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On the interaction of n=0 modes with energetic ions

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n=0 axisymmetric modes

Different types of axisymmetric, n=0 modes:

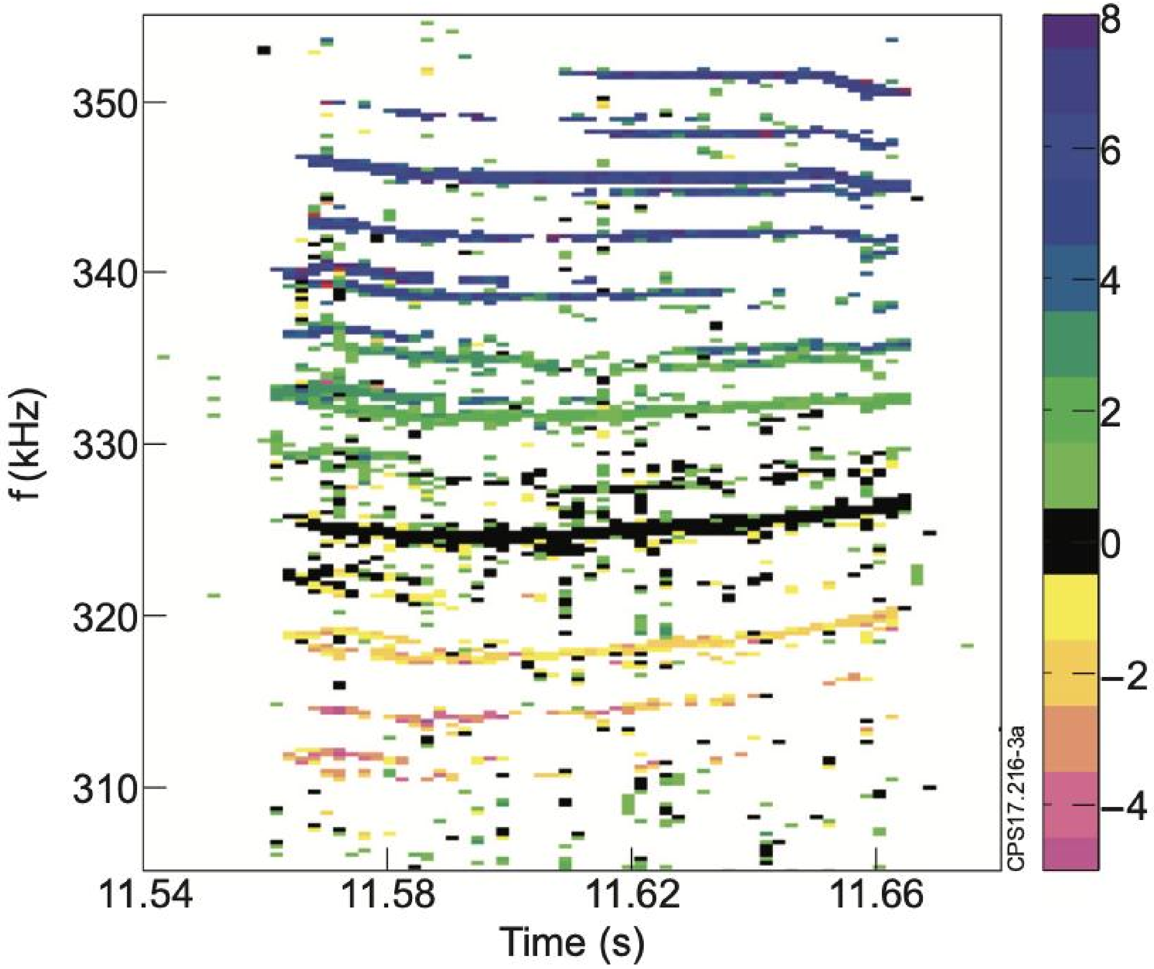
- Geodesic acoustic modes (GAM) -> low frequency (sound)
- Global Alfvén eigenmodes (GAE) -> high (Alfvén) frequency
- Vertical displacements (VDE) -> purely growing, zero frequency
- Vertical Displacement Oscillatory Modes (VDOM) - high (Alfvén) frequency

GAM, GAE and VDOM can all be driven unstable by **resonant** interaction with fast ion orbits.

My talk will focus on VDOM.

JET experimental observations of n=0 modes (2016)

- H. J. C. Oliver et al. 2017 PoP
- Are these GAE or VDOM?



JET experimental observations of n=0 modes (2021)

Nucl. Fusion **61** (2021) 114006 V.G. Kiptily *et al*

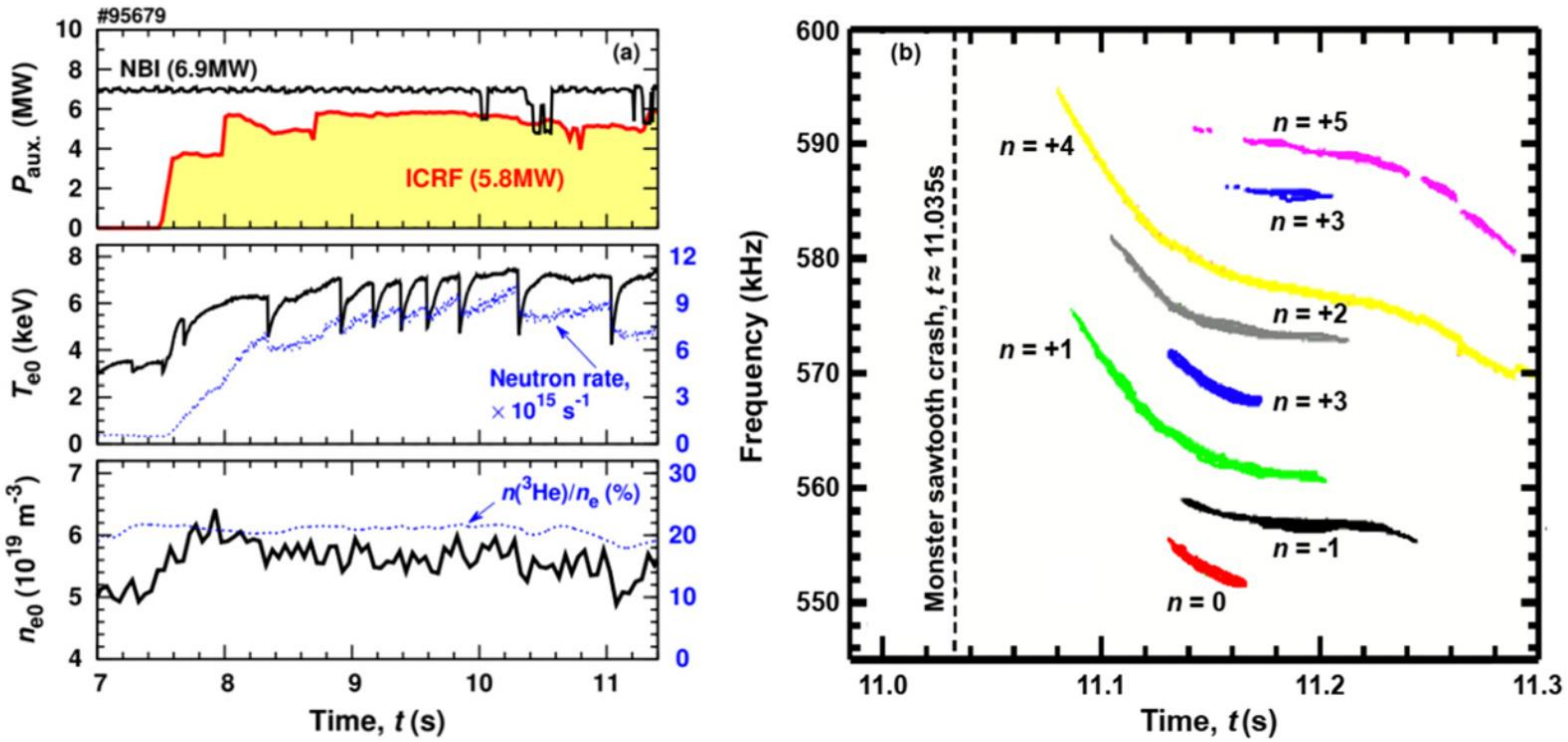
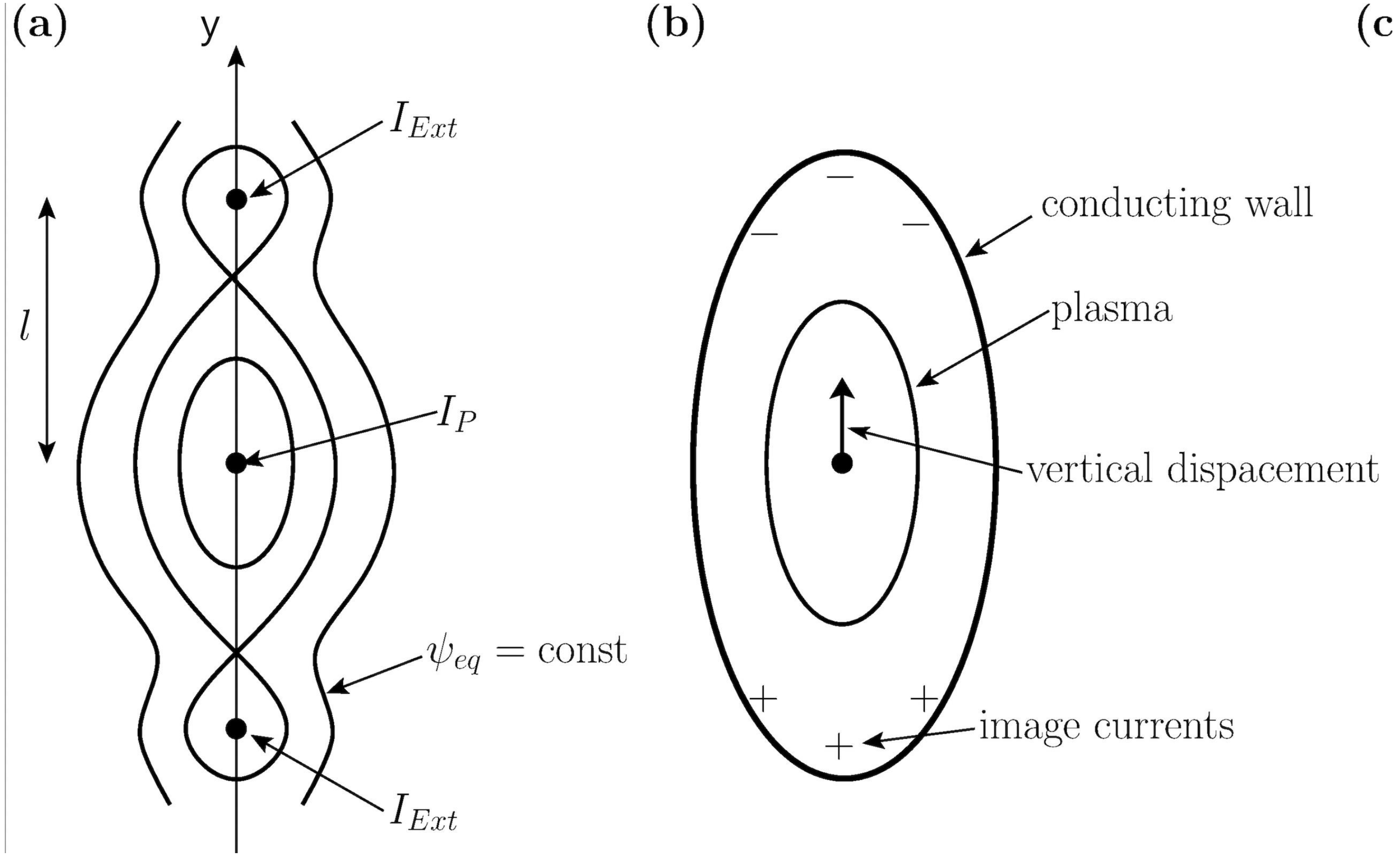


Figure 1. (*a*) Overview of JET pulse #95679 (3.7 T/2.5 MA) in D-^{3}He plasmas with energetic D-ions and fusion-born alpha particles. (*b*) Dynamics of Alfvén activities in the EAE frequency range after the monster sawtooth crash at $t \approx 11.035$ s.

See also more recent JET discharges from recent 2023 DT experimental campaign -> Tritium plasma with D neutral beams produces abundance of alphas from beam-target reactions -> Alpha particles driving VDOM unstable?

What are n=0 Vertical Displacement Oscillatory Modes (VDOM)?

n=0 VDOM are natural modes of oscillation in a tokamak plasma.



Analytic theory - no X-point effects
Resistive Wall – Ideal Plasma - Cubic dispersion relation

$$\gamma^3 + \gamma^2 \frac{1}{\tau_\eta} \frac{1}{1 - \hat{e}_0 D} + \gamma \omega_0^2 + \omega_0^2 \frac{1}{\tau_\eta} \frac{1}{1 - D} = 0$$

$\gamma = -i\omega$

τ_η: resistive wall time

$e_0 = \frac{b^2 - a^2}{b^2 + a^2}$: ellipticity parameter; $\hat{e}_0 = \frac{b}{a+b} e_0$

$D\left(\frac{b}{b_w}, \kappa\right) = \frac{\kappa^2+1}{(\kappa-1)^2} \left\{1 - \left[1 - \frac{\kappa^2-1}{\kappa^2}\left(\frac{b}{b_w}\right)^2\right]^{1/2}\right\}$: geometrical wall parameter; $\kappa = \frac{b}{a}$

$\omega_0 = \left[\frac{D-1}{1 - \hat{e}_0 D}\right]^{\frac{1}{2}} \gamma_\infty$: oscillation frequency (for D>1); $\gamma_\infty = \frac{2ab}{a^2+b^2}\left(1 - \frac{a}{b}\right)^{1/2} \tau_A^{-1}$: no-wall growth rate ($D$=0)

Three roots, D>1 (passive wall stabilization), $\tau_\eta \omega_0 \gg 1$

$$\omega = \pm\omega_0 - \mathrm{i}\frac{1}{2\tau_\eta}\frac{D(1-\hat{e}_0)}{(D-1)(1-\hat{e}_0 D)},$$

$$\gamma = \frac{1}{(D-1)\tau_\eta}.$$

oscillatory solutions (VDOM)

n=0 resistive wall mode – requires active feedback stabilization

- A.Yolbarsop et al, *Analytic theory of ideal-MHD vertical displacements in tokamak plasmas*, Plasma Phys. Contr. Fusion 64, 105002 (2022).
- T. Barberis et al, *Vertical displacement oscillatory modes in tokamak plasma*, Journal of Plasma Physics 88, 905880511 (2022).

A new fast ion instability

One branch of the n=0 dispersion relation corresponds to oscillatory modes with a discrete frequency, *close to the poloidal Alfvén frequency*, not interacting with the Alfvén continuum, weakly damped by wall and/or plasma resistivity.

We find that this mode, dubbed VDOM (Vertical Displacement Oscillatory Mode), can be destabilized by the resonance with fast ion orbits (both passing and trapped)

T. Barberis et al, *Fast ion driven vertical modes in magnetically confined toroidal plasmas*, Nuclear Fusion Letter 2022; DOI: 10.1088/1741-4326/ac5ad0 .

How to distinguish between GAE and VDOM driven unstable by fast ions?

GAE	VDOM
Mostly an internal mode	External mode
Peaking near minimum of continuum spectrum	Nearly rigid vertical shift
Even parity wrt poloidal angle -> ballooning structure favored	Odd parity wrt poloidal angle -> up-down vertical motion
Resonance condition is $\omega = p\omega_{b/t}$, resonance with fast trapped ions important	Resonance condition is $\omega = p\omega_{b/t}$, resonance with fast passing ions important
Frequency depends weakly on ellipticity and wall-plasma distance, more importantly on details of the q profile	Frequency depends importantly on ellipticity and wall-plasma distance, less importantly on details of the q profile

Numerical simulations

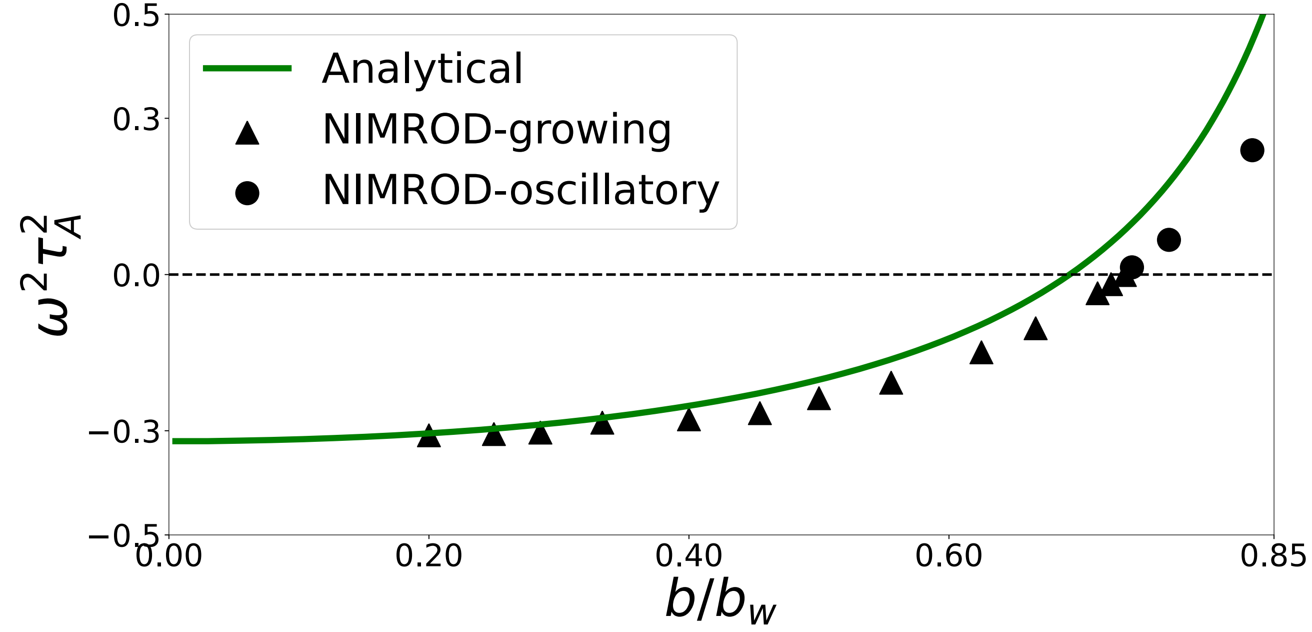
Very few MHD codes can treat the n=0 magnetic X-point resonance correctly -> JOREK, M3D, **NIMROD.**

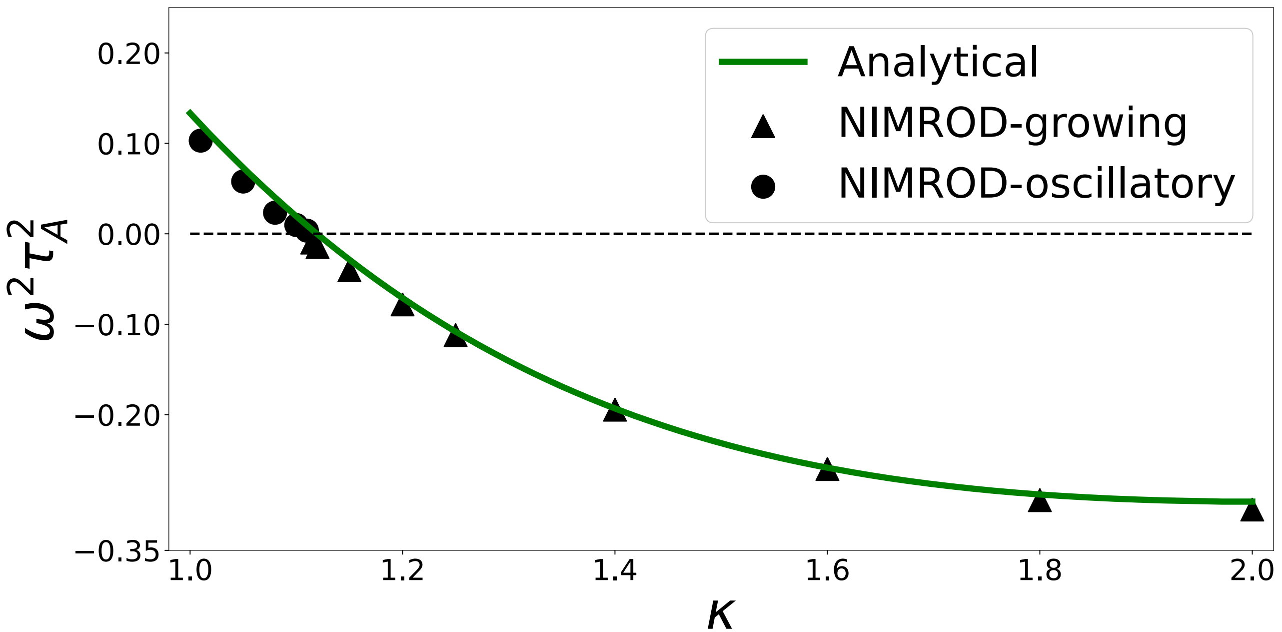
Straight tokamak NIMROD simulation results show very good agreement with analytic theory

Main differences between simulation and analytic theory:

- Wall shape rectangular instead of confocal ellipse
- Low density/High resistivity «halo» plasma instead of vacuum

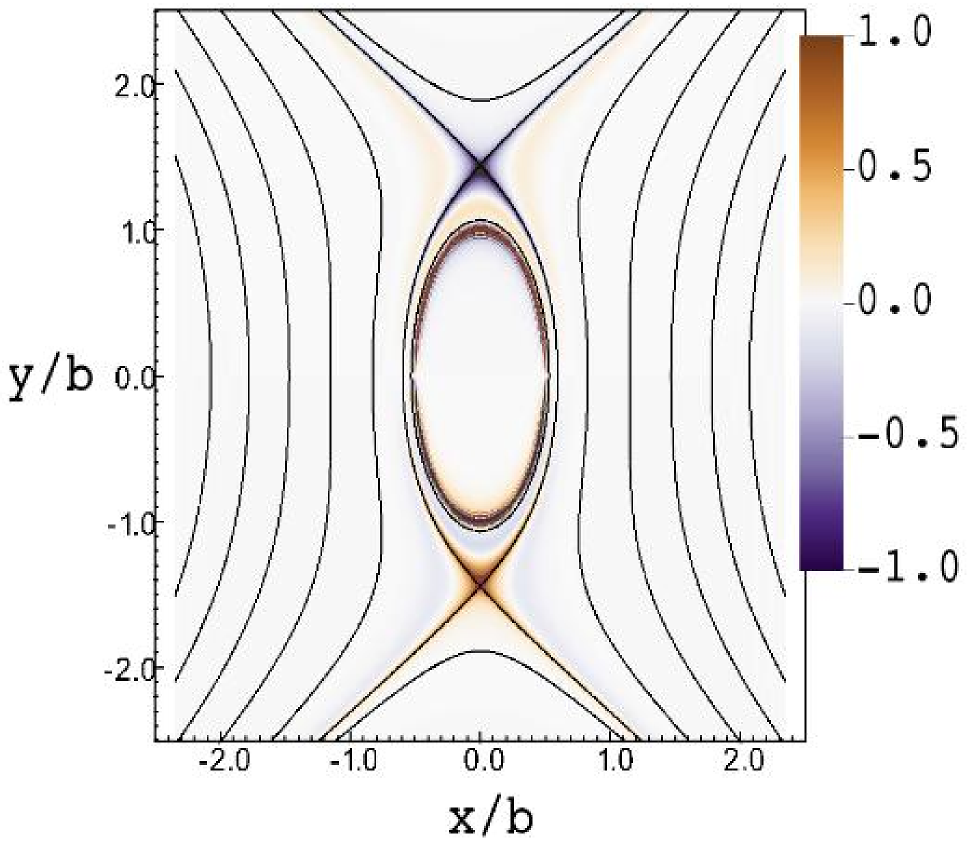
D. Banerjee et al, PoP 2024,
https://doi.org/10.1063/5.0184340



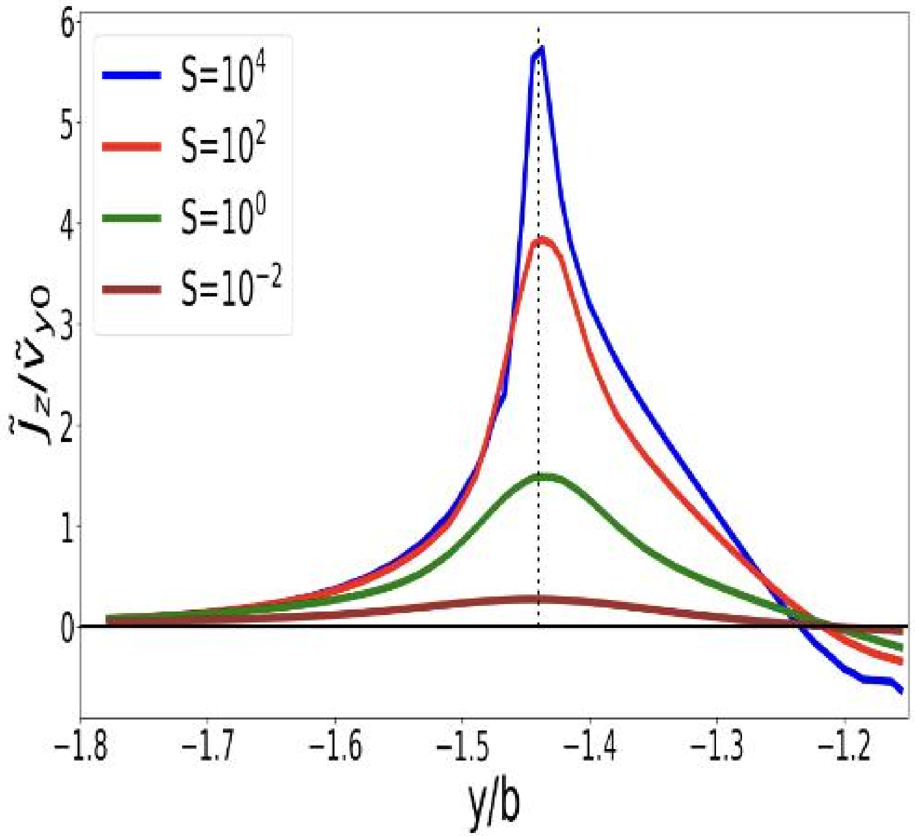


Straight tokamak

X-point currents supported by halo plasma



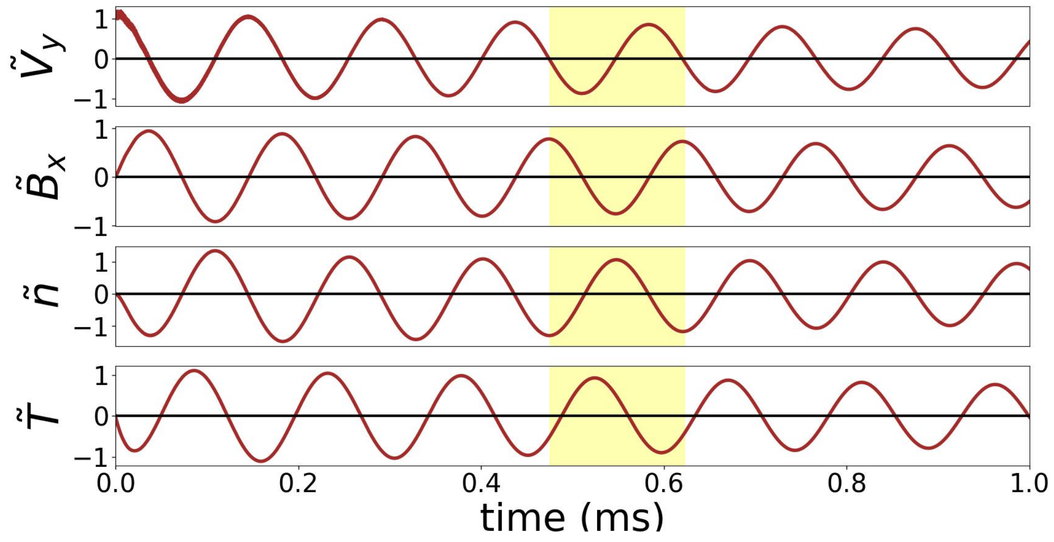
(a) $\tilde{J}_z$

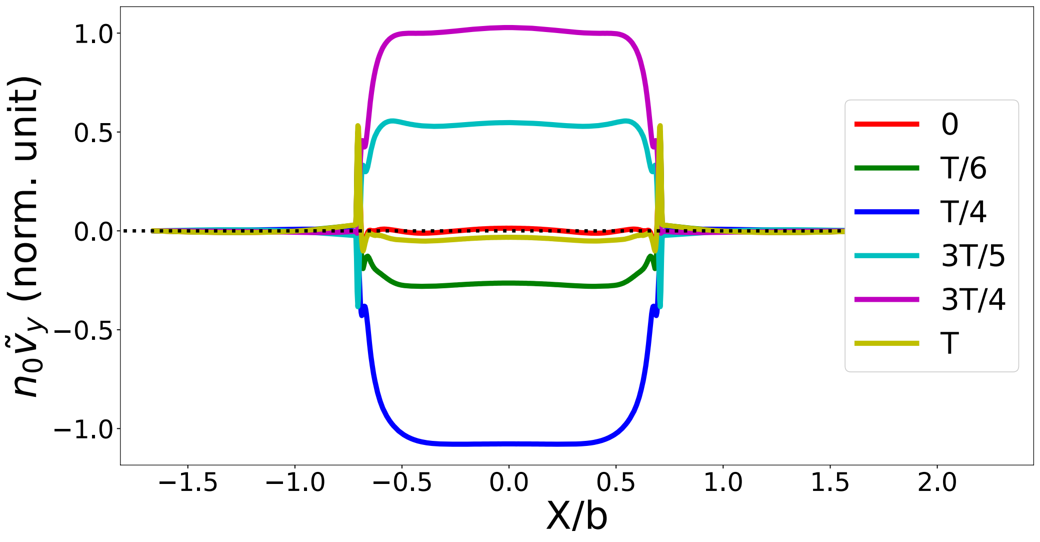


(b) Normalized $\tilde{J}_z$ profile across lower X-point

VDOM stable oscillations characterized by nearly rigid shift mode structure

straight tokamak





NIMROD simulations of JET discharges

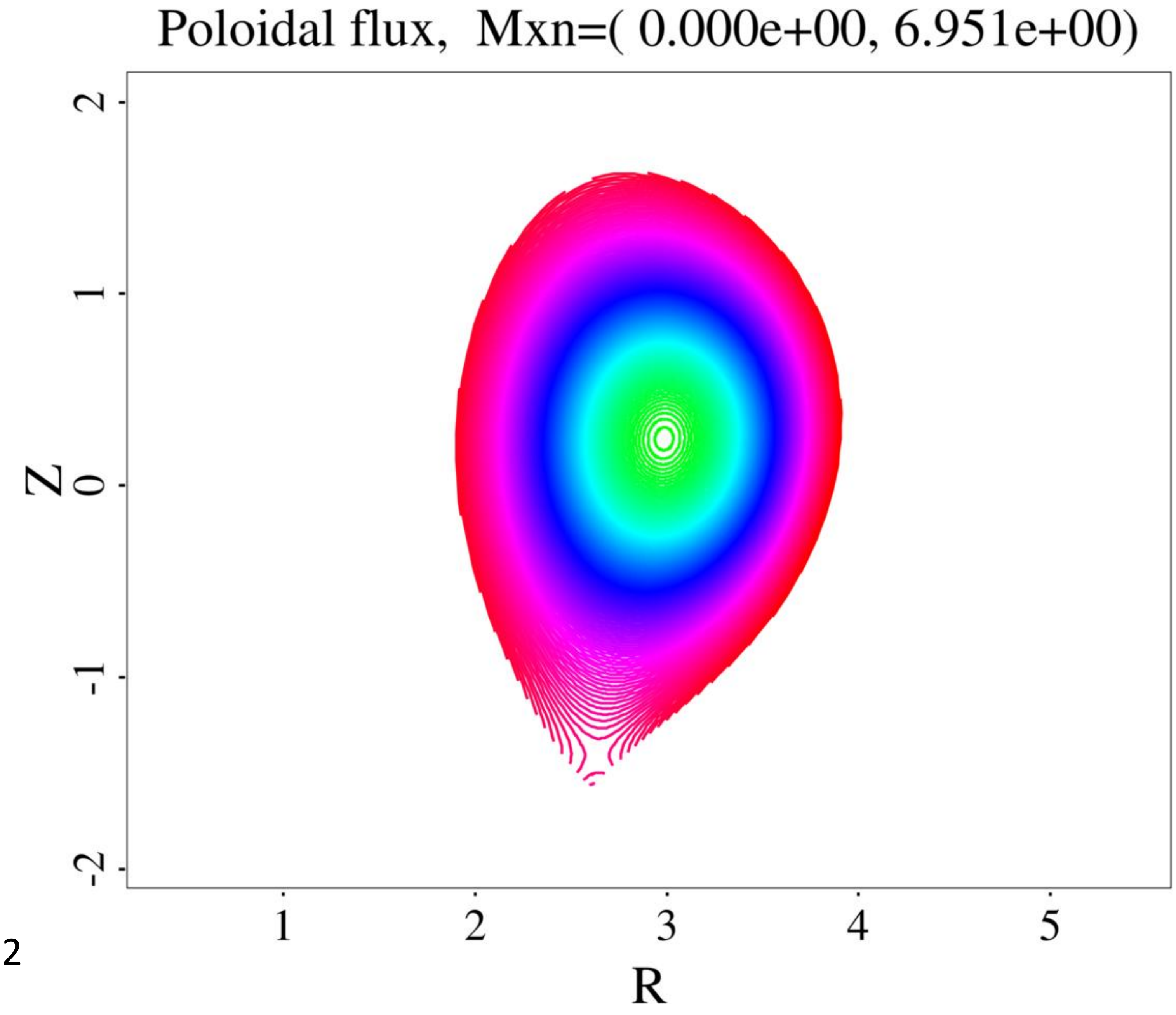
Pressure and current density profiles from EFIT file shot #102371

Ideal wall = simulation boundary

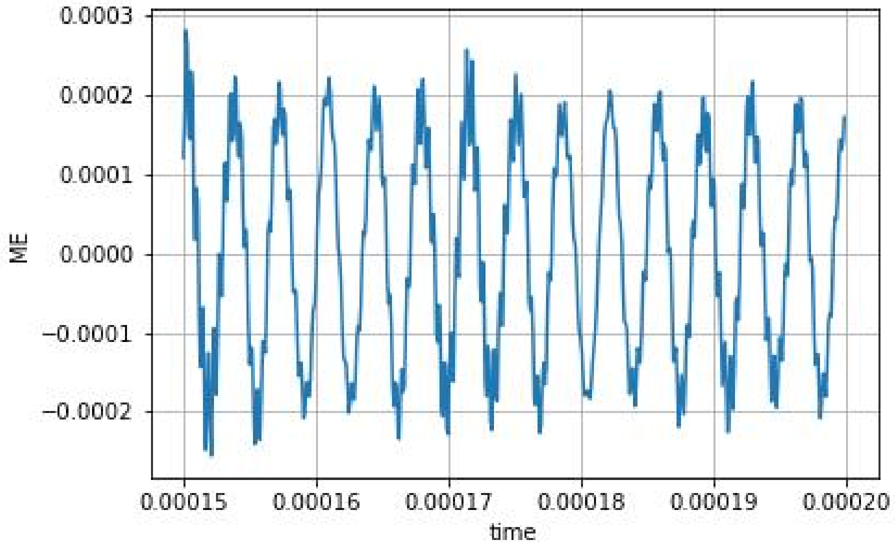
Look for stable oscillations
→ Minimize damping sources

Simulation parameters:
peak $n_{eq} = 5 * 10^{-19}, Z_{eff} = 1.75$

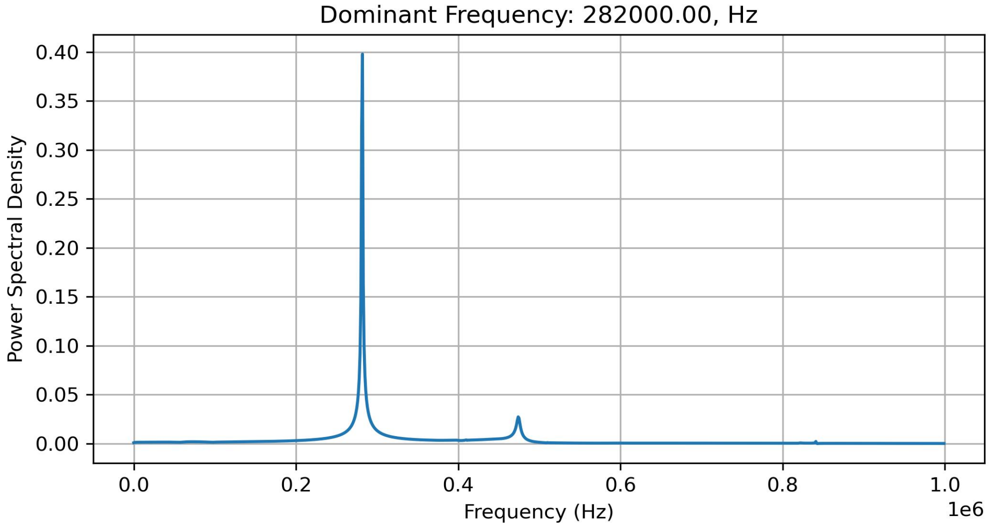
T. Barberis et al, Nuclear Fusion 2024,
https://doi.org/10.1088/1741-4326/ad7ed2



VDOM stable oscillations found when perturbing the equilibrium with a «vertical push» of the plasma



Magnetic energy time trace

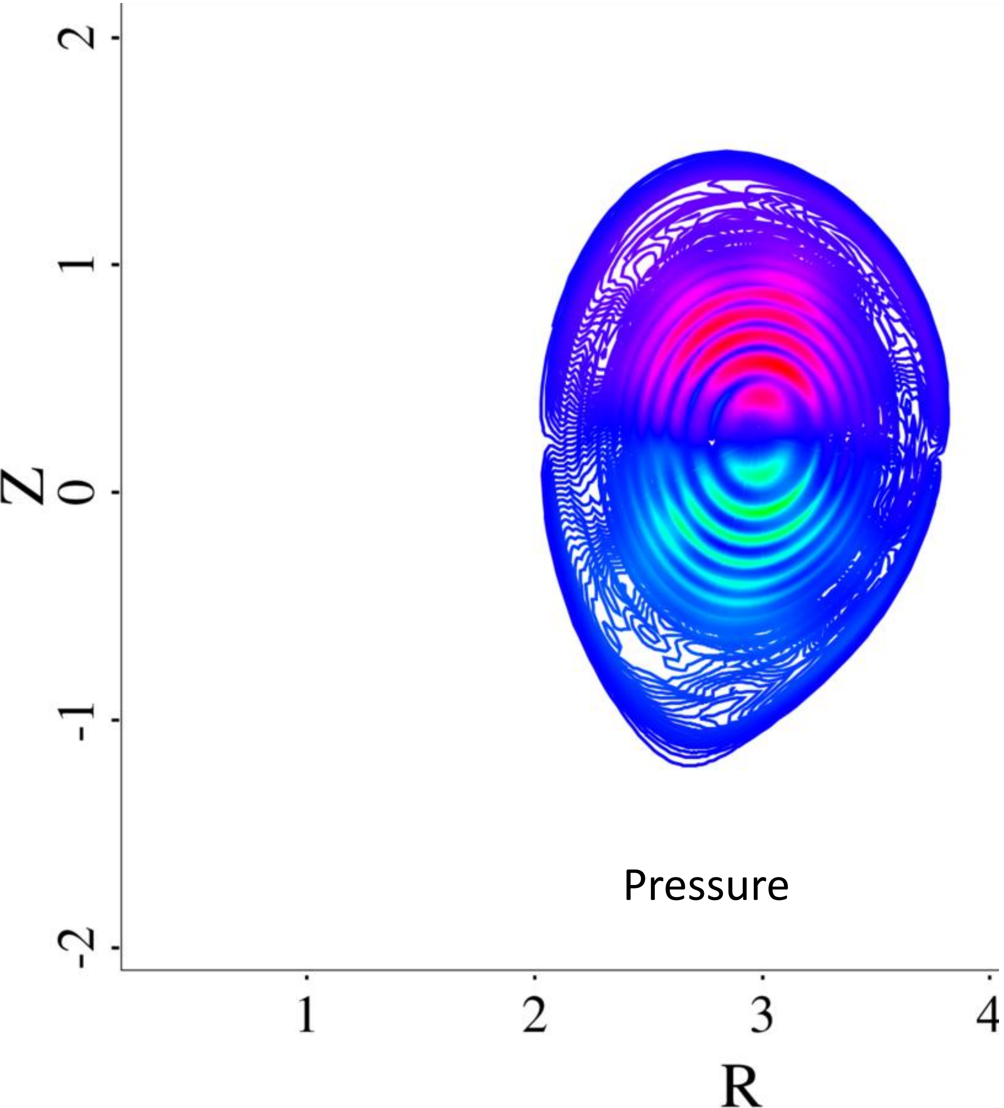


FFT signal shows two distinct peaks:

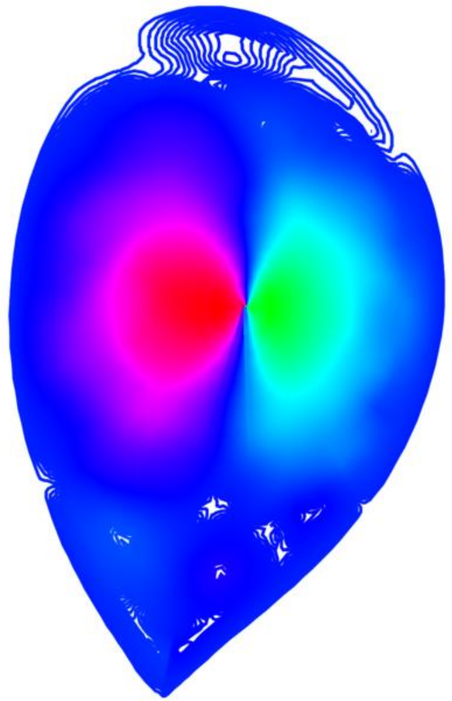
Dominant peak at $\sim 282\ kHz$ and secondary peak at $\sim 474\ kHz$

Initial perturbation → «vertical push»

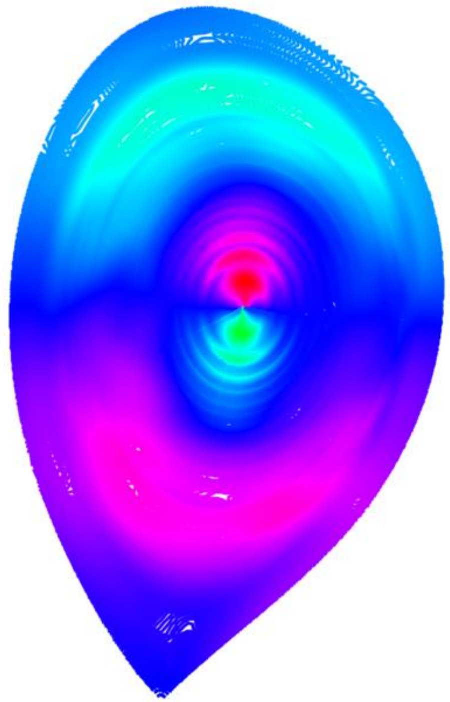
Perturbation contour plots showing the mode structure



Pressure



Perturbation in B norm.

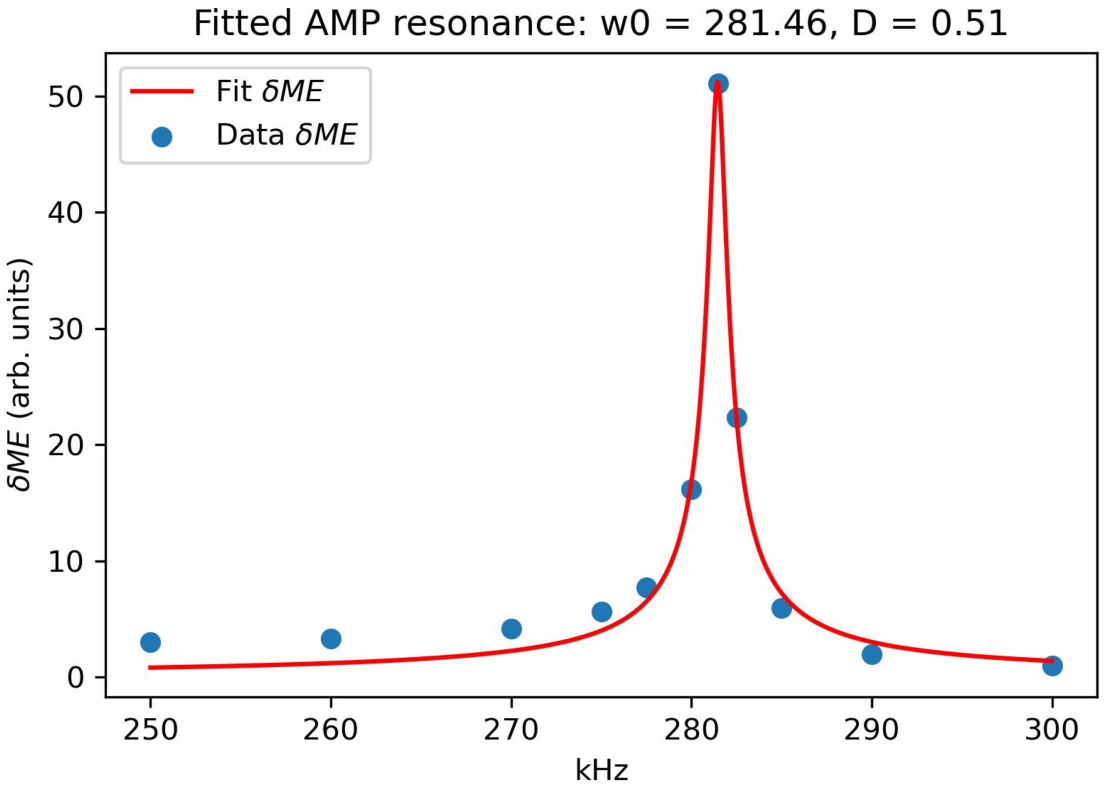


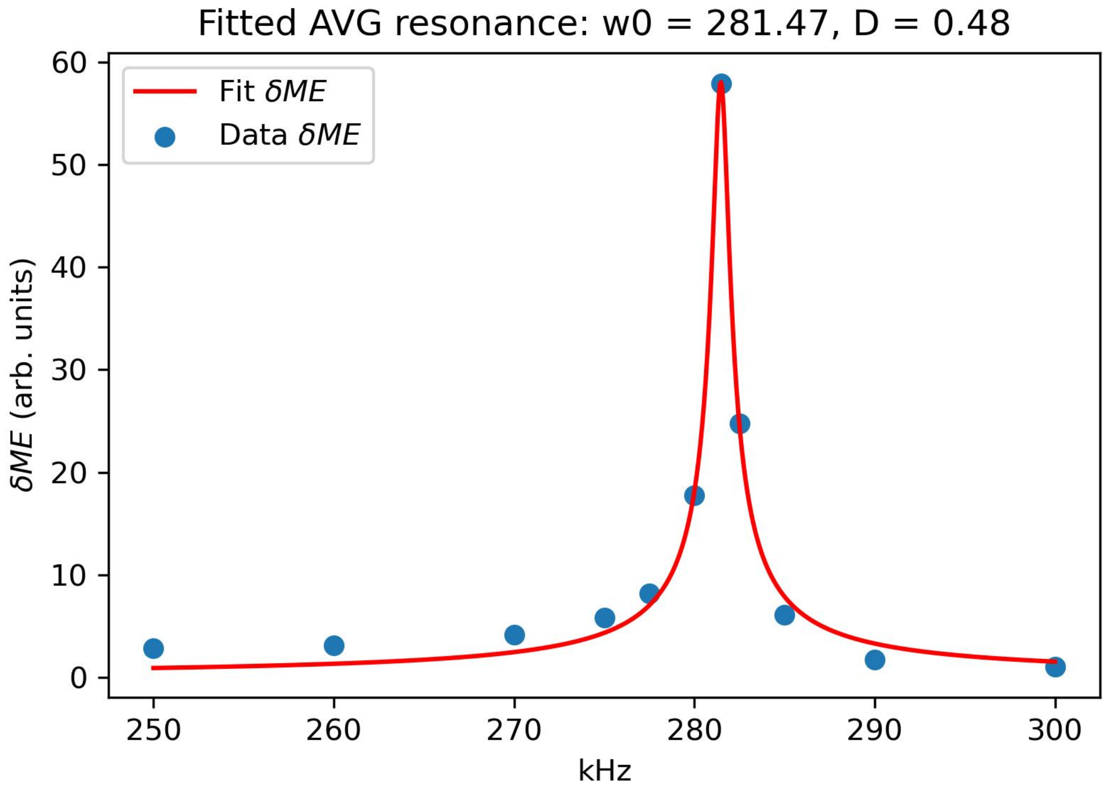
Perturbation in B tang.

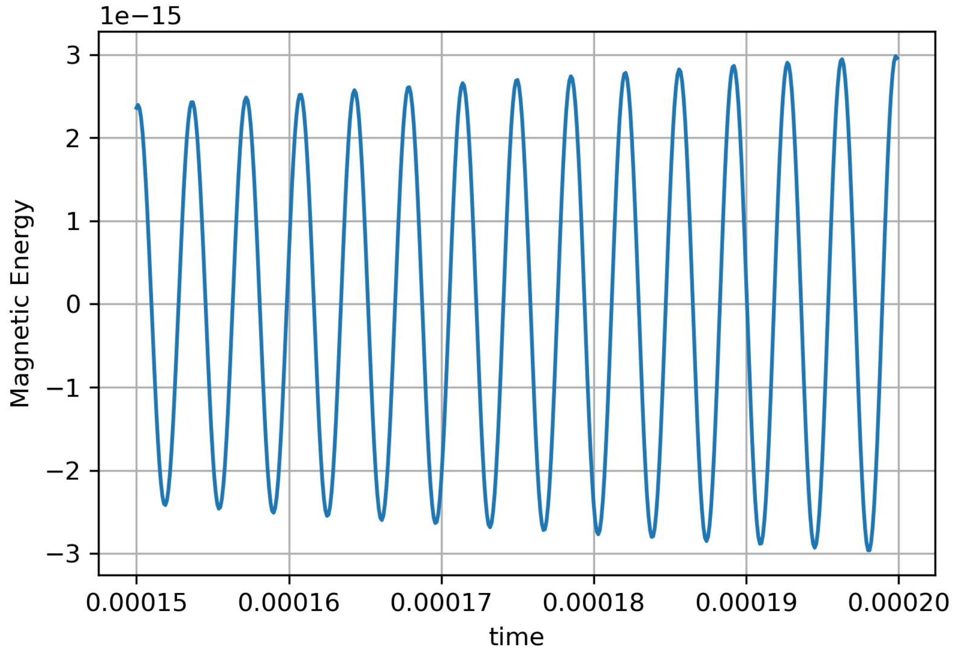
New initial perturbation: localized oscillator in T

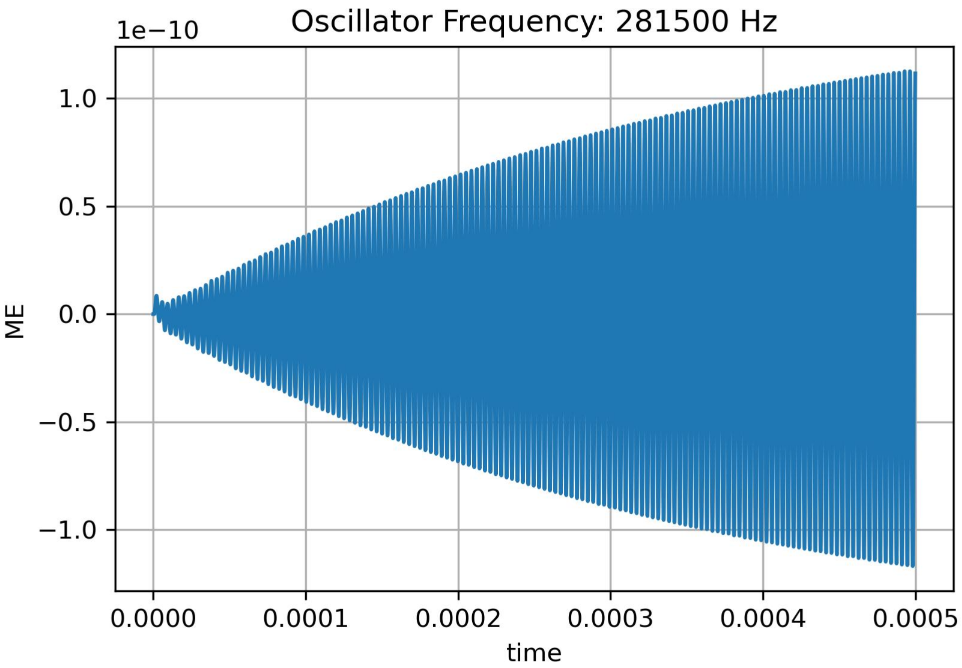
Extra term in the temperature evolution $\propto \sin(\omega_0 t) * \exp(\frac{(r-r_0)^2+(z-z_0)^2}{\Delta^2})$

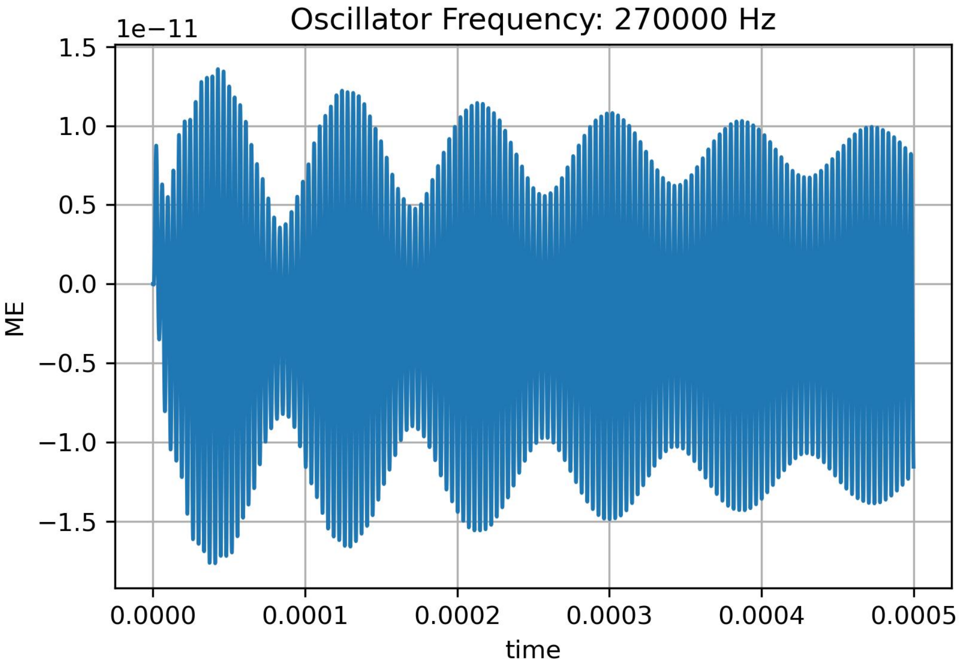
Data fitted with $|Y| \propto \frac{C}{\sqrt{(\omega-\omega_0)^2+D^2}}$ assuming resonance condition $Y \propto \frac{C}{(\omega-\omega_0)+iD}$






1e−15
Magnetic Energy
time
3
2
1
0
−1
−2
−3
0.00015
0.00016
0.00017
0.00018
0.00019
0.00020


Oscillator Frequency: 281500 Hz
1e−10
ME
time
1.0
0.5
0.0
−0.5
−1.0
0.0000
0.0001
0.0002
0.0003
0.0004
0.0005


Oscillator Frequency: 270000 Hz
1e−11
ME
time
1.5
1.0
0.5
0.0
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0.0000
0.0001
0.0002
0.0003
0.0004
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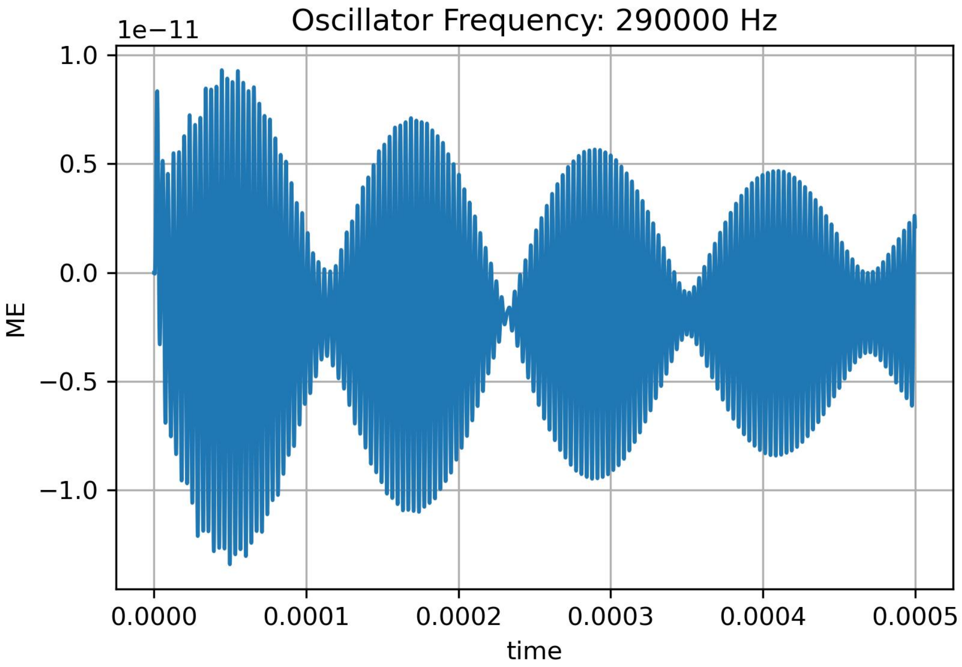
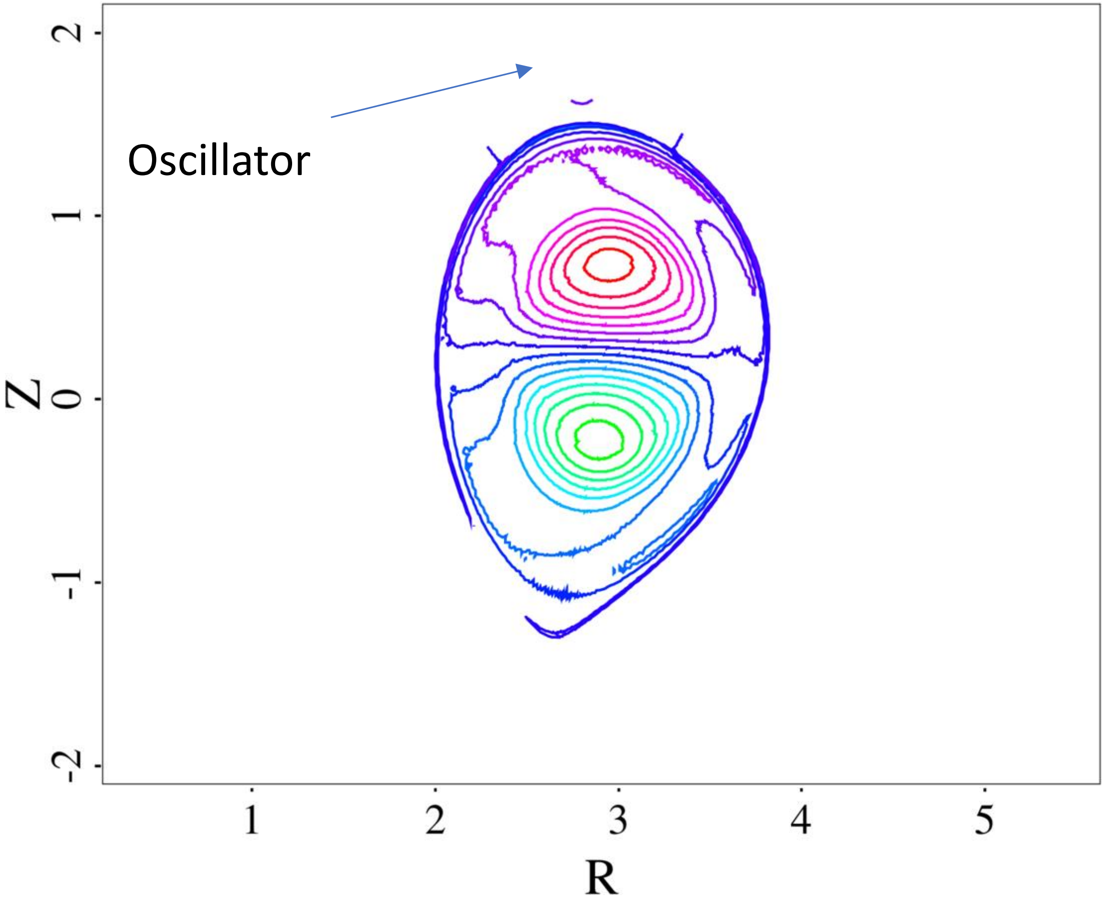
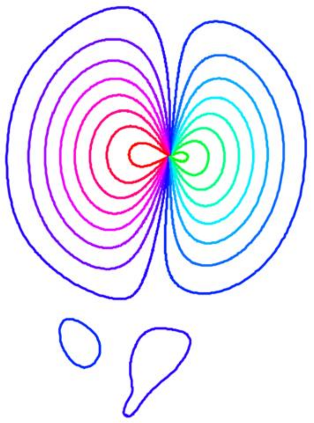

Oscillator Frequency: 290000 Hz
1e−11
ME
time
1.0
0.5
0.0
−0.5
−1.0
0.0000
0.0001
0.0002
0.0003
0.0004
0.0005

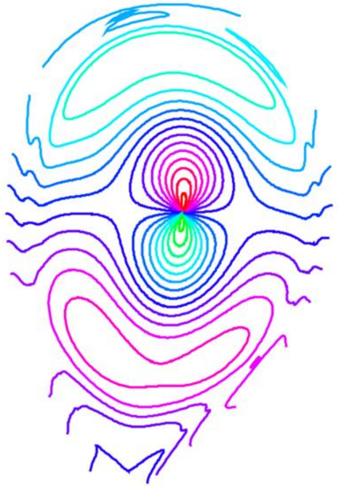
Contour plots: oscillator at 281.5 kHz, «vertical displacement oscillatory mode»: space structure



Pressure

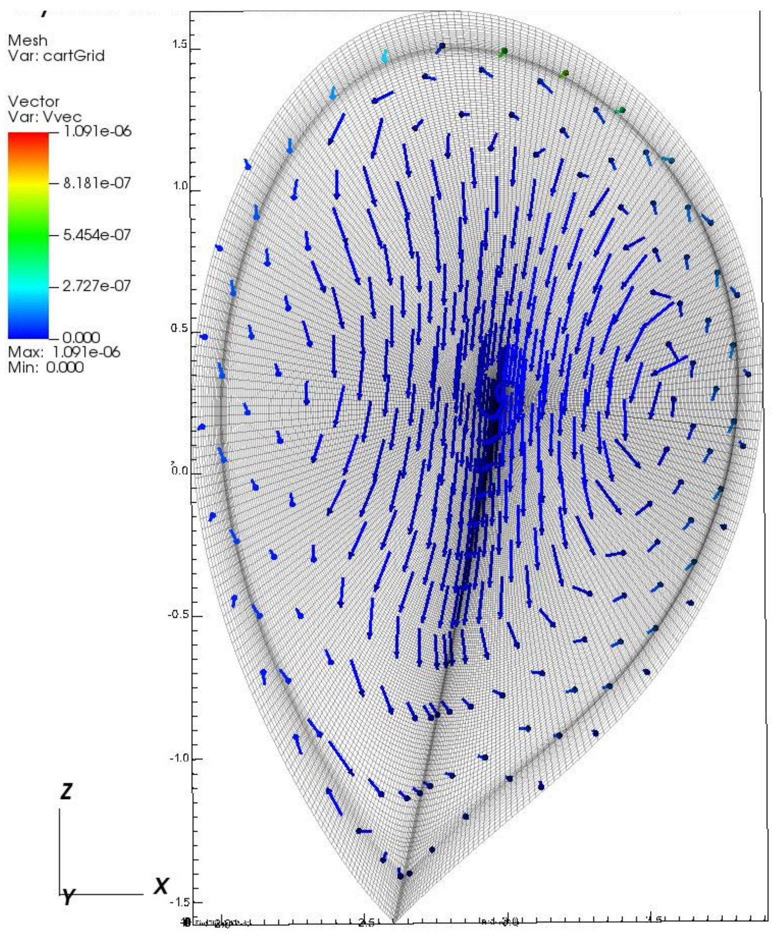


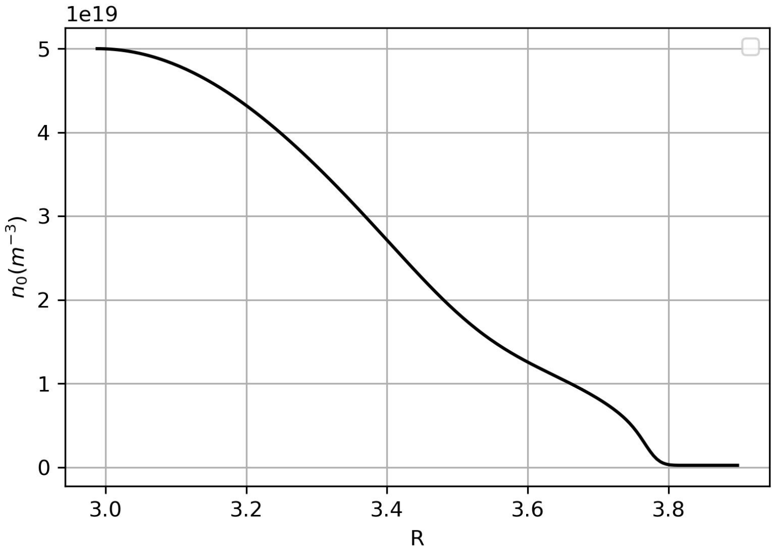
Perturbation in B norm.



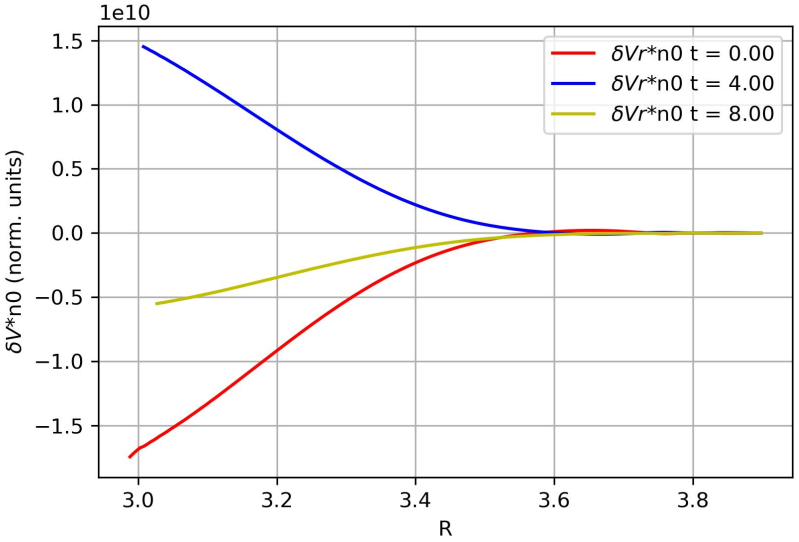
Perturbation in B tang.

Perturbed velocity: oscillator at 281.5 kHz,
«vertical displacement oscillatory mode» space structure

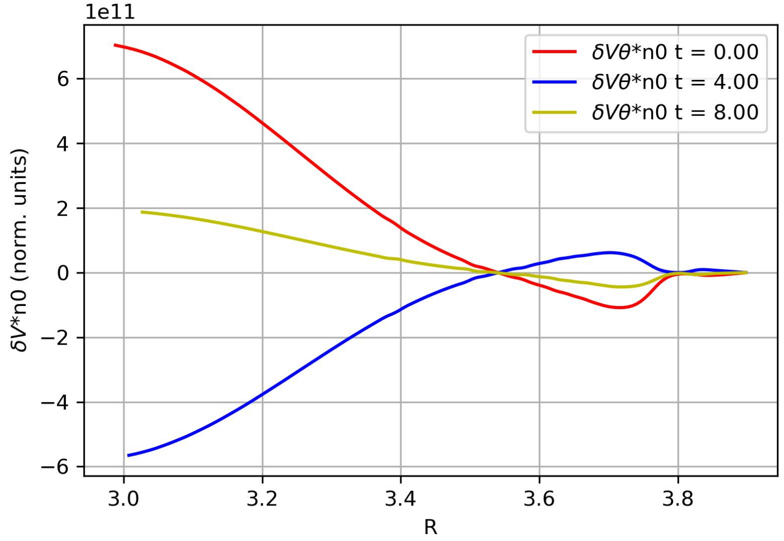




Perturbed momentum in the direction
normal ($\sim r$) to flux surfaces



Perturbed momentum in the direction
Tangential ($\sim \theta$) to flux surfaces



Fast ion destabilization of VDOM

$$-\gamma^2 \delta I = \delta W_{MHD} + \delta W_{fast}$$

- Zeroth order in $\epsilon_h = n_h/n_e \rightarrow$ Ideal MHD solution
- First order in $\epsilon_h \rightarrow$ neglect $O(\epsilon_h)$ correction to the oscillation frequency and keep only the imaginary part of δW_{fast}

Dispersion relation: $\omega = \pm\omega_0 - i2\omega_0\gamma_\eta + i\delta\widehat{W}_h$

Assume $\delta\widehat{W}_h \sim Im(\delta\widehat{W}_{fast}) = i\omega_0^2\lambda_h + O(\gamma^2/\omega_0^2) \rightarrow \quad \omega = \omega_0 + i\gamma_{tot}$ with $\gamma_{tot} = -\gamma_\eta + \frac{1}{2}\omega_0\lambda_h$

Threshold: competition between the damping rate due to wall resistivity and the fast ion drive:

$$\lambda_h > \frac{\gamma_\eta}{2\omega_0} \propto \epsilon_h$$

Fast ion drive

Hybrid kinetic MHD model and dispersion relation

We consider the fast ions effect perturbatively in the framework of the Hybrid Kinetic MHD, where the dispersion relation can be obtained using quadratic forms:

$$-\gamma^2 \delta I = \delta W_{MHD} + \delta W_{fast} \tag{1}$$

After standard manipulations discussed in detail in [2], the fast ion term for toroidal mode number n=0, can be expressed as:

$$\delta W_h = \frac{-2\pi^2 c}{Zem^2} \sum_{\sigma} \int dP_\phi d\mathcal{E} d\mu_\perp \tau_\Omega \omega \left[\frac{\partial F}{\partial \mathcal{E}}\bigg|_\Lambda - \frac{\Lambda}{E}\frac{\partial F}{\partial \Lambda}\bigg|_E \right] \sum_{p=-\infty}^{+\infty} \frac{|\Upsilon_p|^2}{\omega + p\omega_\Omega} \tag{2}$$

Ideal MHD analysis $\rightarrow \gamma^3 + \gamma^2 \frac{1}{\tau_\eta}\frac{1}{1-\hat{e}_0 D} + \gamma\omega_0^2 + \omega_0^2 \frac{1}{\tau_\eta}\frac{1}{1-D} = 0, \quad \omega_0 \approx e_0^{1/2}\tau_A^{-1}\sqrt{D-1}$

With fast ions, defining $\lambda_h \propto \mathcal{I}m(\delta W_h)$

$$\omega = \omega_0 + i\gamma_{tot}, \qquad \gamma_{tot} = \omega_0 \lambda_h/2 - \gamma_d \tag{3}$$

Here we will focus on the term coming from the resonant interaction between the mode and the fast ions, neglecting other sources of damping γ_d.

T. Barberis et al, *Fast ion driven vertical modes in magnetically confined toroidal plasmas*
Nuclear Fusion Letter 2022; DOI: 10.1088/1741-4326/ac5ad0 .

Fast ion resonance

- Mode particle reonance condition $\omega - p\omega_\Omega = 0$ $\rightarrow$ Particles need energies high enough in order to have Alfvénic transit or bounce frequencies.
- Fourier coefficients Υ_p give the amplitude for each harmonic contribution.
- Mode destabilization requires $\left[\left.\frac{\partial F}{\partial \mathcal{E}}\right|_\Lambda - \frac{\Lambda}{E}\left.\frac{\partial F}{\partial \Lambda}\right|_E \right] > 0 \rightarrow$
  - Anisotropy in the velocity space, and thus in pitch angle.
  - Positive slope in energy.

Distribution function

In order to study the effect of the pitch angle anisotropy, the single pitch-angle slowing down distribution function is considered:

$$F(r, \Lambda, v) = C(r)\delta(\Lambda - \Lambda_0)\frac{H(v_b - v)}{v^3 + v_c^3} \qquad (4)$$

v_b and v_c birth and critical velocities. The normalization constant $C(r) \propto n_h$ is different for trapped (bounce orbit, $0 < \Lambda_0 < 1 - \epsilon$) and passing (transit orbit, $1 - \epsilon < \Lambda_0 < 1 + \epsilon$) particles.

With this distribution function $\left[\left.\frac{\partial F}{\partial \mathcal{E}}\right|_\Lambda - \frac{\Lambda}{E}\left.\frac{\partial F}{\partial \Lambda}\right|_E \right]$:

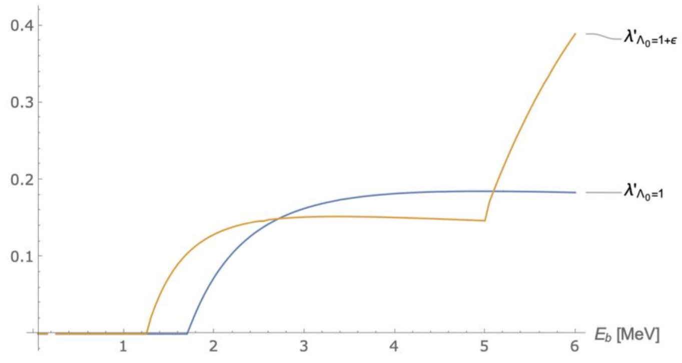
- $\left.\frac{\partial F}{\partial E}\right|_\Lambda < 0$ always stabilizing
- $-\frac{\Lambda}{E}\left.\frac{\partial F}{\partial \Lambda}\right|_E > 0$ potentially destabilizing
- $\rightarrow$ Threshold in Λ_0 for mode destabilization

$\Lambda_0 > \frac{2}{5}$, same threshold found for EGAMs in

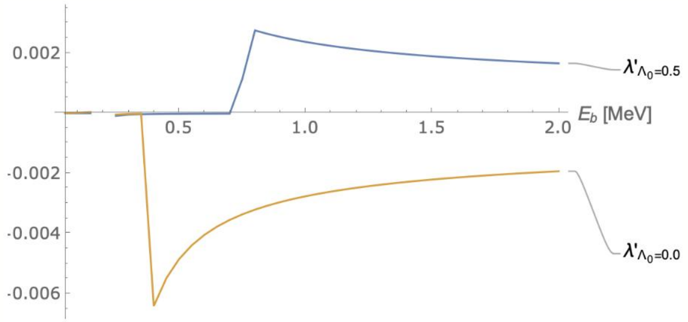
Z. Qiu Z., F. Zonca, and Liu Chen, Plasma Sci. Technol. (2011)

Mode destabilization

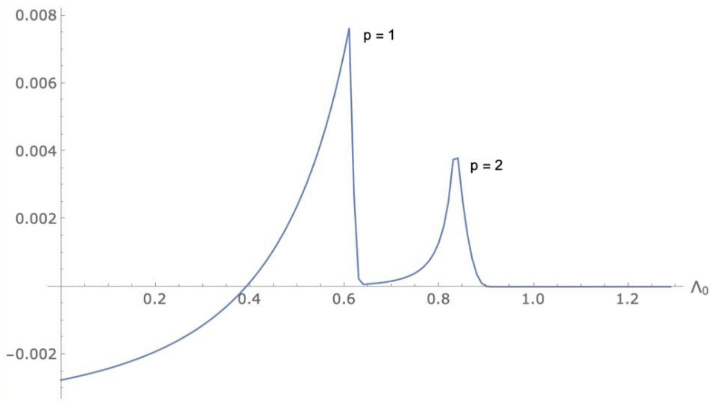
With $\lambda_h = \lambda'_h n_h/n_c$, we can study the resonance for parameters relevant to the observed n=0 modes in JET [3], where n=0 modes with frequency $\omega_0 \approx 300kHz$ were detected in presence of fast deuterium accelerated by auxiliary heating systems ($\sim 1MeV$).



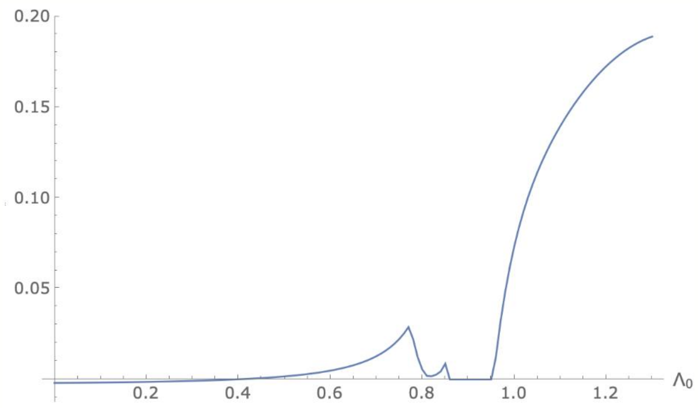
Plot of λ'_h as a function of birth energy E_b for trapped particles ($\Lambda_0 = 1, \Lambda_0 = 1 + \epsilon$).



Plot of λ'_h as a function of birth energy E_b for passing particles ($\Lambda_0 = 0, \Lambda_0 = 0.5$).



Plot of λ'_h as a function of Λ_0 for birth energy $E_b = 1MeV$.



Plot of λ'_h as a function of Λ_0 for birth energy $E_b = 2MeV$.

$\partial f_h / \partial E > 0$ or pitch angle anisotropy required for VDOM instability

Experimental evidence:

M.A. Van Zeeland et al., NF (2021)

Beam Modulation and Bump-on-tail Effects On Alfvén Eigenmode Stability in DIII-D

Insurgence of bump-on-tail fast ion distribution modulating NBI with periods 7-30 ms (reported: $\tau_s \sim$50 ms).

V.G. Kiptily et al., NF (2021)

Self modulation of alpha particles production due to sawtooth oscillations in JET D-He^3 experiments.

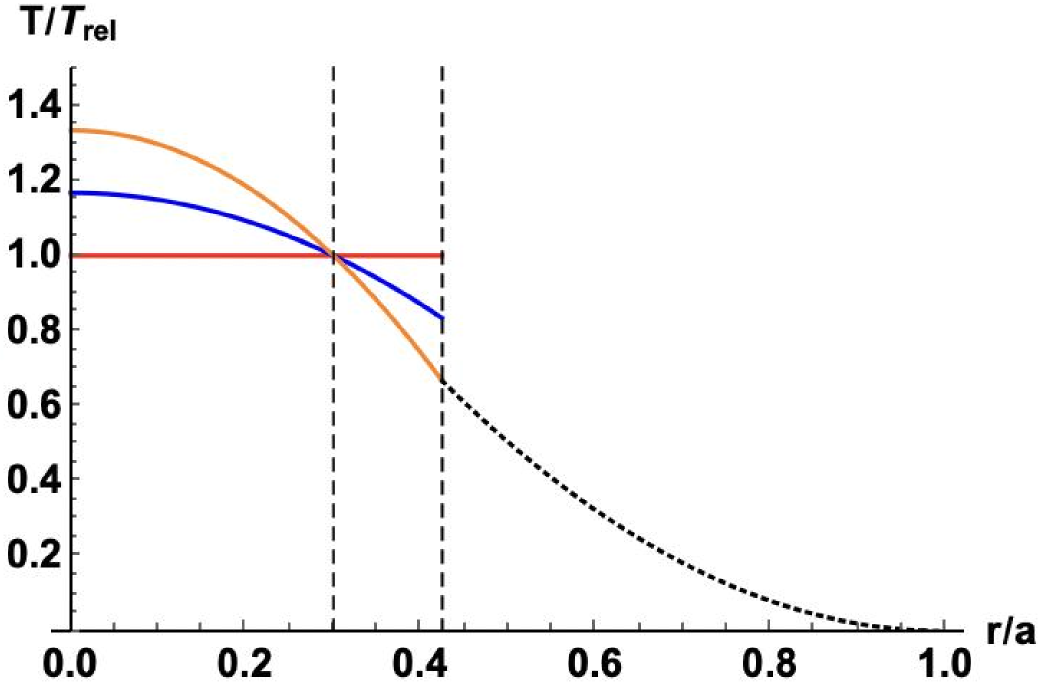
Bump-on-tail distribution for energetic particles when the sawtooth period is relatively short, i.e., $\tau_{saw}/\tau_s < 1$.

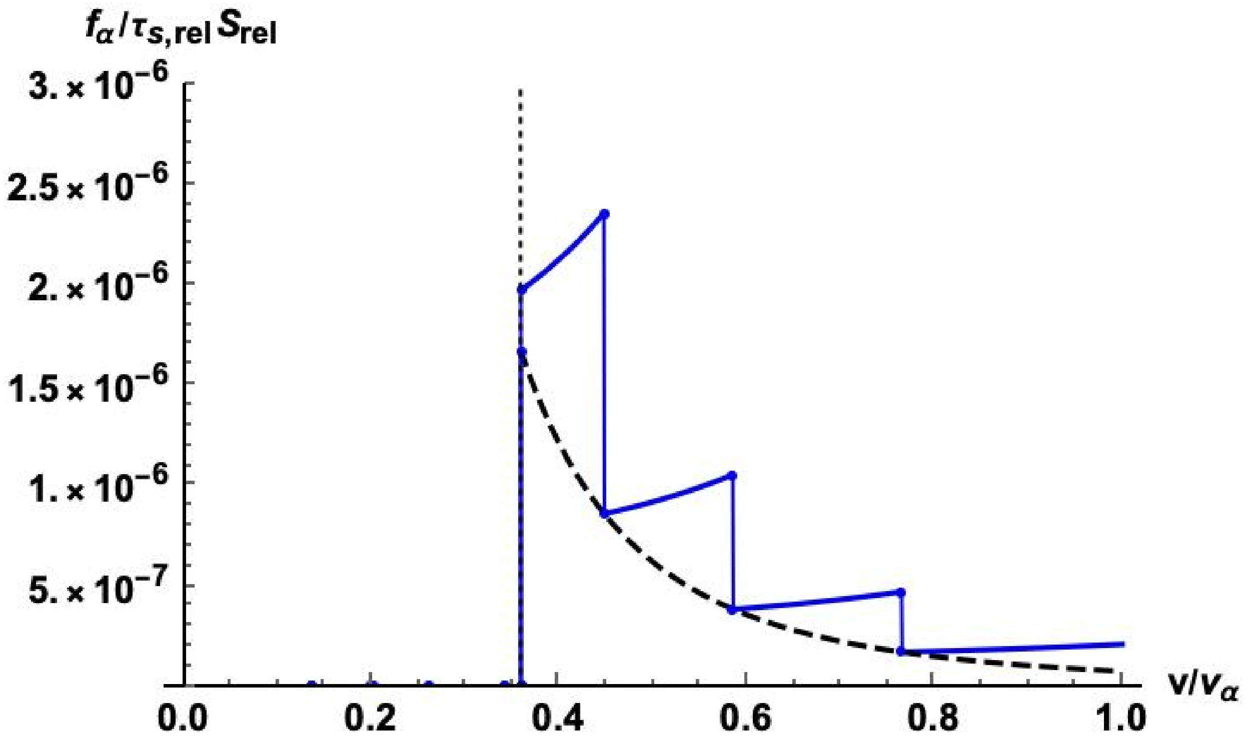
Interpretation:

T. Barberis et al, Plasma Physics and Contr. Fusion (2024)

Velocity space distribution function of fast ions in a sawtoothing plasma

KADOMTSEV RECONNECTION MODEL
A TIME-DEPENDENT ANALYTIC DISTRIBUTION FUNCTION IS OBTAINED





$$f_{\alpha,n}(r,t,v) = S_{rel}\frac{v_\alpha^2}{v^3}H\left[v_\alpha - v\right]$$
$$\tau_{s,rel}\left\{1 + A(r)\left[\frac{\hat{t}_n(r,t,v)}{\tau_{saw}} - n\right]\right\}^{7/2}$$
$$H\left[v - \hat{v}_{n,min}(r,t)\right]H\left[\hat{v}_{n,max}(r,t) - v\right]$$

$$\rightarrow F(t) = \sum_{n=0}^{n_M} f_{\alpha,n}$$

$\partial \mathrm{F}/\partial E > 0$ requires τ_s/t_{saw} of order of, or > 1

T. Barberis et al. 2024 PPCF

Conclusions (I)

1. VDOM analytic theory successfully verified with NIMROD straight tokamak simulations.

2. In JET realistic geometry, VDOM has been confirmed by numerical simulations:

   A mode with frequency around 280 kHz has been identified as VDOM with up-down symmetric density perturbation and nearly rigid shift mode structure.

3. VDOM can be driven unstable by mode-particle resonance with fast ions.

4. Work in progress – A new fast ion distribution function with $\frac{\partial F}{\partial E} > 0$ will be implemented in the PiC module in NIMROD.

Conclusions (II)

This talk contains two separate results:

1 – VDOM driven unstable by fast ions

Adding a new possibility to the catalogue of Energetic Particle modes. **This is becoming a hot topic!**

2 – Impact of X-points *(I haven't talked about it today!)*

Divertor X-points are singular points for axisymmetric modes according to ideal-MHD. Axisymmetric current sheets are shown to be driven near the X-points and along the magnetic separatrix.

1+2 – Joining the two parts together

n=0 modes are global modes that resonate with fast ions in the plasma core, and at magnetic X-points at the plasma edge -> Interesting nonlocal interplay between core and edge plasma → zonal flows?

Bibliography

Analytic theory: Impact of X-points on n=0 stability, X-point current sheets, Vertical Displacement Oscillatory Modes (VDOM):

- A. Yolbarsop et al, *Impact of magnetic X-points on the vertical stability of tokamak plasmas*, Nuclear Fusion Letter (2021).

- A. Yolbarsop et al, *Analytic theory of ideal-MHD vertical displacements in tokamak plasmas*, Plasma Phys. Contr. Fusion (2022).

- T. Barberis et al, *Vertical displacement oscillatory modes in tokamak plasma*, Journal of Plasma Physics (2022).

- F. Porcelli et al, *Vertical displacements close to ideal-MHD marginal stability in tokamak plasmas*, Fundamental Plasma Physics (2023).

- A. Yolbarsop et al, *Axisymmetric oscillatory modes in cylindrical magnetized plasma bounded by a conducting wall*, Physics Letters A (2023).

Analytic theory: Fast-ion-driven n=0 modes:

- T. Barberis et al, *Fast ion driven vertical modes in magnetically confined toroidal plasmas*, Nuclear Fusion Letter (2022).
- T. Barberis et al, Velocity space distribution function of fast ions in a sawtoothing plasma, Plasma Phys. Contr. Fusion (2024).

Linear NIMROD numerical simulations:

- D. Banerjee et al, *Linear NIMROD simulations of n=0 modes for straight tokamak configuration and comparison with analytic results*, Physics of Plasmas (2024).

- T. Barberis et al, *Simulations of VDOM and GAE modes in JET geometry*, Nuclear Fusion (2024).

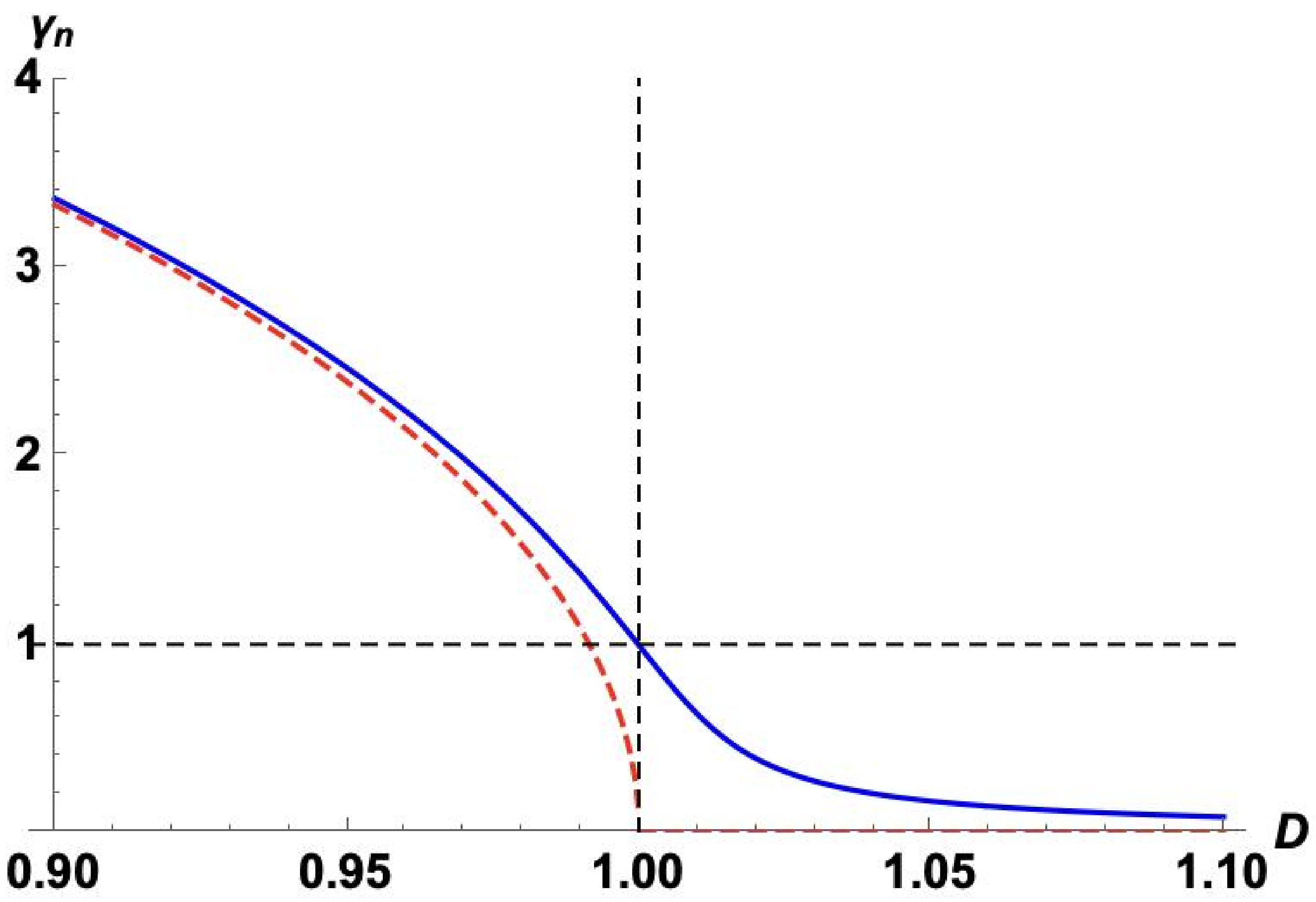
Extra slides

n=0 resistive wall mode near ideal-MHD marginal stability

When a nearby resistive wall is present, the n=0 vertical wall mode can still grow on the relatively slow resistive wall time scale. Active feedback control is then required for complete stabilization. However, it is shown that the resistive growth rate can be significantly faster, **scaling with fractional powers of wall resistivity**, if the wall position satisfies the criterion for ideal-MHD marginal stability, thus posing more stringent conditions for active feedback stabilization.

F. Porcelli et al, *Vertical displacements close to ideal-MHD marginal stability in tokamak plasmas*, Fundamental Plasma Physics (2023); DOI: 10.1016/j.fpp.2023.100017

Effects of a resistive wall



Growth rate $\gamma_n = \gamma(D)/\gamma(D=1)$ for the the thin wall limit, with $\gamma(D=1)$ given in Eq. (16), as function of the ideal wall parameter D close to ideal-MHD marginal stability. The blue curve shows the numerical solution of the full cubic dispersion relation (Eq. 13), while the dashed red line represents the ideal wall solution.

$$\gamma \approx \frac{\gamma_\infty}{(1-\hat{e}_0)^{1/3}(\gamma_\infty \tau_{\eta w})^{1/3}} = \frac{(a_w/\delta_w)^{1/3}\gamma_\infty}{(1-\hat{e}_0)^{1/3}(\gamma_\infty \tau_\eta)^{1/3}}. \tag{16}$$

(thin wall limit)

F. Porcelli et al, *Vertical displacements close to ideal-MHD marginal stability in tokamak plasmas*, Fundamental Plasma Physics (2023); DOI: 10.1016/j.fpp.2023.100017

A separate, but related topic:

n=0 modes are *resonant* at X-points

Impact of magnetic X-points (work in progress)

$\mathbf{B}\cdot\nabla\xi = i(n/R)B_T\,\xi + \; i\,B_P(\psi,\vartheta)\,\Sigma_m\,(m/r)\,\xi_m = 0$

Resonance condition at X-points satisfied for n=0 and any m

because $B_P(\psi,\vartheta)=0$ on the X-points.

The toroidal field line going through a magnetic X-point is resonant to n=0 perturbations!

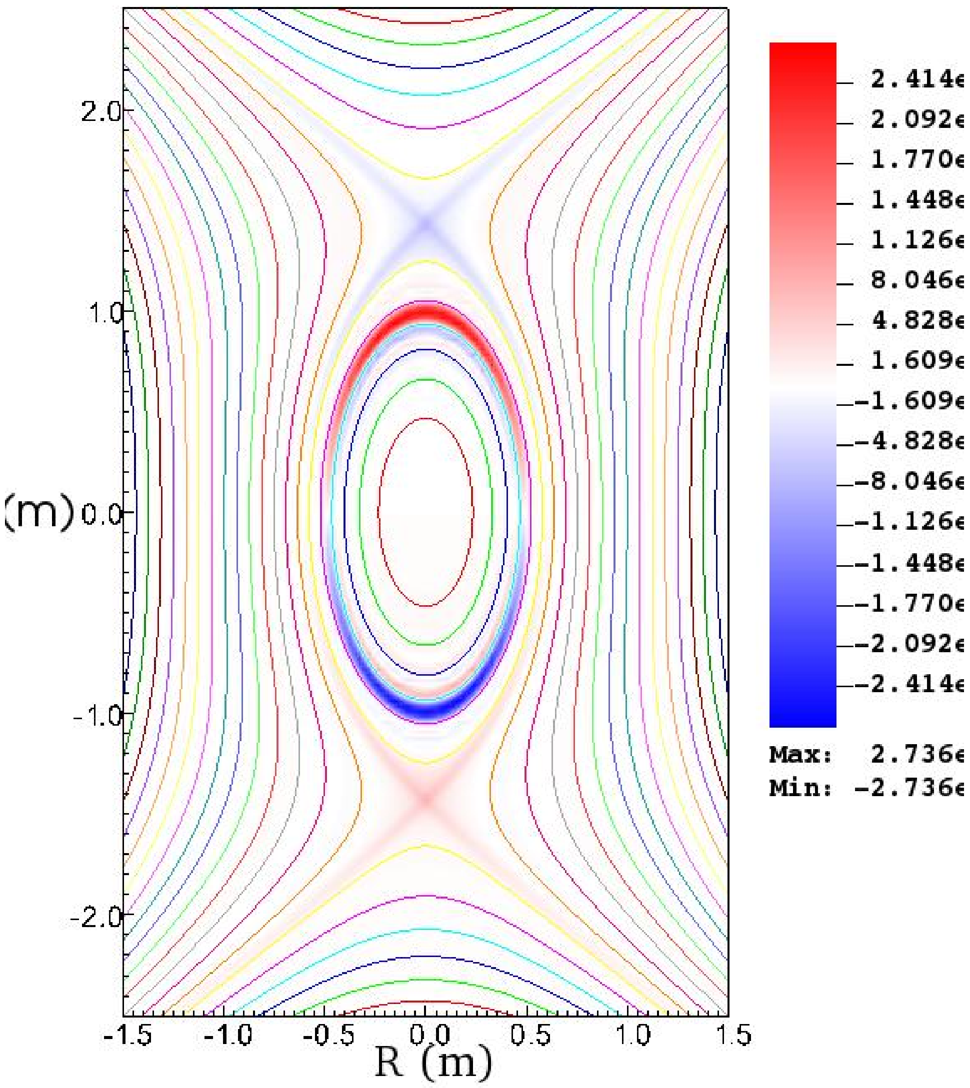
Why is the X-point resonance important?

- The ideal-MHD singularity can be resolved in a variety of ways (for instance, by plasma resistivity).
- However, **current sheets driven near X-points and along the magnetic divertor separatrix are likely to form** -> may have an impact on vertical stability, plasma edge stability, and ELMs.

A.Yolbarsop et al, *Impact of magnetic X-points on the vertical stability of tokamak plasmas*, Nuclear Fusion Letter 2022; DOI: 10.1088/1741-4326/ac27c5 .

X-point current sheets in numerical simulations

NIMROD simulation

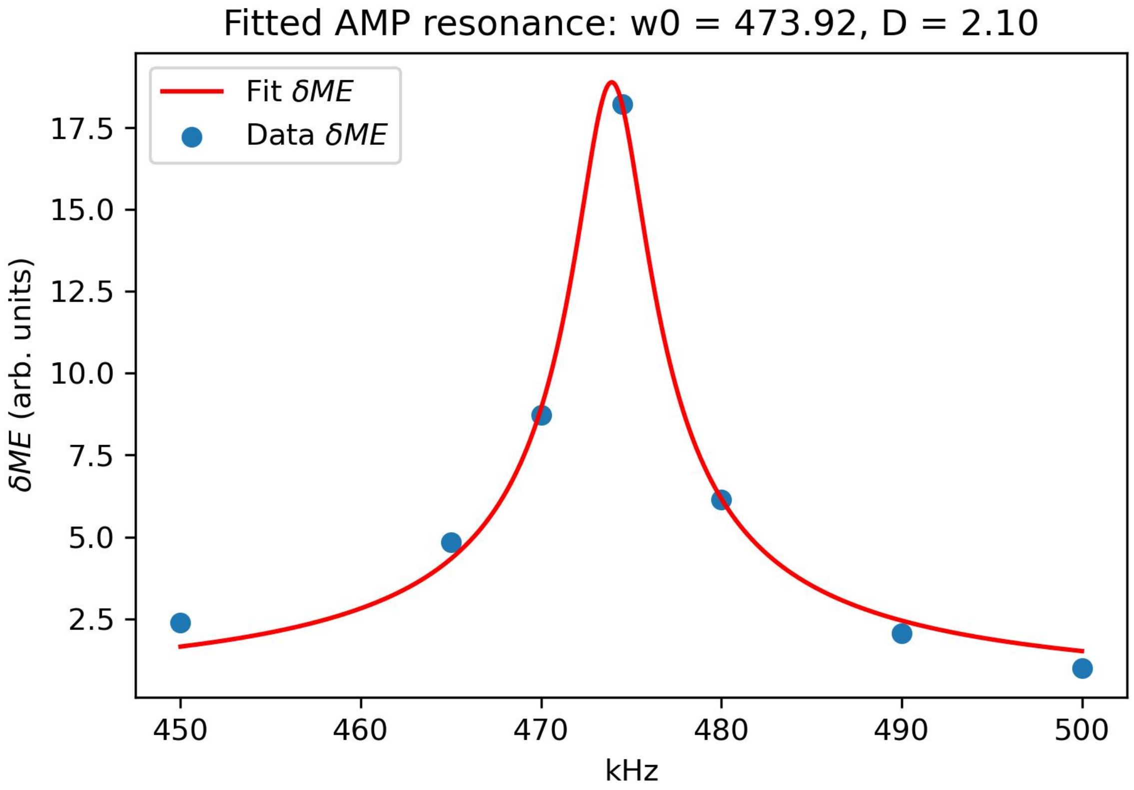


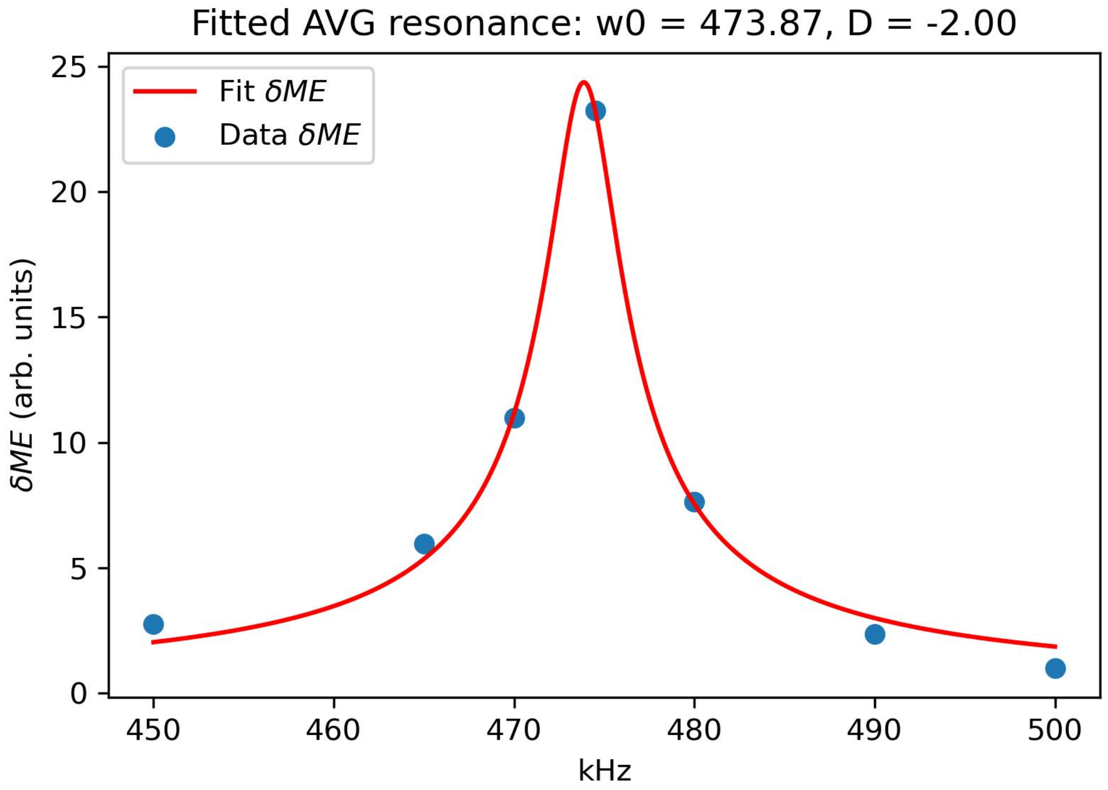
NIMROD simulations of n=0 vertical displacements in an idealized, straight tokamak equilibrium with elliptical cross section.

Experimental evidence of axisymmetric X-point currents

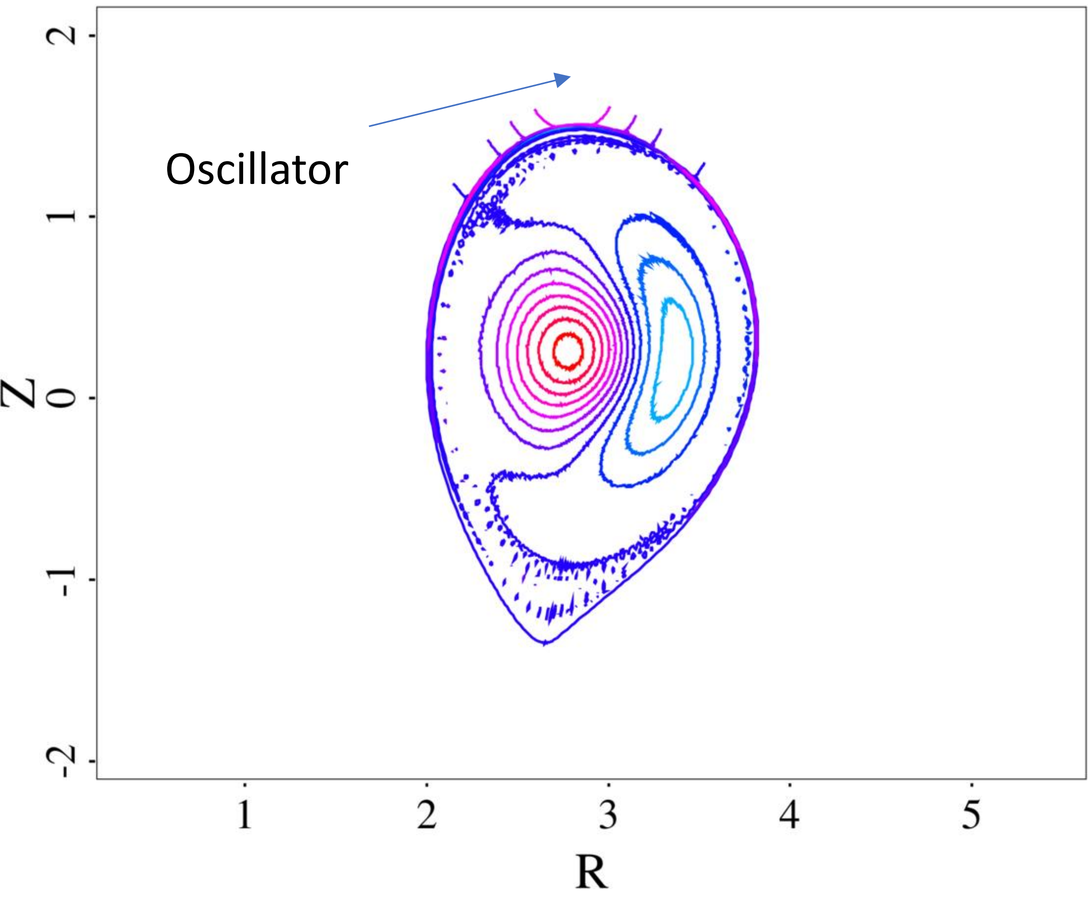
- VDEs and active feedback control, ELM pacing via vertical kicks -> any role for X-point resonance?
- JET -> J. Lingertat et al, *Studies of giant ELM interaction with the divertor target in JET*, J. Nucl. Mat. 1997 -> evidence of axisymmetric X-point currents.
- **X-point currents observed as shifts of the strike points on divertor target plates.**

GAE: High-frequency mode is affected by stronger damping (low viscosity case)

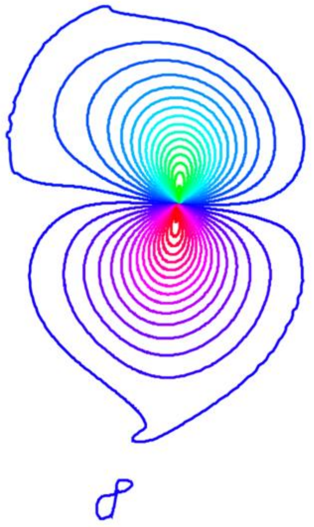




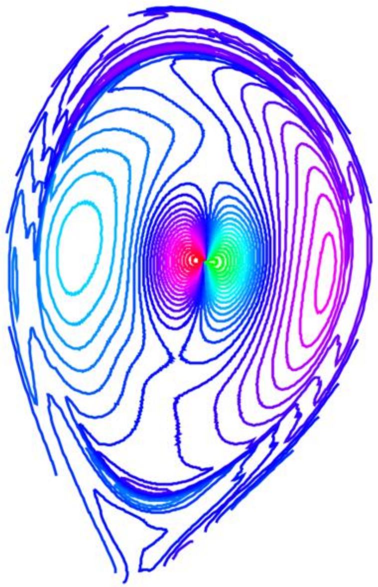
Contour plots: oscillator at 474 kHz, «high frequency mode» space structure
Possibly a Global Alfvén Eigenmode



Pressure

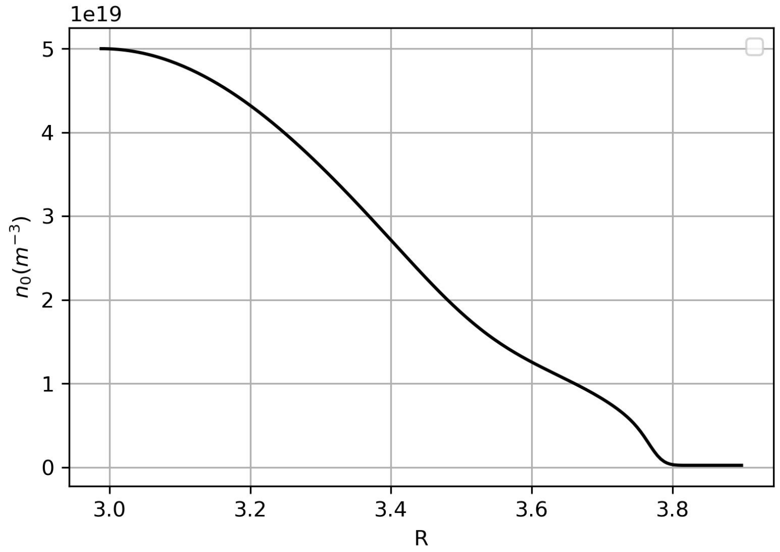


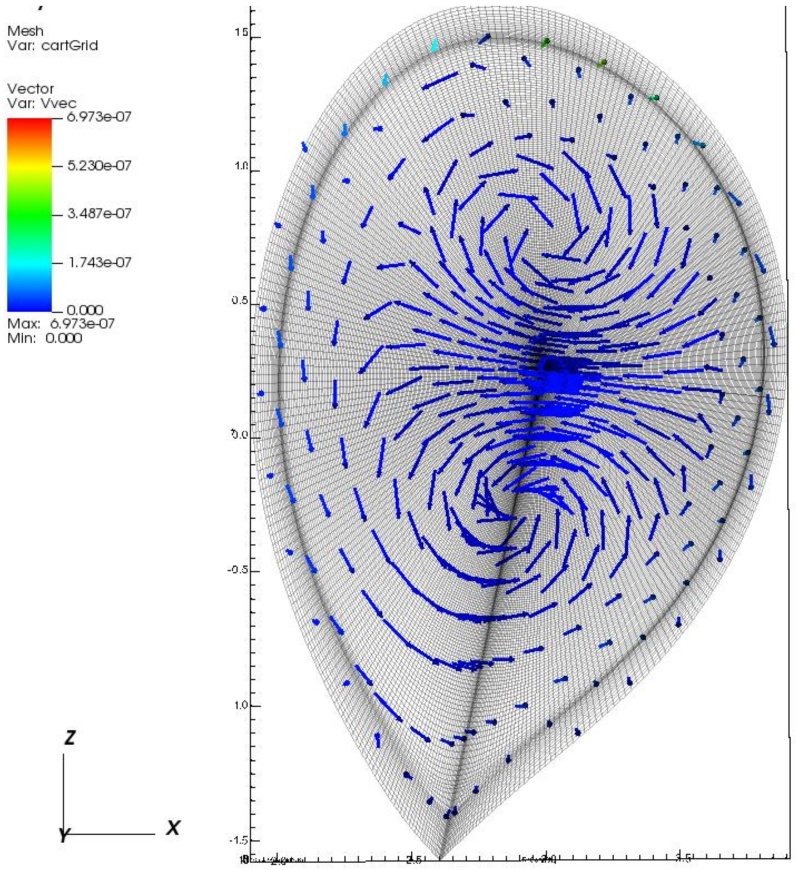
Perturbation in B norm.



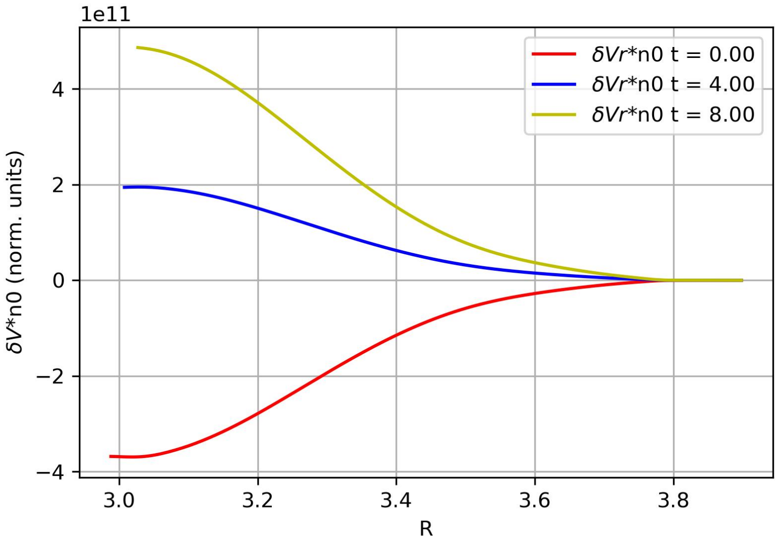
Perturbation in B tang.

Perturbed velocity: oscillator at 474.5 kHz,
«high frequency mode» space structure





Perturbed momentum in the direction normal (∼r) to flux surfaces



Perturbed momentum in the direction tangential (∼θ) to flux surfaces