Electron modeling for MHD simulation in fusion plasmas

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outline

Motivation

1. Electron models for GK code

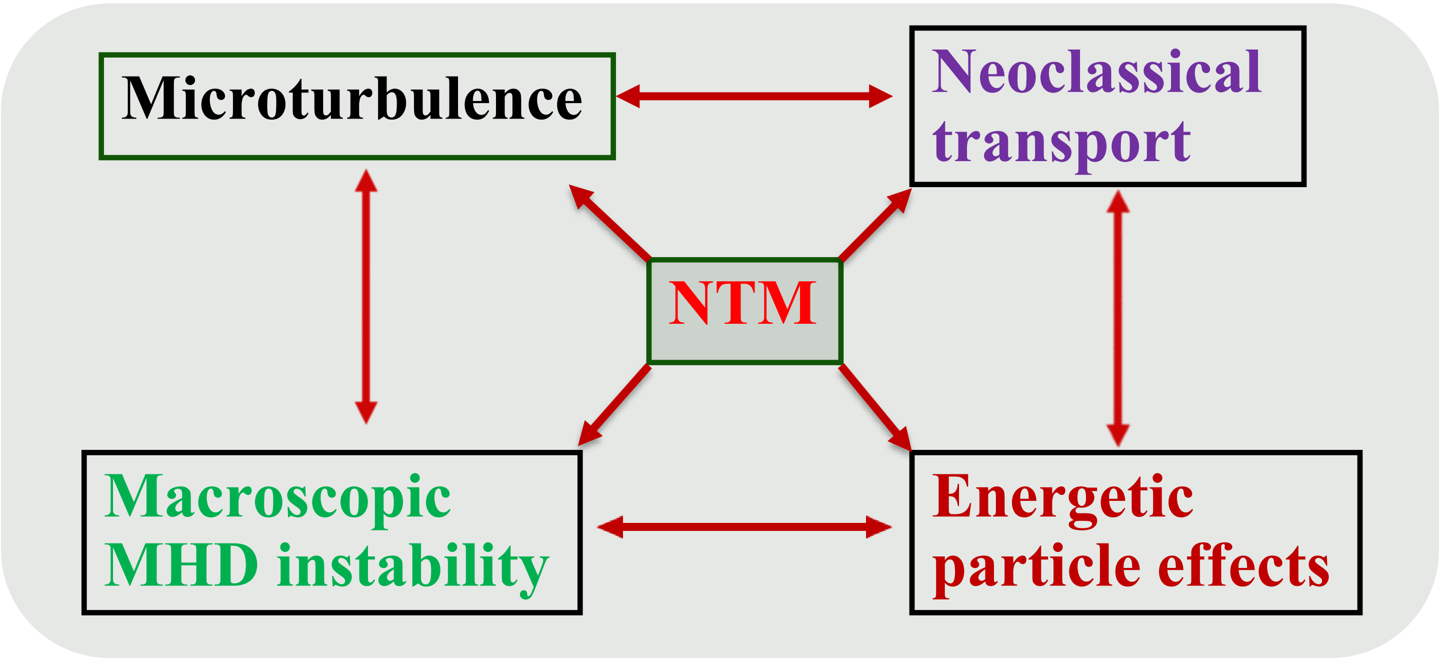
2. East experiment simulation

3. Fine Radial Structure

Summary and discussion

Motivation

- Multiscale instabilities exist in fusion plasmas, macroscopic MHD instabilities: resistive kink, tearing mode, NTM may cause the disruption of plasmas, while microscopic instabilities, ITG、CTEM and MTM caused turbulence transport may limit the performance of confinement in large Tokamaks like ITER. Plenty of experimental evidence shows the strong cross scale coupling among these modes and particles. An example:



- Self-consistent simulation of the above low frequency and multiscale processes require accurate and efficient model of plasma spices, especially electron model which limit the shortest time step of the simulation.

1.1 Hybrid Electron Model for MHD Mode

- For $\omega/k_{||} << v_{te}$, electron response mostly adiabatic (isothermal). First, estimate $\boldsymbol{E}_{||}$ using massless fluid electron

$$E_{||} = -\nabla_{||}(\phi + \phi_{ind}) = -\frac{T_e}{en_0}\nabla_{||}\delta n_e \qquad \frac{e\phi_{ind}}{T_e} = -\frac{e\phi}{T_e} + \frac{\delta n_e}{n_0}$$

  - ✓ Vector potential $A_{||}$ calculated from $E_{||}$ $\qquad \frac{\partial A_{||}}{\partial t} = \nabla_{||}\phi_{ind}$
  - ✓ Perturbed flow δu_e from Ampere's law $\qquad n_0 e\delta u_e = -\nabla_{\perp}^2 A_{||}$
  - ✓ Perturbed density δn_e from continuity equation $\qquad \frac{\partial \delta n_e}{\partial t} = -\nabla_{||} n_0 \delta u_e$
  - ✓ Electrostatic potential ϕ from Poisson equation using perturbed density δn_e

- Then, $\boldsymbol{E}_{||}$ is corrected by kinetic electron non-adiabatic response using split-weight scheme to reduce noise $\quad \delta f_e = f_M e^{e(\phi+\varphi_{ind})/T_e} + \delta g$

- Technical difficulties:

  1. No tearing mode due to the lowest order massless fluid electron,

  2. Electron response, especially passing electron, near the mode rational surface is not accurate.

[Lin and Chen, Phys. Plasmas ***8****, 1447 (2001)*
I. Holod, W. L. Zhang, Y. Xiao, and Z. Lin, Phys. Plasmas 16, 122307 (2009)]

1.2 Fluid Electron Model for MHD Mode

Field equation:

$$\frac{n_0 e}{T_e}\rho_s^2\nabla_\perp^2\delta\phi = \delta n_e - \delta n_i$$

$$\frac{c}{4\pi n_0 e}(\nabla_\perp^2 - \frac{1}{D_e^2})\delta A_{||} = \delta u_{e||c}$$

$$\delta u_{e||} = \frac{c}{4\pi n_0 e}\nabla_\perp^2\delta A_{||} + \delta u_{i||} - \frac{1}{n_0 e}(J_{ECCD} + J_{bs})$$

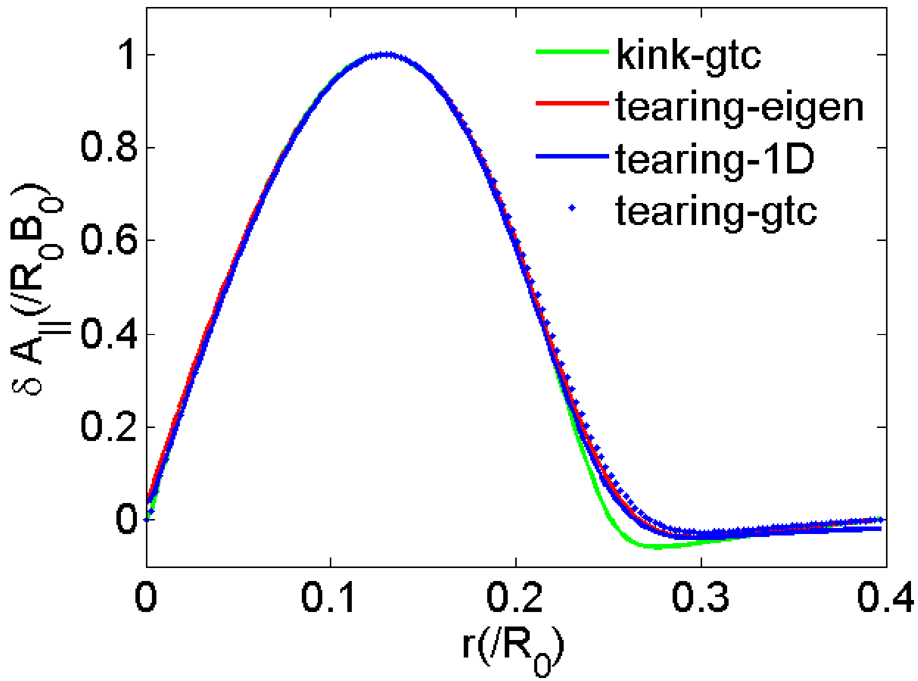
Dynamics equation:

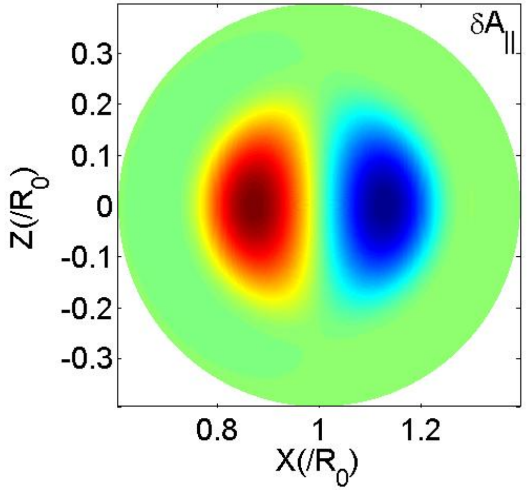
$$\frac{\partial}{\partial t}\delta n_e = -\mathbf{B}_0\cdot\nabla\left(\frac{n_0\delta u_{e||}}{B_0}\right) - \delta\mathbf{u}_E\cdot\nabla n_0 - \frac{\delta\mathbf{B}_\perp}{B_0}\cdot\nabla(n_0 u_{e0})$$

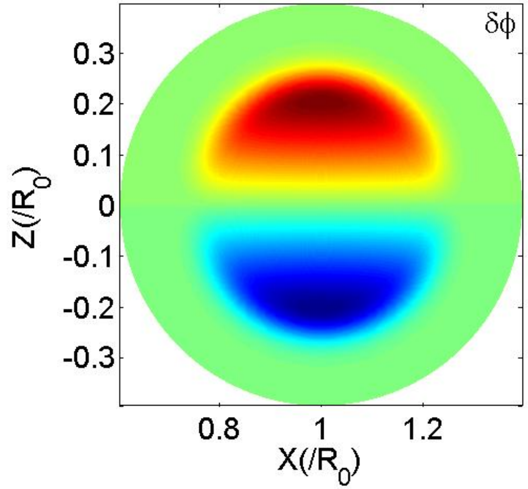
$$n_0 m_e\frac{\partial}{\partial t}\delta u_{e||c} = -\nabla_{||}\delta p_e + n_0 e\nabla_{||}\delta\phi - n_0 m_e\nu_{ei}\delta u_{e||}$$

➢ (1,1) resistive kink tearing mode in cylinder

$q=0.8+3.2*r^2$, $a/R_0=1/2$, $B=1.0\mathbf{T}$, $n_0=10^{20}\ /\mathbf{m^3}$, $T_e=100\mathbf{ev}$

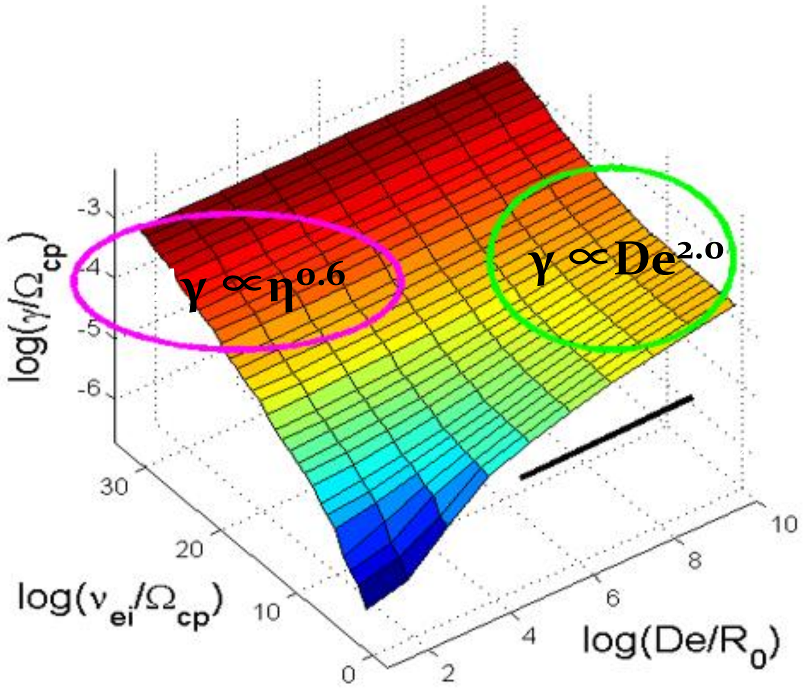


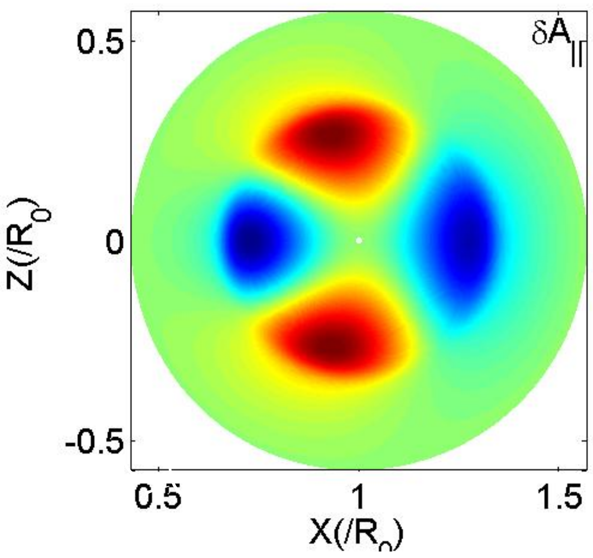


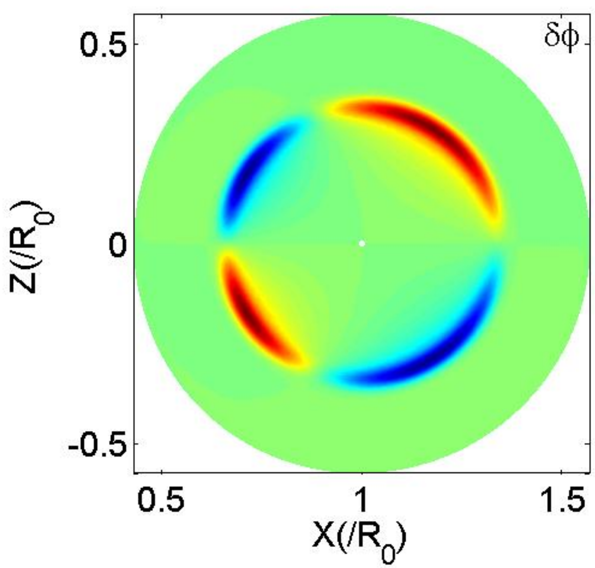


- γ_{GTC}=0.066Ω_A, γ_{eig}=0.074 Ω_A, γ_{init}=0.074 Ω_A
- **Finite resistivity near the mode rational surface breaks the ideal MHD limit and causes finite $E_{||}$**

➢ (2, 1) tearing mode simulation







This scaling agrees well with the FKR prediction. Fluid model can recover both the resistive and the collisionless tearing mode.

[Liu et al, POP, 2014, 2016]

1.3 general kinetic electron model of MHD Modes

- Parallel electric field $\boldsymbol{E}_{||} = -\nabla_{||}\phi - \frac{\partial A_{||}}{\partial t}$
- Need to calculate accurately scalar ϕ and vector potential $A_{||}$

$$\phi - \tilde{\phi} = -\sum_s 4\pi e Z_s \bar{n}_s \qquad \nabla_\perp^2 A_{||} = -\sum_s \frac{4\pi e}{c} Z_s \bar{n}_s u_s$$

- Ideal MHD $E_{||}$~0. $|E_{||}|/|\nabla_{||}\phi| \sim |E_{||}|/|\frac{\partial A_{||}}{\partial t}| \sim (k_\perp \rho_s)^2 \ll 1$
- Physics: cancellation between electrostatic and inductive $E_{||}$
- Canonical momentum used as independent velocity variable to avoid explicit time derivative operation:

$$(\nabla_\perp^2 - \frac{\omega_{pe}^2}{c^2} - \frac{\omega_{pe}^2}{c^2})\frac{\partial \delta A_{||}}{\partial t} = \frac{4\pi}{c}\frac{\partial}{\partial t}(n_{e0} e \delta u_{e||c} - n_{i0} q_i \delta u_{i||c})$$

- LHS: $|1^{st}\ term|/|2^{nd}\ term| \sim (k_\perp d_e)^2 \ll 1$. Small numerical error of RHS leads to large error of $A_{||}$
- Technical difficulty: Infamous "cancellation problem": calculate 2nd-order $\mathcal{O}(k_\perp^2 \rho_s d_e)^2$ term from 0th–order $\mathcal{O}(1)$ equation!

1.3 A Conservative Scheme Solving Exact DKE

- Calculate exact $\boldsymbol{E}_{||}$ by separating $A_{||}$ into adiabatic and non-adiabatic parts: $A_{||} = A_{||}^{A} + A_{||}^{NA}$

$$\frac{\partial A_{||}^{A}}{\partial t} = \nabla_{||}\phi_{ind}$$

$$\frac{e\phi_{ind}}{T_e} = -\frac{e\phi}{T_e} + \frac{\delta n_e}{n_0} - \frac{\delta\psi^A}{n_0}\frac{\partial n_0}{\partial \psi}$$

Alfven wave, IAW, drift wave

- Define electron adiabatic responses using adiabatic $\boldsymbol{E}_{||}$

$$\delta f_A = \frac{e(\phi + \phi_{ind})}{T_e} f_0 + \delta\psi^A \frac{\partial f_0}{\partial \psi}$$

$$L\delta h = -\delta L f_{e0} - L_0 \delta f_a$$

$$= -\left(\mathbf{v}_{\mathbf{E}} + v_{||}\frac{\boldsymbol{\delta}\mathbf{B}_{\perp}^{NA}}{B_0}\right)\cdot\nabla f_{e0} + \frac{e f_{e0}}{T_{e0}}\mathbf{v}_{\mathbf{c}}\cdot\nabla\delta\phi - \frac{\partial \delta f_a}{\partial t} - \mathbf{v}_{\mathbf{d}}\cdot\nabla\delta f_a + \frac{e v_{||} f_{e0}}{c T_{e0}}\frac{\partial \delta A_{||}^{NA}}{\partial t}$$

[Mitigation of the cancellation problem in the gyrokinetic particle-in-cell simulation of the global electromagnetic modes, Alexey Mishchenko, Alberto Bottino, Roman Hatzky, Eric Sonnendrücker, Ralf Kleiber and Axel Könies, Phys. Plasmas, 2017]

[A conservative scheme of drift kinetic electrons for gyrokinetic simulation of kinetic-MHD processes in toroidal plasmas, J. Bao, D. Liu, Z. Lin, Phys. Plasmas, 2017]

1.3 A Conservative Scheme Solving Exact DKE

- Non-adiabatic part of $E_{||}$ calculated via electron parallel momentum equation (generalized Ohm's law) using non-adiabatic response
- Recover collisionless tearing mode with current sheet at d_e scale
- Non-tearing modes: current screened by collisionless skin depth d_e
- No "cancellation problem" due to the adiabatic treatment of most of electron

$$\left(\nabla_\perp^2 - \frac{1}{d_e^2}\right)\frac{\partial A_{||}^{NA}}{\partial t} = \frac{1}{d_e^2}\chi_{||} - c\nabla_\perp^2\left(\nabla_{||}\delta\phi_{ind}\right)$$

$$\chi_{||} = \underbrace{-\frac{c}{en_0}\mathbf{b_0}\cdot\nabla\delta P_{||}^{NA}}_{\{III\}}\underbrace{-\frac{c}{en_0B_0}\boldsymbol{\delta}\mathbf{B}^{NA}\cdot\nabla P_{||0}}_{\{IV\}}\underbrace{-\frac{c}{en_0B_0}\boldsymbol{\delta}\mathbf{B}\cdot\nabla\delta P_{||}^{NA}}_{\{V\}}\underbrace{-\frac{c}{B_0}\boldsymbol{\delta}\mathbf{B}\cdot\nabla\delta\phi_{ind}}_{\{VI\}}$$

$$\underbrace{+\frac{c}{B_0}\boldsymbol{\delta}\mathbf{B}\cdot\nabla\langle\phi\rangle}_{\{VII\}}\underbrace{-\frac{cm_e}{en_0}\nabla\cdot\left[n_0\delta u_{||e}\left(3\mathbf{V}_c+\mathbf{V}_g\right)+n_0u_{||0}\mathbf{V}_E\right]}_{\{VIII\}}\underbrace{-\frac{cm_e}{en_0}\nabla\cdot\left(n_0\delta u_{||e}\mathbf{V}_E\right)}_{\{IX\}}$$

$$\underbrace{+\frac{c}{en_0}\frac{P_{||0}-P_{\perp 0}}{B_0^2}\boldsymbol{\delta}\mathbf{B}\cdot\nabla B_0+\frac{c}{en_0}\frac{\delta P_{||}^{NA}-\delta P_{\perp}^{NA}}{B_0^2}\mathbf{B_0}\cdot\nabla B_0}_{\{X\}}$$

The conservative scheme recovers collisionless tearing mode dispersion relation in drift kinetic electron model

Linearize the physics model of conservative scheme and solve the eigen value problem:

$$\nabla_\perp^4 \omega^2 \delta A_{||} = \nabla_\perp^2 [\frac{1}{D_e^2} \omega^2 \xi_e^2 Z'(\xi_e) \delta A_{||}] - \frac{m_i}{m_e} k_{||} \xi_e^2 Z'(\xi_e)(k_{||} \nabla_\perp^2 - k_{||}'') \delta A_{||}$$

Inner region equation:

$$\nabla_\perp^2 \delta A_{||} = \frac{1}{D_e^2} \xi_e^2 Z'(\xi_e) \delta A_{||}$$

Outer region equation:

$$\left(k_\parallel \nabla_\perp^2 - k_\parallel''\right) \delta A_\parallel = 0$$

Asymptotic matching to CTM dispersion relation: Recover Drake and Lee 1977 result.

$$-i\omega = \gamma = \frac{D_e^2}{\sqrt{\pi}} |k_{||}' v_{te}| \Delta_o'$$

Collisionless tearing mode dispersion relation in finite mass fluid electron model

$$\nabla_\perp^2\left(k_{||}v_{te}^2-\frac{\omega^2}{k_{||}}\right)\nabla_\perp^2\delta A_{||}=\nabla_\perp^2\left(k_{||}''v_{te}^2-\frac{\omega^2\omega_{pe}^2}{k_{||}c^2}\right)\delta A_{||}+\frac{v_{te}^2}{\rho_s^2}(k_{||}\nabla_\perp^2-k_{||}'')\delta A_{||}$$

$$\left(k_{||}^2v_{te}^2-\omega^2\right)\frac{\partial^2}{\partial x^2}\delta A_{||}=\left(-\frac{\omega^2\omega_{pe}^2}{c^2}\right)\delta A_{||}$$

$$\left(k_\parallel\nabla_\perp^2-k_\parallel''\right)\delta A_\parallel=0$$

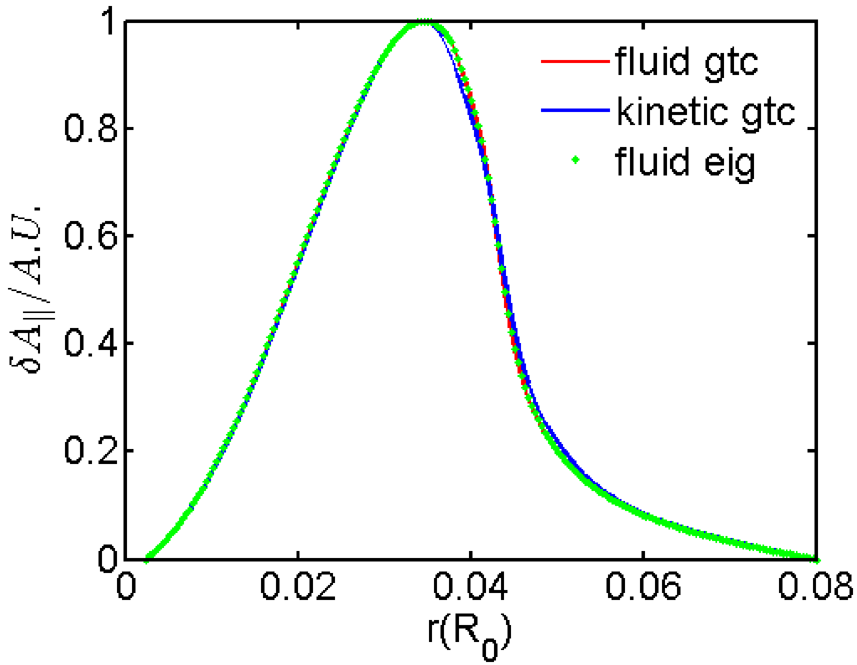
$$\Delta_o'=\lim_{|x|\to 0}\frac{\delta A_{||}'}{\delta A_{||}}=\frac{2}{a}\left(\frac{1}{k_ya}-k_ya\right)=\Delta_I'$$

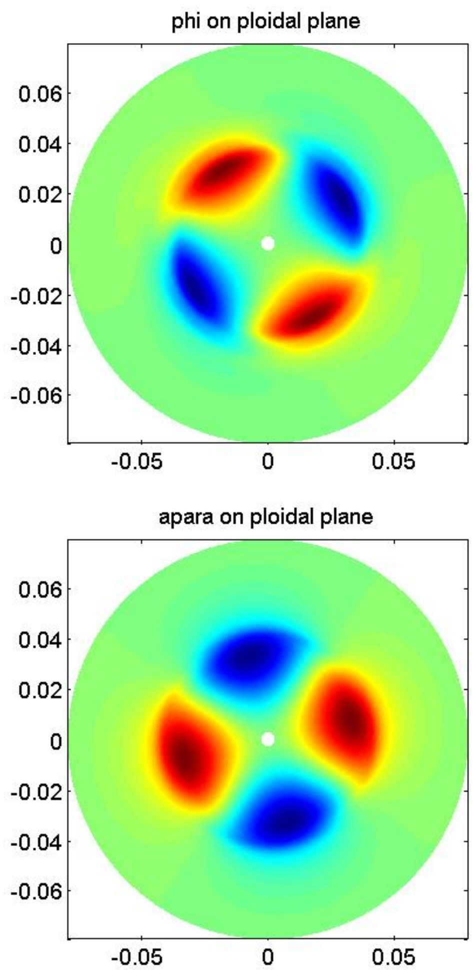
and

$$-i\omega=\gamma=\frac{D_e^2}{\pi}|k_{||}'v_{te}|\Delta_o'$$

- **Kinetic growth rate is $\pi^{1/2}$ times larger than fluid one.**
- **Reason: fluid calculation assumes two counter propagating beam electron background, kinetic model uses shifted Maxwellian electron.**

Kinetic collionless tearing mode simulation

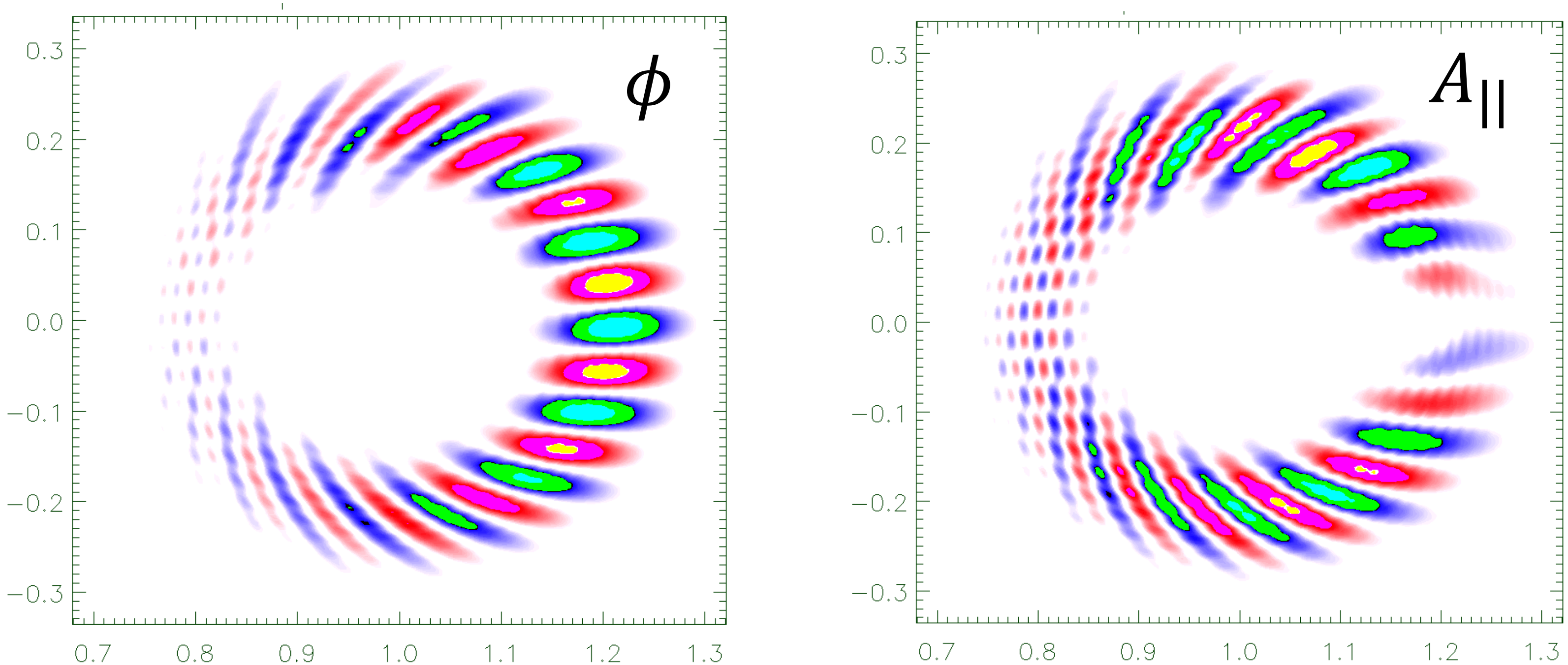




1. γ_{gtc}=0.0031(C_s/R_0) almost the $\pi^{1/2}$ of fluid growth rate (0. 00176) .
2. Kinetic picture of the collisionless tearing mode can be accurately captured by the DKE model.

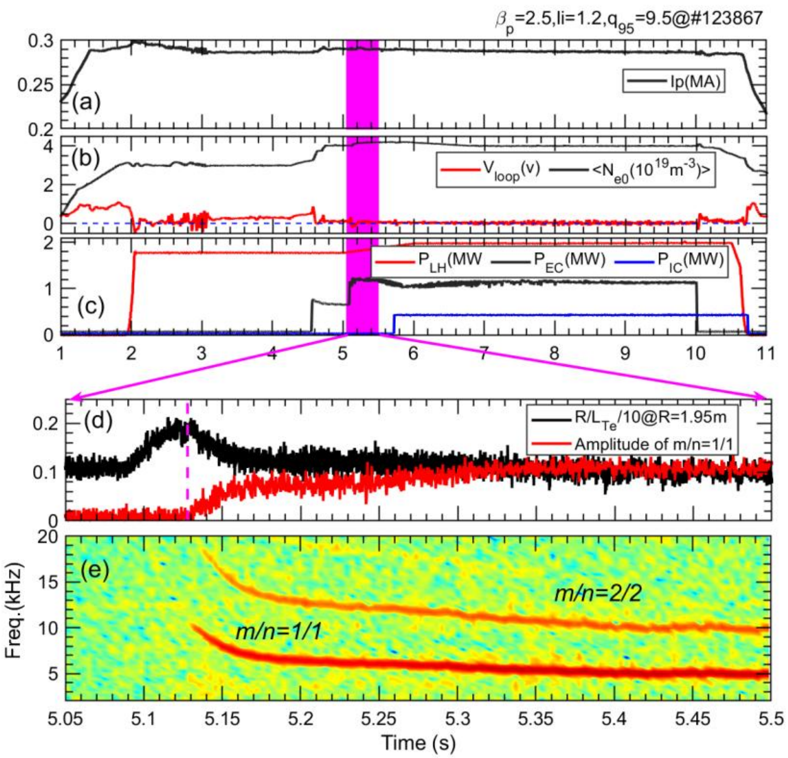
DKE simulation of Kinetic Ballooning Mode

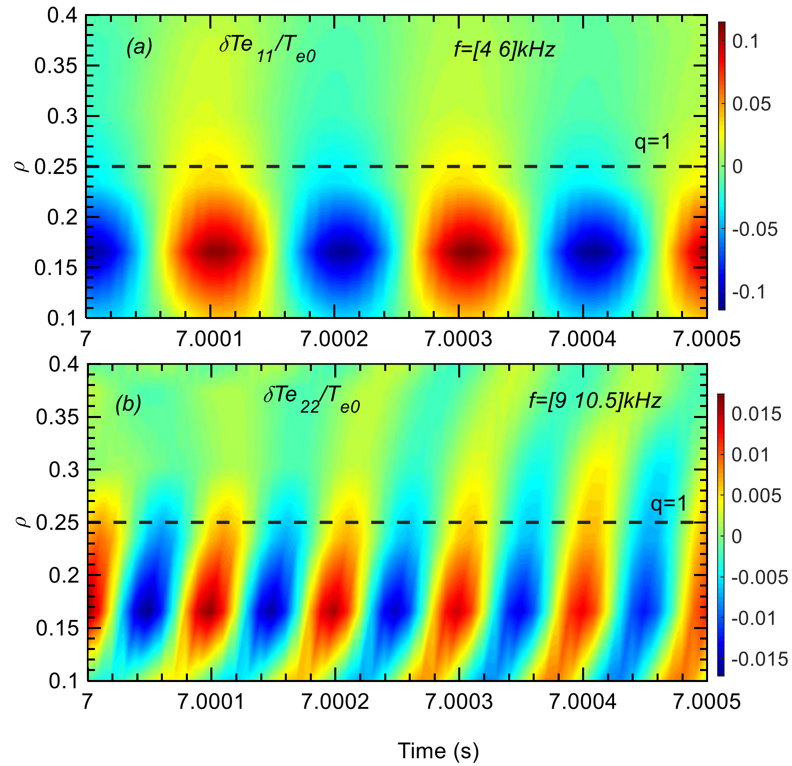
$$\beta_e = 2.0\%$$



The drift kinetic electron faithfully captures kinetic ballooning mode in high β_e plasmas. Near the mode rational surface, accurate electron response is essential for low frequency MHD modes, especially for tearing Mode.

2. Simulation of MHD and microturbulence in EAST

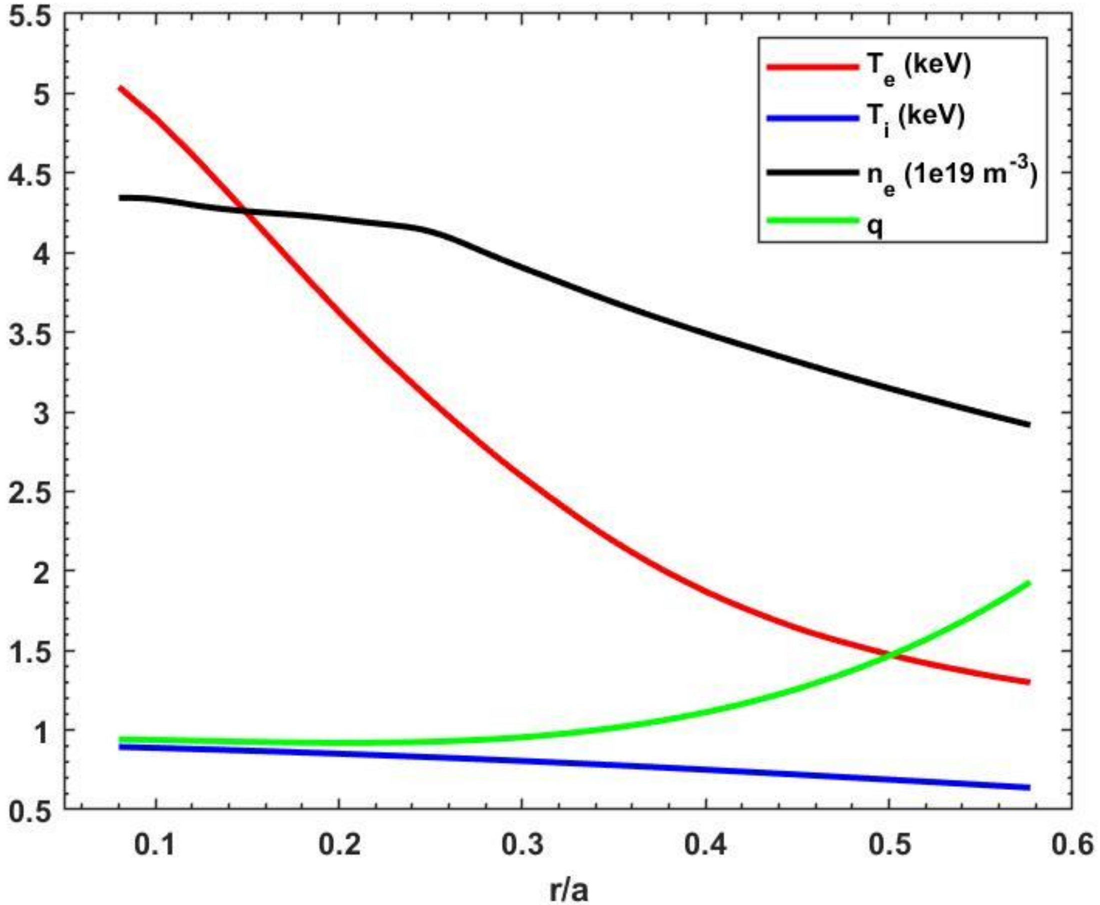




1. Obvious internal kink and microturbulence in EAST experiment.

2. The role of internal kink in e-ITB formation is not clear.

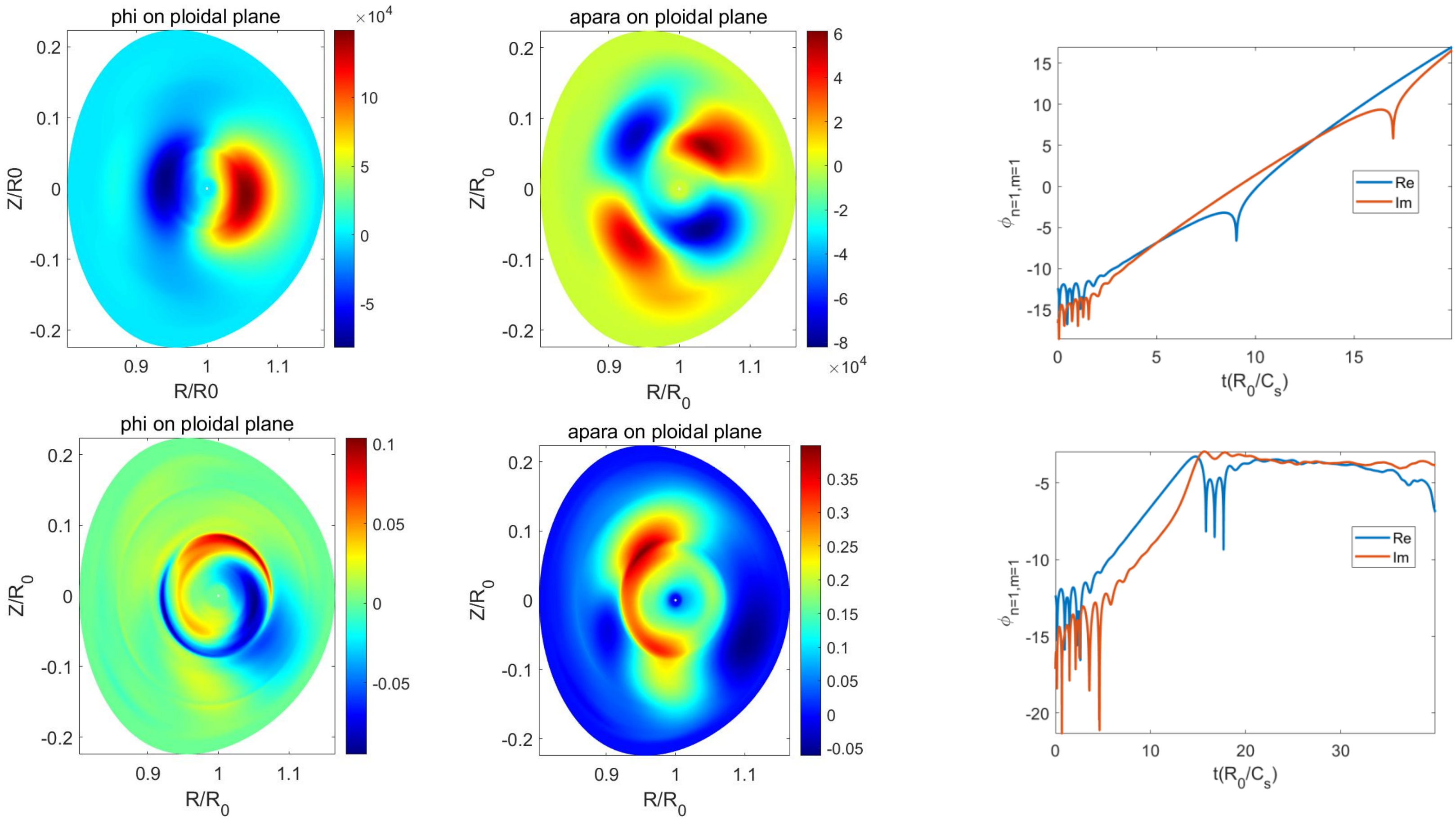
Internal kink simulation



Equilibrium setup:

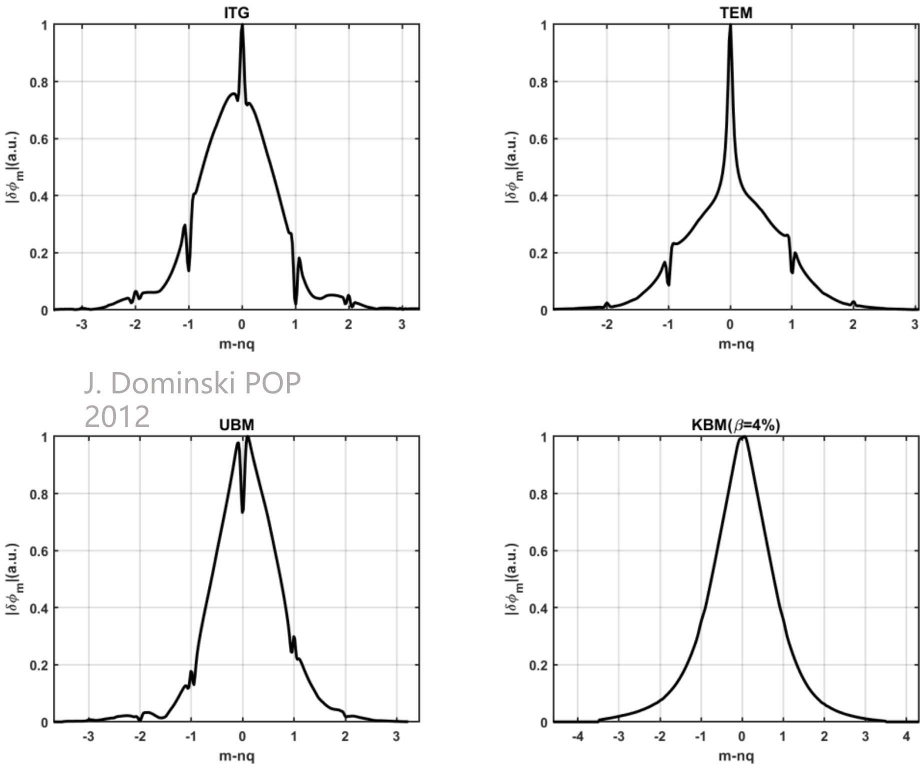
1. **Internal transport barrier exists,**
2. **Low shear and reversed q profile with q_{min}<1.**

Internal kink simulation

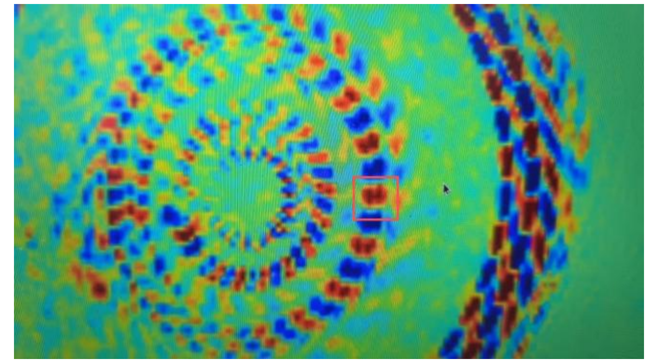


1. Kink mode structure is radially expanded.
2. More obvious toroidal mode coupling due to the finite orbit width of thermal background.
3. Real frequency due to kinetic thermal electron, which is 7.0 kHz, and rotating in electron diamagnetic direction.

3. Passing-electron-induced fine radial structure (FRS)



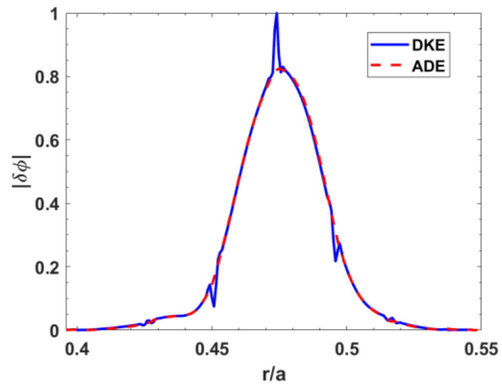
The behavior of FRS in various micro-instabilities



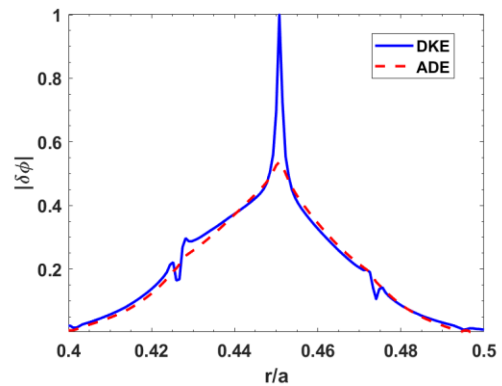
FRS obtained from the fully kinetic code

- **Fully kinetic simulation** demonstrates the physical reality of FRS.
- FRS exhibits distinct **characteristics** in different instabilities.
- The **finite β effect** can suppress the FRS.

Adiabatic passing electrons (ADE) vs. drift-kinetic passing electrons (DKE)



n=20 ITG



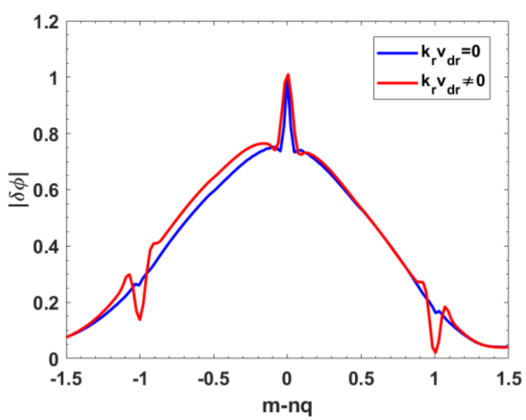
n=20 TEM

Modes(CBC)	Growth rate of ADE	Growth rate of DKE
n=20 ITG	0.478	0.487
n=20 TEM ($\kappa_{Ti} = 1$)	0.401	0.437
n=45 TEM	0.248	0.357

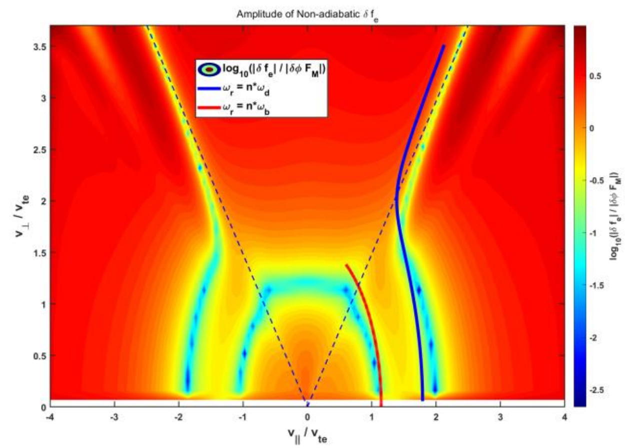
- The **non-adiabatic response of passing electrons** leads to the FRS.
- Non-adiabatic response of passing electrons affects the instability **growth rate** when the FRS width becomes comparable to the mode structure width (Reduction of **adiabatic electron screening)**

Preliminary study of passing electron effects on ITG

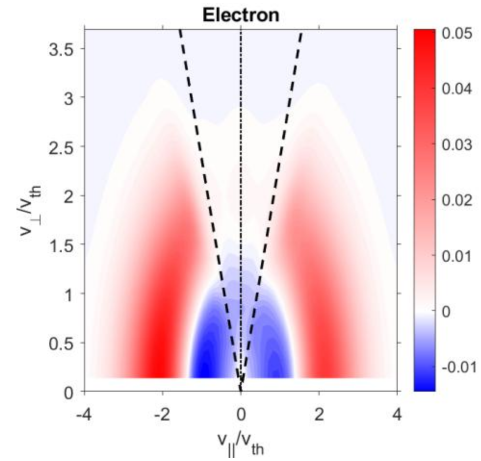
Non-adiabatic condition of passing electrons: $|\omega_r/k_\parallel| \sim v_{the}$



n=20ITG mode structure



Passing electron ITG resonance



Energy transfer from electrons to ITG

- FRS is due to the dynamics of passing electrons near the mode rational surface.
- Magnetic drifts cause the FRS to couple to other harmonics, broadening the FRS.
- Unlike trapped electrons, passing electrons can undergo magnetic drift resonance with ITG, resulting in net energy exchange and driving or damping ITG.

Y. F. Qiu, S. J. Wang AIP Advances 2025

Progresses:

1. A series of electron model, fluid—hybrid—kinetic, is introduced;

2. Kink mode simulation qualitatively agrees with the experiment observation for EAST tokamak.

3. Fine Radial Structure due to the passing electron is physical, it's effect on short wave length mode should be carefully treated.

Future works:

1. More accurate electron model is needed, especially for passing electron which might be singular near the mode rational surface for DKE.

2. Self-consistent cross scale simulation of interaction between MHD instabilities (tearing mode, kink) and micro-turbulence(drift waves) should be carried out for real tokamak physics.

Thanks very much!

师恩难忘，儒师如父，中心培养，
永记心间。

钱塘赤子，究等离子天火，怀珠握瑾育桃李满园；
愿恩师如松鹤长春，与山河同寿，福泽绵长。