

# Global simulation of drift-Alfvénic instability based on Landau fluid-gyrokinetic hybrid model in general geometry

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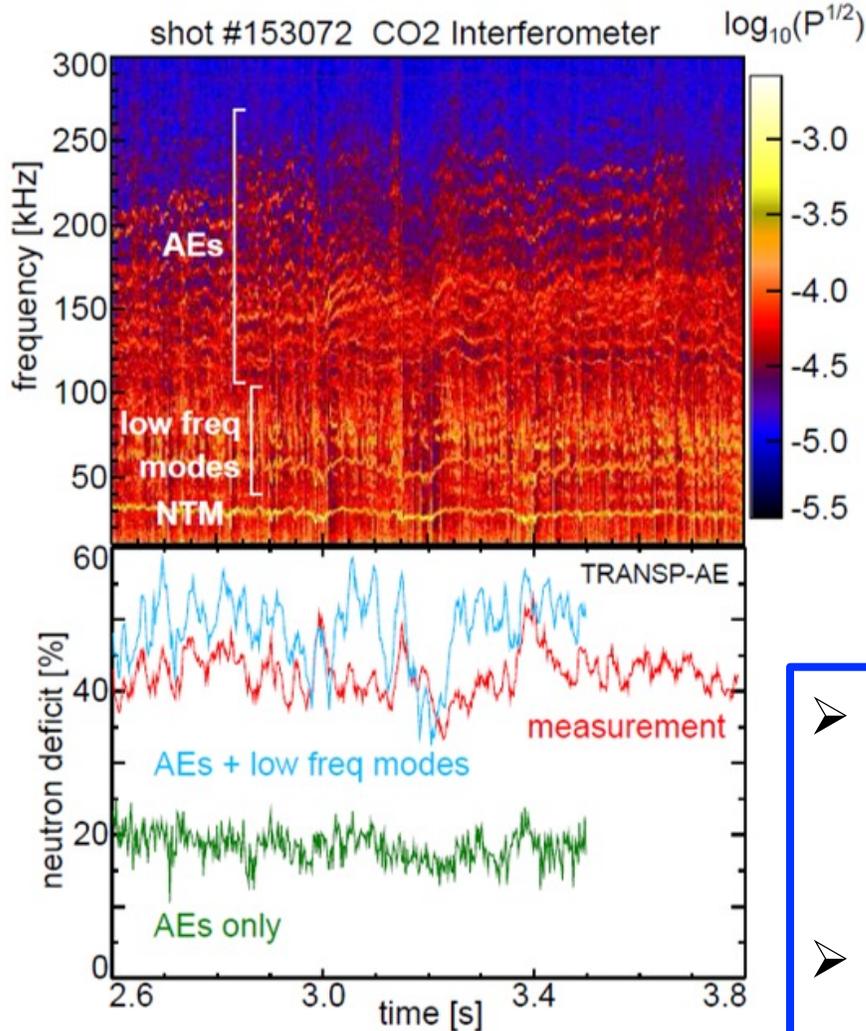
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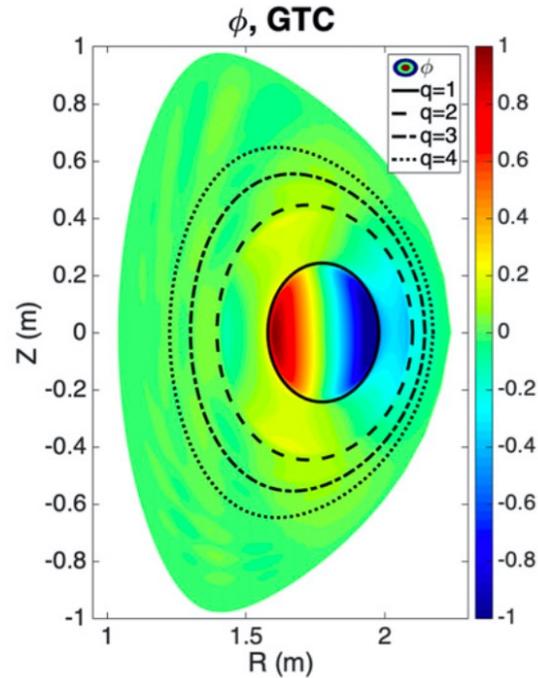
# Outlines

- Background
- Discretized eigenmode
- Continuous spectrum
- Resonance condition in phase space
- Summary

# Background

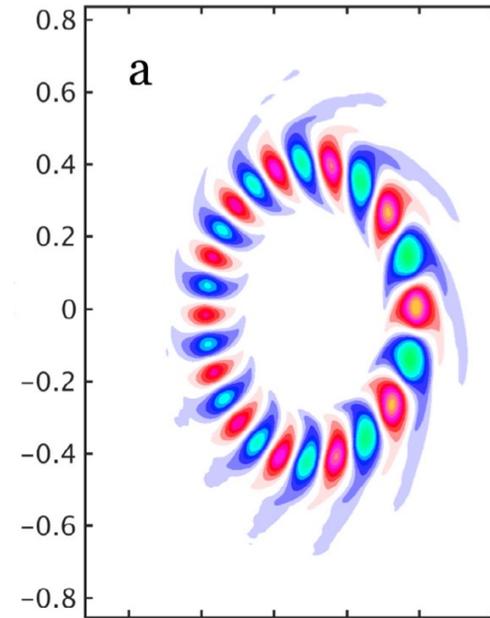


G. Brochard 2022 NF



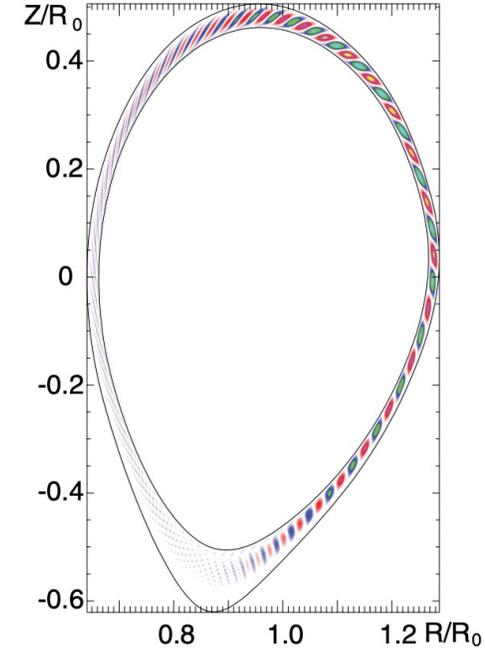
Kink ( $n=1$ )

S. Taimourzadeh 2019 NF



RSAE ( $n=4$ )

I. Holod 2015 NF



KBM ( $n=20$ )

- Drift-Alfvenic instabilities widely exist in burning plasmas: MHD (macro-scale), EP-driven Alfvén eigenmode (meso-scale), drift-wave instability (micro-scale).
- Initial value simulation for nonlinear problem (e.g. GTC, M3D-K).
- Eigenvalue simulation for linear stability analysis (e.g. LIGKA, NOVA-K).

# Main plasma instabilities

## Drift-wave instability

- ES: ITG, CTEM
- EM: IBM/KBM

## MHD mode

- EM: kink, tearing

## Alfven eigenmode

- EM: TAE, RSAE, KBAE
- EM/ES hybrid: BAAE

Reactive (fluid-type)  
Dissipative (kinetic-type)

Modify/Drive

EP kinetic response  
(Wave-particle resonance)

### Goals of MAS eigenvalue code:

- Cross-scale drift-Alfvenic instabilities
- MHD/kinetic continuous spectrum
- Resonance condition in phase space
- Capability of realistic geometry

# Outlines

- Background
- Discretized eigenmode
  - Various bulk plasma instabilities
  - Energetic electron excitation of BAE
  - Energetic ion responses to arbitrary wavelength fluctuations
- Continuous spectrum
- Resonance condition in phase space
- Summary

# Landau fluid model for bulk plasmas

Bao et al, Nucl. Fusion **63** 076021 (2023)

➤ Vorticity equation

$$\frac{\partial}{\partial t} \frac{c}{V_A^2} \nabla_{\perp}^2 \delta\phi + \underbrace{\frac{\partial}{\partial t} (0.75\rho_i^2 \nabla_{\perp}^2)}_{\{Ion-FLR\}} \frac{c}{V_A^2} \nabla_{\perp}^2 \delta\phi + \underbrace{i\omega_{p,i}^* \frac{c}{V_A^2} \nabla_{\perp}^2 \delta\phi}_{\{Drift\}} + \mathbf{B}_0 \cdot \nabla \left( \frac{1}{B_0} \nabla_{\perp}^2 \delta A_{\parallel} \right) - \frac{4\pi}{c} \delta \mathbf{B} \cdot \nabla \left( \frac{J_{\parallel 0}}{B_0} \right) - 8\pi (\nabla \delta P_i + \nabla \delta P_e) \cdot \frac{\mathbf{b}_0 \times \boldsymbol{\kappa}}{B_0} = 0$$

➤ Parallel Ohm's law

$$\frac{\partial \delta A_{\parallel}}{\partial t} = -c \mathbf{b}_0 \cdot \nabla \delta\phi + \underbrace{\frac{c T_{e0}}{en_{e0}} \mathbf{b}_0 \cdot \nabla \delta n_e}_{\{Drift\}} + \underbrace{\frac{c T_{e0}}{en_{e0} B_0} \delta \mathbf{B} \cdot \nabla n_{e0}}_{\{Electron-Landau\}} + \underbrace{\frac{cm_e}{e} \sqrt{\frac{\pi}{2}} v_{the} |k_{\parallel}| \delta u_{\parallel e}}_{\{Resistivity\}} + \frac{c^2}{4\pi} \eta_{\parallel} \nabla_{\perp}^2 \delta A_{\parallel}$$

➤ Thermal ion pressure Eq.

$$\frac{\partial \delta P_i}{\partial t} + \frac{c \mathbf{b}_0 \times \nabla \delta\phi}{B_0} \cdot \nabla P_{i0} + 2\Gamma_{i\perp} P_{i0} c \nabla \delta\phi \cdot \frac{\mathbf{b}_0 \times \boldsymbol{\kappa}}{B_0} + \Gamma_{i\parallel} P_{i0} \mathbf{B}_0 \cdot \nabla \left( \frac{\delta u_{\parallel i}}{B_0} \right) - \underbrace{i\Gamma_{i\perp} \omega_{p,i}^* Z_i n_{i0} \rho_i^2 \nabla_{\perp}^2 \delta\phi}_{\{Ion-FLR\}} + \underbrace{2\Gamma_{i\perp} P_{i0} \frac{c}{Z_i} \nabla \delta T_i \cdot \frac{\mathbf{b}_0 \times \boldsymbol{\kappa}}{B_0} + 2\Gamma_{i\perp} T_{i0} \frac{c}{Z_i} \nabla \delta P_i \cdot \frac{\mathbf{b}_0 \times \boldsymbol{\kappa}}{B_0}}_{\{Drift\}} + \underbrace{n_{i0} \frac{2}{\sqrt{\pi}} \sqrt{2} v_{thi} |k_{\parallel}| \delta T_i}_{\{Ion-Landau\}} = 0$$

➤ Parallel momentum Eq.

$$m_i n_{i0} \frac{\partial \delta u_{\parallel i}}{\partial t} = -\mathbf{b}_0 \cdot \nabla \delta P_e - \frac{1}{B_0} \delta \mathbf{B} \cdot \nabla P_{e0} - \mathbf{b}_0 \cdot \nabla \delta P_i - \frac{1}{B_0} \delta \mathbf{B} \cdot \nabla P_{i0} - \underbrace{Z_i n_{i0} \frac{m_e}{e} \sqrt{\frac{\pi}{2}} v_{the} |k_{\parallel}| \delta u_{\parallel e}}_{\{Electron-Landau\}} - \underbrace{Z_i n_{i0} \eta_{\parallel} \frac{c}{4\pi} \nabla_{\perp}^2 \delta A_{\parallel}}_{\{Resistivity\}}$$

➤ Thermal ion density Eq.

$$\frac{\partial \delta n_i}{\partial t} + \frac{c \mathbf{b}_0 \times \nabla \delta\phi}{B_0} \cdot \nabla n_{i0} + 2cn_{i0} \nabla \delta\phi \cdot \frac{\mathbf{b}_0 \times \boldsymbol{\kappa}}{B_0} + n_{i0} \mathbf{B}_0 \cdot \nabla \left( \frac{\delta u_{\parallel i}}{B_0} \right) - \underbrace{i\omega_{p,i}^* \frac{Z_i n_{i0}}{T_{i0}} \rho_i^2 \nabla_{\perp}^2 \delta\phi}_{\{Ion-FLR\}} + \underbrace{\frac{2c}{Z_i} \nabla \delta P_i \cdot \frac{\mathbf{b}_0 \times \boldsymbol{\kappa}}{B_0}}_{\{Drift\}} = 0$$

## Field equations

$$\delta P_e = \delta n_e T_{e0} + n_{e0} \delta T_e$$

$$e \delta n_e = Z_i \delta n_i + \underbrace{\frac{c^2}{4\pi V_A^2} \nabla_{\perp}^2 \delta\phi}_{\{Drift\}}$$

$$\mathbf{b}_0 \cdot \nabla \delta T_e + \frac{1}{B_0} \delta \mathbf{B} \cdot \nabla T_{e0} = 0$$

$$en_{e0} \delta u_{\parallel e} = Z_i n_{i0} \delta u_{\parallel i} + \underbrace{\frac{c}{4\pi} \nabla_{\perp}^2 \delta A_{\parallel}}_{\{Drift\}}$$

$$\delta T_i = \frac{1}{n_{i0}} (\delta P_i - \delta n_i T_{i0}).$$

# Important features of Landau-fluid model

- Braginskii model using **drift-ordering**
- Kinetic effects on top of full-MHD
  - ✓ Ion/electron diamagnetic drifts
  - ✓ Ion/electron Landau damping (Hammett-Perkins closure)
  - ✓ Ion finite Larmor radius
  - ✓ Parallel electric field
- Reduce to full-MHD by dropping labelled kinetic terms

# Algorithm: eigenvalue approach

➤ Five-field Landau-fluid model can be converted to a generalized eigenvalue problem

- $AX = \omega BX$

- $X = (\delta\phi, \delta A_{||}, \delta P_i, \delta u_{i||}, \delta n_i)^T$

➤ Operator discretization

- Radial: finite difference

- Poloidal/toroidal: Fourier expansion

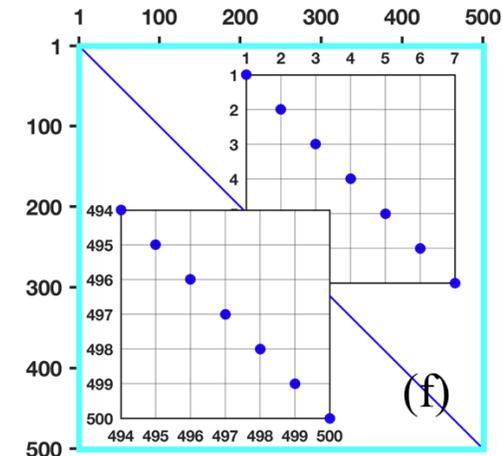
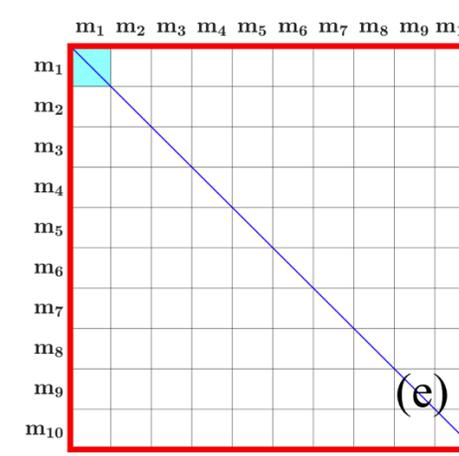
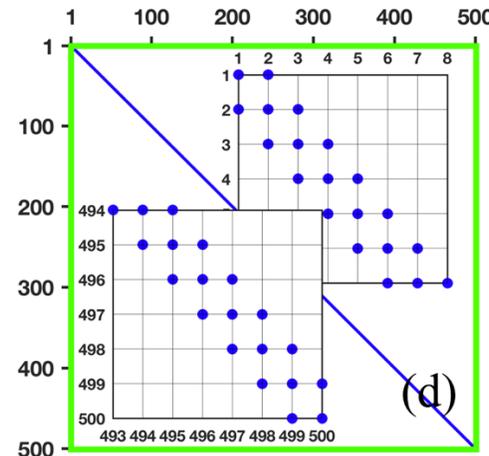
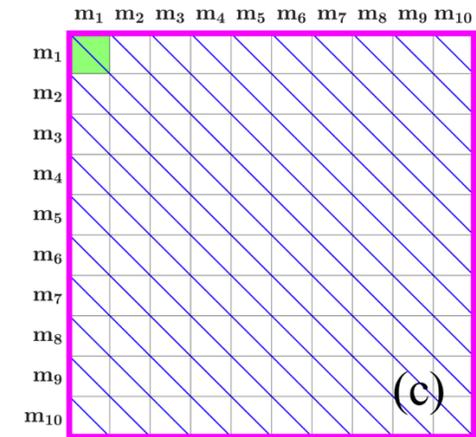
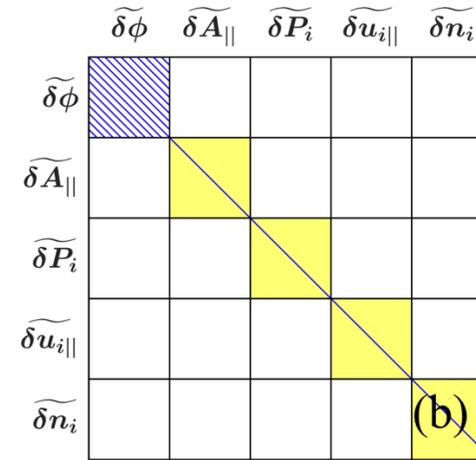
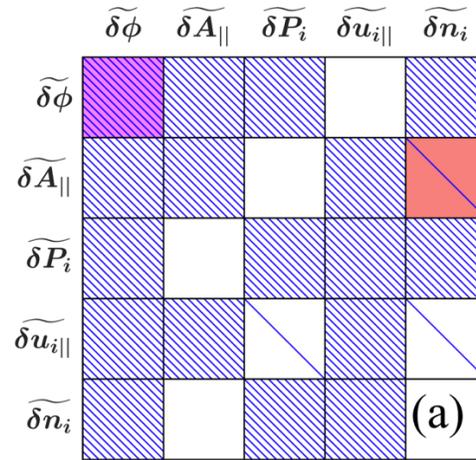
➤ Language/library

- matlab/eigs

➤ Computational speed/cost

- Less than 1 mins for AE problems on Laptop

## Multi-layer block matrices



# Normal modes: Alfvén wave and acoustic wave

LF:

$$\left[ \frac{\omega^2}{k_{\parallel}^2 V_A^2} - 1 \right] \left[ R_e^{\text{LF}}(\xi_e) + \frac{T_{e0} Z_i^2 n_{i0}}{T_{i0} e^2 n_{e0}} R_i^{\text{LF}}(\xi_i) \right] = \frac{Z_i^2 n_{i0}}{e^2 n_{e0}} k_{\perp}^2 \rho_s^2$$

$$R_e^{\text{LF}}(\xi_e) = \frac{1}{1 - i\sqrt{\frac{\pi}{2}}|\xi_e|}$$

$$R_i^{\text{LF}}(\xi_i) = \frac{|\xi_i| + i\frac{2}{\sqrt{\pi}}}{-2\xi_i^2|\xi_i| - i\frac{4}{\sqrt{\pi}}\xi_i^2 + \Gamma_{i\parallel}|\xi_i| + i\frac{2}{\sqrt{\pi}}}$$

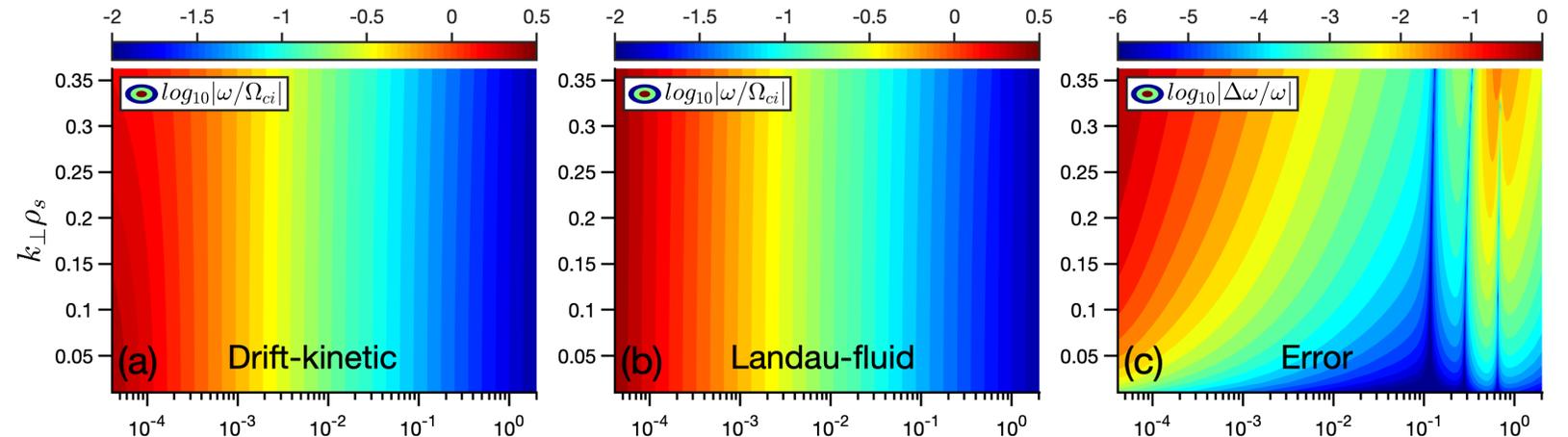
DK:

$$\left[ \frac{\omega^2}{k_{\parallel}^2 V_A^2} - 1 \right] \left[ R_e^{\text{DK}}(\xi_e) + \frac{T_{e0} Z_i^2 n_{i0}}{T_{i0} e^2 n_{e0}} R_i^{\text{DK}}(\xi_i) \right] = \frac{Z_i^2 n_{i0}}{e^2 n_{e0}} k_{\perp}^2 \rho_s^2$$

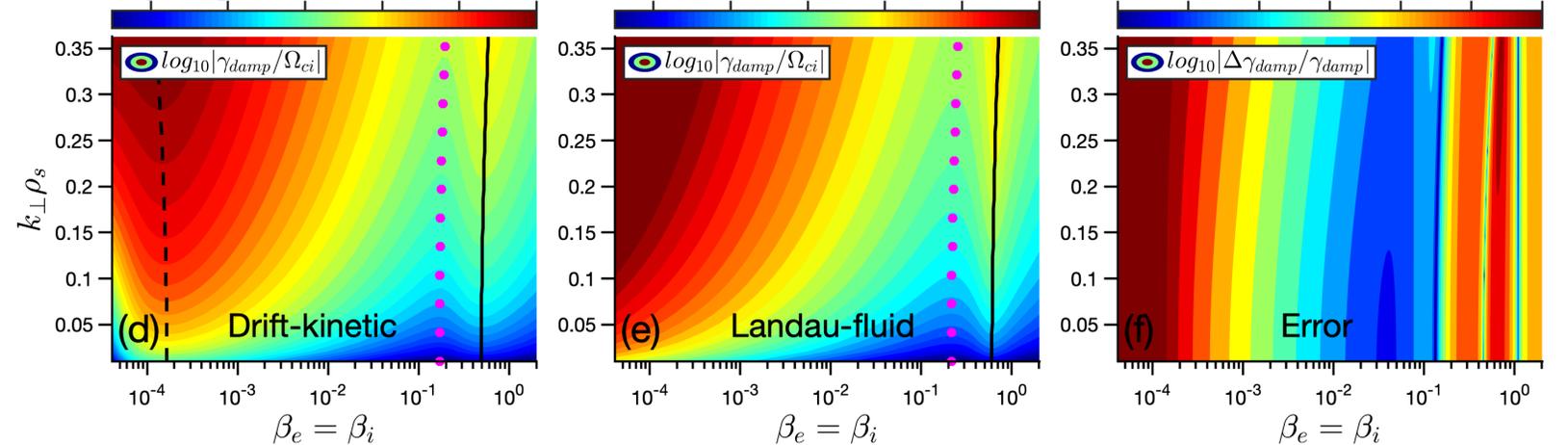
$$R_e^{\text{DK}}(\xi_e) = 1 + \xi_e Z(\xi_e)$$

$$R_i^{\text{DK}}(\xi_i) = 1 + \xi_i Z(\xi_i)$$

Frequency

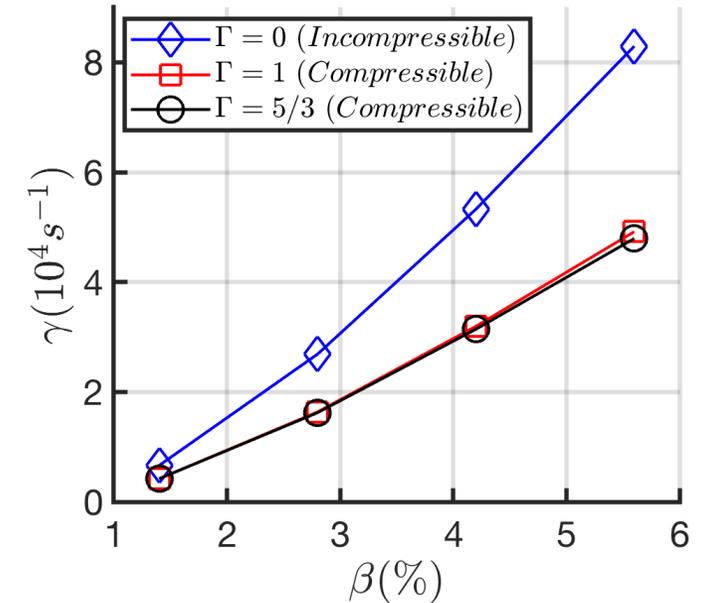
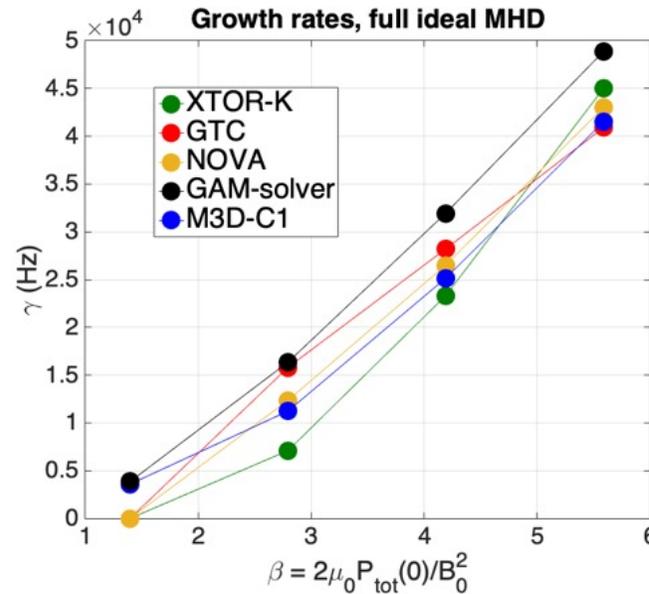
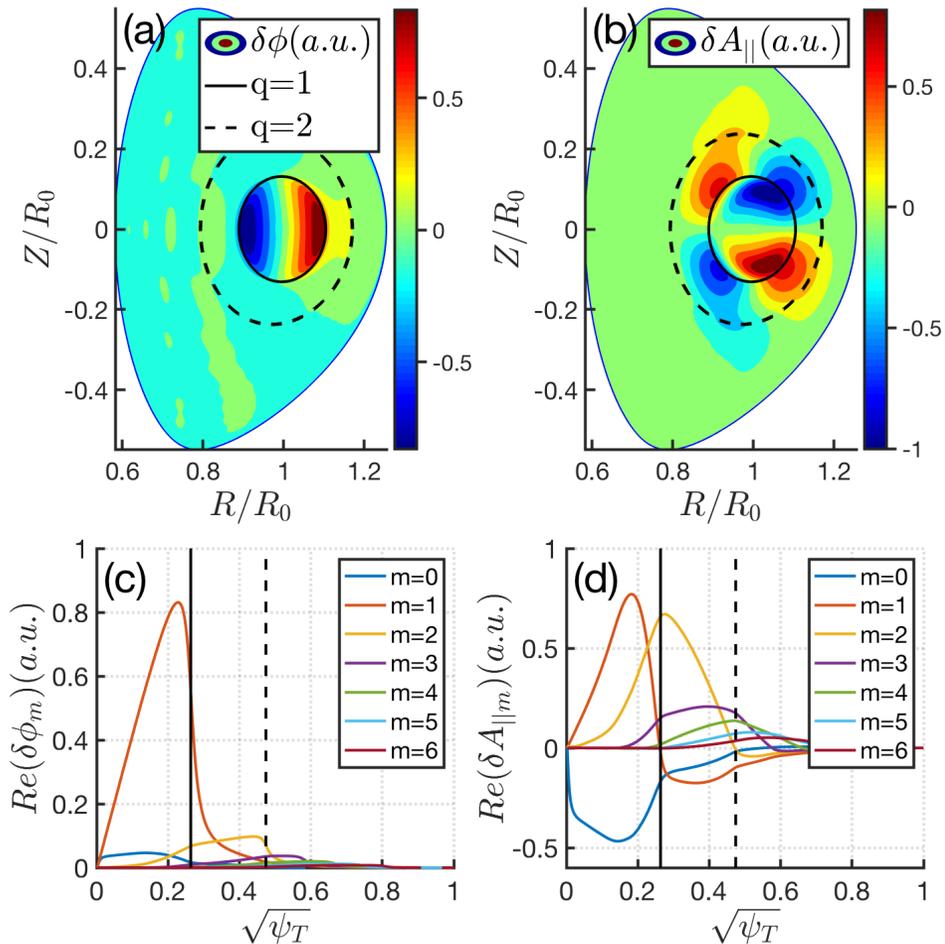


Damping rate



- ✓ Comparison of coupled KAW-ISW dispersion relation between drift-kinetic model and Landau-fluid model, which show good agreement for typical tokamak plasma beta ( $\beta \sim 0.01 - 0.1$ ). 9

# MHD mode: internal kink mode

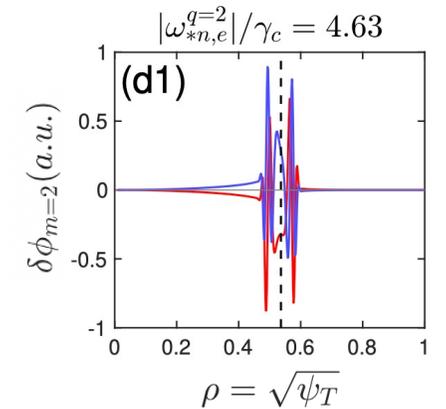
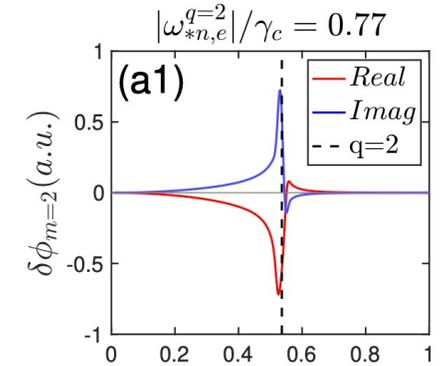
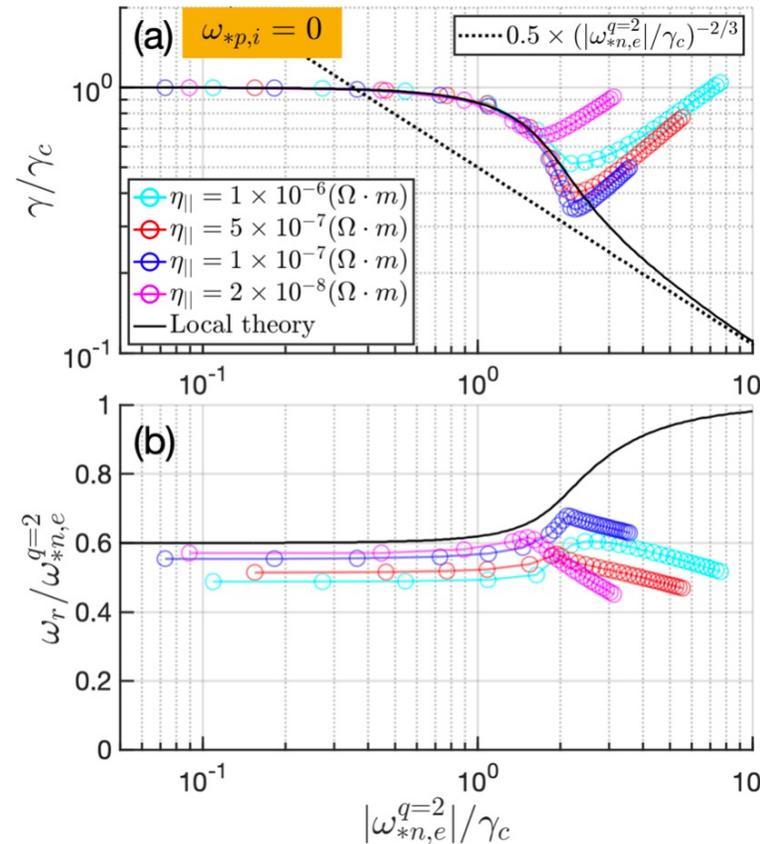
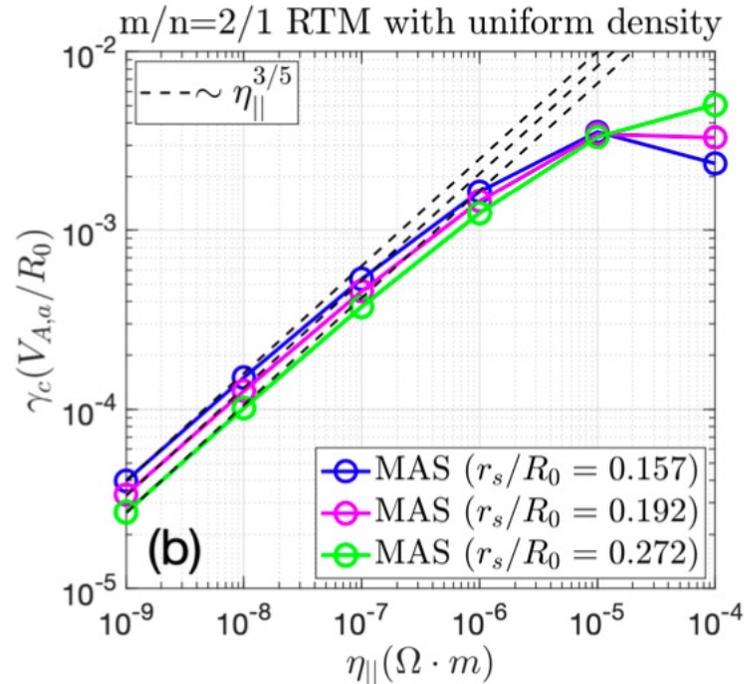


- Cross-code verification of kink mode (NOVA, XTOR-K, M3D-C1, GTC, MAS)
- ✓ Necessity of full-MHD: finite ion acoustic compression stabilization

# MHD mode: resistive-tearing/drift-tearing modes

$$\gamma_c = \eta_{\parallel}^{3/5} \left( \frac{ns}{r} \right)^{2/5} \left[ \Delta' \frac{\Gamma(1/4)}{2\pi\Gamma(3/4)} \right]^{4/5}$$

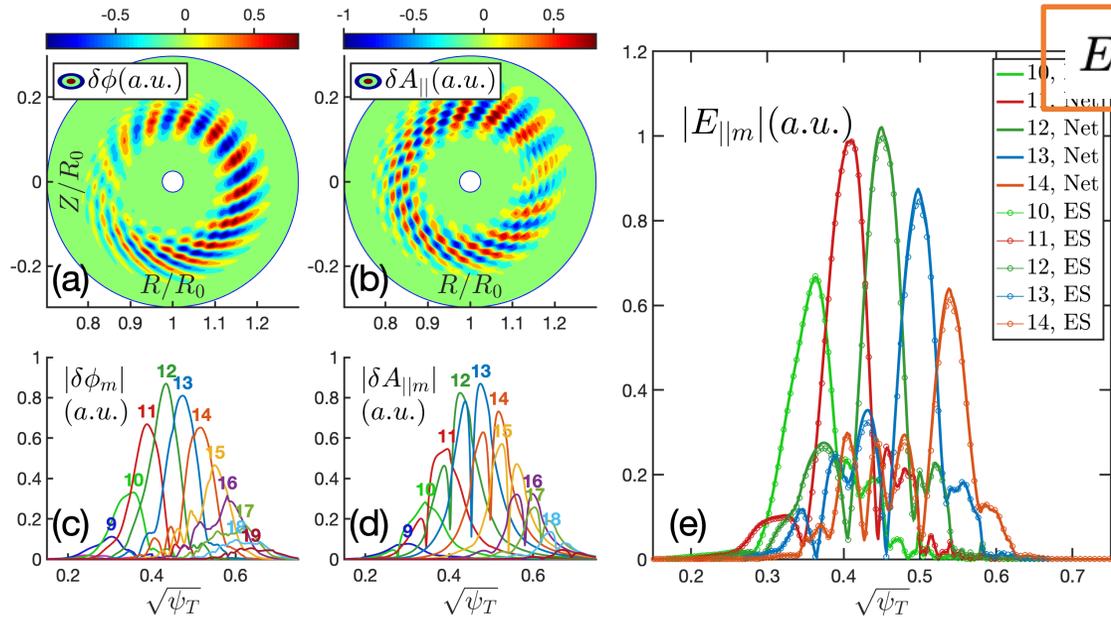
$$\omega(\omega - \omega_{*p,i})(\omega - \omega_{*n,e})^3 = i\gamma_c^5,$$



- ✓ The RTM growth rate scaling is close to  $\gamma_c \sim \eta_{\parallel}^{5/3}$  in the small  $\eta_{\parallel}$  regime.
- ✓ DTM dispersion relation in MAS agrees with local theory in the small  $\omega_e^*$  regime, while deviates from local theory when  $\omega_e^* \sim \gamma_c$  due to the non-local mode structure.

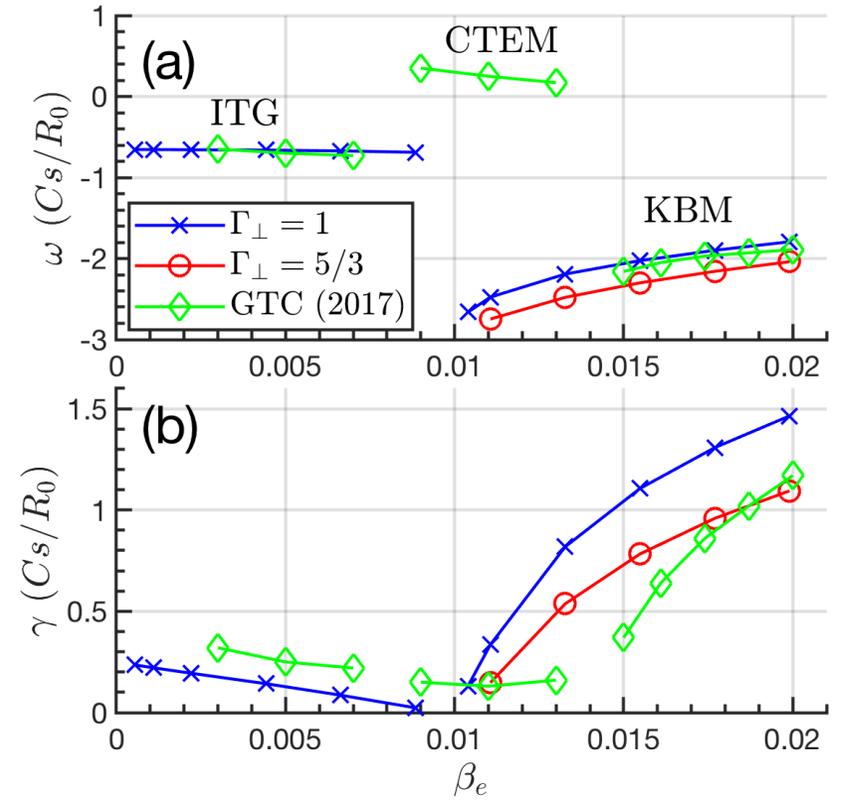
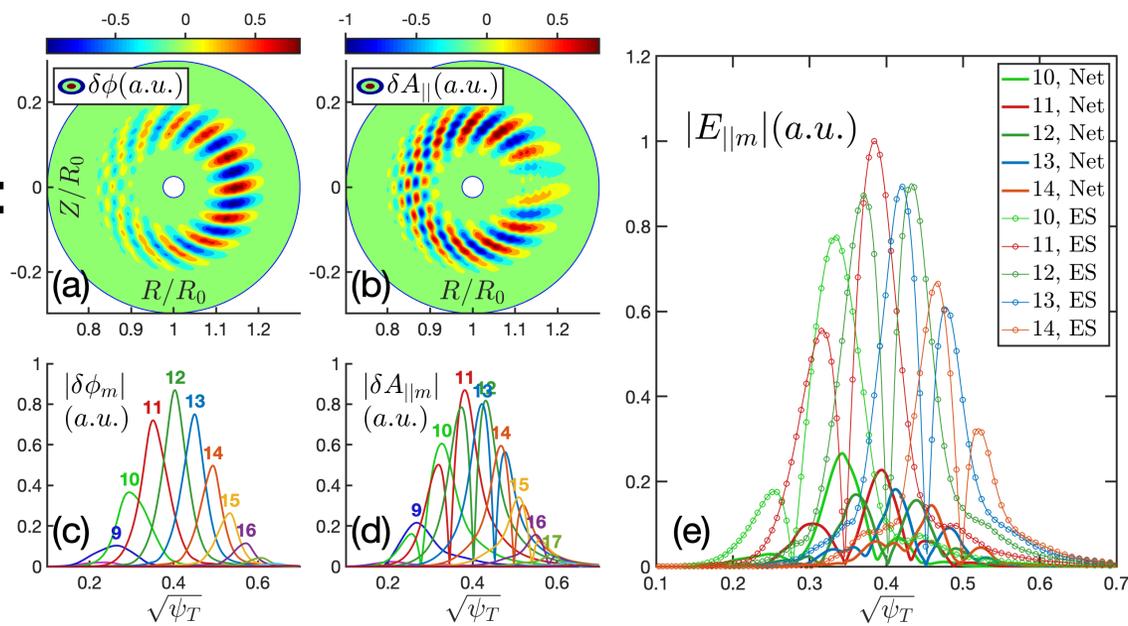
# Drift-wave instabilities: ITG/KBM

ITG:



$$E_{||}^{ES} = -\mathbf{b}_0 \cdot \nabla \delta\phi \quad E_{||}^{Net} = -\mathbf{b}_0 \cdot \nabla \delta\phi - (1/c) \partial_t \delta A_{||}$$

KBM:



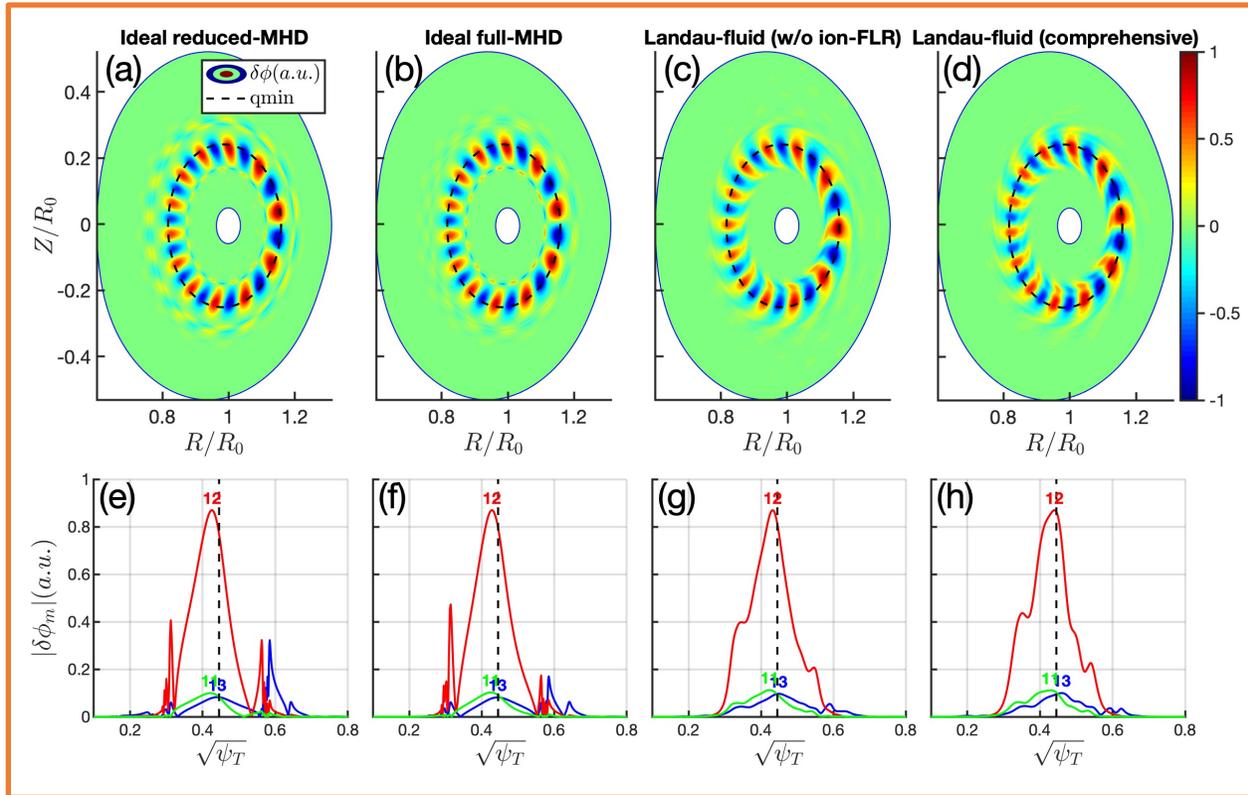
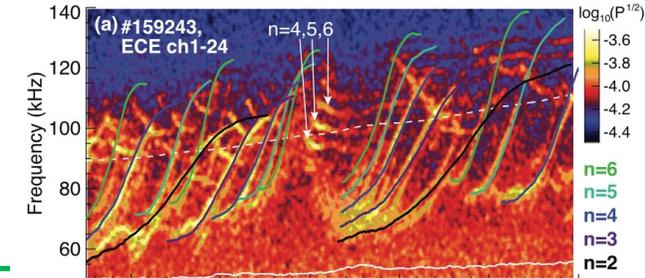
- Cover ITG-KBM transition in CBC case, verify mode polarizations
- ✓ Low beta regime: finite-beta stabilization on ITG
- ✓ High beta regime: KBM onset

# RSAE: upward frequency sweeping

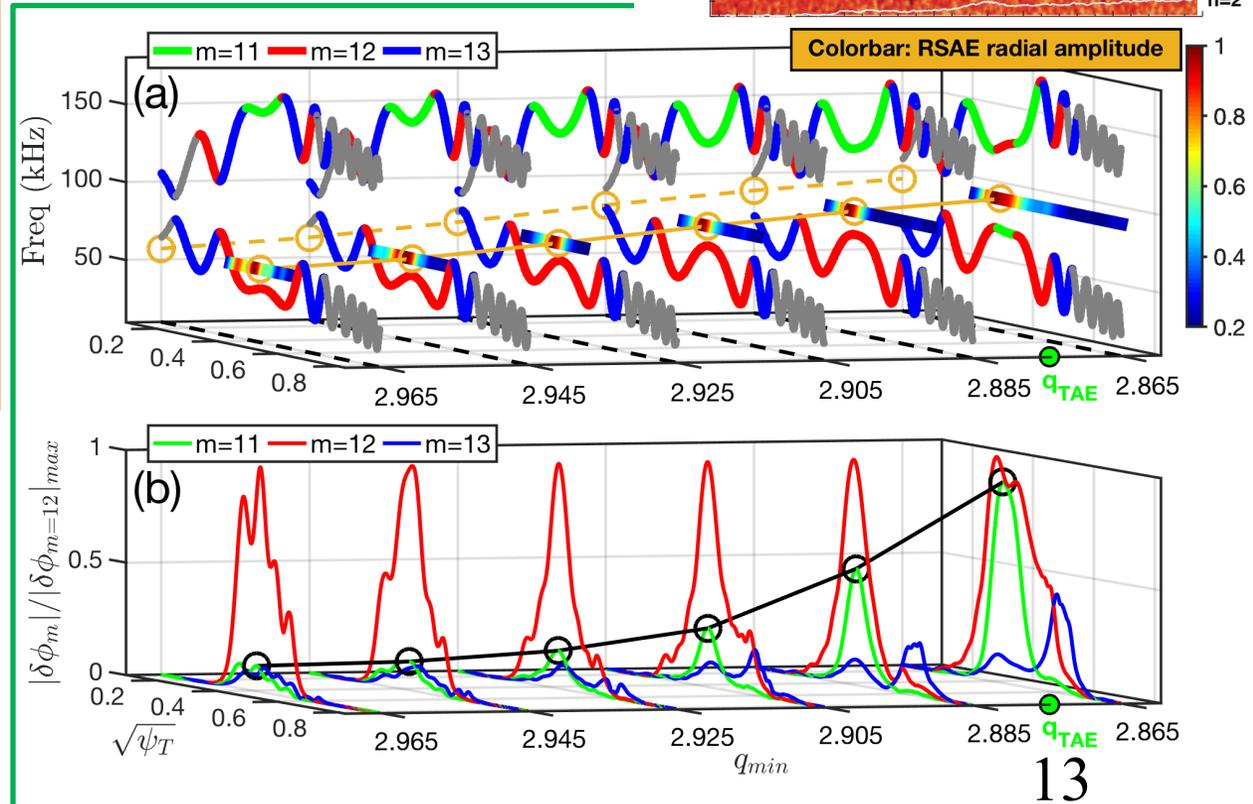
- RSAE frequency increases with decreasing  $q_{min}$
- RSAE-TAE transition:  $q_{min} < q_{TAE} = (2m-1)/2n$

$$\omega_{RSAE} \approx \left[ \left( \frac{m}{q_{min}} - n \right)^2 \frac{V_A^2}{R_0^2} + \omega_{geod}^2 \right]^{1/2} + \delta\omega,$$

$$m - 0.5 < nq_{min} < m$$

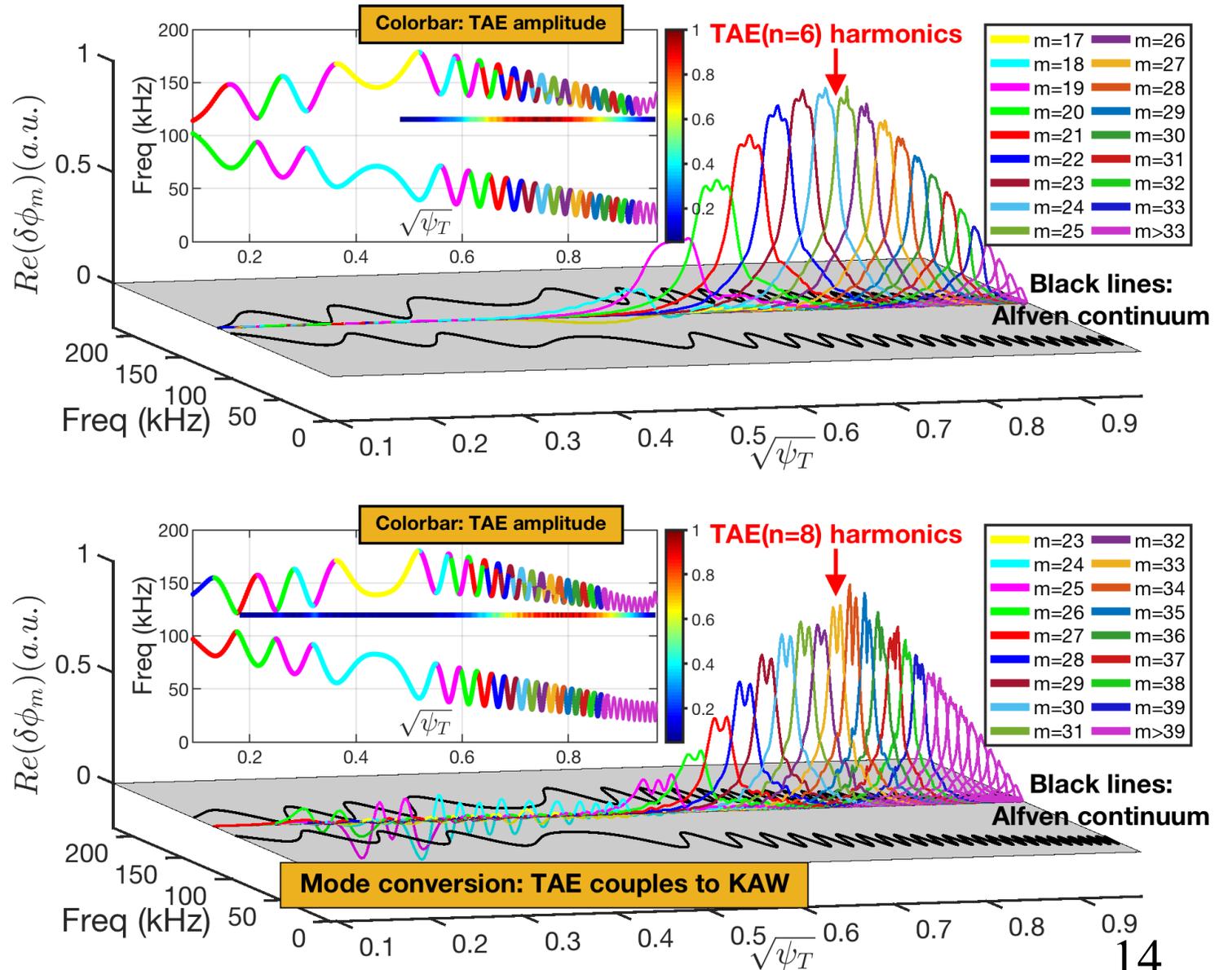


- Comparison of RSAE mode structure between different level physics models



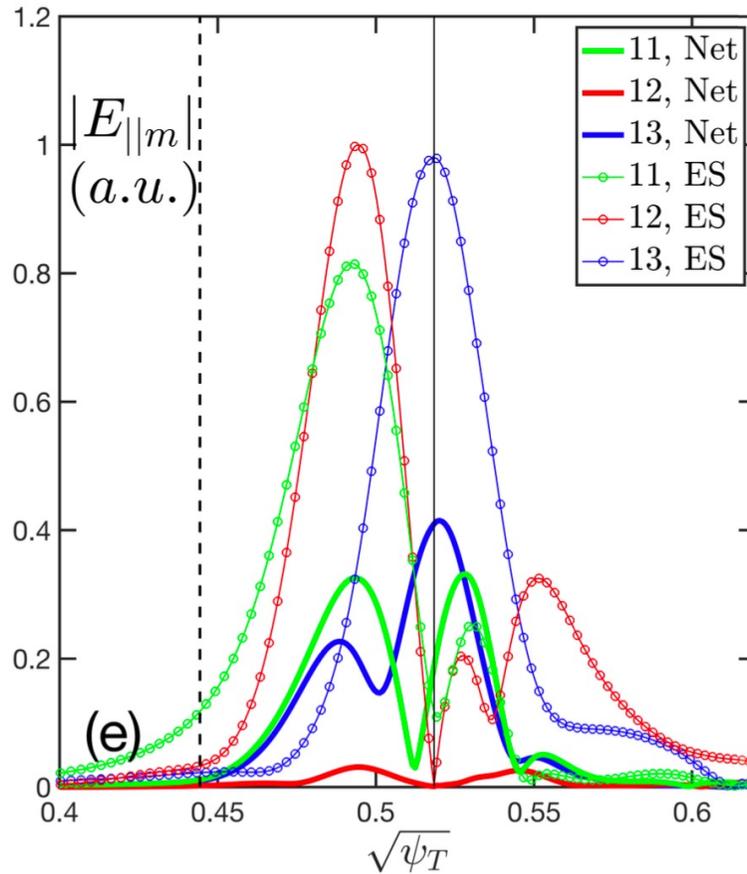
# TAE radiative damping

- Tunneling interaction between TAE and Alfvén continua due to kinetic effects (FLR, finite  $E_{||}$  etc)
- ✓ Short-wavelength KAW arises and couples to TAE, enhanced by increasing  $n$  number
- ✓ Radiative damping

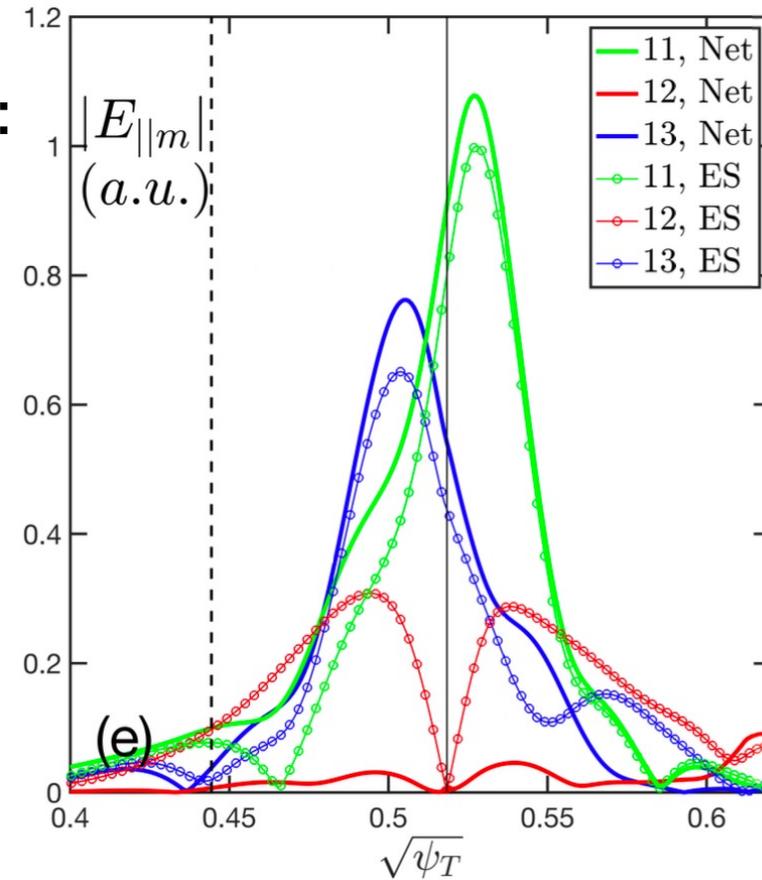


# Polarizations of KBAE and BAAE

**KBAE:**



**BAAE:**



- KBAE : Alfvénic polarization for all poloidal harmonics ( $E_{||}^{Net} \ll E_{||}^{ES}$ ).
- BAAE : Alfvénic polarization ( $E_{||}^{Net} \ll E_{||}^{ES}$ ) for predominant poloidal harmonics, electrostatic polarization ( $E_{||}^{Net} \approx E_{||}^{ES}$ ) for sidebands  $m \pm 1$ .

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# Drift-kinetic energetic electrons

Linearized drift-kinetic equation

$$L_0 \delta f_h + \delta L^L f_{h0} = 0$$

$$L_0 = \frac{\partial}{\partial t} + (v_{\parallel} \mathbf{b}_0 + \mathbf{v}_d) \cdot \nabla - \frac{\mu}{m_e B_0} \mathbf{B}_0^* \cdot \nabla B_0 \frac{\partial}{\partial v_{\parallel}}$$

$$\delta L^L = \left( v_{\parallel} \frac{\delta \mathbf{B}}{B_0} + \mathbf{v}_E \right) \cdot \nabla - \left[ \frac{\mu}{m_e B_0} \delta \mathbf{B} \cdot \nabla B_0 + \frac{q_e}{m_e} \left( \frac{\mathbf{B}_0^*}{B_0} \cdot \nabla \delta \phi + \frac{1}{c} \frac{\partial \delta A_{\parallel}}{\partial t} \right) \right] \frac{\partial}{\partial v_{\parallel}},$$

- Adiabatic response
- ✓ convective effect

$$\delta f_h = \delta f^A + \delta K$$

$$\delta f^A = \underbrace{-\frac{q_e}{T_{h0}} (\delta \phi - \delta \psi) f_{h0}}_{\{I\}} - \underbrace{\frac{q_e}{T_{h0}} \delta \psi \left[ \frac{\omega_{*n,h}}{\omega} + \left( \frac{m_e v_{\parallel}^2 + 2\mu B_0}{2T_{h0}} - \frac{3}{2} \right) \frac{\omega_{*T,h}}{\omega} \right] f_{h0}}_{\{II\}}$$

- Non-adiabatic response
- ✓ Precessional-drift resonance

$$v_{\parallel} \mathbf{b}_0 \cdot \nabla \delta K^p = -i \frac{q_e}{T_{h0}} \omega \left( 1 - \frac{\omega_{*p,h}}{\omega} \right) (\delta \phi - \delta \psi) f_{h0} - i \frac{q_e}{T_{h0}} \omega_d \left( 1 - \frac{\omega_{*p,h}}{\omega} \right) \delta \psi f_{h0}$$

$$\delta K^t \simeq \delta K_b^t \simeq \underbrace{\frac{\omega}{\omega - \bar{\omega}_d} \frac{q_e}{T_{h0}} \left( 1 - \frac{\omega_{*p,h}}{\omega} \right) (\delta \phi - \delta \psi) f_{h0}}_{\{I\}} + \underbrace{\frac{1}{\omega - \bar{\omega}_d} \frac{q_e}{T_{h0}} \left( 1 - \frac{\omega_{*p,h}}{\omega} \right) \bar{\omega}_d \delta \psi f_{h0}}_{\{II\}}$$

# EE moments integrated from kinetic responses

$$\delta n_h^A = \int \delta f^A \mathbf{d}\mathbf{v} = -\frac{q_e n_{h0}}{T_{h0}} (\delta\phi - \delta\psi) - \frac{q_e n_{h0}}{T_{h0}} \frac{\omega_{*n,h}}{\omega} \delta\psi$$

$$\delta P_{||h}^A = \int m_e v_{||}^2 \delta f^A \mathbf{d}\mathbf{v} = -q_e n_{h0} (\delta\phi - \delta\psi) - q_e n_{h0} \left( \frac{\omega_{*n,h}}{\omega} + \frac{\omega_{*T,h}}{\omega} \right) \delta\psi$$

$$\delta P_{\perp h}^A = \int \mu B_0 \delta f^A \mathbf{d}\mathbf{v} = -q_e n_{h0} (\delta\phi - \delta\psi) - q_e n_{h0} \left( \frac{\omega_{*n,h}}{\omega} + \frac{\omega_{*T,h}}{\omega} \right) \delta\psi$$

- Adiabatic response
- ✓ Density
- ✓ Pressure

$$\delta n_h^{NA} = -f_t \frac{q_e n_{h0}}{T_{h0}} \left[ \left( 1 - \frac{\omega_{*n,h}}{\omega} + \frac{3}{2} \frac{\omega_{*T,h}}{\omega} \right) \zeta R_1(\sqrt{\zeta}) - \frac{\omega_{*T,h}}{\omega} \zeta R_3(\sqrt{\zeta}) \right] (\delta\phi - \delta\psi)$$

$$- f_t \frac{q_e n_{h0}}{T_{h0}} \left[ \left( 1 - \frac{\omega_{*n,h}}{\omega} + \frac{3}{2} \frac{\omega_{*T,h}}{\omega} \right) R_3(\sqrt{\zeta}) - \frac{\omega_{*T,h}}{\omega} R_5(\sqrt{\zeta}) \right] \delta\psi$$

$$\delta P_h^{NA} = -\frac{f_t}{2} q_e n_{h0} \left[ \left( 1 - \frac{\omega_{*n,h}}{\omega} + \frac{3}{2} \frac{\omega_{*T,h}}{\omega} \right) \zeta R_3(\sqrt{\zeta}) - \frac{\omega_{*T,h}}{\omega} \zeta R_5(\sqrt{\zeta}) \right] (\delta\phi - \delta\psi)$$

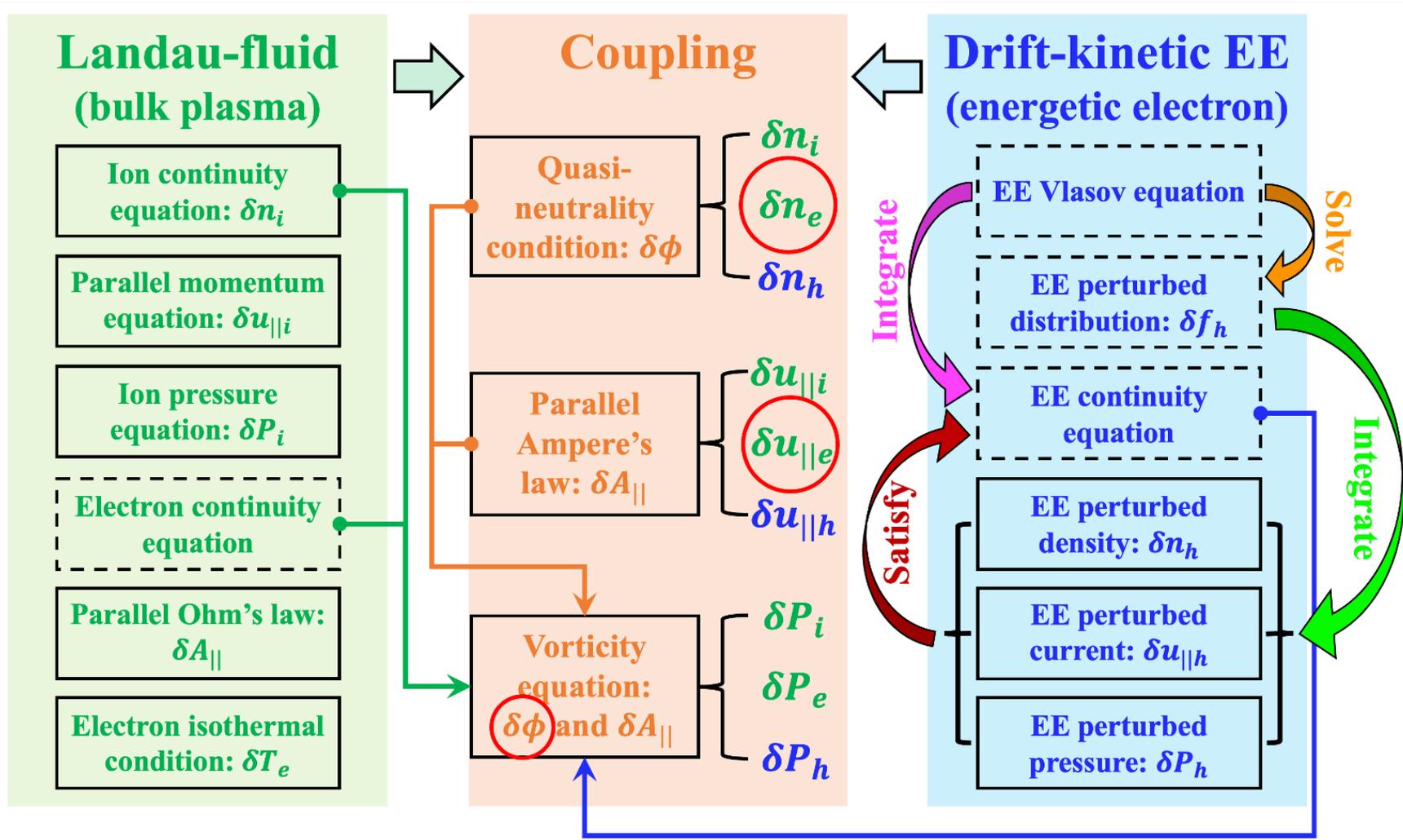
$$- \frac{f_t}{2} q_e n_{h0} \left[ \left( 1 - \frac{\omega_{*n,h}}{\omega} + \frac{3}{2} \frac{\omega_{*T,h}}{\omega} \right) R_5(\sqrt{\zeta}) - \frac{\omega_{*T,h}}{\omega} R_7(\sqrt{\zeta}) \right] \delta\psi$$

- Non-adiabatic response of trapped electrons
- ✓ Density
- ✓ Pressure

$$\delta u_{||h} = \frac{e}{T_{h0}} \frac{\omega}{k_{||}} (1 - f_t) \left( 1 - \frac{\omega_{*n,h}}{\omega} \right) (\delta\phi - \delta\psi) + 2 \frac{e}{T_{h0}} \frac{\omega}{k_{||}} \left( \frac{\omega_d}{\omega} - f_t \frac{3\omega_{D0}}{4\omega} \right) \left( 1 - \frac{\omega_{*n,h}}{\omega} - \frac{\omega_{*T,h}}{\omega} \right) \delta\psi$$

- Non-adiabatic response of passing electrons
- ✓ Parallel velocity

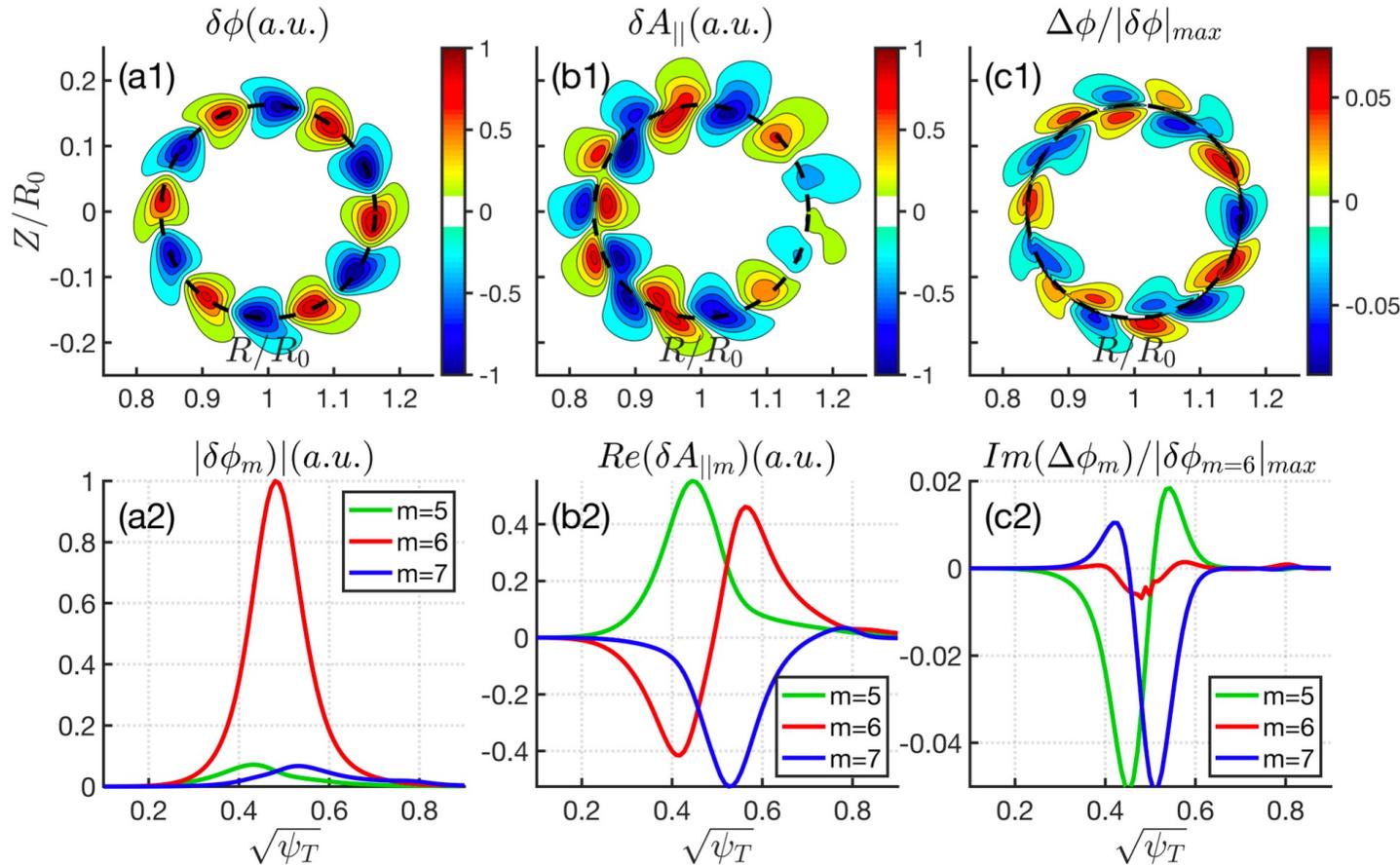
# Coupling scheme for EE and bulk plasmas



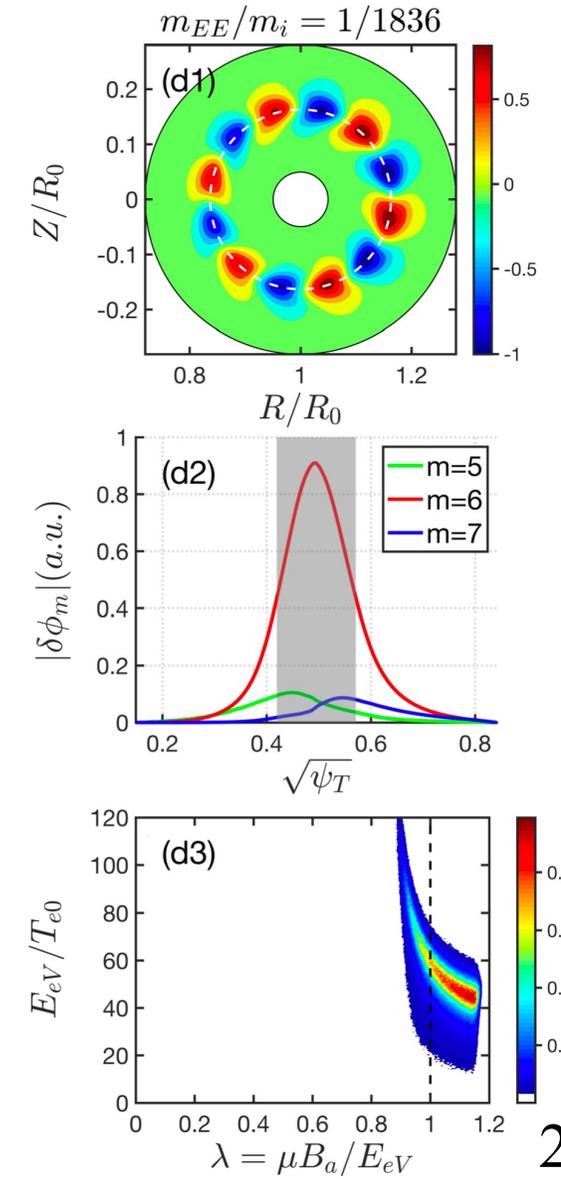
- ✓  $\delta n_h$  modifies quasi-neutrality condition
- ✓  $\delta u_{\parallel h}$  modifies parallel Ohms law
- ✓  $\delta P_h$  modifies vorticity equation
- Enable accuracy for both EM and ES cases through density and pressure coupling

# Verification of EE-driven BAE (e-BAE)

## MAS



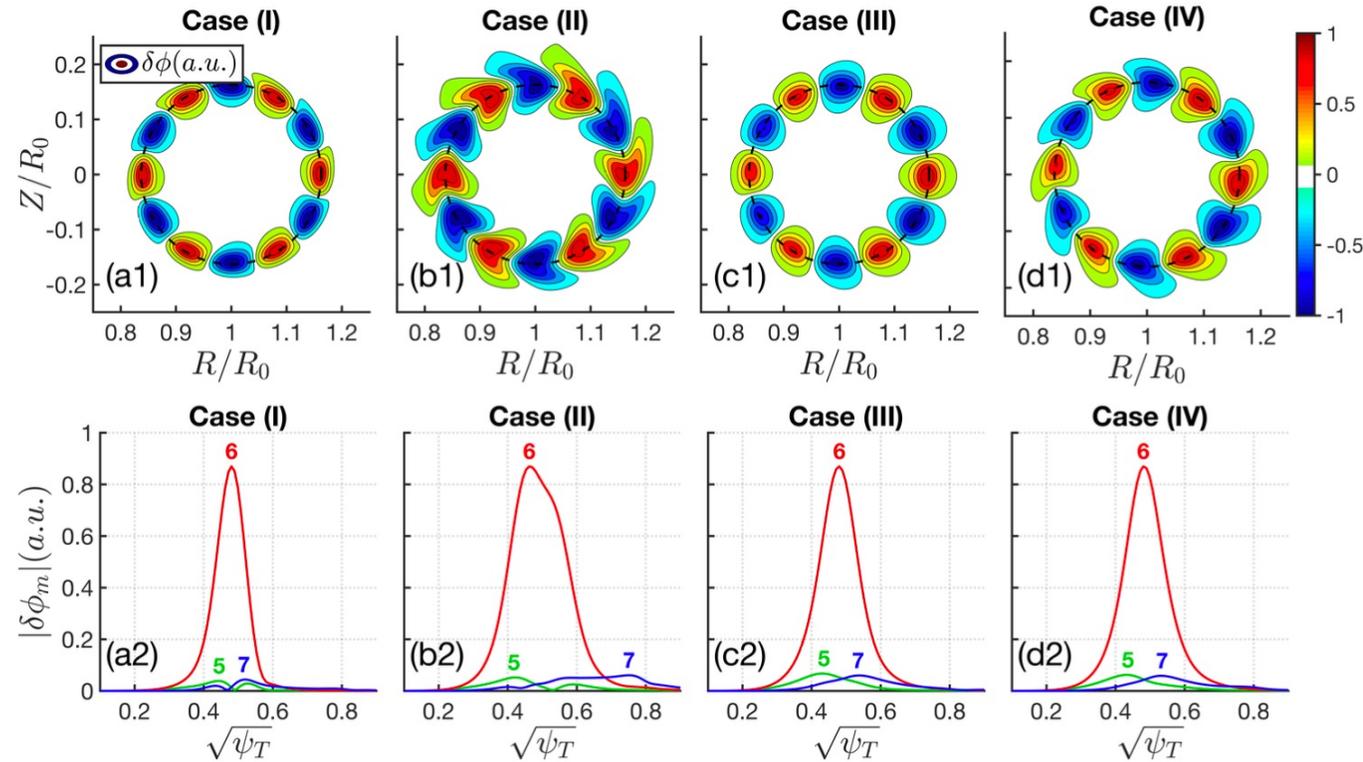
## GTC



- ✓ Weakly ballooning mode structure, finite  $E_{||}$  in sidebands, precessional-drift resonance of deeply-trapped EEs.
- ✓ Good agreements between MAS eigenvalue and GTC initial value results.

# Effects of different EE responses on e-BAE

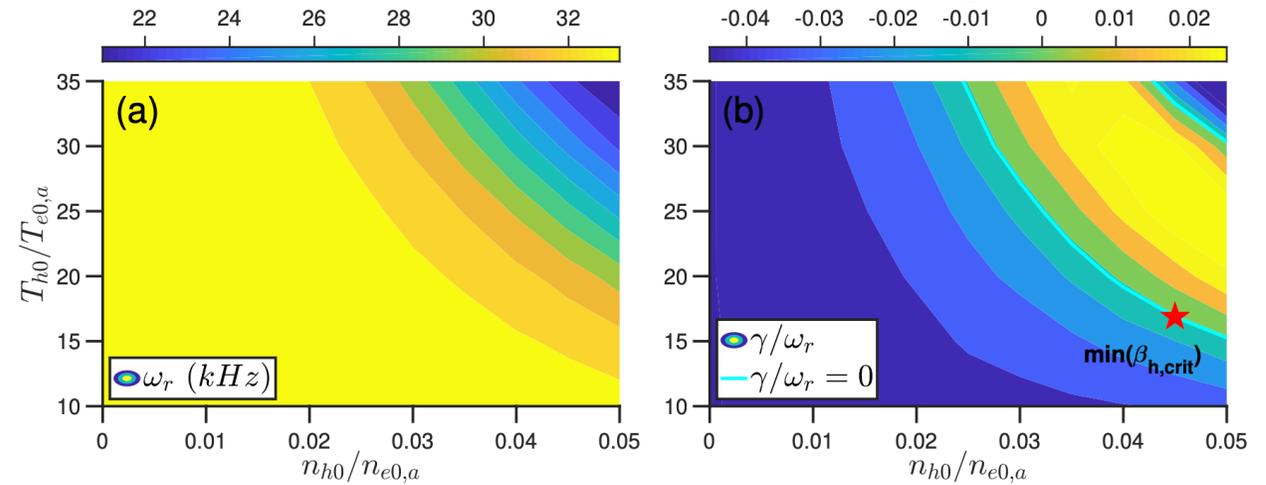
Case	EE-IC	EE-KPC	$\omega_r (V_{Ap}/R_0)$	$\gamma (V_{Ap}/R_0)$
(I)	No	No	0.160	-0.007 07
(II)	No	Yes	0.175	0.004 96
(III)	Yes	No	0.134	-0.009 09
(IV)	Yes	Yes	0.149	0.009 04



- EE-IC (interchange convective response): broaden radial width, decrease frequency
- EE-KPC (kinetic particle compression response): anti-Hermitian contribution to dielectric constant  
induce mode structure poloidal phase variation (triangle shape), increase frequency

# Experimental application in EAST discharges

- Dependences of  $m/n=4/1$  e-BAE  $\omega_r$  and  $\gamma$  on EE density and temperature in EAST shot #82589.

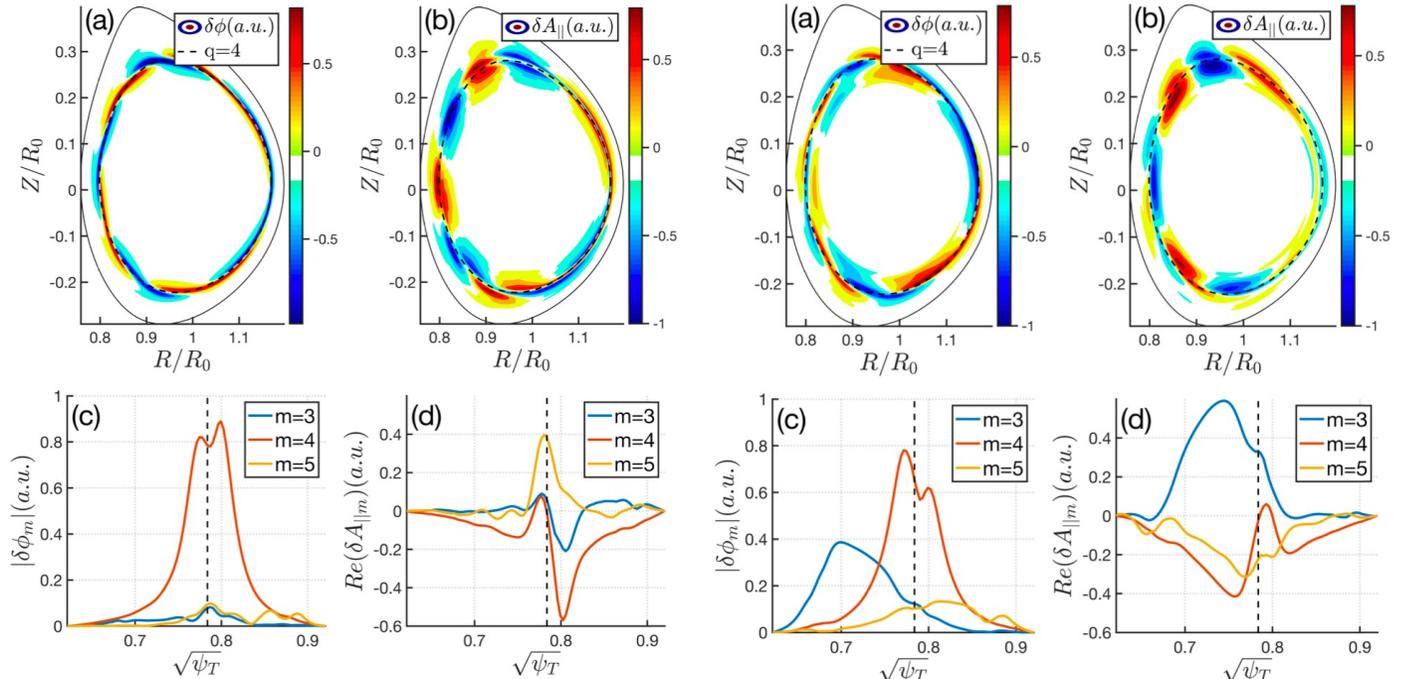


- EE non-perturbative effects

➤ Decrease  $\omega_r$

➤ Increase e-BAE sideband amplitudes

- Identify the  $\beta_h$  threshold for EE excitation of BAE.



# Outlines

- Background
- Discretized eigenmode
  - Various bulk plasma instabilities
  - Energetic electron excitation of BAE
  - Energetic ion responses to arbitrary wavelength fluctuations
- Continuous spectrum
- Resonance condition in phase space
- Summary

# Gyrokinetic energetic ions

Gyrokinetic equation for non-adiabatic response  $\left(\frac{v_{\parallel}}{JB}\partial_{\theta} + i\frac{nqv_{\parallel}}{JB} - i(\omega - \omega_{p,h}^*)\right)\delta K_h = -i\frac{Z_h}{T_h}(\omega - \omega_{p,h}^*)J_0 f_{h0}(\Delta\phi + \frac{\omega_d}{\omega}\delta\psi)$

Solution1:  
well-circulating particles

$$J_0\delta K_h = -\frac{Z_h}{T_h}\left(1 - \frac{\omega_{p,h}^*}{\omega}\right)J_0^2 f_{h0}\Sigma_{p,s,m}J_p(\lambda_h)J_s(\lambda_h)i^{p-s}e^{i(p-s)(\theta+\theta_r)}e^{-im\theta} \times$$

$$\left\{\frac{\Delta\phi_m}{R_{N+s}} + \frac{(k_0v_d/\omega)\delta\psi_m}{R_{N+s}} + \frac{(k_1v_d/\omega)\delta\psi_me^{i\theta}}{R_{N+s+1}} + \frac{(k_{-1}v_d/\omega)\delta\psi_me^{-i\theta}}{R_{N+s-1}}\right\}$$

$$= \mathbb{J}_p \times \mathbb{R} \times \mathbb{J}_s \times (\Delta\phi - i\mathbb{A} \times \delta\psi)$$

where  $\lambda_h = k_f \frac{JBv_d}{v_{\parallel}}$ ,  $k_f = 2\sqrt{k_1k_{-1}}$  and  $R_N = \frac{k_{\parallel}v_{\parallel}}{\omega} + \frac{k_0v_d}{\omega} - 1$ ,  $k_{\parallel} = \frac{N}{JB} = \frac{nq-m}{JB}$

$$\mathbb{J}_p = \Sigma_p J_p(\lambda_h) i^p e^{ip(\theta+\theta_r)} \quad \mathbb{J}_s = \Sigma_s J_s(\lambda_h) i^{-s} e^{-is(\theta+\theta_r)} = \mathbb{J}_p^{\dagger}$$

$$\mathbb{R} = -\frac{Z_h}{T_h}\left(1 - \frac{\omega_{p,h}^*}{\omega}\right)J_0^2 f_{h0} \frac{v_d}{\omega} \frac{1}{R_N} \quad \mathbb{A} = \frac{R_0}{JB_0} \{(I\kappa_{\zeta} - g\kappa_{\theta})\partial_{\psi} - (\delta\kappa_{\zeta} - g\kappa_{\psi})\partial_{\theta} + (\delta\kappa_{\theta} - I\kappa_{\psi})\partial_{\zeta}\}$$

Solution2:  
deeply trapped particles

$$J_0\delta K_h = \frac{Z_h}{T_h}\left(1 - \frac{\omega_{p,h}^*}{\omega}\right)J_0^2 f_{h0}\Sigma_{p,s,m}J_p(\lambda_{Bh})J_s(\lambda_{Bh})i^{p-s}e^{i(p-s)\eta}e^{-i(m-nq)\theta} \times$$

$$\left\{\Delta\phi_m T_s + \delta\psi_m \frac{\bar{\omega}_d}{\omega} T_s + \delta\psi_m \frac{-\frac{i}{2}\omega_d^{(1)}e^{i\eta}}{\omega} T_{s-1} + \delta\psi_m \frac{\frac{i}{2}\omega_d^{(1)}e^{-i\eta}}{\omega} T_{s+1}\right\}$$

where  $T_s = \frac{\omega}{\omega - \bar{\omega}_d + s\omega_b}$ ,  $\omega_d^{(1)} = \bar{\omega}_d \theta_b \xi$  and  $\lambda_{Bh} = \frac{\bar{\omega}_d}{\omega_b} \theta_b \xi$

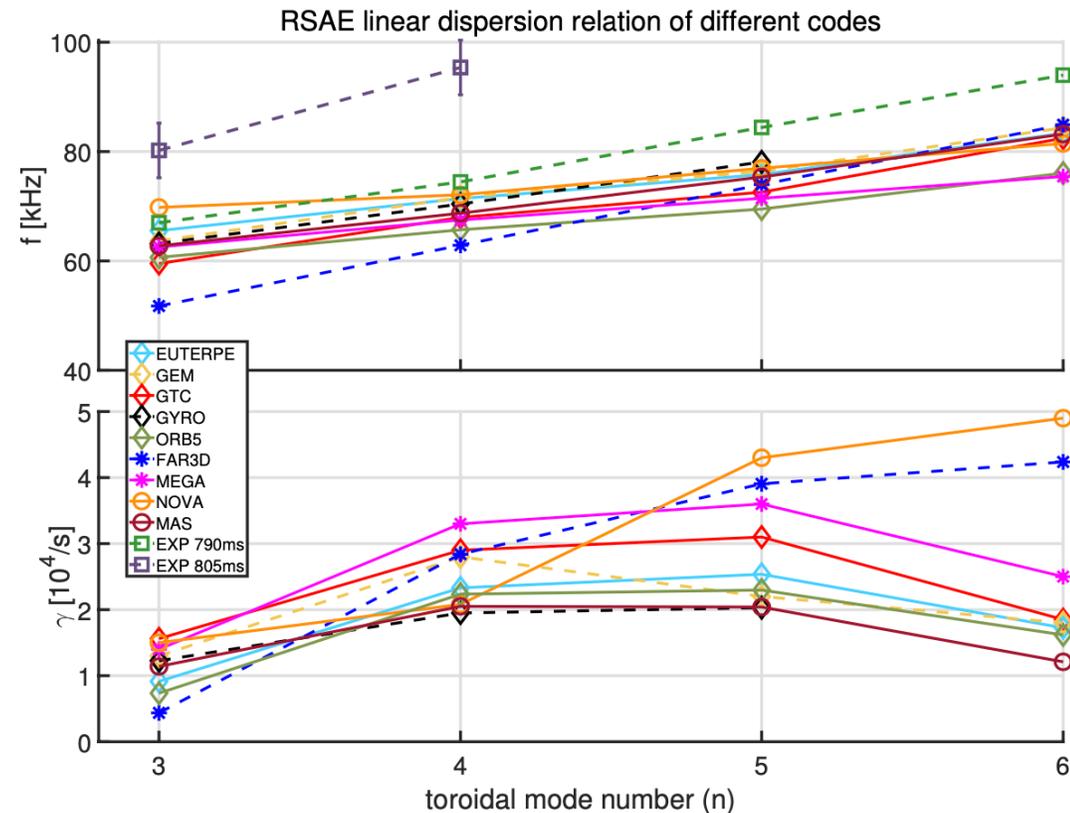
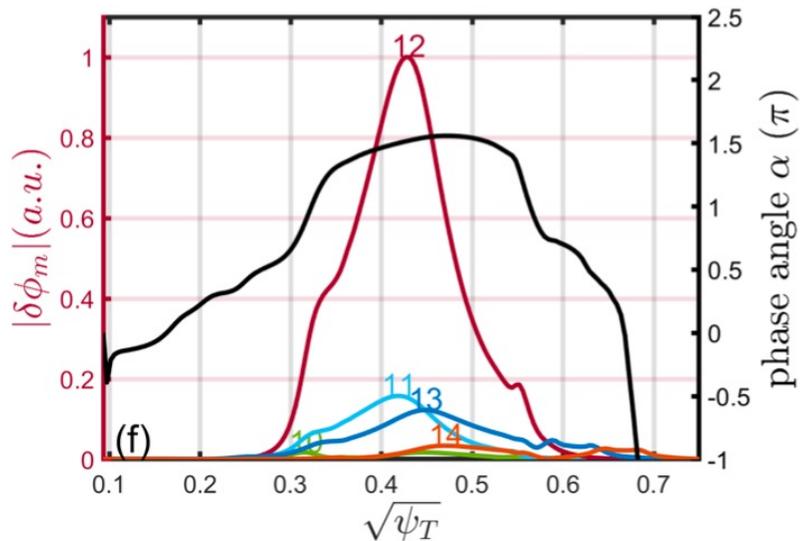
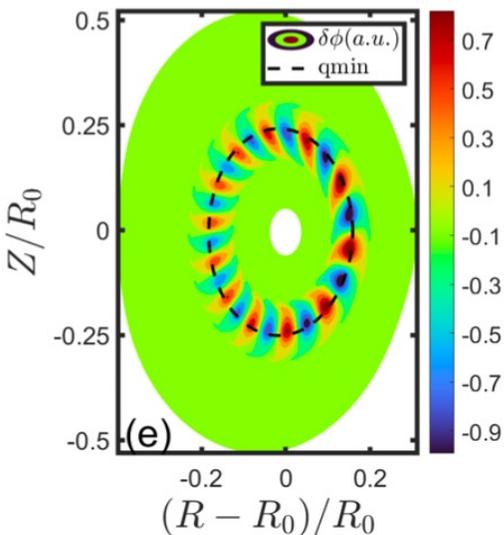
➤ Numerically integrate E1 moments in velocity space from perturbed distributions with Bessel function

coefficients

- ✓ Transit resonance
- ✓ Precessional drift resonance
- ✓ Finite Larmor radius
- ✓ Finite orbit width

X. R. Xu et al 2024,  
submitted to PPCF

# Verification of EI-driven RSAE



MAS simulation of EI-driven RSAE in DIII-D shot #159243 equilibrium

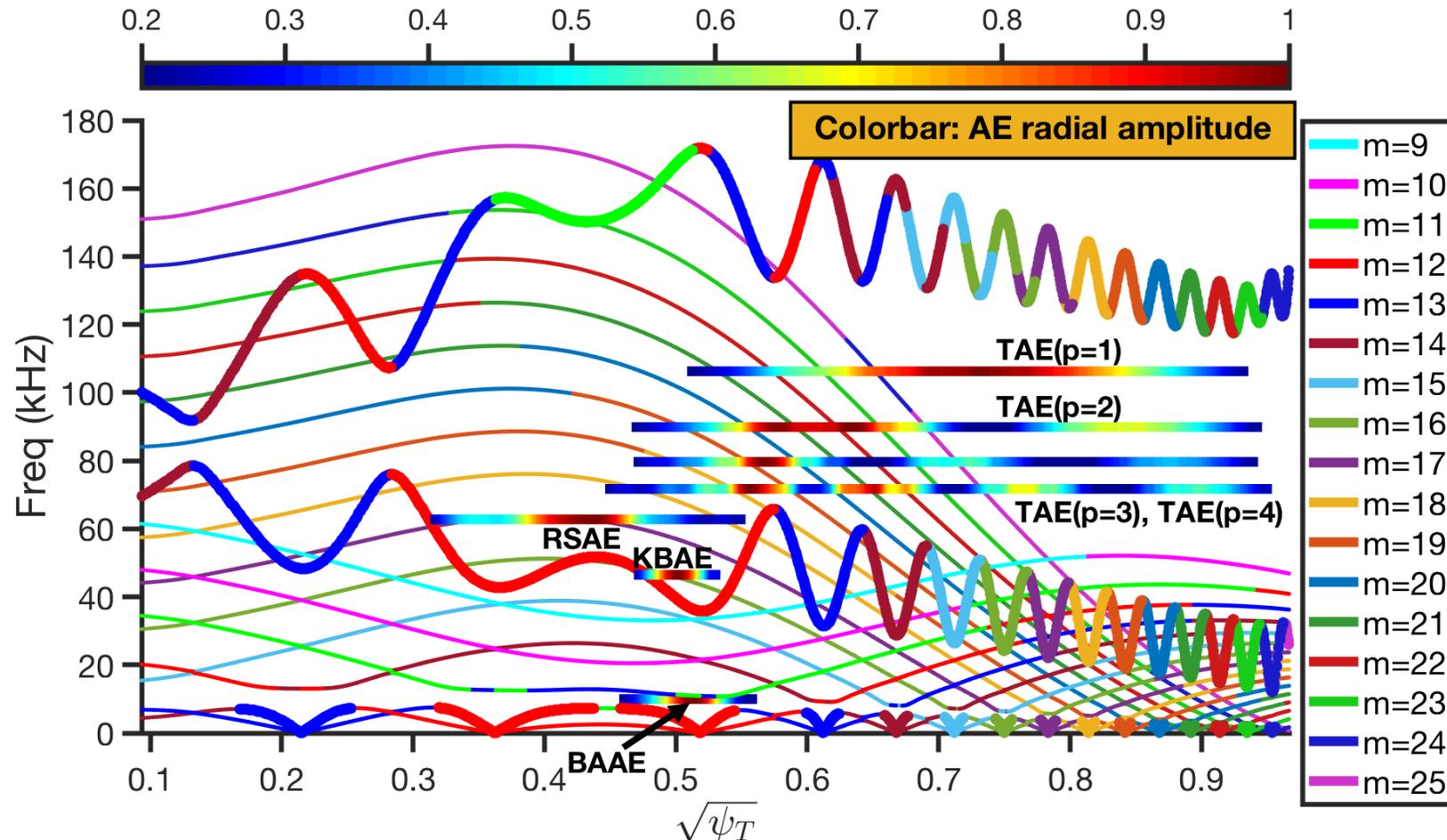
- EI non-perturbatively modifies the RSAE mode structure with radially varied poloidal phase angle (i.e., triangle shape mode structure).
- ✓ **FOW stabilization of RSAE in high-n regime**, good agreements on RSAE dispersion relation with other codes.

# Outlines

- Background
- Discretized eigenmode
- Continuous spectrum
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- Summary

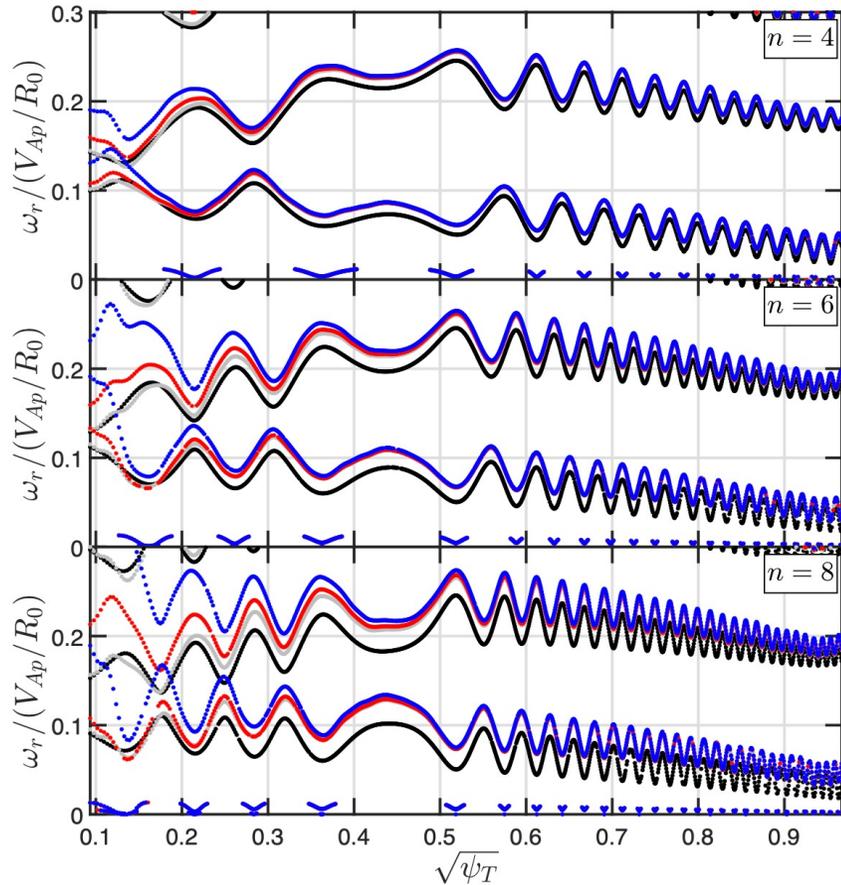
# Full-MHD results of n=4 continuum in DIII-D shot #159243

- Independent continuum module has been developed in MAS framework.
- ✓ Full-MHD calculations of Alfvénic and acoustic continua, with carefully identifying polarization and poloidal mode numbers.

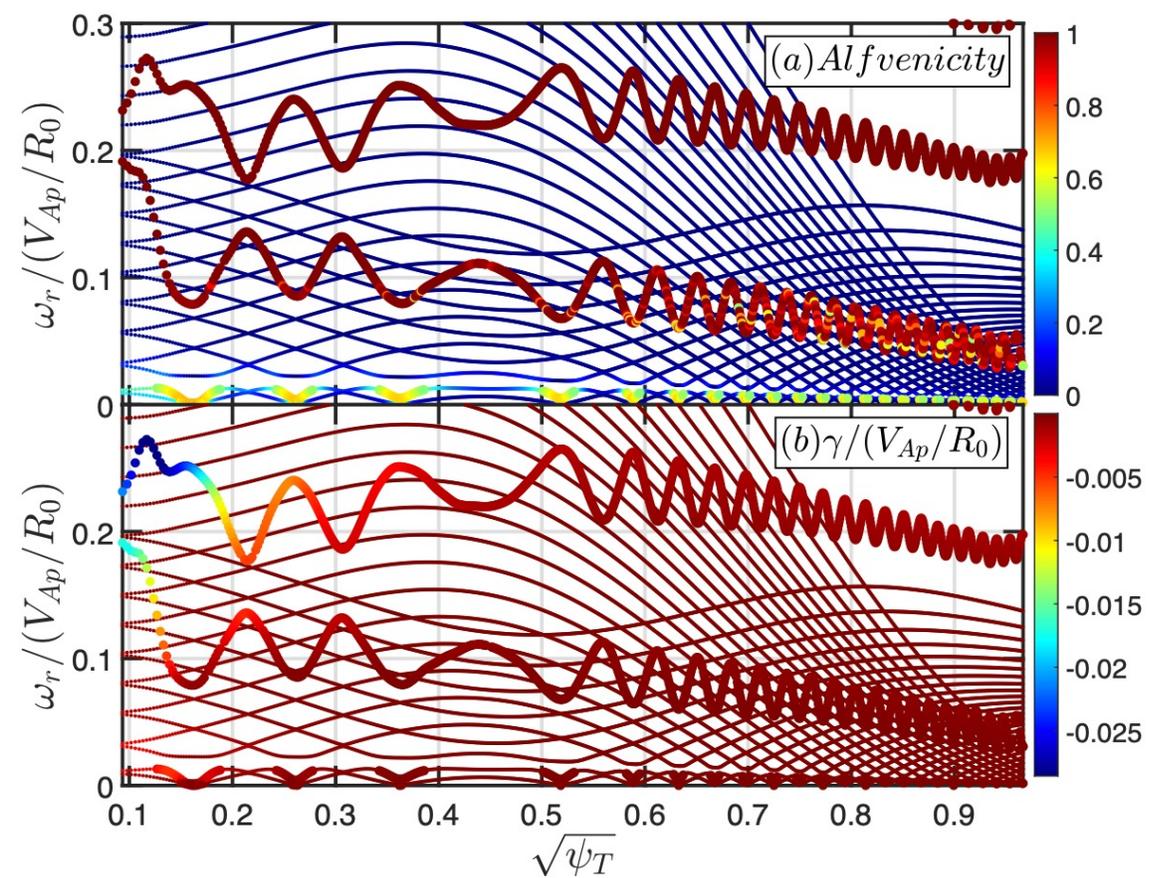


# Landau-fluid results with kinetic effects

Ion diamagnetic drift upshifts  
continuum frequency



Upper panel: polarization indicated by Alfvénicity  
Lower panel: damping rate



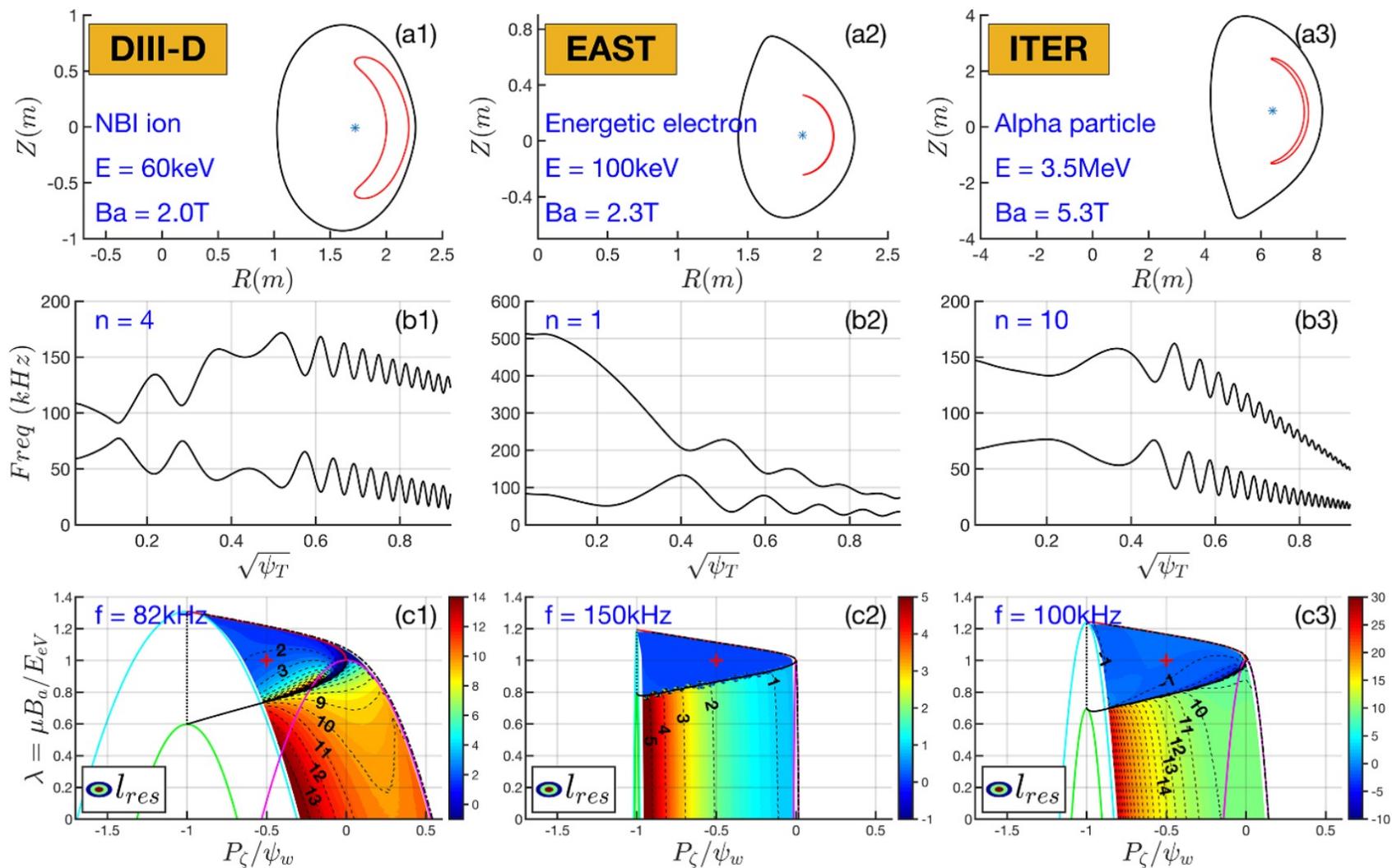
- ✓ Effects of ion diamagnetic drifts
- ✓ Landau damping and radiative damping from thermal plasmas

W. J. Sun, to submit

# Outlines

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# Resonance condition of typical EPs in phase space



$$\Omega(\mathcal{E}, P_\zeta, \mu) = n\langle\omega_\zeta\rangle - l\langle\omega_\theta\rangle - \omega_n = 0,$$

MAS compute poloidal and toroidal frequencies ( $\omega_\zeta$  and  $\omega_\theta$ ) by tracing the particle motion.

- **Test particle module** has been developed for calculating EP characteristic frequencies in general geometry.
- The small dimensionless orbit width of EEs in present-day tokamak (i.e., EAST) is close to alpha particles in future fusion reactor (i.e., ITER), which mainly interact with AEs through precessional-drift resonance.

# Outlines

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# Summary on MAS capability

- ✓ Five-field Landau-fluid model for bulk plasmas
  - Cover cross-scale plasma modes: low- $n$  MHD, mediate- $n$  AE, high- $n$  drift wave instability
  - Diamagnetic drift, Landau damping, FLR, finite parallel electric field  $E_{\parallel}$
- ✓ Drift-kinetic EE and gyrokinetic EI
  - Precessional-drift resonance, transit resonance, FOW, FLR
- ✓ Continuous spectra
  - Ideal full-MHD continua: SAW and ISW
  - Landau-fluid continua: KAW and ISW (ion diamagnetic drift, Landau and radiative damping)
- ✓ Resonance condition in phase space
  - Numerical calculation of characteristic frequencies by tracing the particle orbit
  - Resonance line calculation for each harmonics
- ✓ Wide applications for AE stability analysis in EAST, HL-2A/3 and DIII-D experiments.

# MAS research activities

## • Code developments and physical applications

- [1] [Bao J.](#), Zhang W.L., Li D., Lin Z. et al, MAS: A versatile Landau-fluid eigenvalue code for plasma stability analysis in general geometry [Nucl. Fusion 63 076021 \(2023\)](#)
- [2] [Bao J.](#), Zhang W.L., Li D., Lin Z. et al, Global simulations of kinetic-magnetohydrodynamic processes with energetic electrons in tokamak plasmas [Nucl. Fusion 64 016004 \(2024\)](#)
- [3] [Bao J.](#), Zhang W. L., Li D. and Lin Z. Effects of plasma diamagnetic drift on Alfvén continua and discrete eigenmodes in tokamaks [Journal of Fusion Energy 39 382-389 \(2021\)](#)
- [4] [Bao J.](#), Zhang W. L. and Li D. Global simulations of energetic electron excitation of beta-induced Alfvén eigenmodes [Acta Physica Sinica 72\(21\) 215216 \(2023\)](#)
- [5] [Bao J.](#), Zhang W. L., Lin Z., Cai H. S. et al, Global destabilization of drift-tearing mode with coupling to discretized electron drift wave instability <https://arxiv.org/abs/2407.10613>, under review in [Nucl. Fusion \(2024\)](#)
- [6] Xu X. R., Guo L. Z., Sun W. J., [Bao J.](#) et al., Gyrokinetic modelling of energetic ion response to arbitrary wavelength electromagnetic fluctuations in magnetized plasmas under review in [Plasma Phys. Control. Fusion \(2024\)](#)

## • V&V collaboration

- [7] Brochard G., [Bao J.](#), Liu C. et al Verification and validation of gyrokinetic and kinetic-MHD simulations for internal kink instability in DIII-D tokamak [Nucl. Fusion 62 036021 \(2022\)](#)
- [8] Jiang P. Y., Liu Z. Y., Liu S. Y., [Bao J.](#) and Fu G. Y. 2024 Development of a gyrokinetic-MHD energetic particle simulation code. I. MHD version [Physics of Plasmas 31 \(7\) \(2024\)](#)

## • Experimental collaboration

- [9] Zhao N., [Bao J.](#), Chen W. et al Multiple Alfvén eigenmodes induced by energetic electrons and nonlinear mode couplings in EAST radio-frequency heated H-mode plasmas [Nucl. Fusion 61 046013 \(2021\)](#)
- [10] Su P., Lan H., Zhou C. [Bao J.](#) et al, Bursting core-localized ellipticity-induced Alfvén eigenmodes driven by energetic electrons during EAST ohmic discharges [Nucl. Fusion 64 036019 \(2024\)](#)
- [11] Chen W., Yu L. M., Shi P. W., Hou Y. M., Shi Z. B., [Bao J.](#), Qiu Z. et al Nonlinear Dynamics and Effects of Fast-ion Driven Instabilities in HL-2A NBI Heating High- $\beta_N$  H-mode [Plasmas Physics Letters A, 129983 \(2024\)](#)
- [12] Zhu X., Qiu Z. Y., [Bao J.](#) et al, Toroidal Alfvén eigenmodes excited by energetic electrons in EAST low-density ohmic plasmas [Nucl. Fusion 64 126023 \(2024\)](#)

**Thank you for your attention!**