Universal behaviour of frequency chirping fluctuations in magnetized plasmas

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Outline

- Frequency-chirping fluctuations are ubiquitous and are one of the most studied problems in plasma physics:
  - What was done: self consistent solution of chirped wave packet with narrow spectrum
  - What is novel: solution of the nonlinear phase space structure evolution, consistent with chirped wave packet
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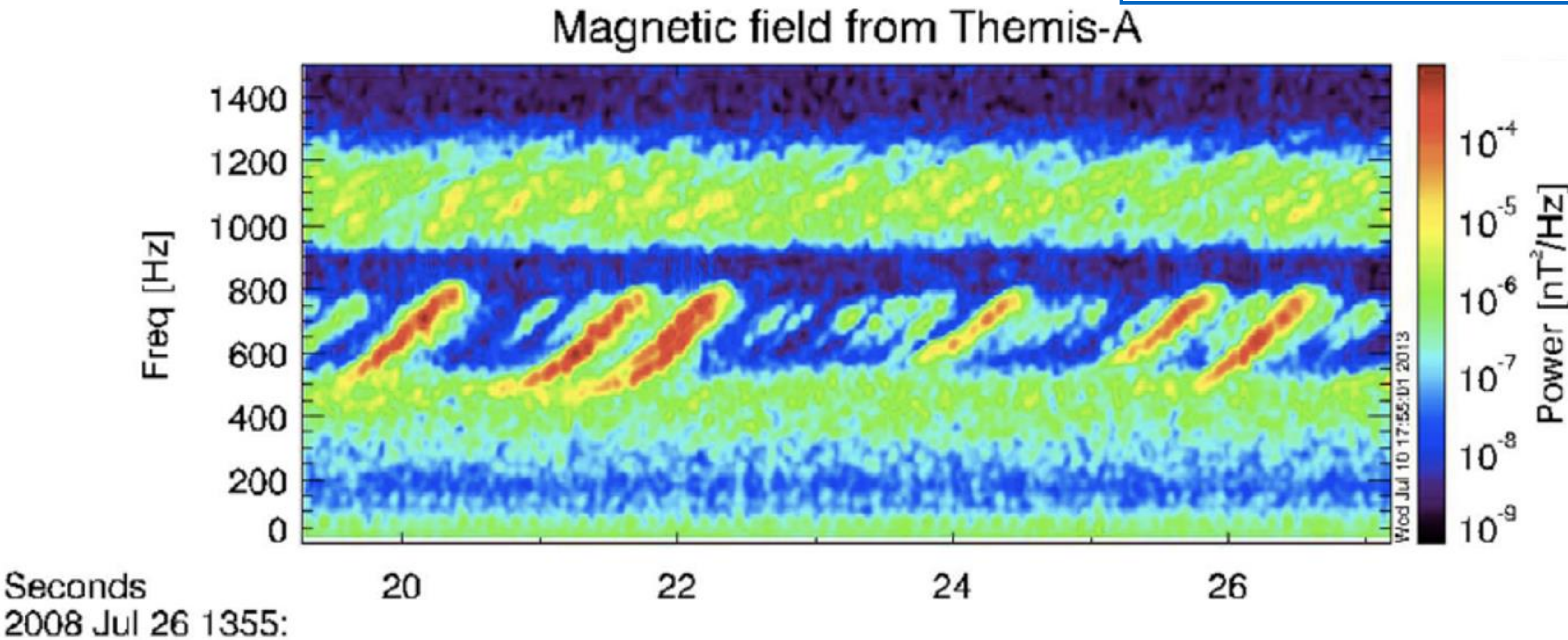
Ubiquitous chirping in plasmas

- Frequency-chirping fluctuations are ubiquitous in magnetized plasmas and are routinely observed in space and laboratory environments:
  - Space plasmas: whistler mode chorus and electromagnetic ion cyclotron (EMIC) waves in the Earth's magnetosphere
  - Laboratory fusion plasmas: fishbone oscillations and energetic particle modes (EPM)
  - Quasi coherent spectrum: not turbulence, with important role of wave-particle resonances

Chorus chirping

- Earth's magnetosphere can, in certain circumstances, amplify e.m. wave frequency bursts, which are known as chorus for their characteristic chirping

Angelopoulos V. 2008.
Space Sci Rev, 141: 5–34



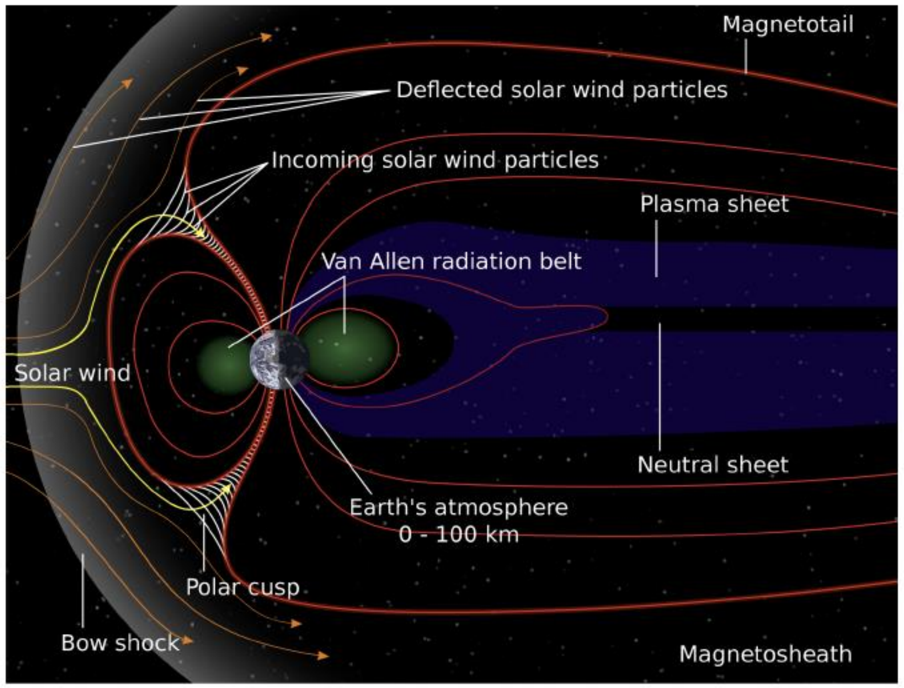
Importance of frequency chirping

- Whistler mode chorus excitation and nonlinear dynamics is one of the long-studied physics problems of the Earth's magnetosphere due to its implications for particle acceleration and distribution in the radiation belts

- In fusion plasmas, as fishbone/EPM mode frequency sweeps, energetic particles (EP) are transported outward to maximize wave-EP power transfer (maximized mode growth and EP transport)

Earth's radiation belts

Diagram of Earth's magnetosphere



Source: https://en.wikipedia.org/wiki/Magnetosphere
Original: NASA Vector: Aaron Kaase, Medium69 - Structure of the magnetosphere numbered.svg

- ❑ Significant MeV electron population
- ❑ Formation mechanism?

Earth's radiation belts

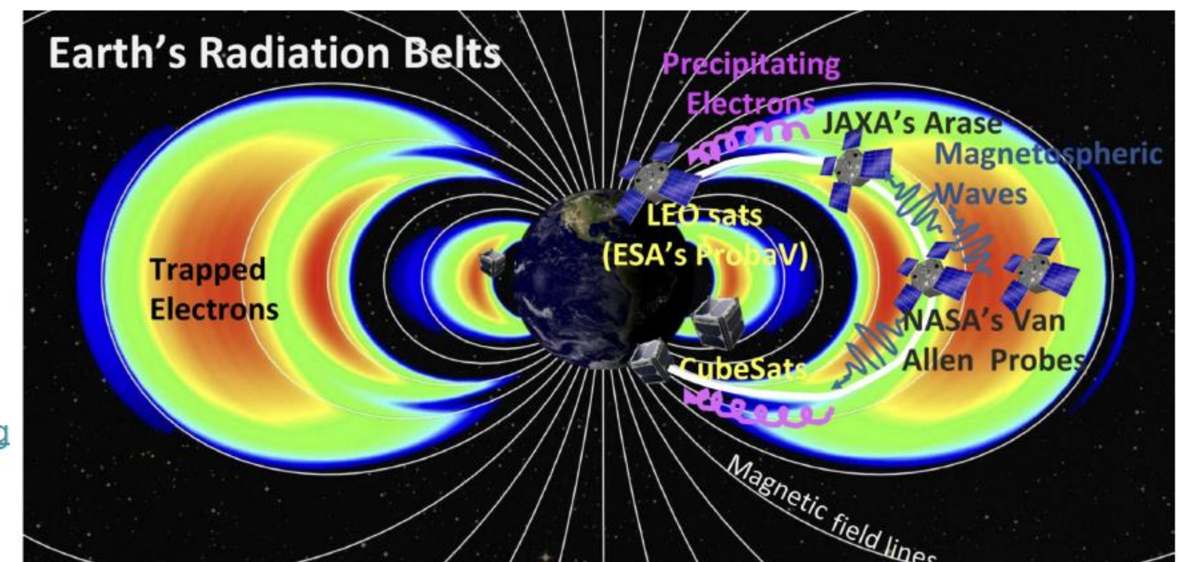


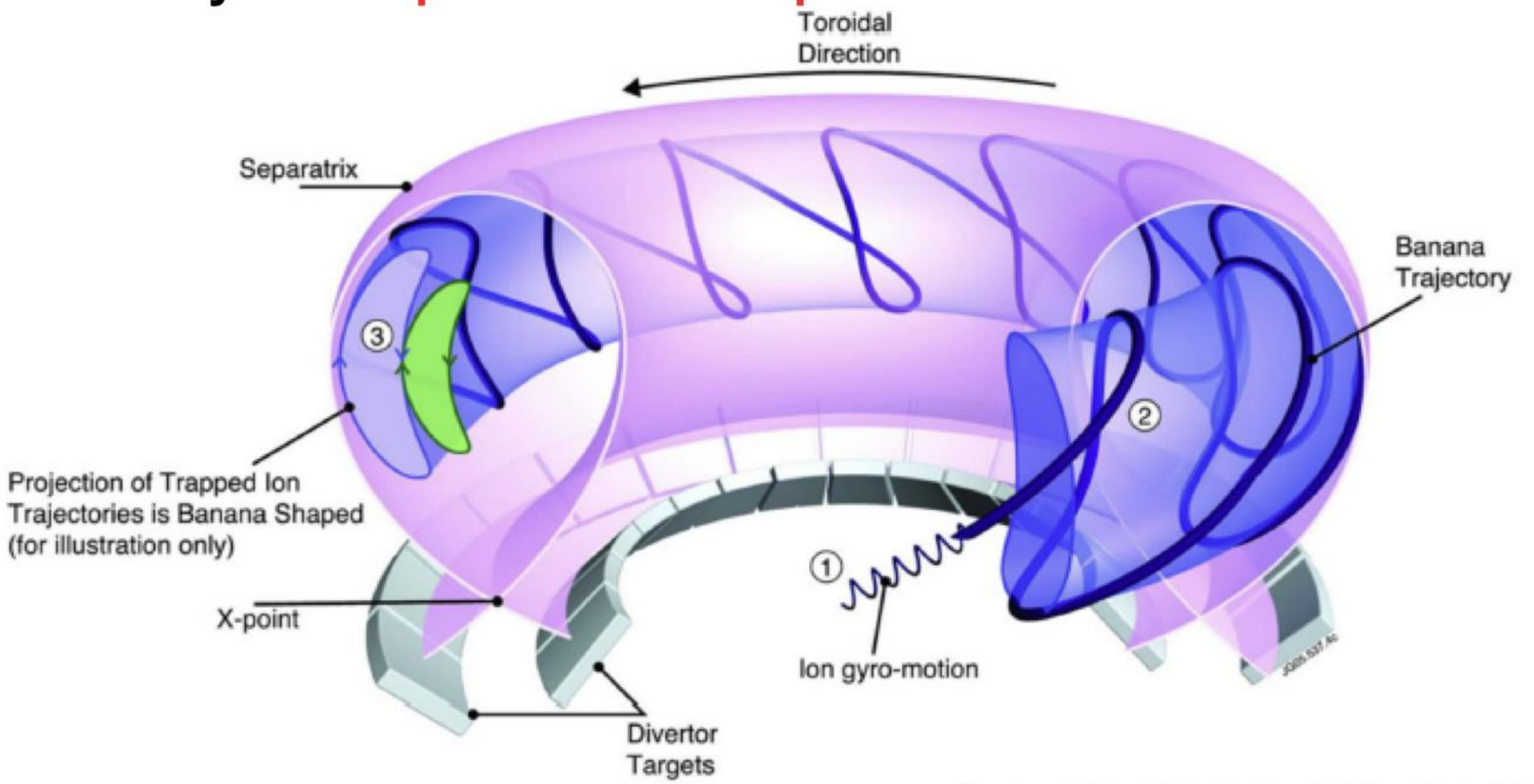
Figure: Latidunal repartition of the Arase, Proba V (LEO), Van Allen Probes satellites within the Earth's inner magnetosphere and radiation belts. Here belts are illustrated with a newly found and unusual 3 Belt structure.

Source: https://www.issibern.ch/teams/radbeltphysics/

- ❑ Whistler mode chorus is responsible for electron acceleration to MeV energy in Earth's radiation belts (R. B. Horne, Nature 2005)

Resonant transport in laboratory

- Crucial role of resonant transport in collision less burning plasmas, particularly of supra-thermal particles



[from Pitts, Buttery & Pinches, PhysicsWorld 2006]

- Loss of MeV particles (fusion alphas, supra-thermal), naturally resonating with || propagating Alfvén waves ($v \simeq \omega/k_{||}$), may impact material walls and:

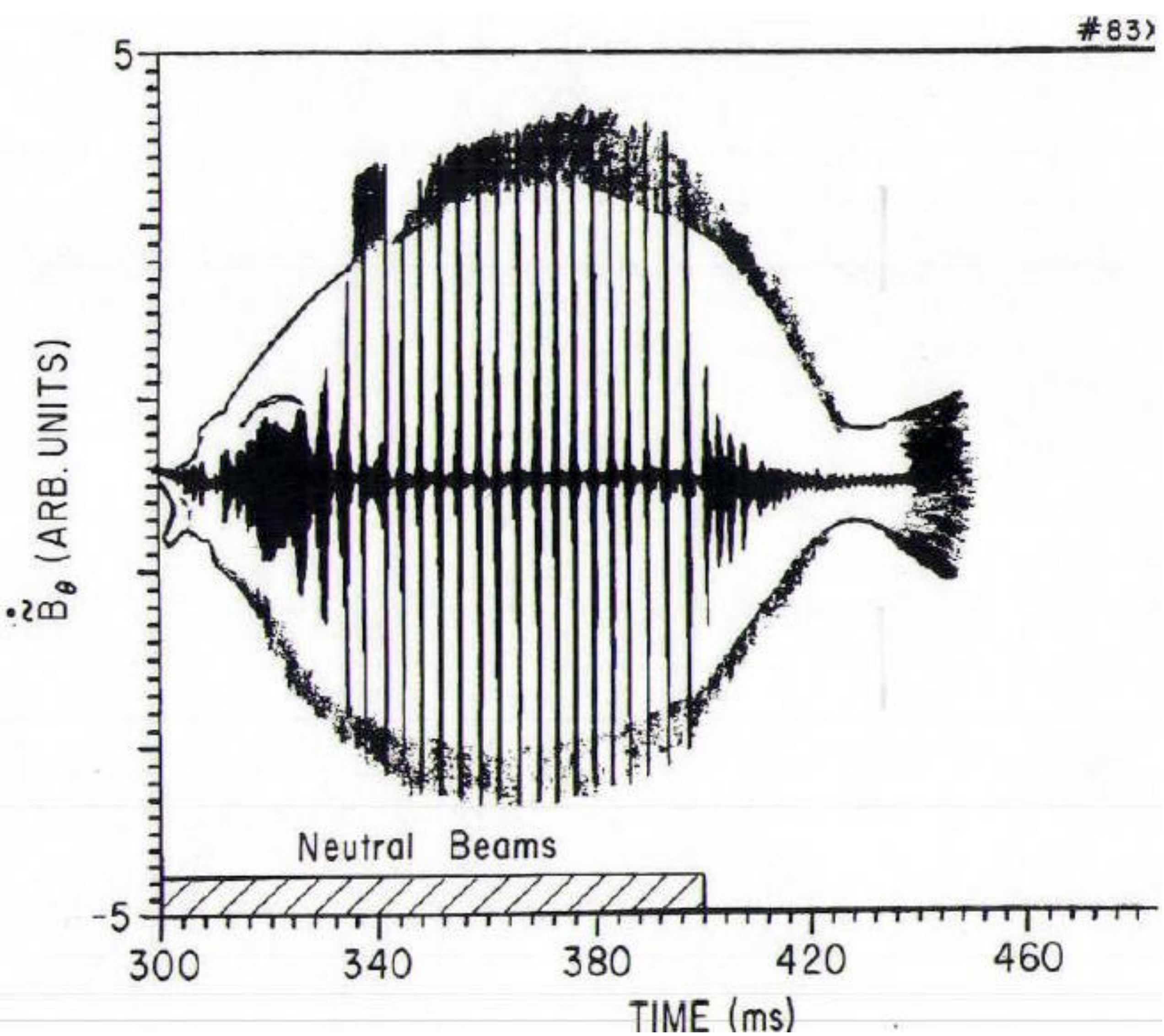
Key Issues!!

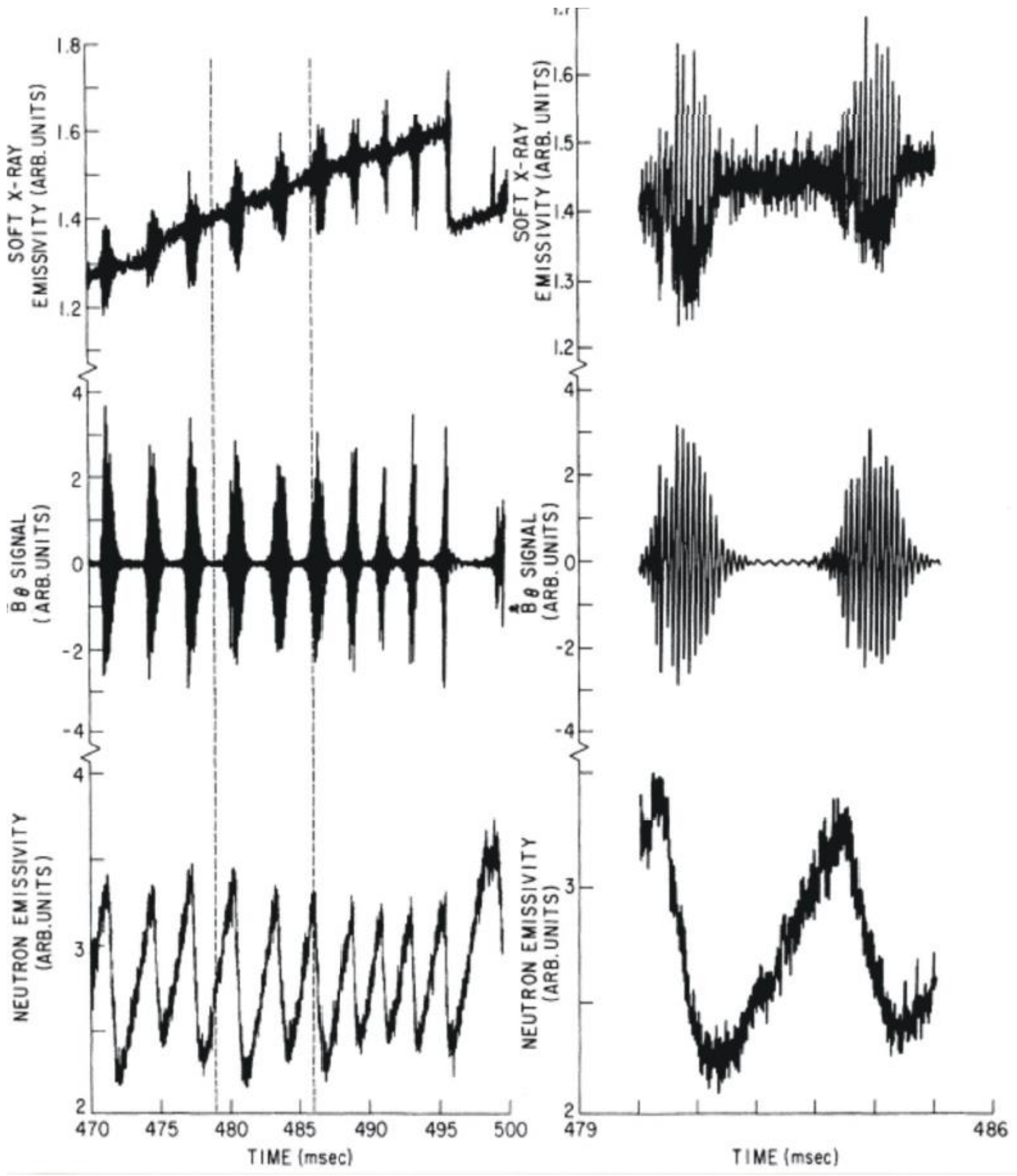
➡ Reduce fusion reactivity

➡ Damage plasma facing components

Fishbone observation

- Experimental observation of fishbones in PDX [McGuire et al. 83] with macroscopic losses of ⊥ injected fast ions

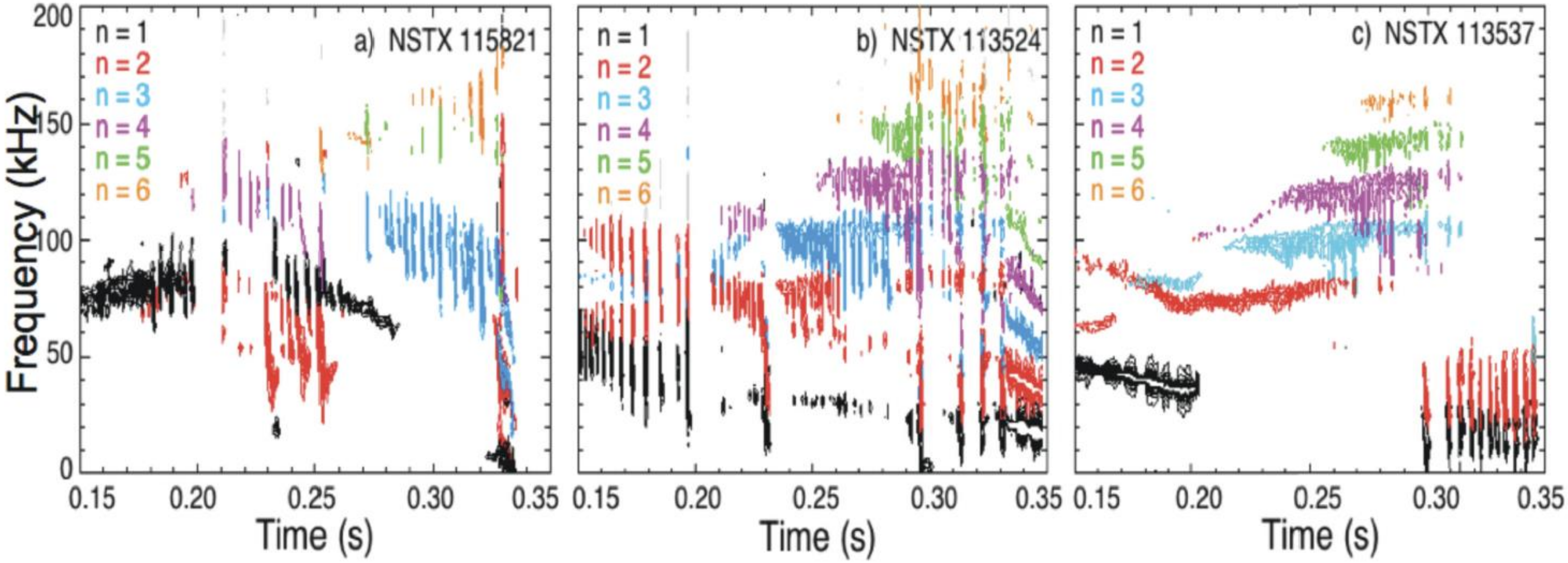




Transport enhanced by EPM

- Toroidal Alfvén Eigenmodes (TAEs) [Cheng, Chen and Chance 1985] and Energetic Particle Modes (EPMs) [Chen 1994] observed in toroidal devices

NSTX $\delta \boldsymbol{B}$ [Courtesy of Fredrickson *et al.* POP **13**, 056109 (2006)]



- On left, bursting, chirping EPM-like modes. ➔ Enhanced transport
- Evolutions to nearly coherent, TAE-like modes on right.

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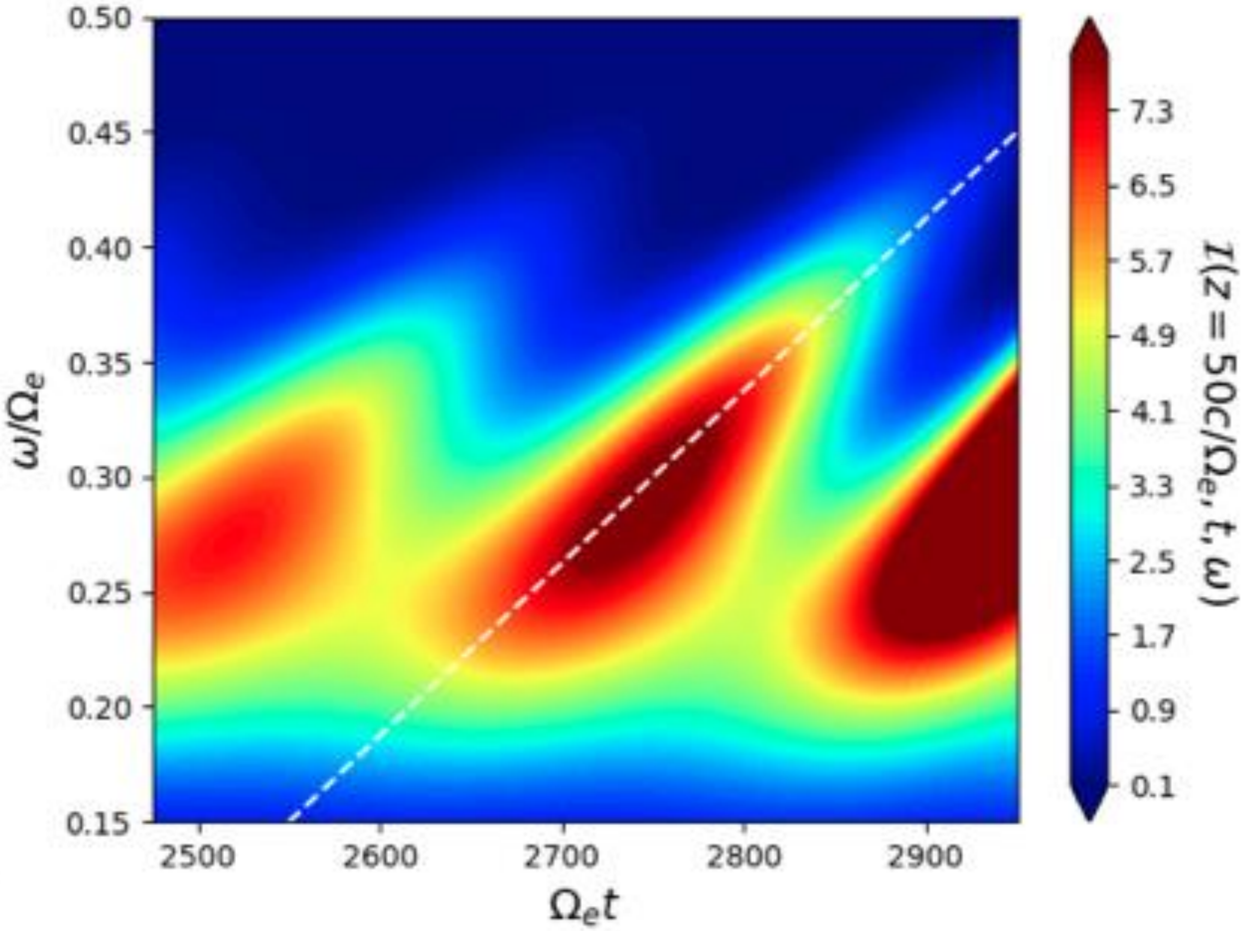
Earth's chorus chirping

- Adopt the general approach to construct the nonlinear growth rate and frequency shift [FZ et al, RMPP/JGR]



(b)

$$\frac{\partial \omega}{\partial t} = \pm \frac{1}{2} \frac{\left\langle\left\langle \omega_{\mathrm{tr}k}^4 \right\rangle\right\rangle^{1/2}}{\left(1 - v_{r\omega}/v_{g\omega}\right)^2}$$

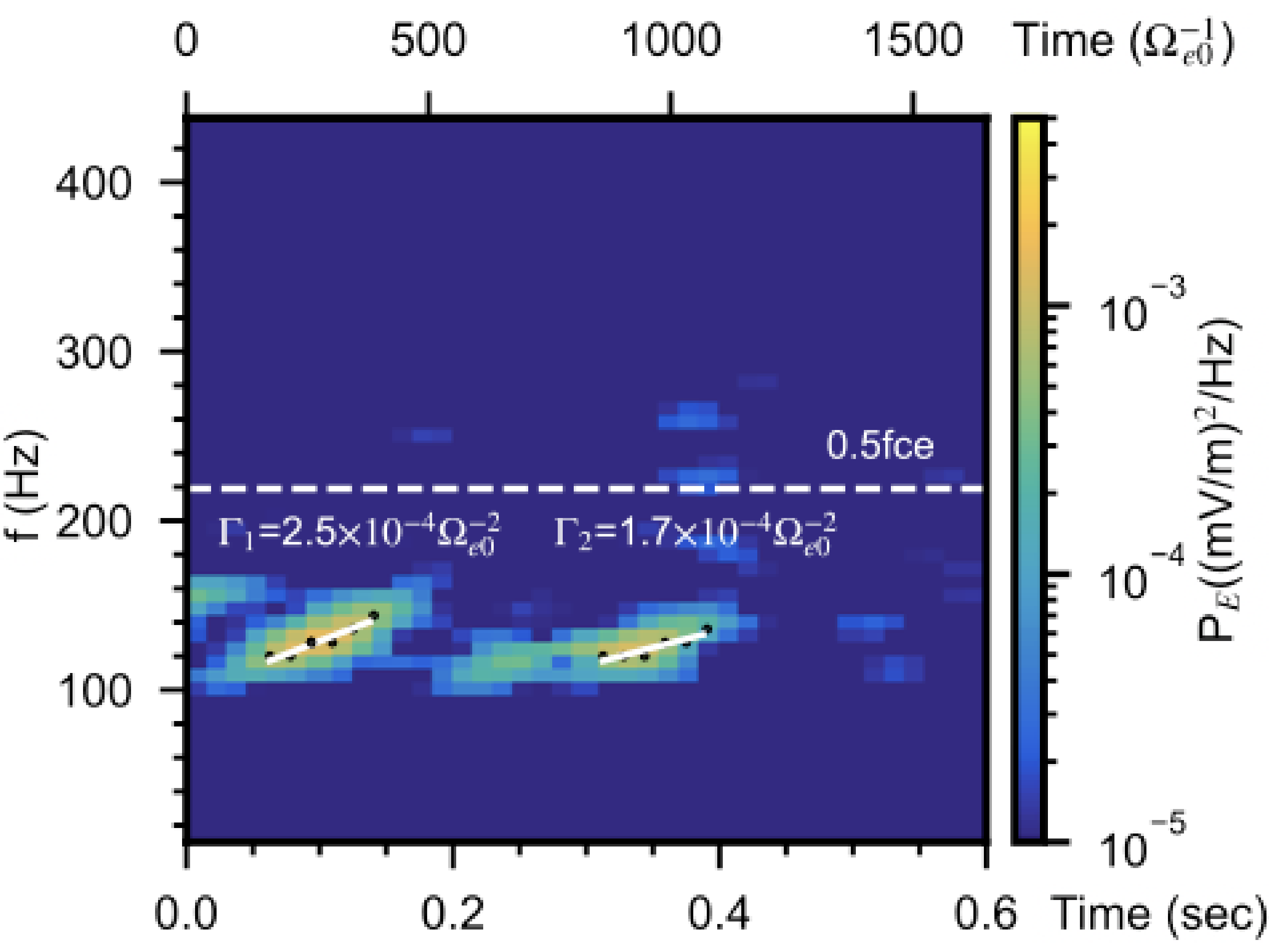
[Vomvoridis et al 1982]

X. Tao, F. Zonca, L. Chen, A "Trap-Release-Amplify" model of Chorus Waves, JGR: Space Physics, 126, e2021JA029585

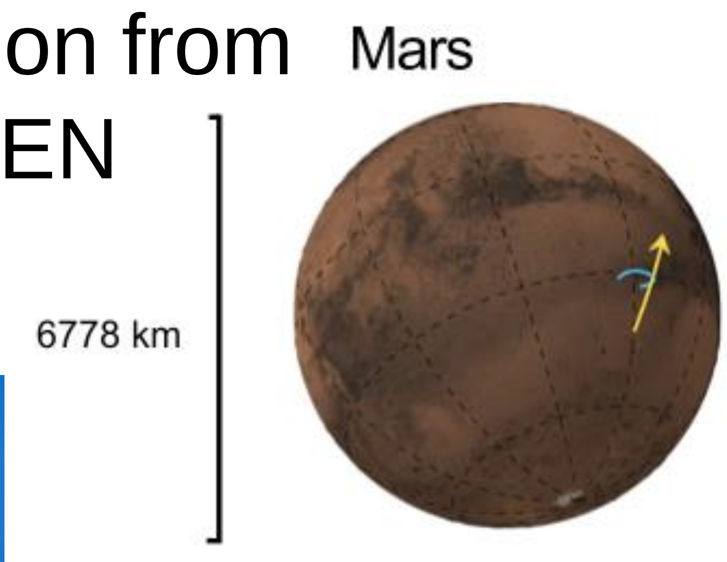
F. Zonca, X. Tao, L. Chen, Nonlinear dynamics and phase space transport by chorus emission RMPP 5, 8; A theoretical framework of chorus wave excitation, JGR 127, e2021JA029760

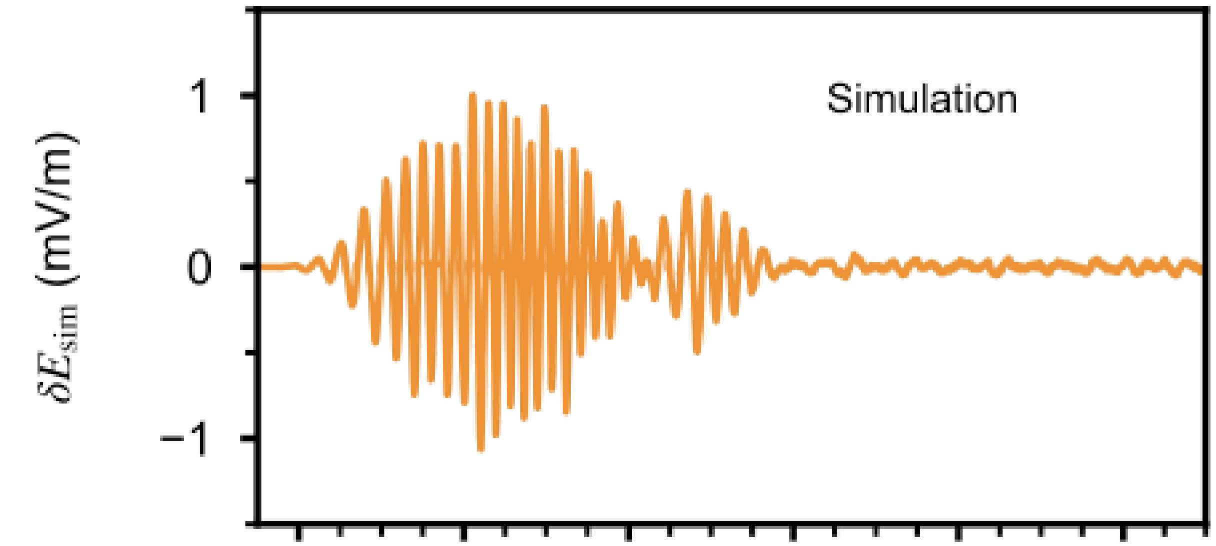
Chorus chirping at Mars

- PIC simulations based on the same theoretical framework predict chorus chirping at MARS [Teng et al, Nat. Comm. 2023]

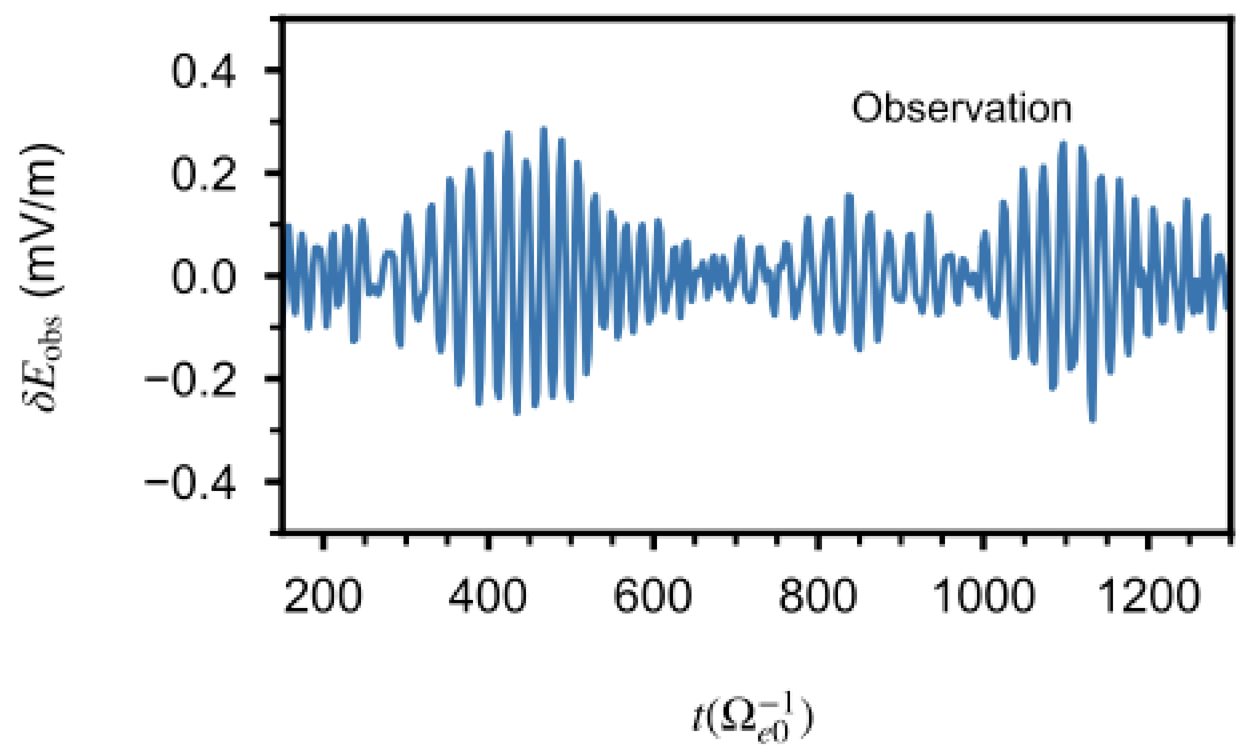


Observation from MAVEN





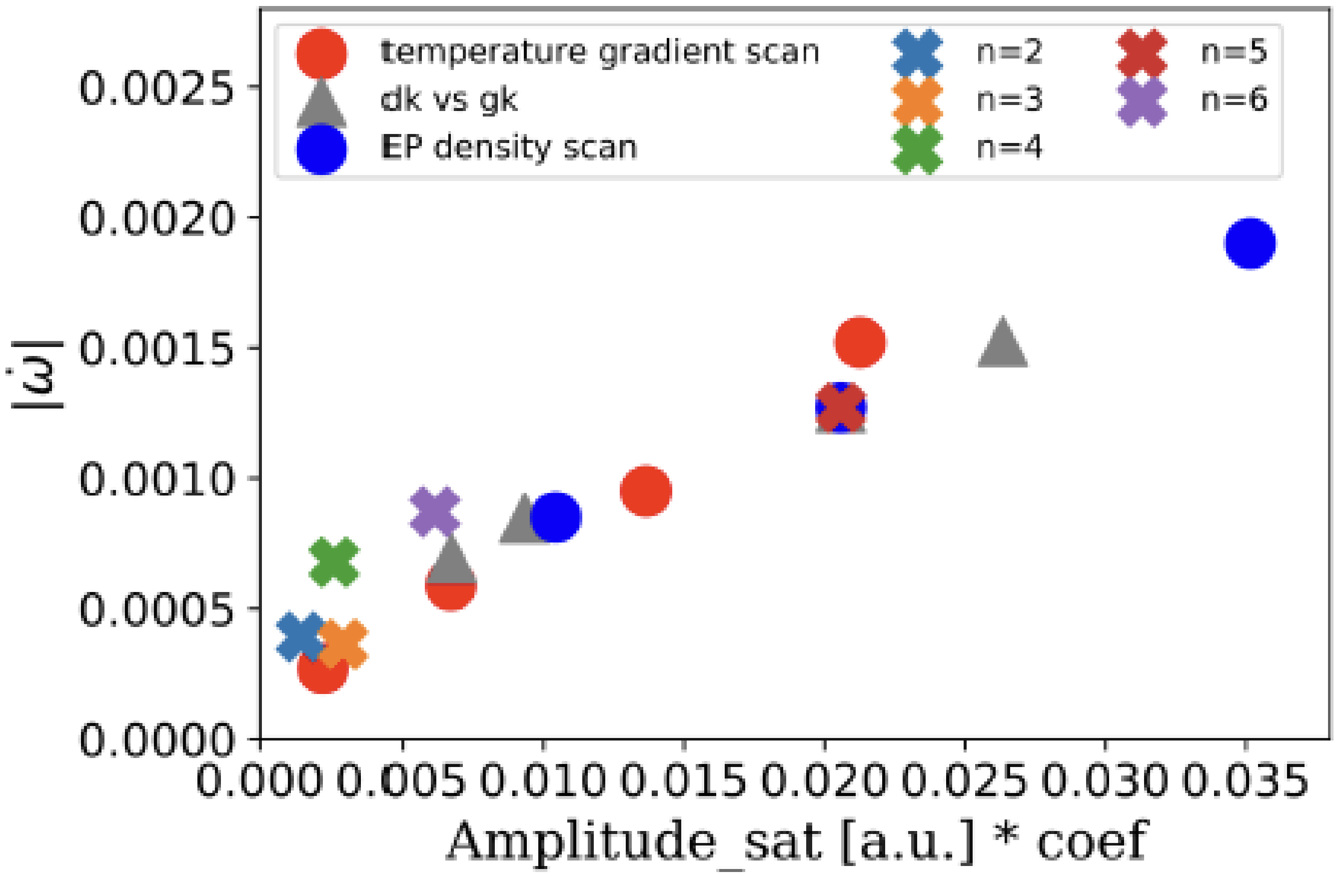
https://doi.org/10.1038/s41467-023-38776-z

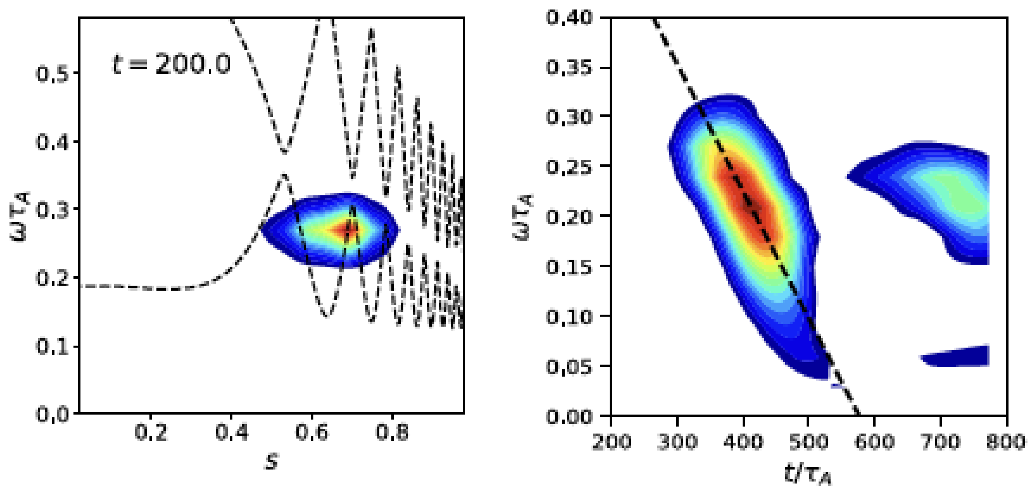


EPM chirping rate

- PIC simulations of EPM in tokamaks show linear scaling of chirping rate with amplitude [X. Wang et al, EPS-DPP invited 2023]

Chirping rate vs. Saturation amplitude





intensity contour plot

$$\dot{\omega} \simeq \delta \dot{X}_{\perp} \cdot \nabla \omega_{res}$$

← theory prediction

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Solution of the Dyson-like equation

- The whistler chorus DSE (as illustration) reads

$$\partial_\tau f_0 = \omega_{tr}^2\omega/(2k^2)\bar{\partial}_{\mathcal{E}}\partial_\tau\left[(\omega-\omega_{res})^2+\partial_\tau^2\right]^{-1}\bar{\partial}_{\mathcal{E}}(\omega_{tr}^2\omega/k^2)f_0$$

- Here, $\partial_\tau = (1 - v_r/v_g)\partial_t$, $\bar{\partial}_{\mathcal{E}} = (k/\omega)\partial_{v_\parallel} + (1 - kv_\parallel/\omega)/v_\perp \partial_{v_\perp}$ and ω_{res} is the resonance frequency. This equation has 1degree of freedom as $B\omega\dot{\mu} = \Omega\dot{\mathcal{E}}$, with $\mathcal{E} = v^2/2$, and a nonlinear invariant exists.

- From existing theory, a wave packet solution of the wave equation can be constructed, satisfying the chorus chirping expression, provided that

$$\mathcal{E}_{res} = \mathcal{E}_{res,0} + \int_0^\tau R\omega_{tr}^2\omega/k^2 d\tau'$$

- The DSE can be solved for weakly varying wave packet amplitude, changing variables from $(\mathcal{E},\tau)$to (x,T) (moving in the wave packet moving frame)

Solution of the Dyson-like equation

$$x = \frac{k^2}{\omega\omega_{tr}}\left(\frac{2}{(2-4R^2)^{1/2}}\right)^{1/2}\left(\mathcal{E}-\mathcal{E}_{res,0}-\int_0^\tau R\omega_{tr}^2\omega/k^2 d\tau'\right)$$

$$T = \omega_{tr}\tau\frac{(2-4R^2)^{1/4}}{2^{1/2}}$$

- The solution is expressed as series of orthonormal Hermite functions $\psi_n(x)$

$$f_0(x,T) = \bar{f}_0 + \sum_{n=0}^{\infty}\{\kappa_n[\varphi_n(x,T)-\varphi_n(x_0,0)] + c.c.\}$$

$$\sum_{n=0}^{\infty}(C_{m,n}\kappa_n + c.c.) = \int_{-\infty}^{\infty}\bar{f}_0\psi_m(x_0)dx_0$$

$$C_{m,n} = \int_{-\infty}^{\infty}\varphi_n(x_0,0)\psi_m(x_0)dx_0$$

$$\varphi_n(x,T) = \int_0^x\frac{dx'}{2R}\left[\psi_n'(x') - \frac{2Rb_n}{(2-4R^2)^{1/2}}\psi_n(x')\right] \times \exp\left[b_n\left(T+\frac{(2-4R^2)^{1/2}x}{2R}-\frac{x'}{(2-4R^2)^{1/2}R}\right)\right]$$

$$b_n = i(n+1/2)^{1/2}(2-4R^2)^{1/2}$$

- Phase space structure rotation is slowed down by chirping
  ➔ PHASE LOCKING
- Wave particle power exchange is maximized for $R \cong 1/2$, consistent with previous analysis of wave packet propagation.

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Universal behavior of frequency chirping

- Use action angle coordinates for general tokamak geometry: q_c and ζ_c such that $\omega_b = \dot{\theta}_c$ and $\overline{\omega}_d = \dot{\zeta}_c$ are, respectively, the bounce/transit and the magnetic drift precession frequency; $\tilde{\Xi}_c$ parameterizing the equilibrium particle motion as $\zeta = \zeta_c + \tilde{\Xi}_c$ at constant actions (μ, J, P_ϕ)

- Use the notion of nonlinear equilibrium in the presence of flows to self-consistently compute wave-particle resonant interaction with EPM/fishbone

$$\dot{P}_\phi = en \left| \overline{e^{-in\zeta - im\bar{\theta}_c + i\bar{Q}} \frac{\omega_{dn}}{\omega} \langle \delta\psi_{ng} \rangle} \right| \sin(\Theta + \beta)$$

$$\dot{E} = e\omega \left| \overline{e^{-in\zeta - im\bar{\theta}_c + i\bar{Q}} \frac{\omega_{dn}}{\omega} \langle \delta\psi_{ng} \rangle} \right| \sin(\Theta + \beta)$$

$$i\bar{Q} = \frac{RB_\phi}{d\psi/dr} \frac{v_\parallel}{\Omega} \frac{\partial}{\partial r} + \tilde{\Xi}_c \frac{\partial}{\partial \zeta} \; ; \qquad \overline{(\ldots)} = \frac{\omega_b}{2\pi} \oint (\ldots) \frac{d\theta}{\dot{\theta}}$$

Universal behavior of frequency chirping

- Near resonance of *(m,n)* poloidal harmonics ← phase locking

$$\Theta = n\zeta_c - m\bar{\theta}_c + \frac{1}{\omega_b}\int^{\theta_c} \Delta_1 d\theta'_c - \int^t \omega dt'$$

$$\dot{\Theta} = \omega_{\mathrm{res}} - \omega = n\bar{\omega}_d + n\bar{q}\sigma\omega_b - m\dot{\bar{\theta}}_c + \Delta_1 - \omega$$

$$\ddot{\Theta} = -\dot{\omega} + \frac{\partial \omega_{\mathrm{res}}}{\partial P_\phi}\dot{P}_\phi + \frac{\partial \omega_{\mathrm{res}}}{\partial E}\dot{E} \simeq 0$$ ← phase locking

- Predicted frequency chirping for EPM/fishbones scales linearly with fluctuation amplitude. Effect of zonal flows is embedded in Δ_1.

$$\Delta_1 = -i\overline{\left[e^{i\bar{Q}}\left(\delta\dot{\mathbf{X}}_z \cdot \nabla + \delta\dot{\mathcal{E}}_z \partial_{\mathcal{E}}\right)\right]},$$

$$\dot{\omega} \simeq \omega_{\mathrm{tr}}^2/2 = \frac{1}{2}\left|\left(en\frac{\partial \omega_{\mathrm{res}}}{\partial P_\phi} + e\omega\frac{\partial \omega_{\mathrm{res}}}{\partial E}\right)\overline{e^{-in\zeta - im\bar{\theta}_c + i\bar{Q}}\frac{\omega_{dn}}{\omega}\langle\delta\psi_{ng}\rangle}\right.$$

Concluding remarks and discussion

- Explicit expression of frequency chirping is derived, showing it is a consequence of maximized wave-particle power transfer and phase locking.

- Explicit expression of frequency chirping illuminates the important role of zonal field structures.

- Explicit expression of chirping rate also shows linear scaling with fluctuation amplitude, demonstrating the **universal behavior of frequency chirping in space and laboratory plas**mas, consistent with the Vomvoridis expression.

- Detailed quantitative numerical verifications of these predictions are in progress.

General bibliography: https://doi.org/10.1103/RevModPhys.88.015008
https://doi.org/10.1007/s41614-021-00057-x
https://doi.org/10.1038/s41467-023-38776-z