Indirect nonlinear interaction between TAE and ITG mediated by zonal structures

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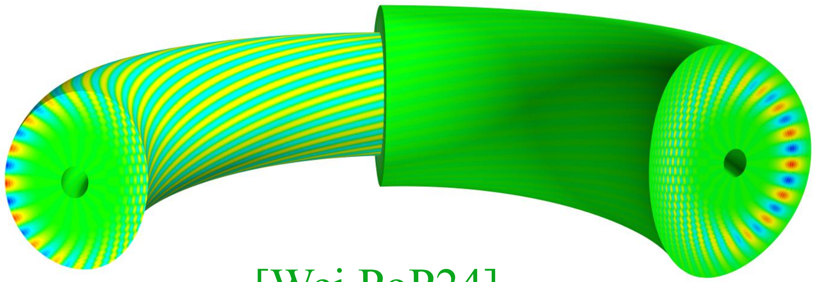
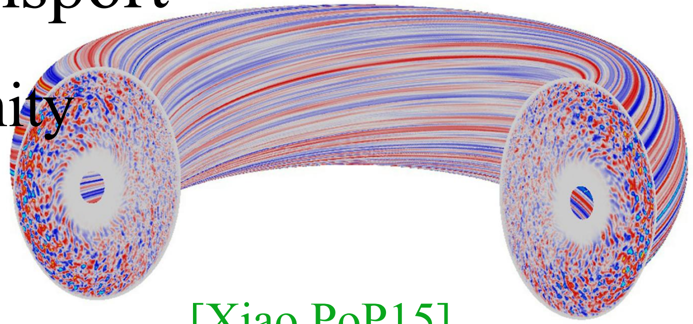
Two categories of low frequency fluctuations

- Shear Alfvén wave (SAW) instabilities: crucial in energetic particle dynamics
  - typically meso-scale ($\sim\rho_h$) electromagnetic oscillation
  - excite as various Alfven eigenmodes due to equilibrium magnetic geometry
  - driven unstable by energetic particles
  - lead to energetic particle transport loss



[Wei PoP24]

- Drift wave turbulence (DW): crucial in bulk plasma transport
  - micro-scale ($\sim\rho_i$) turbulence excited by bulk plasma nonuniformity
  - cause negligible direct transport of energetic particles
  - can be regulated by EPs due to, e.g., dilution

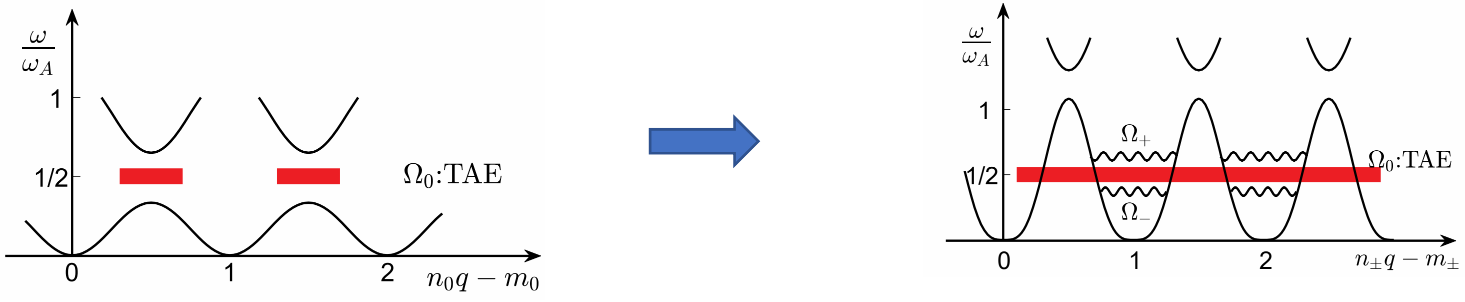


[Xiao PoP15]

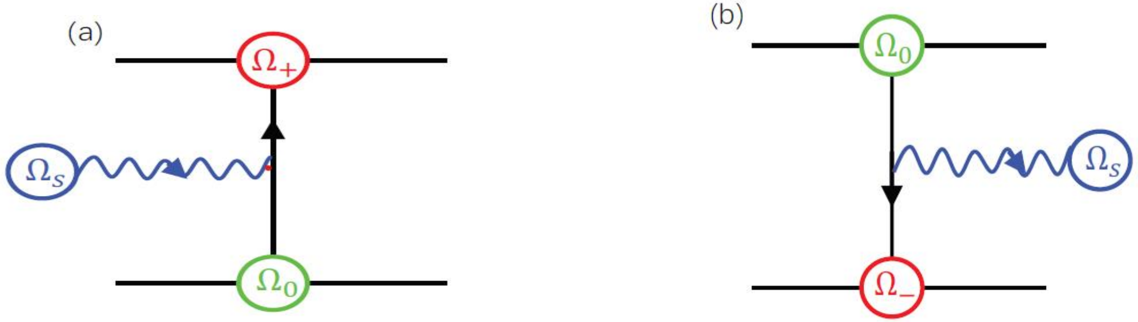
- Direct/indirect interaction between AE and DWs used to interpret improved bulk plasma confinement in the existence of EPs

We show in previous works

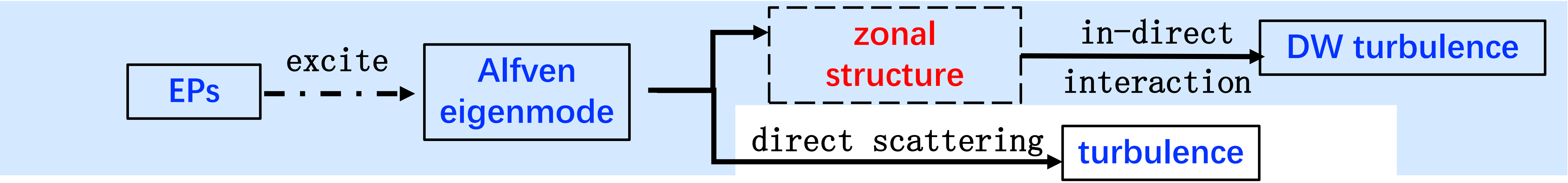
- direct scattering of ambient DWs significantly regulate even suppress TAE [Chen NF2022]



- direct scattering of TAE have small effects on DW stability: scattering to Ω_+ and Ω_- leads to stimulated absorption (damping) and spontaneous emission (growth) of DW [Chen NF2023]

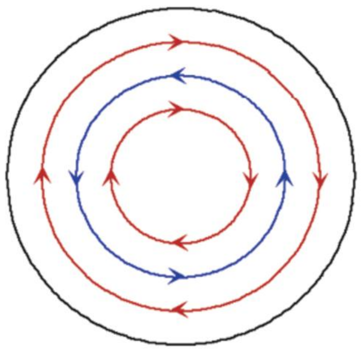


⇒ Importance of in-direct coupling mediated by zonal structure?

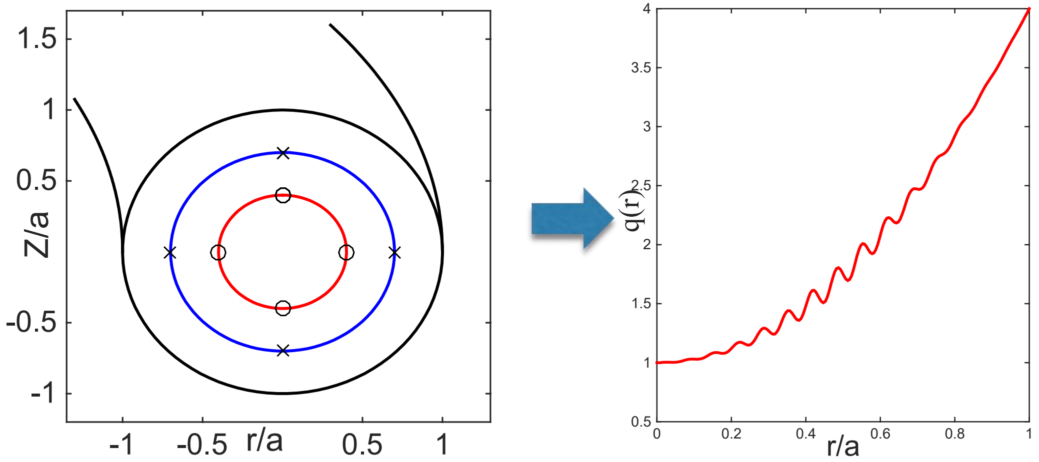


In-direct modulation of ITG by TAE mediated by zonal structures

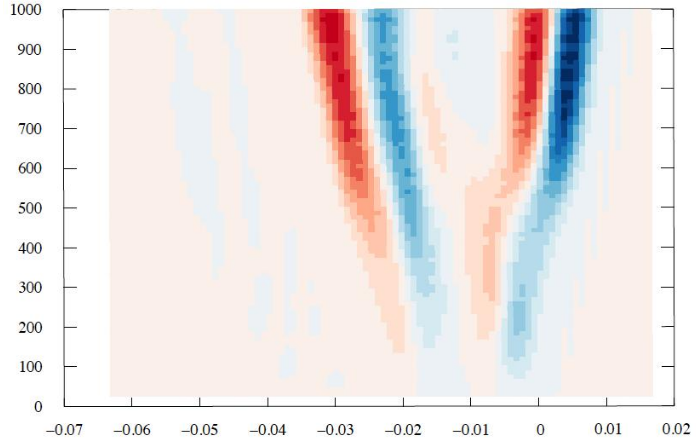
- Zonal structures: toroidally/poloidally symmetric radial corrugations
  - Linearly stable to expansion free energy
  - Nonlinear excitation by DWs/DAWs ⇒ stabilize DW/DAW
  - zonal electromagnetic fields (zonal flow + zonal current), phase space zonal structures



zonal flow



zonal current



phase space zonal structure
[Lauber 22]

- Effects of TAE driven zonal structure on ITG stability? Local theory + beat-driven ZS for now
- ZS generation due to thermal plasma contribution

Gyrokinetic theoretical model

- Gyrokinetic theory: systematic removal of fast gyro motion ⇒ powerful in studying low frequency dynamics
- $\delta f_j = -(e/T)_j \delta\phi_k F_{Mj} + \exp(-\rho\cdot\nabla)\delta H_j$, δH from NL gyrokinetic equation [Frieman&Chen PoF82]

$$\left(\partial_t + v_\parallel \mathbf{b}\cdot\nabla + \mathbf{v}_d\cdot\nabla\right)\delta H_k = \frac{q}{T}F_M(\partial_t + i\omega_*)J_k\delta L_k - \sum_{\mathbf{k}=\mathbf{k}'+\mathbf{k}''} \Lambda^{k'}_{k''} J_{k'}\delta L_{k'}\delta H_{k''}$$

- Filed variables $\delta\phi$ and $\delta A_\parallel$ ($\Rightarrow \delta B_\perp$) used: $\beta \ll 1$
- Field equations derived from quasineutrality condition

$$\frac{n_0 e^2}{T_i}\left(1+\frac{T_i}{T_e}\right)\delta\phi_k = \sum_j \langle q J_k \delta H_j\rangle_v$$

Nonlinear gyrokinetic vorticity equation

$$\frac{c^2}{4\pi\omega^2}B\frac{\partial}{\partial l}\frac{k_\perp^2}{B}\frac{\partial}{\partial l}\delta\psi_k + \frac{e^2}{T_i}\langle(1-J_k^2)F_0\rangle\delta\phi_k - \sum_s\left\langle\frac{q}{\omega}J_k\omega_d\delta H_s\right\rangle$$

$$= -i\frac{c}{B\omega}\sum\hat{\mathbf{b}}\cdot\mathbf{k}''\times\mathbf{k}'\left[\langle(J_kJ_{k'}-J_{k''})\delta L_{k'}\delta H_{k''}\rangle + \frac{k_\perp''^2c^2}{4\pi}\frac{1}{\omega_{k'}\omega_{k''}}\partial_l\delta\psi_{k'}\partial_l\delta\psi_{k''}\right]$$

- derived from GKE + Q.N. + parallel Ampére's law
- LHS: field line bending, inertia, ballooning-interchange
- RHS: gyrokinetic Reynolds stress (RS, $\delta v\cdot\nabla\delta v$), Maxwell stress (MX, $\delta j\times\delta B/c$).

RS and MX dominate NL W-W coupling in the kinetic regime with $k_\perp\rho_i\sim O(1)$

⇒ powerful and mandatory in studying NL W-W couplings [Qiu RMPP23]

Indirect interaction: work flow

- beta-drive ZS by TAE [Chen WLIS2023, NF2024 accepted]

+

- finite amplitude ZFS+PSZS on ITG stability [Chen PoP2021]

↓

- in-direct interaction [Fang NF2024 submitted]

Zonal structure beat-driven by TAE

- ZS ($\delta\phi_Z$, $\delta A_{\parallel Z}$, δH_Z^{NL}) beat driven by TAE Ω_0

$$(\partial_t + v_\parallel \partial_l + i\omega_D)\delta H_k = -i\frac{q}{T}\omega_k F_M J_k \delta L_k - \sum_{\mathbf{k}=\mathbf{k}'+\mathbf{k}''} \Lambda_{k''}^{k'} J_{k'} \delta L_{k'} \delta H_{k''}$$

- Linear particle response to ZS

$$\overline{\delta H_{Z,e}^L} \cong -\frac{e}{T_e} F_{Me}\left(\delta\phi - \frac{v_\parallel}{c}\delta A\right)_Z, \quad \overline{\delta H_{Z,i}^L} = \frac{e}{T_i} F_{Mi} J_Z \left|\overline{e^{-i\lambda_Z}}\right|^2 \left(\delta\phi - \frac{v_\parallel}{c}\delta A\right)_Z$$

$e^{i\lambda_Z}$: operator for drift/banana orbit transformation

- Nonlinear particle response to ZS (δH_Z^{NL}, PSZS)

$$\overline{\delta H_{Z,e}^{NL}} \cong -\frac{c}{B_0} k_{\theta 0} \frac{e}{T_e} F_{Me}\left(\frac{\omega_{*e,0}^t}{\omega_0^2} - \frac{k_{\parallel 0} v_\parallel}{\omega_0^2}\right), \quad \overline{\delta H_{Z,i}^L} = \frac{c}{B_0}\left|\overline{e^{-i\lambda_Z}}\right|^2 k_{\theta 0} \frac{e}{T_i} F_{Mi} J_0^2 \frac{\omega_{*i,0}^t}{\omega_0^2} \partial_r |\delta\phi_0|^2$$

ZS beat-driven by toroidal Alfven eigenmode

- Zonal flow ($\delta\phi_Z$) derived from quasi-neutrality condition [Chen NF2024]

$$\left.\begin{array}{c} \overline{\delta H^L_{Z,e}},\ \overline{\delta H^L_{Z,i}},\ \overline{\delta H^{NL}_{Z,e}},\ \overline{\delta H^{NL}_{Z,i}} \\ + \\ \frac{Ne^2}{T_i}\left(1+\frac{T_i}{T_e}\right)\delta\phi_k = \sum_s \langle e_s J_k \delta H_k \rangle \end{array}\right\} \delta\phi_Z = -\frac{c}{B_0} k_{\theta 0} \frac{\omega_{*i,0}}{\omega_0^2}(1+\eta_i)\partial_r|\delta\phi_0|^2$$

- Zonal current ($\delta A_{\parallel Z}$) from parallel Ampere's law [Chen NF2024]

$$\left.\begin{array}{c} \delta\phi_Z,\ \overline{\delta H^L_{Z,e}}, \\ \overline{\delta H^{NL}_{Z,e}} \quad + \\ -\frac{c}{4\pi}\nabla_\perp^2 \delta A_\parallel = \delta J_\parallel = \langle -e v_\parallel \delta f_e \rangle \end{array}\right\} \delta A_{\parallel Z} \cong \frac{c^2}{B_0} k_{\theta 0} \frac{k_{\parallel 0}}{\omega_0^2}\partial_r|\delta\phi_0|^2$$

- ($\delta\phi_Z$, $\delta A_{\parallel Z}$, δH_Z^{NL}) will be used in deriving NL particle response to ITG

Effects of beat-driven ZS on ITG stability

$$(\partial_t + v_\parallel \partial_l + i\omega_D)\delta H_k = -i\frac{q}{T}(\omega - \omega_*^t)F_M J_k \delta L_k - \sum_{\mathbf{k}=\mathbf{k}'+\mathbf{k}''} \Lambda_{k''}^{k'} J_{k'} \delta L_{k'} \delta H_{k''}$$

- Linear particle response to ITG ⇒ linear ITG D.R. (adiabatic electron)

$$\boldsymbol{\delta H_{I,i}^L} = \frac{e}{T_i}\left(1 - \frac{\omega_{*i}^t}{\omega}\right)\left(1 + \frac{k_\parallel v_\parallel}{\omega} + \frac{k_\parallel^2 v_\parallel^2}{\omega^2} + \frac{\omega_{Di}}{\omega}\right) F_{Mi} J_0 \delta\phi_I$$

- Nonlinear particle response to ITG

$$(\partial_t + v_\parallel \partial_l + i\omega_d)\,\delta H_{Ii}^{NL} = \Lambda\left[J_I \delta\phi_I \left(\delta H_{Zi}^L + \delta H_{Zi}^{NL}\right) - J_Z\left(\delta\phi_Z - v_\parallel \delta A_{\parallel Z}\right)\delta H_{Ii}^L\right]$$

PSZS zonal flow zonal current

$$\Rightarrow \boldsymbol{\delta H_{I,i}^{NL}} = \frac{e}{T_i} F_{Mi}\left[\left(\left|\overline{e^{-i\lambda_Z}}\right|^2 - 1 + \frac{\omega_{*i}^t}{\omega}\right)\frac{\omega_{*pi}}{\omega} - \frac{\omega_{*i}^t}{\omega}\left|\overline{e^{-i\lambda_Z}}\right|^2\right]\delta\hat{\phi}_0^{\,2}\,\delta\phi_I \text{ with } \delta\hat{\phi}_0^{\,2} \equiv \left|\frac{ck_{\theta 0}}{B_0\omega_0}\right|^2 \partial_r^2 |\delta\phi_0|^2$$

- ITG D.R. in the WKB limit

$$\left[\frac{\omega}{\tau(\omega - \omega_{*pi})} + \frac{\omega_{*i}}{(\omega - \omega_{*pi})} + b_\perp - \frac{k_\parallel^2 v_{ti}^2}{2\omega^2} - \frac{2\omega_{di} C}{\omega} + \frac{\omega_{*i}}{\omega}\,\delta\hat{\phi}_0^{\,2}\right]\delta\phi_I = 0$$

ITG dispersion function in ballooning space

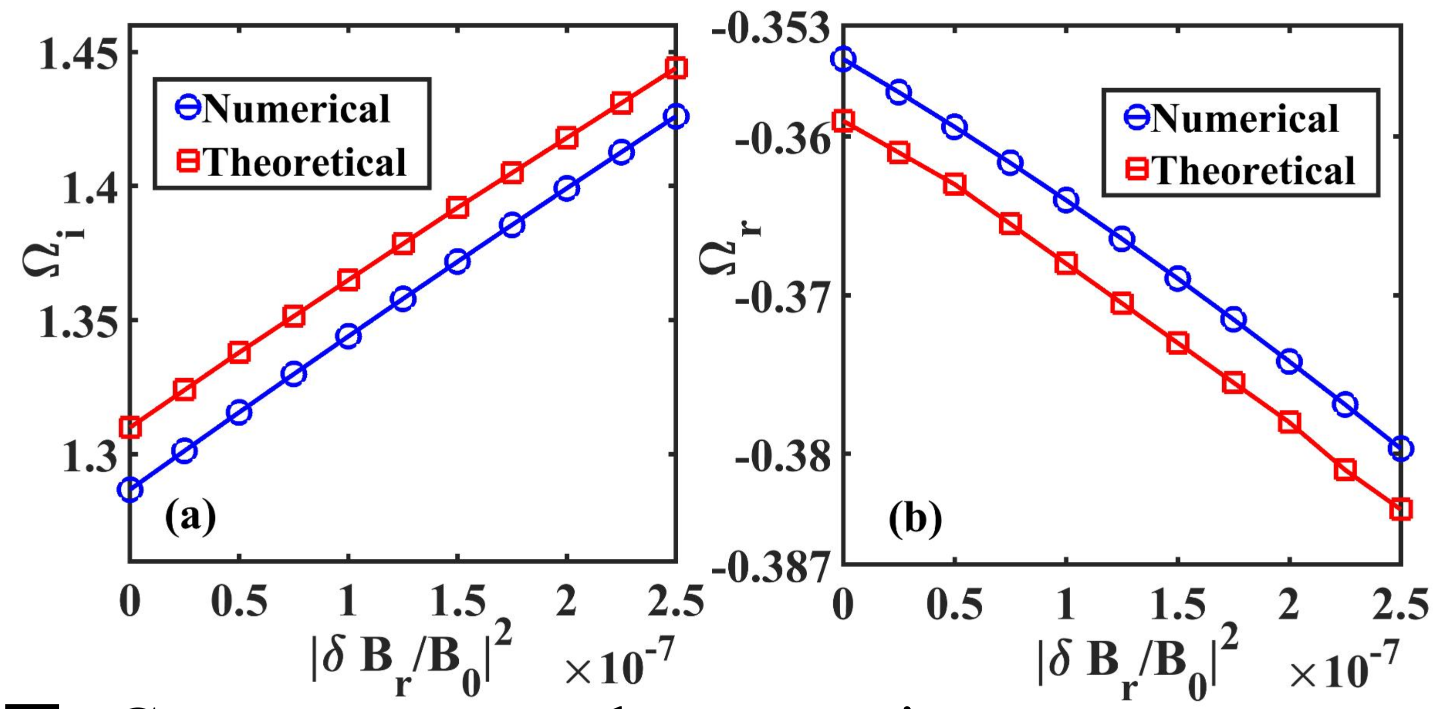
$$\frac{d^2\Phi(\eta)}{d\eta^2} + q^2\Omega^2 b\left[\frac{\tau\Omega}{1+\tau\Omega\epsilon_{pi}^{1/2}} - \frac{\tau}{(1+\tau\Omega\epsilon_{pi}^{1/2})(1+\eta_i)\epsilon_{pi}^{1/2}} + b(1+\hat{s}^2\eta^2) + \frac{2}{\Omega}(cos\eta + \hat{s}\eta sin\eta) - \frac{1}{\Omega\epsilon_{pi}(1+\eta_i)}\delta\hat{\phi}_0{}^2\right]\Phi(\eta) = 0$$

- Solved in various limits for effects of ZS on ITG stability
  - Short-wavelength limit: uniform/nonuniform “ZS” [Guzdar PoF83]
  - long-wavelength limit [Chen PoFB91]

Short-wavelength limit: weakly destabilizing

$$\frac{d^2\Phi(\eta)}{d\eta^2} + q^2\Omega^2 b\left[\frac{\tau\Omega}{1+\tau\Omega\epsilon_{pi}^{1/2}} - \frac{\tau}{(1+\tau\Omega\epsilon_{pi}^{1/2})(1+\eta_i)\epsilon_{pi}^{1/2}} + b(1+\hat{s}^2\eta^2) + \frac{2}{\Omega}(cos\eta + \hat{s}\eta sin\eta) - \frac{1}{\Omega\epsilon_{pi}(1+\eta_i)}\delta\hat{\phi}_0^{\,2}\right]\Phi(\eta) = 0$$

- Short-wavelength limit (strong coupling): mode localized round $\eta \ll 1$
- "Uniform"-ZS: TAE scale being much larger than ITG: relevant to reactor



- good agreement between analytical and numerical results
- ITG growth rate increase with TAE (ZS) amplitude
- Small correction within relevant parameter regime
- artificially change the sign/amplitude of the NL term
- $\frac{e\delta\phi_Z}{T} \leq 10^{-4}$ for typical TAE

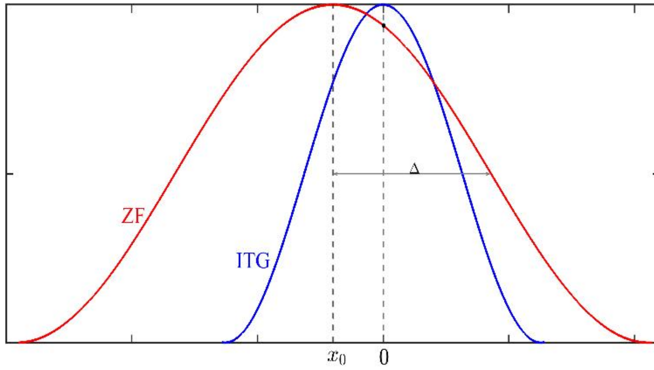
- Contrary to usual expectations

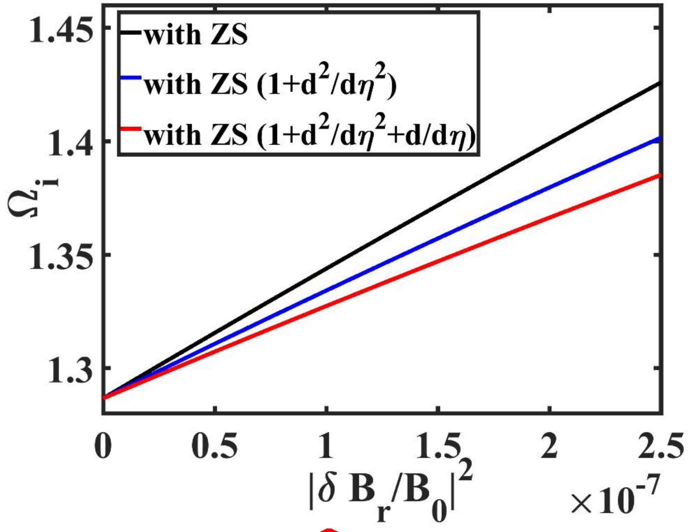
Short-wavelength "nonuniform-ZS" limit: weakly destabilizing

- "Nonuniform"-ZS: TAE mode variation not negligible for experiments with $T_h/T_i \sim O(10)$

$$\left[\frac{\omega}{\tau(\omega-\omega_{*pi})}+\frac{\omega_{*i}}{(\omega-\omega_{*pi})}+b_\perp-\frac{k_\parallel^2 v_{ti}^2}{2\omega^2}-\frac{2\omega_{di}C}{\omega}+\frac{\omega_{*i}}{\omega}\delta\hat{\phi}_0{}^2 e^{-\frac{(x-x_0)^2}{\Delta^2}}\right]\delta\phi_I=0$$

$$\frac{d^2\Phi(\eta)}{d\eta^2}+q^2\Omega^2 b\left[\frac{\tau\Omega}{1+\tau\Omega\epsilon_{pi}^{1/2}}-\frac{\tau}{(1+\tau\Omega\epsilon_{pi}^{1/2})(1+\eta_i)\epsilon_{pi}^{1/2}}+b(1+\hat{s}^2\eta^2)\right.$$
$$\left.+\frac{2}{\Omega}(cos\eta+\hat{s}\eta sin\eta)-\frac{1}{\Omega\epsilon_{pi}(1+\eta_i)}\delta\hat{\phi}_0{}^2\left(1+i\frac{2x_0}{\Delta^2}\frac{d}{d\eta}+\frac{\boldsymbol{\Delta^2-2x_0^2}}{\boldsymbol{\Delta^4}}\frac{\boldsymbol{d^2}}{\boldsymbol{d\eta^2}}\right)e^{-\frac{x_0^2}{\Delta^2}}\right]\Phi(\eta)=0$$





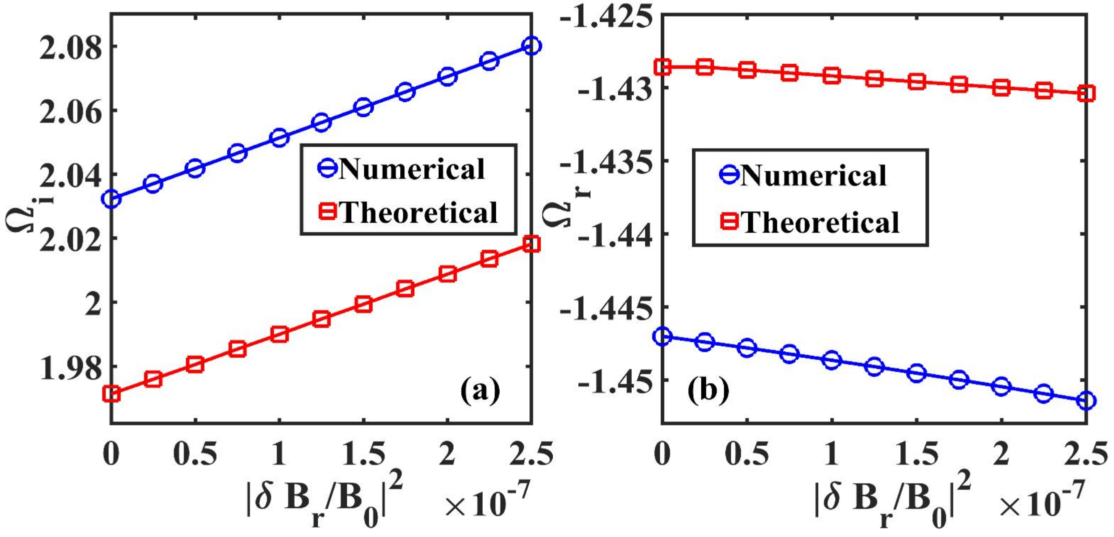
- inclusion of "nonuniform"-ZS: even weaker destabilization
- nonuniform ZS + mis-alignment
- qualitative picture unchanged

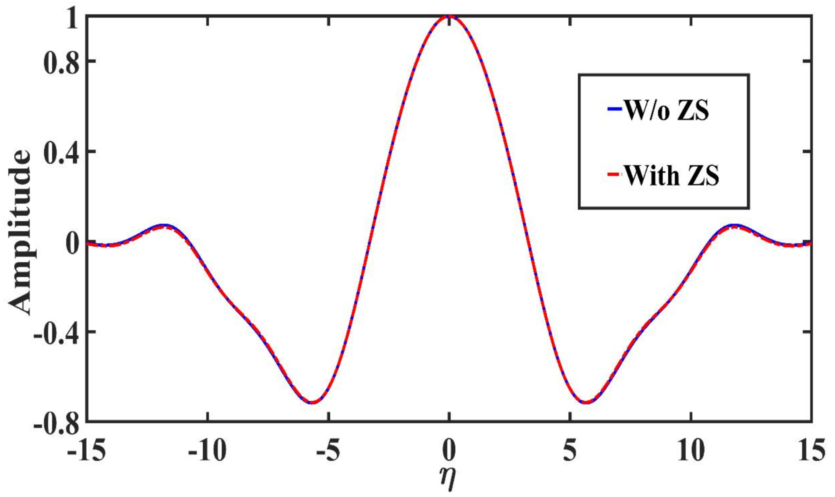
- sign of $\delta\hat{\phi}_0^2$ could changing with x ⇒ weak stabilization of ITG by TAE beat-driven ZS

Long-wavelength limit: weakly destabilizing

$$\frac{d^2\Phi(\eta)}{d\eta^2} + q^2\Omega^2 b\left[\frac{\tau\Omega}{1+\tau\Omega\epsilon_{pi}^{1/2}} - \frac{\tau}{(1+\tau\Omega\epsilon_{pi}^{1/2})(1+\eta_i)\epsilon_{pi}^{1/2}} + b(1+\hat{s}^2\eta^2) + \frac{2}{\Omega}(cos\eta + \hat{s}\eta sin\eta) - \frac{1}{\Omega\epsilon_{pi}(1+\eta_i)}\delta\hat{\phi}_0{}^2\right]\Phi(\eta) = 0$$

- long-wavelength limit (moderate coupling)
- reduce into Mathieu's equation: $\Phi = A(\sigma\eta)cos\eta/2 + B(\sigma\eta)sin\eta/2$





- weakly destabilizing effect of TAE beat-driven ZS on ITG

Summary and Discussions

- Indirect regulation of ITG by TAE formulated to understand enhanced thermal plasma confinement in the presence of EPs: beat-driven ZS + local theory
- Weak destabilization of ITG by TAE beat-driven ZS: contrary to usual expectations

  [Fang NF2024 submitted]
- Both direct and in-direct scattering by TAE have negligible effects on DW stability

Assumptions in the indirect scattering case:

- local ITG stability
- beat-driven zonal structure

⇒Radial envelope modulation? Spontaneously excited ZS?