}|v_{ti}$, and, again, we have $|\omega_0| \sim |\omega_{A-r}| \gg |k_{\parallel 0}|v_{ti} \sim |k_{\parallel A-}|v_{ti}$, the Ω_- KAW mode is, again, a backscattered KAW normal mode with frequency lower than the pump wave frequency ω_0 .

Note, from Eqs. (61) and (62), that the parametric decay instability growth rates increase with the nonlinear coupling coefficient, $|C_k|\Phi_0|^2$ of Eq. (52); which can be readily shown to scale with $|k_{\perp}\rho_i|^4|\delta\mathbf{B}_{\perp 0}/B_0|^2$ for $|k_{\perp}\rho_i|^2 \ll 1$ and $|\delta\mathbf{B}_{\perp 0}/B_0|^2/|k_{\perp}\rho_i|^2$ for $|k_{\perp}\rho_i|^2 \gg 1$. The decay instabilities are, thus, strongest when $|k_{\perp}\rho_i| \sim \mathcal{O}(1)$; and it clearly demonstrates the necessity of keeping FILR kinetic effects in dealing with the decay instabilities of KAW.

Finally, it is illuminating to compare the decay instabilities of KAWs versus those of SAWs in the MHD regime [52]. In a nutshell, employing the ideal MHD fluid theory, the PDI dispersion relation takes the form similar to the KAW PDI dispersion relation, Eq. (52), with KAW terms replaced by corresponding SAW terms; *e.g.*, ϵ_{Ak-} by ϵ_{A-} *etc.* The more fundamental change lies in the nonlinear coupling term; that is, C_k is replaced by C_I given as

$$\begin{aligned} C_I &= |\mathbf{k}_{\perp 0}\rho_s \cdot \mathbf{k}_{\perp -}\rho_s|^2 / [b_{s-}(1 + \Gamma_i T_i/T_e)] \\ &= [b_{s0}/(1 + \Gamma_i T_i/T_e)] \cos^2 \theta_0 . \end{aligned} \quad (64)$$

Here, θ_0 is the angle between $\mathbf{k}_{\perp 0}$ and $\mathbf{k}_{\perp -}$, and Γ_i is the ion ratio of specific heats. C_k , meanwhile, can be expressed as

$$C_k = (\Omega_i/\omega_0)^2 (b_{s0}/\sigma_-) H^2 \sin^2 \theta_0 . \quad (65)$$

We then have

$$|C_k|/|C_I| \sim \mathcal{O}(|\Omega_i H/\omega_0|^2) ; \quad (66)$$

which becomes

$$|C_k|/|C_I| \sim \mathcal{O}(|\Omega_i/\omega_0|^2)|k_{\perp}\rho_i|^4 ; \quad \text{for } |k_{\perp}\rho_i|^2 \ll 1 , \quad (67)$$

and

$$|C_k|/|C_I| \sim \mathcal{O}(|\Omega_i/\omega_0|^2) ; \quad \text{for } |k_{\perp}\rho_i| \sim \mathcal{O}(1) . \quad (68)$$

Equation (67) indicates that, for $1 > |k_{\perp}\rho_i|^2 > |\omega_0/\Omega_i|$, nonlinear couplings via kinetic effects dominate. Noting that $|\omega_0/\Omega_i| \sim \mathcal{O}(10^{-3})$ in typical laboratory plasmas, the validity regime of MHD fluid theory for the SAW nonlinear physics is rather limited. Furthermore, at the $|k_{\perp}\rho_i| \sim \mathcal{O}(1)$ regime where KAW nonlinear effects maximize, we have $|H| \sim \mathcal{O}(1)$ and $|C_k|/|C_I| \sim \mathcal{O}(|\Omega_i/\omega_0|^2) \sim \mathcal{O}(10^6)$ for typical parameters.

In addition to the significantly enhanced PDI growth rates, there is, perhaps, more significant qualitative difference between KAW and SAW PDI in terms of the wave vector of the scattered daughter wave with respect to that of the pump wave. Note, from Eq. (64), $C_I \propto \cos^2 \theta_0$ and, thus, the SAW scattering maximizes around $\theta_0 = 0$ and π ; *i.e.*, when $\mathbf{k}_{\perp -}$ is parallel or anti-parallel to $\mathbf{k}_{\perp 0}$; or \mathbf{k}_0 and \mathbf{k}_- are co-planar. In contrast, we have, from Eq. (65), $C_k \propto \sin^2 \theta_0$ and, thus, the KAW scattering maximizes around $\theta_0 = \pm\pi/2$;

i.e., $\mathbf{k}_{\perp 0}$ and $\mathbf{k}_{\perp -}$ are orthogonal. This difference not only affects, as might be expected, the nonlinear evolution of KAW turbulence, but also, as we will argue further below and perhaps more significantly, charged particle transports induced by the KAW decay processes.

Let us consider the pump wave be the mode-converted KAW at the Earth's dayside magnetopause; thus, $\mathbf{k}_{\perp 0} = k_{\perp 0} \hat{\mathbf{r}}$ with $\hat{\mathbf{r}}$ being in the Sun-Earth radial direction. Now, according to the ideal MHD theory, the decay wave tends to have $\mathbf{k}_{\perp -} = k_{\perp -} \hat{\mathbf{r}}$ and, thus, the East-West azimuthal symmetry is in general kept. In other words, charged particle's East-West azimuthal generalized momentum, P_ϕ , is conserved, which implies no or little radial transport [10]. On the other hand, in the KAW regime, the decay wave would have wave vector in the East-West azimuthal direction; *i.e.*, $\mathbf{k}_{\perp -} = k_{\perp -} \hat{\phi}$ and, hence, the East-West azimuthal symmetry is broken by the daughter wave and, consequently, P_ϕ is no longer conserved and finite radial transports could occur [10]. These features are observed in the numerical simulations to be discussed below. In addition, the MHD fluid theory would suggest that the turbulence in the perpendicular to \mathbf{B}_0 plane to be preferentially anisotropic in the $\hat{\mathbf{r}}$ direction; while KAW turbulence would tend to be more isotropic.

Insights to the above qualitative and quantitative transitions in the nonlinear coupling coefficient between the long-wavelength MHD fluid and the short-wavelength KAW regimes can be also gained by examining the responsible nonlinear coupling mechanisms. More specifically, while in the MHD regime, ion-sound fluctuations are nonlinearly generated by the $(\delta \mathbf{J}_\perp \times \delta \mathbf{B}_\perp) \cdot \mathbf{b}_0/c$ parallel (to \mathbf{B}_0) force; in the KAW regime, the nonlinear force is due to the $m_i n_i (\delta \mathbf{u} \cdot \nabla) \delta u_\parallel$ convective nonlinear term. Similarly, while in the MHD regime, scatterings of the SAW by the ion sound fluctuations occur via the $\delta n_s (\partial \delta \mathbf{u}_0 / \partial t)$ nonlinear ion density modulation; scatterings of the KAW occurs, again, via the $n_i (\delta \mathbf{u} \cdot \nabla) \delta \mathbf{u}_0$ convective nonlinearity.

Numerical simulations on the linear mode conversion of KAW and the ensuing nonlinear wave generations were carried out by Lin et al. [38] using a three-dimensional hybrid model, in which ions are treated as fully kinetic particles and electrons are treated as a massless fluid. Readers are referred to the original work for details. Here, we summarize and discuss the essentials. Specifically, consider a slab plasma with $\mathbf{B}_0 = B_0 \hat{\mathbf{z}}$ and nonuniformities in the x (radial) direction. Simulations demonstrated that an incoming fast compressional Alfvén wave mode converted into a short-wavelength KAW with $|k_x \rho_i| \sim \mathcal{O}(1)$ localized about the Alfvén resonance point. This mode-converted KAW then serves as a pump KAW and nonlinearly excited secondary KAWs with, preferentially, short azimuthal wavelengths; *i.e.*, $|k_\perp \rho_i| \sim |k_y \rho_i| \sim \mathcal{O}(1)$. In order to analyze the nonlinear wave generation mechanism in more details, Lin et al. [38] further carried out dedicated simulations with a prescribed pump KAW in a uniform plasma. The resultant (k_\parallel, ω) spectra of δB_x and δE_\parallel are shown in Fig. 10. In the right plot of δE_\parallel , we can see the pump KAW at $\Omega_0 = (k_{\parallel 0} = 0.2, \omega_0 = 0.6)$ and the ion sound wave at $\Omega_s = (k_{\parallel s} \simeq 2k_{\parallel 0} = 0.4, \omega_s = 0.2)$. Correspondingly, in the left plot of δB_x , we see the backscattered decay KAW with $\Omega_- = (k_{\parallel -} = -0.2, \omega_0 = 0.4)$.

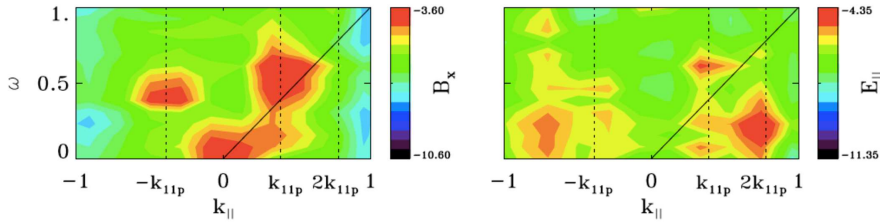


Fig. 10: k_{\parallel} - ω spectra of δB_x and δE_{\parallel} obtained from the simulation of decay of an initial pump KAW in a uniform plasma. The solid black line indicates the dispersion relation of the MHD shear Alfvén mode for reference. Multiples of the parallel pump KAW wave number, $k_{\parallel p}$, are also shown. [Reproduced from Ref. [38]].

Note, since $T_e/T_i \simeq 0.4$, the ion sound wave, Ω_s , is a heavily ion Landau damped quasi mode, and the PDI corresponds to the nonlinear ion induced scattering. Both the Ω_- and Ω_s modes have preferentially short wavelengths in the \hat{y} direction; *i.e.*, $|k_{\perp s} \rho_i| \sim |k_{ys} \rho_i| \sim |k_{y-} \rho_i| \sim \mathcal{O}(1)$. The simulation results are, thus, consistent with analytical theories discussed above.

As noted by Lin et al. [38], the δB_x spectrum also showed excitations around ($k_{\parallel} \approx 0, \omega \approx 0$) with $|k_{ys} \rho_i| \sim \mathcal{O}(1)$ short wavelengths; which, as suggested, correspond to magnetostatic convective cells [20]. The nonlinear excitations of convective cells also explain the appearance of δB_x fluctuations at $k_{\parallel 0}$ and ω_0 ; since the pump KAW with $k_{y0} \simeq 0$ has $\delta B_{x0} \simeq 0$. These interesting simulation results, thus, naturally lead to the following gyrokinetic analytic theory on excitations of convective cells via the modulational instabilities of a KAW pump wave.

3.3 Nonlinear excitations of convective cells

Convective cells have been of theoretical interests since the 1970's [20, 37, 42, 59]; since they lead to vortex dynamics perpendicular to the confining magnetic field and, consequently, carry significant implications to the cross-field transport [53]. Historically, convective cells have been classified into two categories; the electrostatic convective cells (ESCC) with $\delta \mathbf{E} = \delta \mathbf{E}_{\perp}$ [42, 59] and the magnetostatic convective cells (MSCC) with $\delta \mathbf{B} = \delta \mathbf{B}_{\perp}$ [20]. In recent years, there has been renewed interest in convective cells since they may be regarded as paradigms of the so-called zonal structures in laboratory fusion plasmas [16, 68]. Zonal structures are fluctuations with $\mathbf{k} \cdot \mathbf{B}_0 = 0$ and varying only in the radial direction [30, 31]. Zonal structures may have frequencies either around $\omega = 0$ or a finite frequency (*i.e.*, the so-called geodesic acoustic mode [62]). The $\omega = 0$ zero-frequency zonal structures could be either zonal flow or zonal field/current; corresponding, respectively, to ESCC and MSCC. In this respect, zonal structures may be regarded as subset of convective cells.

Since convective cells have $\omega \approx 0$, they are nominally damped by either viscosity and/or resistivity; and, thus, generally require nonlinear excitations in order to achieve finite intensities. In laboratory fusion plasmas, nonlinear excitations of convective cells (*i.e.*, zonal structures) usually occur via mode-mode couplings of ambient drift-wave and/or Alfvén-wave instabilities. In this respect, zonal structures may be regarded as spontaneous growth of corrugations of the radial equilibrium profiles; which, in turn, scatter the ambient instabilities into the radially short-wavelength stable domain. Zonal structures, therefore, provide self-regulatory mechanisms for the ambient turbulences and the associated transports. We refer to the recent review [17] for readers interested in this important topic.

In the present review, we will focus on nonlinear excitations of convective cells by KAWs in uniform plasmas in order to explore in sufficient details the underlying physics mechanisms. Since convective cells have $\mathbf{k} \cdot \mathbf{B}_0 = 0$, their nonlinear excitations involve couplings between co-propagating SAWs with the same k_{\parallel} ; which vanishes in the ideal MHD limit due to the cancellation between the Reynolds and Maxwell stresses; *i.e.*, the pure Alfvénic state [1, 2, 60]. It, thus, has long been recognized that only non-ideal MHD fluctuations, such as KAW, can nonlinearly excite convective cells [16, 41, 43, 44, 45, 46, 65]. Furthermore, since having $\omega = 0$, it is also recognized that it takes the form of modulational instabilities for the spontaneous excitations of convective cells by KAWs; that is of the reinforcement by nonlinearity of the deviation from wave periodic behavior, which may lead to spectral sidebands and possibly to breaking of the periodic fluctuation into modulated pulses [4, 16, 68]. Previous theoretical studies, however, suffer from two limiting considerations; (1) employing two-fluid or drift-kinetic descriptions, and (2) assuming that ESCC and MSCC are decoupled. By (1), effects due to finite ion Larmor radii (FILR) are ignored. Both limiting considerations have been adopted in order to simplify the theoretical analysis and, as will be shown here, lead to erroneous conclusions on the nonlinear excitation mechanisms. Here, we will employ the nonlinear gyrokinetic equation and demonstrate that both the FILR as well as the finite coupling between ESCC and MSCC play qualitatively crucial roles in the dynamics of the modulational excitations of convective cells. Only key points of the theoretical analysis and results will be highlighted here. Readers are referred to the original works for details.

We consider a uniform Maxwellian plasma immersed in a confining magnetic field, $\mathbf{B}_0 = B_0 \hat{\mathbf{z}}$. Furthermore, we assume $1 \gg \beta_e, \beta_i \gg m_e/m_i$ and ignore the compressional Alfvén wave; *i.e.*, $\delta B_{\parallel} \approx 0$. Denoting $\Omega_0 = (\omega_0, \mathbf{k}_0)$ as the finite-amplitude pump KAW and $\Omega_z = (\omega_z, \mathbf{k}_z)$ as the convective cell (CC) mode, four-wave modulational instability then involves couplings with the upper and lower KAW sidebands denoted, respectively, as $\Omega_+ = (\omega_+ = \omega_z + \omega_0, \mathbf{k}_+ = \mathbf{k}_z + \mathbf{k}_0)$ and $\Omega_- = (\omega_- = \omega_z - \omega_0, \mathbf{k}_- = \mathbf{k}_z - \mathbf{k}_0)$. With compressional Alfvén wave suppressed due to frequency separation, the field variables are $\delta\phi_{\mathbf{k}}$ and $\delta A_{\parallel\mathbf{k}}$ with $\mathbf{k} = 0, z, \pm$ corresponding to the Ω_0, Ω_z and Ω_{\pm} fluctuations. The governing equations, meanwhile, are the nonlinear

gyrokinetic equation, Eq. (45), the quasi-neutrality condition, Eq. (48), and the nonlinear gyrokinetic vorticity equation, Eq. (49).

Carrying out the standard perturbative analysis to $\mathcal{O}(|\delta\phi_0|^2)$, we then derive, after some straightforward but lengthy algebra [68], the following coupled equations between $\delta\phi_z$ and $\delta\psi_z \equiv \omega_0 \delta A_{\parallel z} / (k_{\parallel} c)$

$$\begin{aligned} \left[\gamma_z^2 + \frac{\Delta^2}{1 + \Delta/\omega_0} \right] \delta\phi_z &= -\alpha_\phi (\delta\phi_z - \delta\psi_z) + \beta_\phi \delta\psi_z, \\ \left[\gamma_z^2 + \frac{\Delta^2}{1 + \Delta/\omega_0} \right] \delta\psi_z &= -\alpha_\psi (\delta\phi_z - \delta\psi_z) + \beta_\psi \delta\psi_z. \end{aligned} \quad (69)$$

Here, we have let $\omega_z = i\gamma_z$ and $\mathbf{k}_z \perp \mathbf{k}_0$ to maximize the nonlinear couplings,

$$\frac{\Delta}{\omega_0} = \frac{b_+ \sigma_+ (1 - \Gamma_0) - b_0 \sigma_0 (1 - \Gamma_+)}{2b_0 \sigma_0 (1 - \Gamma_+)} \quad (70)$$

is the frequency mismatch between the normal-mode frequency of KAW at \mathbf{k}_\pm and ω_0 ; $\Delta > 0$ and we have noted that $b_- = b_+$ as well as $\Gamma_- = \Gamma_+$ for $\mathbf{k}_z \cdot \mathbf{k}_0 = 0$. Furthermore,

$$\begin{aligned} \alpha_\phi &= \left| \frac{c}{B_0} k_z k_{\perp 0} \delta\phi_0 \right|^2 \frac{1}{1 - \Gamma_+} \left[\frac{\Gamma_0 - \Gamma_+}{1 - \Gamma_z} \left(\Gamma_0 - \Gamma_z \right. \right. \\ &\quad \left. \left. - \frac{b_+}{b_0} \frac{1 - \Gamma_0}{1 + \Delta/\omega_0} \right) + \frac{b_z (1 - \Gamma_0)}{b_0 (1 - \Gamma_z)} \left((1 - \Gamma_+) \sigma_0 \right. \right. \\ &\quad \left. \left. - \frac{(\Gamma_0 - \Gamma_z) \sigma_+}{1 + \Delta/\omega_0} \right) \right], \end{aligned} \quad (71)$$

$$\begin{aligned} \beta_\phi &= \left| \frac{c}{B_0} k_z k_{\perp 0} \delta\phi_0 \right|^2 \frac{1}{1 - \Gamma_+} \left[\frac{b_z (1 - \Gamma_0)}{b_0 (1 - \Gamma_z)} \frac{\sigma_+}{1 + \Delta/\omega_0} \right. \\ &\quad \left. - \frac{\Gamma_0 - \Gamma_+}{1 - \Gamma_z} \right] \left(1 - \Gamma_z - b_z \frac{1 - \Gamma_0}{b_0} \right), \end{aligned} \quad (72)$$

$$\begin{aligned} \alpha_\psi &= \left| \frac{c}{B_0} k_z k_{\perp 0} \delta\phi_0 \right|^2 \frac{\sigma_0}{1 - \Gamma_+} \frac{\Delta/\omega_0}{1 + \Delta/\omega_0} \\ &\quad \times [(1 - \Gamma_+) \sigma_0 - (\Gamma_0 - \Gamma_z) \sigma_+], \end{aligned} \quad (73)$$

and

$$\begin{aligned} \beta_\psi &= \left| \frac{c}{B_0} k_z k_{\perp 0} \delta\phi_0 \right|^2 \frac{\sigma_0}{1 - \Gamma_+} \frac{\Delta/\omega_0}{1 + \Delta/\omega_0} \\ &\quad \times \sigma_+ \left(1 - \Gamma_z - b_z \frac{1 - \Gamma_0}{b_0} \right). \end{aligned} \quad (74)$$

Equation (69) clearly indicates that $\delta\phi_z$ (ESCC) and $\delta\psi_z \propto \delta A_{\parallel z}$ (MSCC) are intrinsically coupled. From Eq. (69), one readily obtains the following

modulational instability dispersion relation for the spontaneous excitations of the CCs by the Ω_0 pump KAW;

$$Y^2 + Y(\alpha_\phi - \alpha_\psi - \beta_\psi) + (\alpha_\psi\beta_\phi - \alpha_\phi\beta_\psi) = 0 ; \quad (75)$$

where

$$Y = \gamma_z^2 + \Delta^2/(1 + \Delta/\omega_0) . \quad (76)$$

Equation (75), in general, needs to be solved numerically and the numerical results will be presented later (cf. also Appendix A for further details). It is, however, instructive to examine the stability properties in two limiting cases. First, let us consider the long wavelength limit; where $|b_{\mathbf{k}}| \ll 1$. Straightforward algebra then readily shows that the unstable (or least stable) branch of the modulational instability dispersion relation, Eq. (75), is given by

$$\begin{aligned} Y &\simeq \gamma_z^2 + \left[\left(\frac{3}{4} + \tau \right) \frac{b_z}{2} \omega_0 \right]^2 \simeq \frac{(\alpha_\psi\beta_\phi - \alpha_\phi\beta_\psi)}{(\alpha_\psi - \alpha_\phi)} \\ &\simeq \frac{3}{32} \left| \frac{c}{B_0} k_z k_{\perp 0} \delta\phi_0 \right|^2 \left(\frac{3}{4} + \tau \right)^2 b_z^3 \left(1 - \frac{b_z^2}{b_0^2} \right) ; \end{aligned} \quad (77)$$

where we have applied the $|b_{\mathbf{k}}| \ll 1$ limits of Eqs. (71) – (74). Equation (77) indicates that a necessary condition for instability is $b_z < b_0$ and that the corresponding threshold condition is

$$\left| \frac{\delta B_{\perp 0}}{B_0} \right|_{th}^2 = \frac{8}{3} \left(\frac{b_0^2/b_z^2}{1 - b_z^2/b_0^2} \right) \frac{k_{\parallel 0}^2 \rho_i^2}{b_0^2} . \quad (78)$$

Here, we have noted $\delta\phi_0 = \delta\psi_0/\sigma_0 \simeq \omega_0 \delta A_{\parallel 0}/(k_{\parallel 0} c)$, $k_{\perp 0} \delta A_{\parallel 0} = \delta B_{\perp 0}$, and expressed the amplitude in terms of $\delta B_{\perp 0}$; which is more convenient for comparisons with simulations. Eq. (78) indicates that, as $|k_{\perp} \rho_i|^2 \ll 1$, $|\delta B_{\perp 0}/B_0|_{th}$ rapidly increases as $|k_{\perp 0} \rho_i|^{-2}$ and, hence, finite $|k_{\perp} \rho_i|$ effects are necessary of the instability to set in. Well above the threshold condition, we have

$$\left(\frac{\gamma}{\Omega_i} \right) \simeq \frac{\sqrt{3}}{4\beta_i^{1/2}} \left(\frac{3}{4} + \tau \right) b_z^2 \sqrt{1 - \frac{b_z^2}{b_0^2}} \left| \frac{\delta B_{\perp 0}}{B_0} \right| . \quad (79)$$

Furthermore, since

$$|\delta\psi_z|/|\delta\phi_z| \simeq 1 ; \quad (80)$$

ESCC and MSCC are, indeed, strongly coupled, and arbitrary de-coupling assumptions could lead to erroneous conclusions on the stability. It is readily seen that the threshold in Eq. (78) is minimized for $b_0 = \sqrt{2}b_z$ that yields $|\delta B_{\perp 0}/B_0|_{th, \min}^2 = (32/3)k_{\parallel 0}^2 \rho_i^2/b_0^2$. Thus, $b_0^2 = (32/3)k_{\parallel 0}^2 \rho_i^2/|\delta B_{\perp 0}/B_0|_{th, \min}^2 \ll 1$ for effective mode excitation, which is hard to meet at long wavelength. For this reason, in the original works on CC nonlinear excitation by KAW via modulational instability [16, 68], it was noted that nonlinear excitations of convective cells by KAW are always suppressed in the long wavelength limit,

although only the $Y \simeq (\alpha_\psi - \alpha_\phi) < 0$ root was discussed therein and in the recent review on this subject [17].

The other limit is the short-wavelength limit; *i.e.*, $|b_{\mathbf{k}}| \gg 1$, where FILR effects exhibit distinctively. Taking this limit and, to further simplify the analysis, assuming $b_z \ll b_0$; we can readily show that Eq. (75) yields the following unstable solution

$$Y \simeq \gamma_z^2 + \omega_0^2 \left(\frac{b_z}{2b_0} \right)^2 \simeq \tau \alpha_\phi, \quad (81)$$

where

$$\alpha_\phi \simeq |(c/B_0)k_z k_{\perp 0} \delta\phi_0|^2 (b_z/b_0)(1 + \tau). \quad (82)$$

Equation (81) indicates that, in this $|b_{\mathbf{k}}| \gg 1$ short wavelength limit, convective cells can be modulationally excited when the pump KAW amplitude exceeds the following threshold value, noting $\delta\phi_0 \simeq \delta\psi_0/(1 + \tau)$,

$$\left| \frac{\delta B_{\perp 0}}{B_0} \right|_{th}^2 = \frac{(1 + \tau)}{4\tau} \left(\frac{k_{\parallel 0}}{k_{\perp 0}} \right)^2. \quad (83)$$

Well above the threshold value, we have

$$\left| \frac{\gamma_z}{\omega_0} \right| \simeq \left(\frac{\tau}{1 + \tau} \right)^{1/2} \left| \frac{k_{\perp 0}}{k_{\parallel 0}} \right| \left(\frac{b_z}{b_0} \right)^{3/2} \left| \frac{\delta B_{\perp 0}}{B_0} \right|. \quad (84)$$

Meanwhile, ESCC and MSCC remain strongly coupled;

$$|\delta\psi_z|/|\delta\phi_z| \simeq (1 + \tau)/2. \quad (85)$$

We emphasize that the above two limiting analyses clearly demonstrate that finite $|k_{\perp} \rho_i|$ effects are necessary for the modulational excitations of convective cells and that ESCC and MSCC are intrinsically coupled. Taking $\mathbf{k}_{\perp 0} = \hat{\mathbf{x}}k_x$, $\mathbf{k}_{\perp z} = \hat{\mathbf{y}}k_y$ and $\delta\mathbf{B}_{\perp 0} = \hat{\mathbf{y}}\delta B_y \sin(\omega_0 t - k_x x - k_{\parallel 0} z)$, we shall assume $\delta B_y/B_0 = 2\delta B_{\perp 0}/B_0$ in the comparison of numerical simulation results with theoretical predictions discussed above. The complete dispersion relation is numerically solved in the $(k_x \rho_i, k_y \rho_i)$ -plane for fixed $k_{\parallel 0} \rho_i = 0.02$, $\tau = 1$ and $\beta_e = \beta_i = 0.2$ and different values of $\delta B_y/B_0$. Figure 11 shows the marginal stability curves. It clearly demonstrates, consistent with the above analytical predictions, the crucial roles of the finite $k_{\perp} \rho_i$ effects in the stability properties. Marginal stability curves demonstrate the existence of a necessary condition for instability, $b_z > k_{y\ell}^2 \rho_i^2 \equiv b_{z\ell}$, given by [68]

$$\frac{b_{z\ell}(1 - \Gamma_{z\ell})}{2\Gamma_{z\ell} - \tau(1 - \Gamma_{z\ell})} = \frac{4k_{\parallel 0}^2 \rho_i^2}{|\delta B_y/B_0|^2}, \quad (86)$$

where $\Gamma_{z\ell} \equiv \Gamma_z(b_z = b_{z\ell})$, which holds for $b_0 \gg 1$ and arbitrary $b_{z\ell}$. Figure 12, meanwhile, plots the calculated growth rates vs. $\delta B_y/B_0$ for $(k_x \rho_i, k_y \rho_i) = (0.8, 0.6)$ and $(k_x \rho_i, k_y \rho_i) = (1.0, 0.8)$. Corresponding hybrid simulations have also been carried out to investigate the nonlinear excitations of convective cells by a pump KAW [68]. The observed growth rates, as shown in Fig. 12,

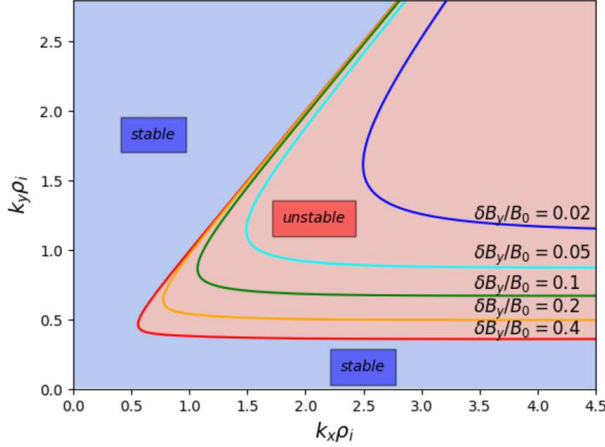


Fig. 11: Marginal stability curves in the $(k_x \rho_i, k_y \rho_i)$ -plane as a function of the pump KAW amplitude $\delta B_y / B_0$. Fixed parameters are $k_{\parallel 0} \rho_i = 0.02$, $\tau = 1$, and $\beta_e = \beta_i = 0.2$ [From original figure in Ref. [68]].

agree reasonably well with the theoretically predicted values. Meanwhile, simulations also show that, for $\mathbf{k}_z = \hat{\mathbf{y}}k_y$, the ESCC (δE_{yz}) and MSCC (δB_{xz}) are coupled and both are spontaneously excited; consistent, again, with the theoretical predictions. For $\gamma_z / \omega_0 = \mathcal{O}(1)$, in general, it is necessary to solve for CC dispersion relation and polarization from Eq. (120) in the Appendix A, which allows determining both CC magnetic perturbation as well as the corresponding inductive electric field; that is,

$$\left| \frac{\delta B_{xz}}{\delta B_{y0}} \right| = \frac{|k_y \rho_i|}{2\sigma_0 |k_x \rho_i|} \frac{|\delta \psi_z|}{|\delta \phi_0|}, \quad (87)$$

$$\left| \frac{c\delta E_{\parallel z}}{v_A \delta B_{y0}} \right| = \frac{\sqrt{\beta_i/2}}{2\sigma_0 |k_x \rho_i|} \left| \frac{\gamma_z}{\Omega_i} \right| \frac{|\delta \psi_z|}{|\delta \phi_0|}. \quad (88)$$

3.4 Quasilinear transports induced by KAWs

In the presence of finite δE_{\parallel} , KAWs can exchange energy and generalized momenta with charged particles when the wave-particle resonance condition is satisfied. Such energy-momentum exchanges, thus, could lead to efficient acceleration/heating, current/flow, as well as cross-field transports; that is, wave-induced collisionless transports in the charged particle's phase space. The self-consistent analysis of charged particle's phase-space dynamics and the corresponding dynamics of collective electromagnetic fluctuations represents, indeed, fundamental and complex investigations on the frontier of

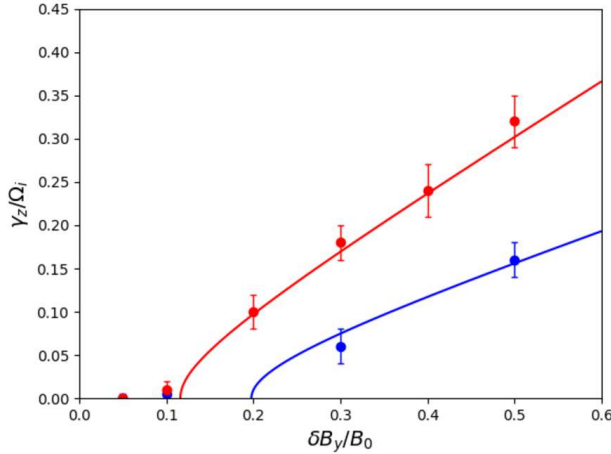


Fig. 12: Modulational instability growth rates (continuous lines), including finite γ_z/ω_0 (cf. Appendix A), vs. $\delta B_y/B_0$ are compared with hybrid simulation results (circles; error bars are a measure of discrete particle noise) for $(k_x \rho_i, k_y \rho_i) = (0.8, 0.6)$ (ble) and $(k_x \rho_i, k_y \rho_i) = (1.0, 0.8)$ (red). Fixed parameters are the same as in Fig. 11 [From original figure in Ref. [68]].

plasma physics research. Such complexities, in one aspect, are associated with the complexities of phase-space dynamics of charged particles in the presence of electromagnetic fluctuations that vary, self-consistently, in space and time. Detailed analyses on this topic are beyond the intended scope of this review. Interested readers are referred ref.[17] for further discussions. In this review, we assume the fluctuations have sufficiently broad spectral widths and finite but small intensities; such that charged particles diffuse stochastically in the phase space and we may employ the quasilinear description. Furthermore, we will limit our considerations to KAWs in a slab plasma. More general analyses in realistic geometries can be found in Chen[10].

Let x be the nonuniformity (radial) direction and $\mathbf{B}_0 = B_0(x)\hat{z}$. Assuming $\beta \ll 1$, B_0 is then approximately constant. The particle distribution function, f , can be decoupled into an “equilibrium” component, F_0 , and a fluctuating component, δf ; i.e.,

$$f = F_0(x, \mu, v_{\parallel}, \epsilon t) + \delta f(\mathbf{x}, t); \quad (89)$$

where ϵt with $\epsilon \ll 1$ denotes that F_0 is slowly varying in time and vary spatially only in the nonuniformity x direction. δf , meanwhile, is given by the linear gyrokinetic equations, Eqs. (14) and (15) with, however, F_M and $(-qF_M/T)$ replaced, respectively, by F_0 and $(q/mv_{\parallel})(\partial F_0/\partial v_{\parallel})$. Employing the linear gyrokinetic equations[27, 5], it is then straightforward to show that

F_0 satisfies the following quasi-linear gyrokinetic equations[10]

$$\frac{\partial}{\partial t}(B_0 F_0) + \frac{\partial}{\partial x}(B_0 \overline{\delta \dot{x} \delta G_{res}}) + \frac{\partial}{\partial v_{\parallel}}(B_0 \overline{\delta \dot{v}_{\parallel} \delta G_{res}}) = 0, \quad (90)$$

where

$$\delta \dot{x} = \left(\frac{c \mathbf{b}}{B_0} \times \nabla \langle \delta L_g \rangle_{\alpha} \right) \cdot \hat{\mathbf{x}}, \quad (91)$$

δL_g is given by Eqs. (16) and (17) ; i.e.,

$$\langle \delta L_g \rangle_{\alpha} = \langle e^{\rho \cdot \nabla} (\delta \phi - v_{\parallel} \delta A_{\parallel} / c) \rangle_{\alpha}, \quad (92)$$

$$\delta \dot{v}_{\parallel} = -\frac{q}{m} \mathbf{b} \cdot \nabla \langle \delta L_g \rangle_{\alpha}, \quad (93)$$

and $\mathbf{b} = \hat{\mathbf{z}}$.

Meanwhile, δG_{res} in Eq. (90) represents the contribution of resonant particles to δf [10]; i.e.,

$$\delta G_{res} = i\pi \frac{q}{m} \delta \left[-i(v_{\parallel} \mathbf{b} \cdot \nabla + \frac{\partial}{\partial t}) \right] Q F_0 \langle \delta L_g \rangle_{\alpha}, \quad (94)$$

and

$$Q F_0 \langle \delta L_g \rangle_{\alpha} = i \left[\frac{1}{v_{\parallel}} \frac{\partial F_0}{\partial v_{\parallel}} \frac{\partial}{\partial t} + \nabla F_0 \cdot \left(\frac{\mathbf{b}}{\Omega} \right) \times \nabla \right] \langle \delta L_g \rangle_{\alpha}. \quad (95)$$

Finally, in Eq. (90) , $\overline{(\dots)}$ denotes averaging over the (fast) wave periods.

Taking perturbations to be of the following form

$$\delta \phi(\mathbf{x}, t) = \frac{1}{2} \sum_{\mathbf{k}} \{ \delta \hat{\phi}_{\mathbf{k}}(x) \exp [i(\mathbf{k} \cdot \mathbf{x} - \omega_{\mathbf{k}} t)] + c.c. \}, \quad (96)$$

with $\mathbf{k} = k_y \hat{\mathbf{y}} + k_{\parallel} \mathbf{b}$, and $\hat{\mathbf{y}}$ corresponds to the azimuthal (east-west) direction. We then have

$$\delta \dot{x} = -i \frac{c}{2B} \sum_{\mathbf{k}} \{ k_y J_0(k_{\perp} \rho) \delta \hat{L}_{\mathbf{k}} \exp [i(\mathbf{k} \cdot \mathbf{x} - \omega_{\mathbf{k}} t)] + c.c. \}, \quad (97)$$

$$\delta \dot{v}_{\parallel} = -i \frac{q}{2m} \sum_{\mathbf{k}} \{ k_{\parallel} J_0(k_{\perp} \rho) \delta \hat{L}_{\mathbf{k}} \exp [i(\mathbf{k} \cdot \mathbf{x} - \omega_{\mathbf{k}} t)] + c.c. \}, \quad (98)$$

and

$$\delta \hat{L}_{\mathbf{k}} = (\delta \hat{\phi} - v_{\parallel} \delta \hat{A}_{\parallel} / c)_{\mathbf{k}}. \quad (99)$$

Note that, for resonant particles,

$$\delta \left[-i(v_{\parallel} \mathbf{b} \cdot \nabla + \frac{\partial}{\partial t}) \right] = \delta(k_{\parallel} v_{\parallel} - \omega_{\mathbf{k}}), \quad (100)$$

and, hence,

$$\delta \hat{L}_{k, res} = (\delta \hat{\phi} - \delta \hat{\Psi})_{\mathbf{k}} \equiv \delta \hat{\phi}_{\parallel \mathbf{k}}. \quad (101)$$

Here, we recall Eq. (20), $\delta\hat{\Psi}_k = (\omega\delta\hat{A}_{\parallel}/ck_{\parallel})_k$, and Eq. (21), $\delta\hat{E}_{\parallel k} = -ik_{\parallel}\delta\hat{\phi}_{\parallel k}$. δG_{res} can then be expressed, correspondingly, as

$$\delta G_{res} = \frac{1}{2}\{\delta\hat{G}_{k,res} \exp[i(\mathbf{k}\cdot\mathbf{x} - \omega_k t)] + c.c.\}, \quad (102)$$

where

$$\delta\hat{G}_{k,res} = i\pi\frac{q}{m}\delta(k_{\parallel}v_{\parallel} - \omega_k)\hat{Q}_k F_0 J_0(k_{\perp}\rho)\delta\hat{\phi}_{\parallel k}, \quad (103)$$

and

$$\hat{Q}_k F_0 = (\omega_k \frac{\partial}{v_{\parallel} \partial v_{\parallel}} + \frac{k_y}{\Omega} \frac{\partial}{\partial x}) F_0. \quad (104)$$

Substituting Eqs. (98), (101), and (102) into Eq. (90), we can readily derive the expression of the quasi-linear gyrokinetic equation in terms of $\delta\hat{\phi}_{\parallel k}$. More specifically, we have

$$\overline{\delta\dot{x}\delta G_{res}} = -\left(\frac{\pi}{2}\right)\left(\frac{c}{B_0}\right)\left(\frac{q}{m}\right) \sum_{\mathbf{k}} \left[\delta(k_{\parallel}v_{\parallel} - \omega_k) k_y J_0^2 \left| \delta\hat{\phi}_{\parallel k} \right|^2 \hat{Q}_k F_0 \right], \quad (105)$$

and, similarly, obtain

$$\overline{\delta\dot{v}_{\parallel}\delta G_{res}} = -\left(\frac{\pi}{2}\right)\left(\frac{q}{mc}\right)^2 \sum_{\mathbf{k}} \left[\delta(k_{\parallel}v_{\parallel} - \omega_k) k_{\parallel} J_0^2 \left| \delta\hat{\phi}_{\parallel k} \right|^2 \hat{Q}_k F_0 \right]. \quad (106)$$

Taking the various moments of Eq. (90), we then obtain the transport equations for density, parallel momentum/current, and energy. Specifically, defining the slowly varying ‘‘equilibrium’’ density, $N(x, \epsilon t)$, as

$$N(x, \epsilon t) = 2\pi \int B_0 d\mu dv_{\parallel} F_0 \equiv \langle F_0 \rangle_v, \quad (107)$$

Eq. (90) along with Eq. (105) then yield the following particle transport equation[10, 30, 36]

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial x} \Gamma_x = 0, \quad (108)$$

where

$$\Gamma_x = \Gamma_{xc} + \Gamma_{xd}, \quad (109)$$

$$\Gamma_{xc} = -\left(\frac{\pi}{2}\right)\left(\frac{q}{m}\right)\left(\frac{c}{B_0}\right) \sum_{\mathbf{k}} \langle \delta(k_{\parallel}v_{\parallel} - \omega_k) J_0^2 \left| \delta\hat{\phi}_{\parallel k} \right|^2 k_y \omega_k \left(\frac{\partial}{v_{\parallel} \partial v_{\parallel}}\right) F_0 \rangle_v, \quad (110)$$

$$\Gamma_{xd} = -\left(\frac{\pi}{2}\right)\left(\frac{q}{m}\right)\left(\frac{c}{B_0}\right) \sum_{\mathbf{k}} \langle \delta(k_{\parallel}v_{\parallel} - \omega_k) J_0^2 \left| \delta\hat{\phi}_{\parallel k} \right|^2 \left(\frac{k_y^2}{\Omega}\right) \left(\frac{\partial}{\partial x}\right) F_0 \rangle_v. \quad (111)$$

Equations (109)-(111) demonstrate that the particle flux, Γ_x , intrinsically consists of a convective, Γ_{xc} , and a diffusive, Γ_{xd} component; even though charged particles diffuse stochastically in the phase space. Note also that $|\Gamma_{xc}|$ and $|\Gamma_{xd}|$ scale, respectively, with k_y and k_y^2 . Thus, no transport occurs if $k_y = 0$.

This, of course, is expected, since for $k_y = 0$, $P_y = mv_y + qA_y/c$ is conserved. As $\langle P_y \rangle_\alpha = qA_y(X_{gc})/c$ with X_{gc} being the guiding center position in x , long-time transport will occur only if P_y conservation is broken by finite- k_y symmetry-breaking perturbations. Equations (110) and (111) also indicates that the relative magnitudes between Γ_{xc} and Γ_{xd} depend on the detailed spectral properties of $|\delta\hat{\phi}_{\parallel k}|^2$. We can also employ Eq. (90) to derive the equation for energy transport and heating. Letting $K = \langle mv^2 F_0/2 \rangle_v$, it is then straightforward to show[10]

$$\frac{\partial K}{\partial t} + \frac{\partial}{\partial x} q_x = S, \quad (112)$$

where $q_x = q_{xc} + q_{xd}$ is the energy flux with

$$q_{xc} = -\left(\frac{\pi}{2}\right)\left(\frac{cq}{B_0}\right) \sum_{\mathbf{k}} \langle \delta(k_{\parallel} v_{\parallel} - \omega_k) \left(\frac{v^2}{2}\right) J_0^2 |\delta\hat{\phi}_{\parallel k}|^2 (k_y \omega_k) \left(\frac{\partial}{v_{\parallel} \partial v_{\parallel}}\right) F_0 \rangle_v, \quad (113)$$

$$q_{xd} = -\left(\frac{\pi}{2}\right)\left(\frac{cq}{B_0}\right) \sum_{\mathbf{k}} \langle \delta(k_{\parallel} v_{\parallel} - \omega_k) \left(\frac{v^2}{2}\right) J_0^2 |\delta\hat{\phi}_{\parallel k}|^2 \left(\frac{k_y^2}{\Omega}\right) \left(\frac{\partial}{\partial x}\right) F_0 \rangle_v, \quad (114)$$

and S is the local heating rate;

$$S = -\left(\frac{\pi}{2}\right)\left(\frac{q^2}{m}\right) \sum_{\mathbf{k}} \omega_k \langle \delta(k_{\parallel} v_{\parallel} - \omega_k) J_0^2 |\delta\hat{\phi}_{\parallel k}|^2 \left(\omega_k \frac{\partial}{v_{\parallel} \partial v_{\parallel}} + \frac{k_y}{\Omega} \frac{\partial}{\partial x}\right) F_0 \rangle_v. \quad (115)$$

Similarly, we can derive the following equation for parallel momentum transport and generation[10];

$$\frac{\partial}{\partial t} P_{\parallel} + \frac{\partial}{\partial x} \Pi_x = F_{\parallel}, \quad (116)$$

where $P_{\parallel} = \langle mv_{\parallel} F_0 \rangle_v$, $\Pi_x = \Pi_{xc} + \Pi_{xd}$ is the parallel momentum flux,

$$\Pi_{xc} = -\left(\frac{\pi}{2}\right)\left(\frac{qc}{B_0}\right) \sum_{\mathbf{k}} \langle \delta(k_{\parallel} v_{\parallel} - \omega_k) (v_{\parallel}) J_0^2 |\delta\hat{\phi}_{\parallel k}|^2 (k_y \omega_k) \left(\frac{\partial}{v_{\parallel} \partial v_{\parallel}}\right) F_0 \rangle_v, \quad (117)$$

$$\Pi_{xd} = -\left(\frac{\pi}{2}\right)\left(\frac{qc}{B_0}\right) \sum_{\mathbf{k}} \langle \delta(k_{\parallel} v_{\parallel} - \omega_k) (v_{\parallel}) J_0^2 |\delta\hat{\phi}_{\parallel k}|^2 \left(\frac{k_y^2}{\Omega}\right) \left(\frac{\partial}{\partial x}\right) F_0 \rangle_v; \quad (118)$$

and

$$F_{\parallel} = -\left(\frac{\pi}{2}\right)\left(\frac{q^2}{m}\right) \sum_{\mathbf{k}} k_{\parallel} \langle \delta(k_{\parallel} v_{\parallel} - \omega_k) J_0^2 |\delta\hat{\phi}_{\parallel k}|^2 \left(\omega_k \frac{\partial}{v_{\parallel} \partial v_{\parallel}} + \frac{k_y}{\Omega} \frac{\partial}{\partial x}\right) F_0 \rangle_v \quad (119)$$

is the local effective parallel force due to KAWs. Multiplying Eq. (116) by (q/m) naturally leads to the equation of current transport and generation.

As noted in Chen[10], the transport equations derived above have the appealing physical pictures that transports as well as acceleration/heating may be viewed as ‘‘collisions’’ between charged particles of energy $= \epsilon = mv^2/2$

and generalized momentum = $\mathbf{P} = m\mathbf{v} + q\mathbf{A}/c$ with wave packets or quasi-particles of energy = ω and momentum = \mathbf{k} . The transport equations also clearly demonstrated that, in collisionless plasmas, wave-particle resonances are responsible for phase-space transports, and the transports consist of both convective and diffusive components with coefficients depending critically on the spectral properties of, in this case, KAWs.

4 Conclusions and discussions

In this paper, we argue that short-wavelength KAWs are ubiquitous in realistic nonuniform magnetized plasmas due to the very existence of SAW continuous spectra. Employing the powerful theoretical tool of gyrokinetic equations, we then re-examine and explore further the linear and nonlinear physics of KAWs. Our analyses clearly demonstrate that kinetic effects due to; e.g., finite ion Larmor radii, can qualitatively and quantitatively modify the nonlinear processes. More specifically, we show that in contrast to the MHD fluid description, the FLR effects lead to the significantly enhanced electron-ion decoupling; which, in turn, leads to significantly enhanced nonlinear coupling coefficients. Our analyses, furthermore, suggest that the KAW turbulence spectra will be more isotropic than those according to the MHD description. In additions, convective cells could be nonlinearly excited only in the short-wavelength regime. These spectral properties obviously carry important implications to the symmetry-breaking wave-induced transports of charged particles. In other words, based on our theoretical studies, we submit that one needs to employ first-principle-based self-consistent kinetic or gyrokinetic theories in order to develop a reliable and accurate understanding of KAW physics; especially, when effects associated with nonlinearities, realistic nonuniformities, and geometries are considered.

Since the primary aim of the present paper is to illuminate physics of KAWs based on the gyrokinetic theory approach, our focus, therefore, has been on the fundamental processes. This paper, thus, is not, and, indeed never intends to be a comprehensive review of all aspects of the rich KAW physics. For complimentary readings, we refer to the monograph by Wu et al. [64], the review article by Chen and Zonca[17], and the recent works by Qiu et al. [50] for KAWs in fusion tokamak plasmas.

As we, hopefully, have demonstrated, there are many interesting issues; especially, in the nonlinear regime, associated with the KAW physics. Some of them remain little explored; for example, the phase-space dynamics of nonlinear wave-particle interactions as well as the physics of the fully developed KAW turbulence, including frequency/wave number cascading and, possibly, filamentary structures via nonlinear excitations of convective cells. Obviously, careful studies of these physics issues employing the powerful gyrokinetic approach analytically and/or via numerical simulation will make significant impacts to our deep understandings of the charged particle dynamics and Alfvén wave turbulences in nature and laboratory plasmas.

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A Appendix: Equations for strongly excited convective cells

The equations for coupled ESCC and MSCC excitations derived in Sec. 3.3, that is Eqs. (69), give only the leading order terms in γ_z^2/ω_0^2 , for simplicity, and are adopted for computing marginal stability curves of the modulational instability as shown in Fig. 11. Exact equations, *i.e.* including higher order corrections in γ_z^2/ω_0^2 , are given below for interested readers and are used for computing the finite growth rate of the modulational instability (cf. Fig. 12) away from marginal stability [68]. The complete equations for coupled ESCC and MSCC excitations can be cast as follows

$$\begin{aligned} \left[\gamma_z^2 + \frac{\Delta^2}{1 + \Delta/\omega_0} + \frac{\gamma_z^4}{4\omega_0^2(1 + \Delta/\omega_0)} \right] \delta\phi_z &= -\alpha_\phi(\delta\phi_z - \delta\psi_z) + \beta_\phi\delta\psi_z \\ &+ \frac{\gamma_z^2}{2\omega_0^2(1 + \Delta/\omega_0)} \left[-\hat{\alpha}_\phi(\delta\phi_z - \delta\psi_z) + \hat{\beta}_\phi\delta\psi_z \right], \\ \left[\gamma_z^2 + \frac{\Delta^2}{1 + \Delta/\omega_0} + \frac{\gamma_z^4}{4\omega_0^2(1 + \Delta/\omega_0)} \right] \delta\psi_z &= -\alpha_\psi(\delta\phi_z - \delta\psi_z) + \beta_\psi\delta\psi_z \\ &+ \frac{\gamma_z^2}{2\omega_0^2(1 + \Delta/\omega_0)} \left[-\hat{\alpha}_\psi(\delta\phi_z - \delta\psi_z) + \hat{\beta}_\psi\delta\psi_z \right], \end{aligned} \quad (120)$$

where

$$\begin{aligned} \hat{\alpha}_\phi &= \left| \frac{c}{B_0} k_z k_{\perp 0} \delta\phi_0 \right|^2 \frac{1}{1 - \Gamma_+} \left[\frac{\Gamma_0 - \Gamma_+}{1 - \Gamma_z} (\Gamma_0 - \Gamma_z) \right. \\ &\left. + \frac{b_z(1 - \Gamma_0)}{b_0(1 - \Gamma_z)} (1 - \Gamma_+) \sigma_0 \right], \end{aligned} \quad (121)$$

$$\begin{aligned} \hat{\beta}_\phi &= - \left| \frac{c}{B_0} k_z k_{\perp 0} \delta\phi_0 \right|^2 \frac{1}{1 - \Gamma_+} \left(\frac{\Gamma_0 - \Gamma_+}{1 - \Gamma_z} \right) \\ &\times \left(1 - \Gamma_z - b_z \frac{1 - \Gamma_0}{b_0} \right); \end{aligned} \quad (122)$$

and

$$\begin{aligned} \hat{\alpha}_\psi &= - \left| \frac{c}{B_0} k_z k_{\perp 0} \delta\phi_0 \right|^2 \frac{\sigma_0}{1 - \Gamma_+} \\ &\times [(1 - \Gamma_+) \sigma_0 + (\Gamma_0 - \Gamma_z) \sigma_+], \end{aligned} \quad (123)$$

$$\begin{aligned} \hat{\beta}_\psi &= \left| \frac{c}{B_0} k_z k_{\perp 0} \delta\phi_0 \right|^2 \frac{\sigma_0}{1 - \Gamma_+} \\ &\times \sigma_+ \left(1 - \Gamma_z - b_z \frac{1 - \Gamma_0}{b_0} \right). \end{aligned} \quad (124)$$

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