Toward Predictive Energetic Particle Transport

Matteo Valerio Falessi[1,2], P. Lauber[3], F. Zonca[1,4], L. Chen[4,5], Z. Qiu[4]

[1] Center for Nonlinear Plasma Science and C.R. ENEA Frascati, C.P. 65, 00044 Frascati, Italy
[2] National Institute for Nuclear Physics, Rome Department, P.le A. Moro, 5, 00185 Roma, Italy
[3] Max-Planck-Institut fur Plasma Physics, 95748 Garching, Germany
[4] Inst. for Fusion Theory and Simulation and Department of Physics Zhejiang University, Hangzhou 310027, China
[5] Dept. Physics and Astronomy, University of California, Irvine, CA 92697- 4575, USA

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E LO SVILUPPO ECONOMICO SOSTENIBILE

Transport processes in burning plasmas

- Predicting the dynamics of a burning plasma on long time scales, comparable with the energy confinement time or longer, is essential in order to understand next generation fusion experiments, e.g., ITER;
- the crucial role of energetic particles Zonca et al. 2015; Chen and Zonca 2016, must be properly described;

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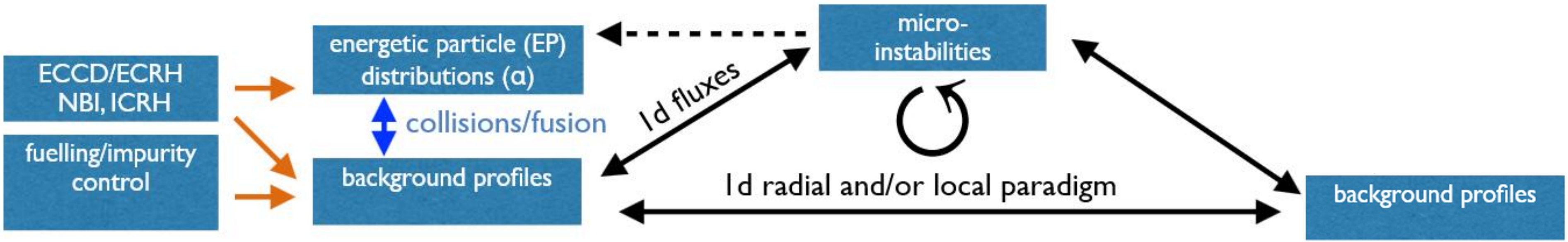
- a first-principle-based, self-consistent approach is crucial;
- extending gyrokinetic simulations to these time scales is a challenging task from the computational resource point of view, i.e., $\sim 10^{24}$grid points;
- simplifying assumptions based on physics understanding and first principles must be introduced;

Transport processes in burning plasmas



Transport processes in burning plasmas

- Transport equations define the evolution of radial profiles

$$\langle \partial_t n \rangle_\psi = - \langle \boldsymbol{\nabla} \cdot (n\boldsymbol{V}) \rangle_\psi \, ;$$

- consistent with a slowly evolving ($\omega^{-1}\partial_t \log p_0 \sim \delta^2$) equilibrium distribution function;

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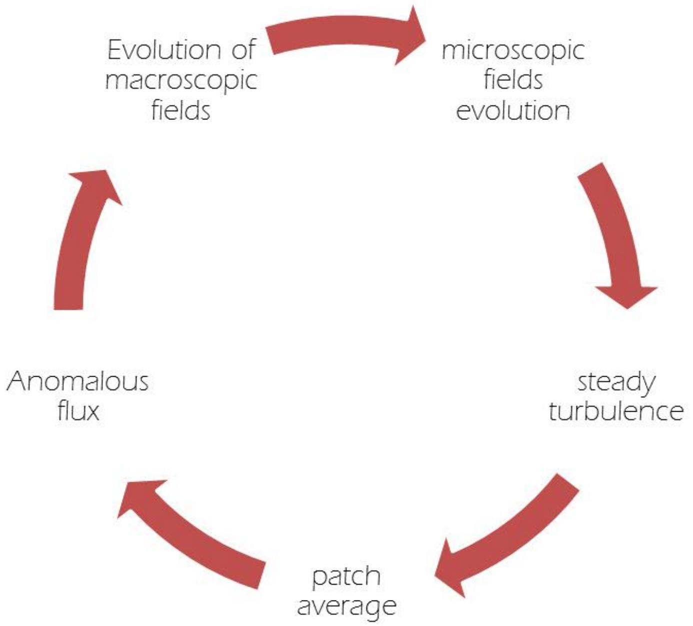

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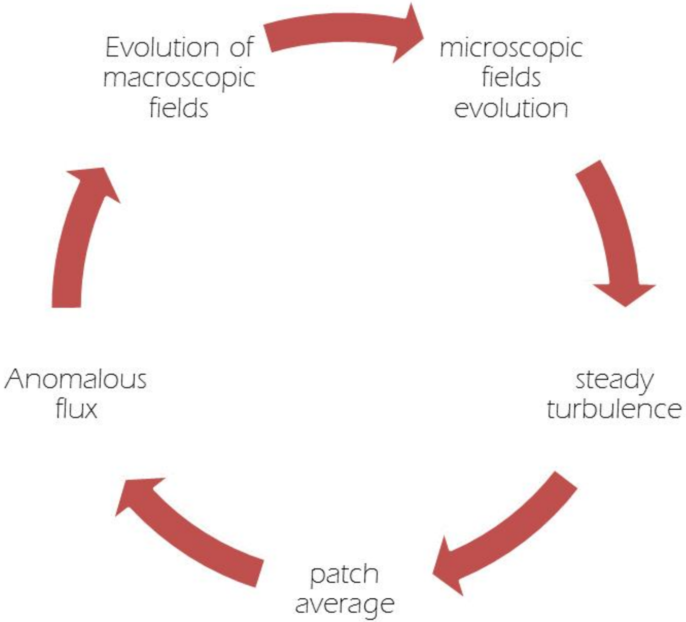
- consistent with a slowly evolving ($\omega^{-1}\partial_t \log p_0 \sim \delta^2$) equilibrium distribution function;
- Implicit separation of scales betwee equilibrium and fluctuations;
- Local Maxwellian is assumed;

Need for generalization!!!



Purpose statement

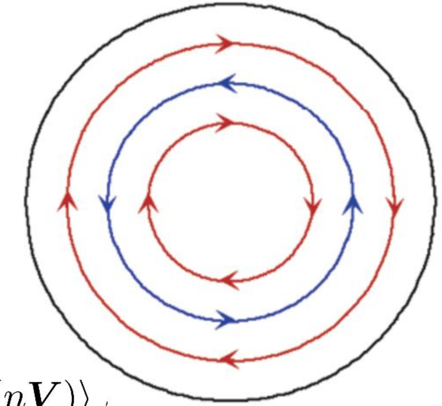
We aim at 1. providing the general expressions describing EPs (plasma) dynamics on long time scales (transport) and 2. introducing a framework to solve these equations within different levels of reduced dynamics.

By means of this approach it is possible to:

- define the concept of nonlinear equilibrium;
- describe the physics of burning plasmas where alpha particles will play a key role in transport studies by interacting with thermal components;
- the derivation, see Falessi 2017; Falessi and Zonca 2019, is based on the Phase space zonal structures theory, see Chen and Zonca 2016.

Phase Space Zonal Structures

- Coupling between fluctuating fields can generate zonal flows and zonal fields;
- Crucial elements for regulating turbulent fluxes e.g., by scattering instability turbulence to shorter radial wavelength stable domain…



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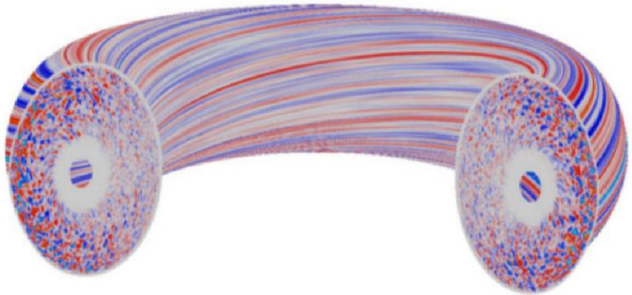

Figure: Courtesy of Y. Xiao et al., PoP 2015.

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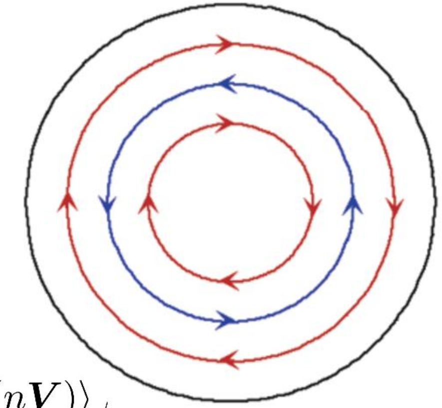

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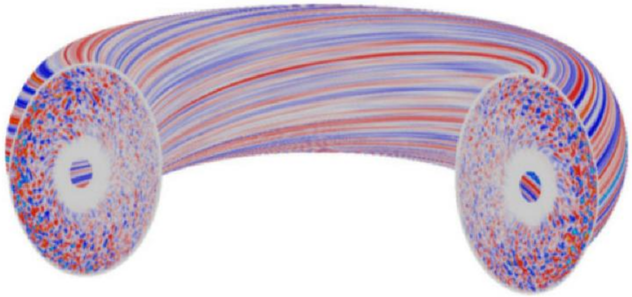

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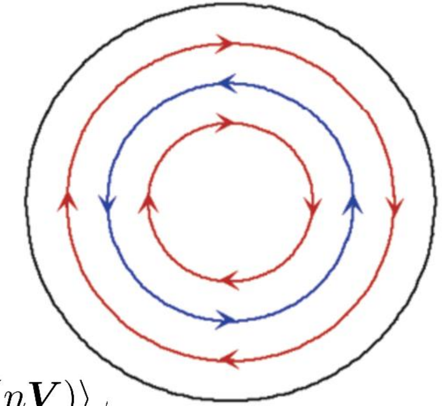
- Crucial elements for regulating turbulent fluxes e.g., by scattering instability turbulence to shorter radial wavelength stable domain…
- zonal structures in the density and temperature profiles are unaffected by rapid collision-less dissipation;
- collision-less undamped fluctuations in the phase space are called phase space zonal structures Chen and Zonca 2016, Falessi and Zonca 2019;



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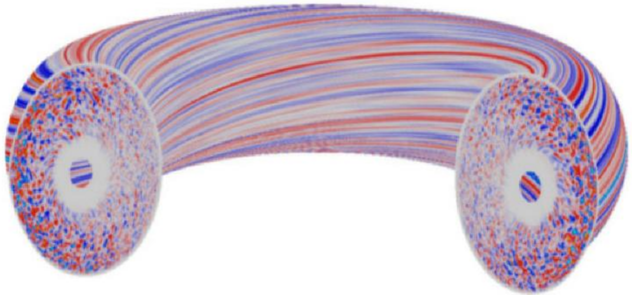

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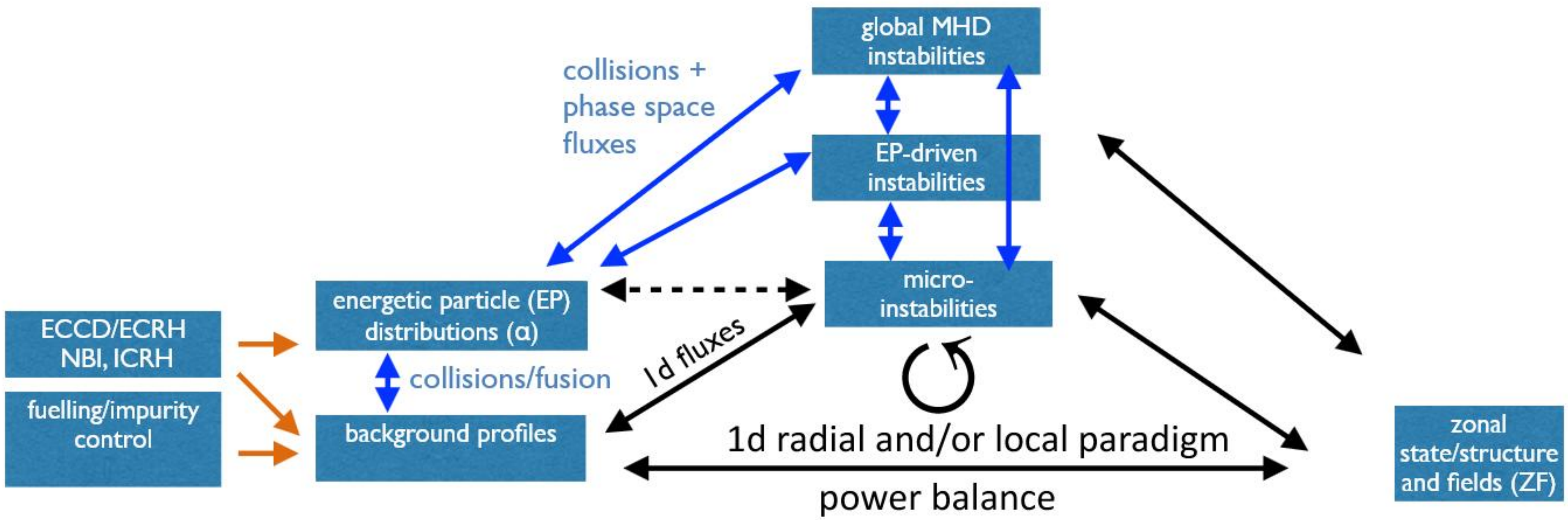
Transport processes in burning plasmas



Zonal Structures

$$\langle \partial_t n \rangle_\psi = - \langle \nabla \cdot (n\boldsymbol{V}) \rangle_\psi$$

- We can re-write the low frequency (transport) component of δf_z in term of only $\delta \overline{G}_B$;
- taking the time derivative of the surface averaged velocity integral we obtain:

$$\partial_t \langle\langle \delta f_z \rangle_v \rangle_\psi = \frac{e}{m} \left\langle \left[1 - \overline{\left(e^{-iQ_z} J_0\right)}\, \overline{\left(e^{iQ_z} J_0\right)} \right] \frac{\partial F_0}{\partial \mathcal{E}} \partial_t \delta\phi_z \right\rangle_v + \frac{1}{V'} \frac{\partial}{\partial \psi} \left\langle \left\langle V' \overline{\left(e^{-iQ_z} J_0\right) \left[c e^{iQ_z} R^2 \nabla\phi \cdot \nabla \langle \delta L_g \rangle \delta G \right]} \right\rangle_v \right\rangle_\psi$$

- This equation describes the radial oscillations on any length-scale of the density profile in the absence of collisions and assuming GK ordering Falessi and Zonca 2019;
- mesoscales are spontaneously created by the turbulence;
- This naturally leads to define a reference distribution function F_0 at each instant of

Orbit averaging

- Particle motion in the reference magnetic field is characterized by three integrals of motion, i.e. $P_\phi, \mu, \mathcal{E}$;

Orbit averaging

- Particle motion in the reference magnetic field is characterized by three integrals of motion, i.e. $P_\phi, \mu, \mathcal{E}$;
- Phase Space Zonal Structures equation is connected with the macro-/meso- scopic component, i.e. $[\ldots]_S$, unperturbed orbit averaged distribution function (Falessi et al NJP 2023);

$$\partial_t \overline{F_{z0}} + \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b \delta\dot{P}_\phi \delta F\right)_z} + \frac{\partial}{\partial \mathcal{E}}\overline{\left(\tau_b \delta\dot{\mathcal{E}} \delta F\right)_z}\right]_S = \overline{\left(\sum_b C_b^g\left[F, F_b\right] + \mathcal{S}\right)}_{zS}$$

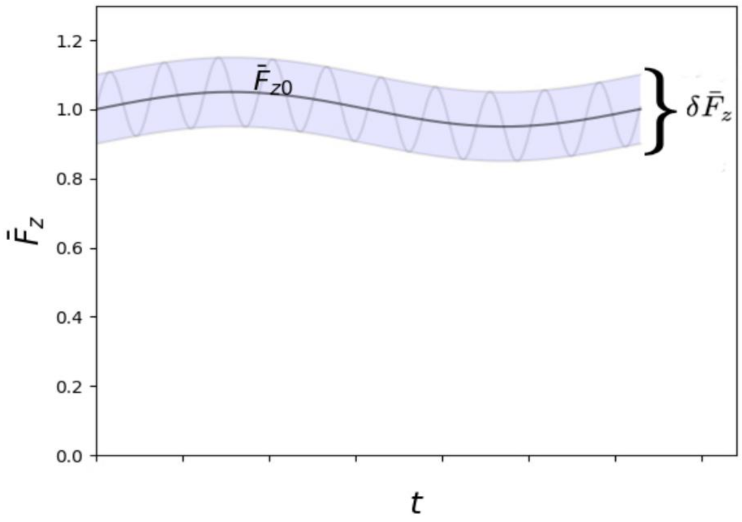
where $\overline{(\ldots)} = \tau_b^{-1}\oint d\theta/\dot{\theta}\,(\ldots) = \oint d\theta/\dot{\theta}\, e^{iQ_z}(\ldots)(\bar{\psi}, \theta)$, with $\tau_b = \oint d\theta/\dot{\theta}$ and $\psi = \overline{\psi} + \delta\widetilde{\psi}(\theta)$;

- equivalent to bounce/transit averaging a quantity shifted with the e^{iQ_z} operator;
- This expression describe transport processes in the phase space due to fluctuations, collisions and sources.

Neighboring nonlinear equilibria

- Having defined Phase Space Zonal Structures, we can decompose the toroidally symmetric distribution function;
- $\overline{F_{z0}}$ describe macro- & meso-scales;
- micro-scales are accounted by $\delta\overline{F}_z$;

$$F_z = \overline{F_{z0}} + \delta\bar{F}_z + \delta\tilde{F}_z$$



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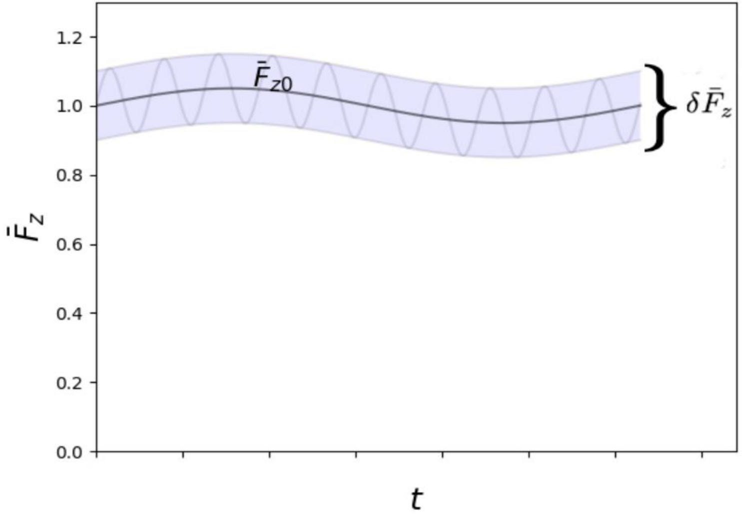
- $\overline{F_{z0}}$ describe macro- & meso-scales;
- micro-scales are accounted by $\delta \overline{F}_z$;
- they describe system transitions between neighboring nonlinear equilibria, see Chen and Zonca 2007; Falessi and Zonca 2019;
- nonlinear equilibria, together with zonal fields, form a zonal state.

$$F_z = \overline{F_{z0}} + \delta \bar{F}_z + \delta \tilde{F}_z$$



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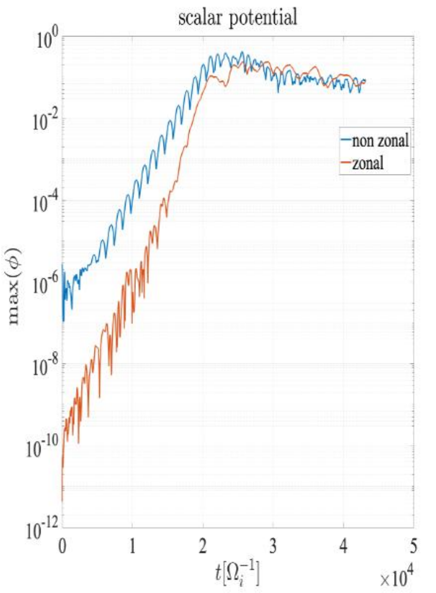
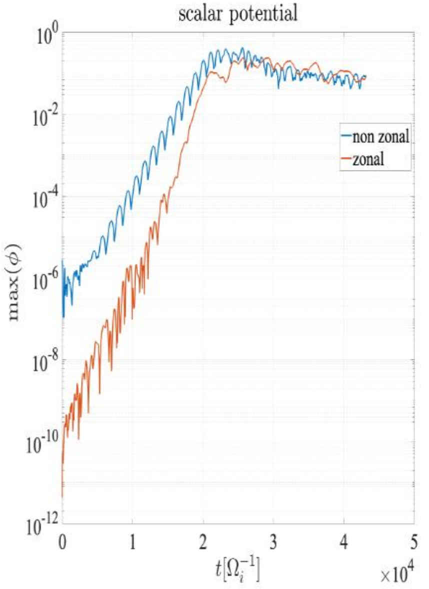


Figure 10: BAE time evolution

Neighboring nonlinear equilibria



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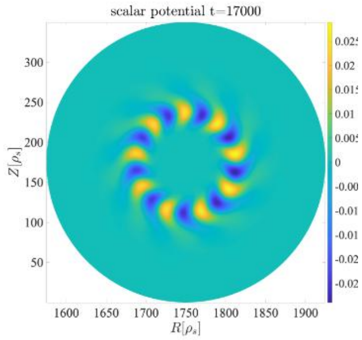

J. Sama Varenna 2024.
See also the talk by A. Biancalani

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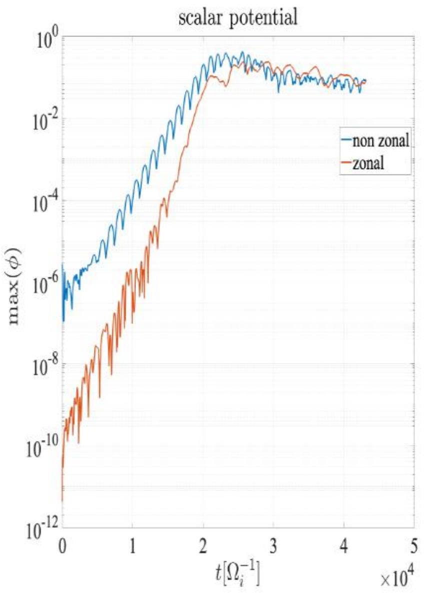

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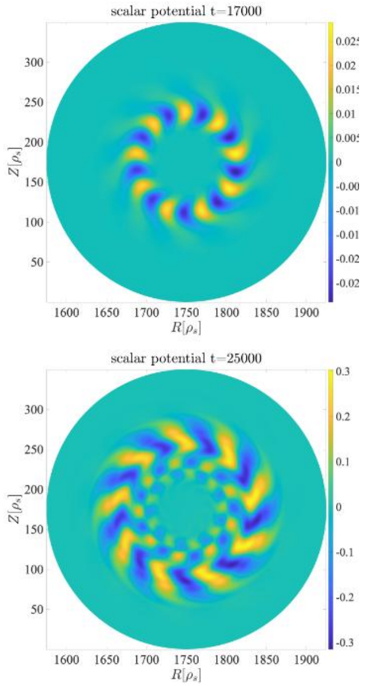

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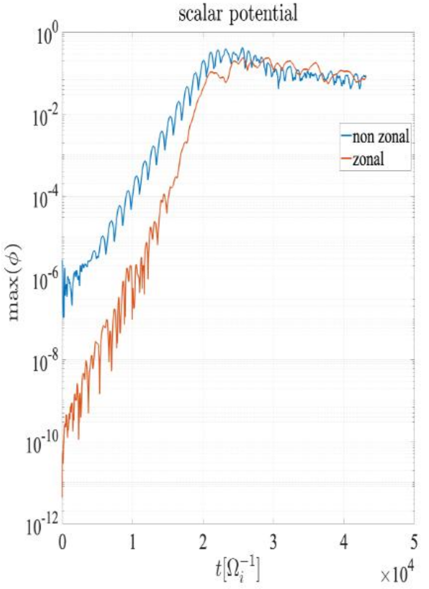

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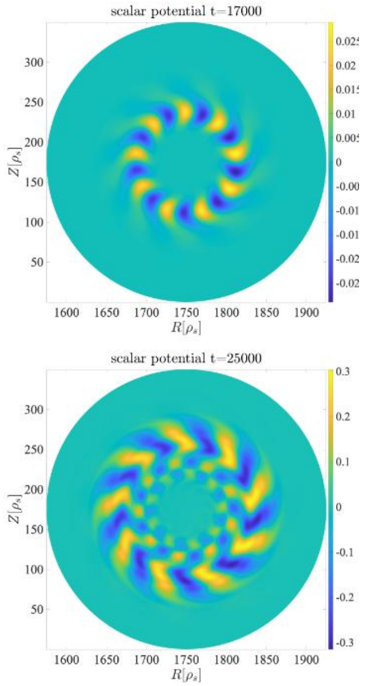

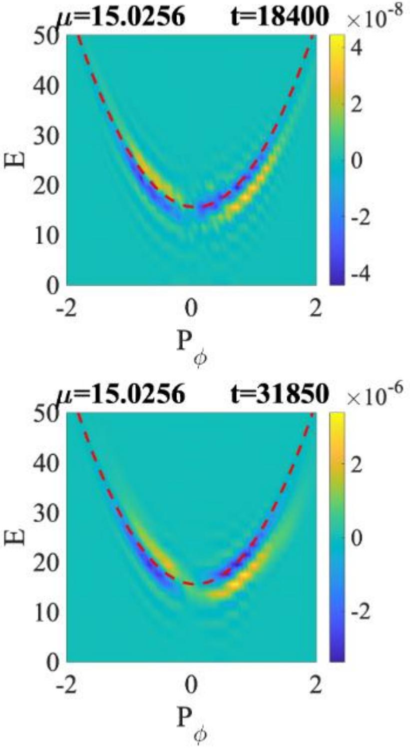

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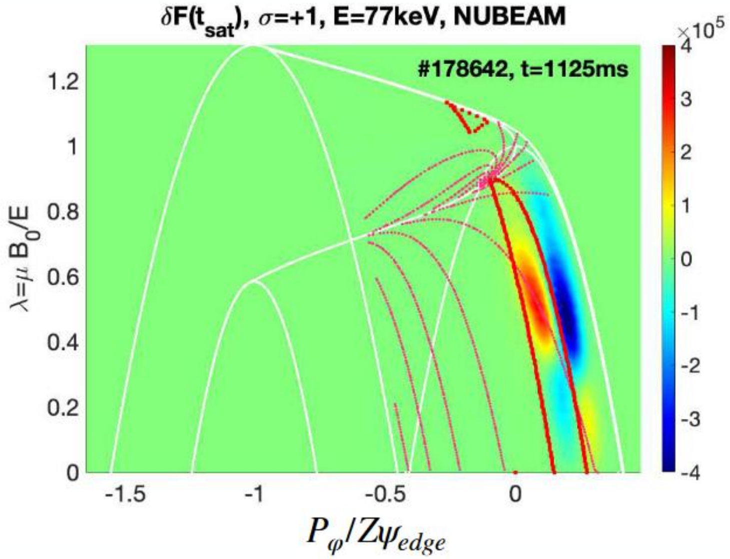
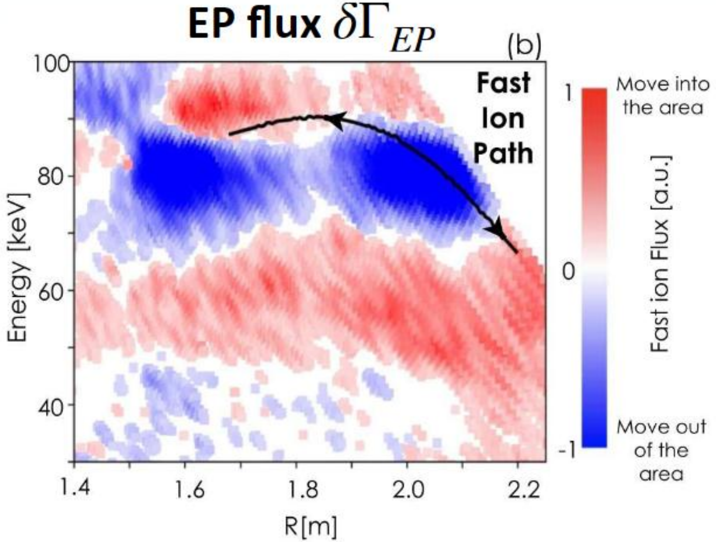
New EP28 : BAE V&V in DIII-D plasmas

➢**Step #4**: Perform a **V&V analysis** on the PSZS [1] observed (i.e. orbit averaged δF_{EP} or $\delta \dot{P}_{\varphi} = \delta \dot{X}_{GC} \cdot \nabla P_{\varphi}$)

◉**Numerical benchmark** on #178642 at early BAE saturation



◉**Experimental validation** on #179415 using the **INPA measurements** [2,3], connected to **EP24**



◉$\delta\Gamma_{EP}$ can be computed in **simulations** from **projection** of $\delta F_{EP}\delta\dot{P}_{\varphi}$ in **CoM space**, and compared through synthetic diagnostic [4]

[1] M. V. Falessi et al. 2023, *New J. Phys.*
[2] X. D. Du et al. 2021 *Phys. Rev. Lett.*
[3] X. D. Du et al. 2023 *Nucl. Fusion*
[4] J. Gonzalez-Martin et al. 2022, *Nucl. Fusion*

Reduced transport models

- PSZS transport is studied by means of a hierarchy of verified and validated reduced models;
- explicit expression of EP fluxes in PSZS equations have been calculated within the following hierarchy of simplifying assumptions:

- the zeroth level of simplification consist in the gyrokinetics description of plasma dynamics;
- the first level of simplification consist in assuming $|\omega| \gg \tau_{NL}^{-1} \sim \gamma_L$, Zonca et al 2021;
- the second and final level of simplification is the quasilinear model

ATEP: kick model limit

$$\partial_t \overline{F_{z0}} + \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b \delta \dot{P}_\phi \delta F\right)_z} + \frac{\partial}{\partial \mathcal{E}}\overline{\left(\tau_b \delta \dot{\mathcal{E}} \delta F\right)_z}\right]_S = \overline{\left(\sum_b C_b^g\left[F, F_b\right] + \mathcal{S}\right)_{z\,S}}$$

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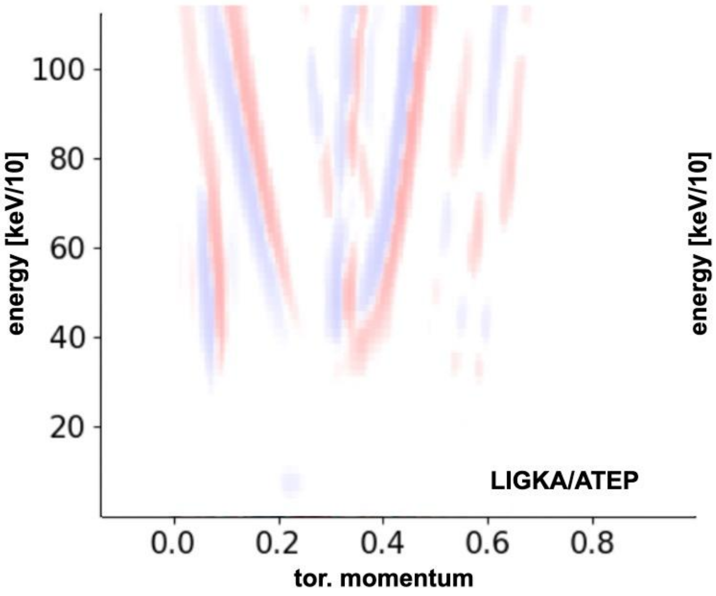
$$\partial_t \overline{F_z^{(O)}} = -\frac{\partial}{\partial P_\phi}\left(\left\langle\frac{dP_\phi}{dt}\right\rangle \overline{F_z^{(O)}}\right) - \frac{\partial}{\partial E}\left(\left\langle\frac{dE}{dt}\right\rangle \overline{F_z^{(O)}}\right) - \frac{\partial}{\partial \Lambda}\left(\left\langle\frac{d\Lambda}{dt}\right\rangle \overline{F_z^{(O)}}\right)$$

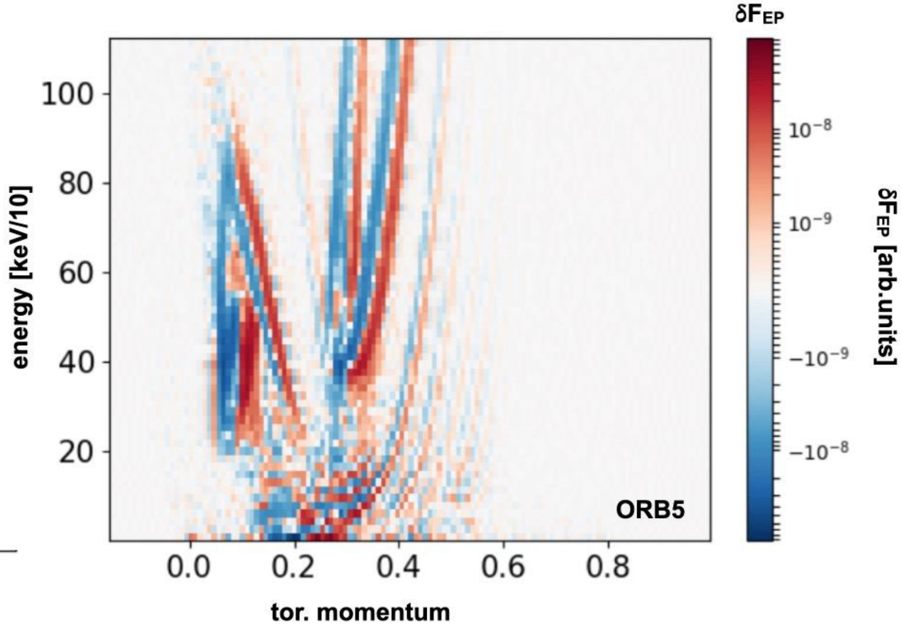
$$-\frac{\partial}{\partial P_\phi}\left(D_{P_\phi P_\phi}\frac{\partial \overline{F_z^{(O)}}}{\partial P_\phi}\right) - \frac{\partial}{\partial E}\left(D_{EE}\frac{\partial \overline{F_z^{(O)}}}{\partial E}\right) - \frac{\partial}{\partial \Lambda}\left(D_{\Lambda\Lambda}\frac{\partial \overline{F_z^{(O)}}}{\partial \Lambda}\right) + \overline{C}_S^{(O)} + \overline{S}_S^{(O)}$$

$$\frac{d}{dt}\left(E_p + \sum_k W_k\right) = -2\sum_k \gamma_{d,k} W_k$$

PSZS moments

- Phase-space fluxes computed with ATEP 3D and ORB5 demonstrate a good level of consistency





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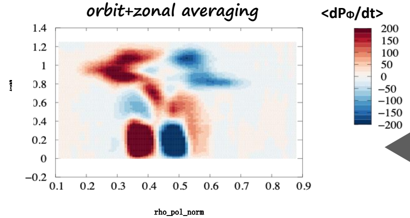

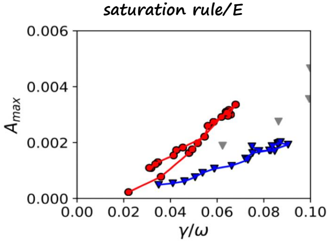

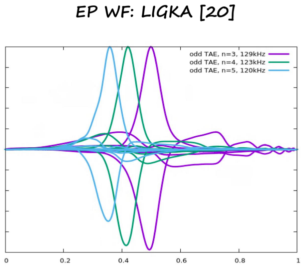

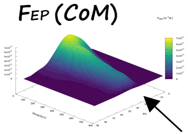

transport code

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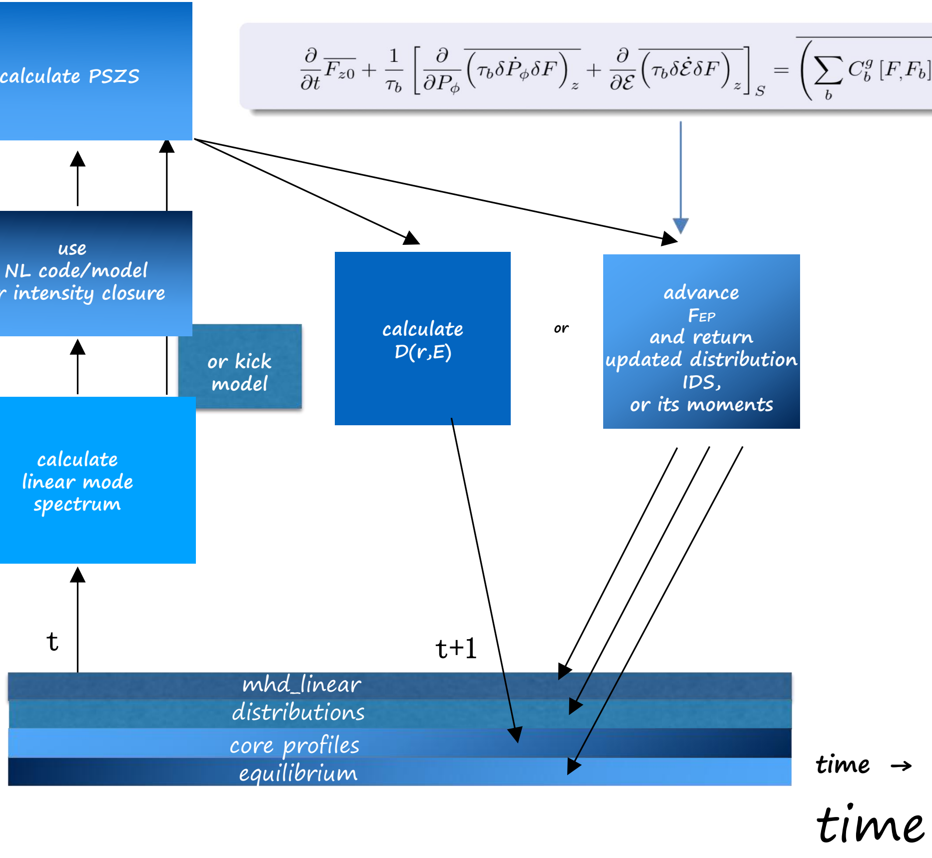


Reduced transport models

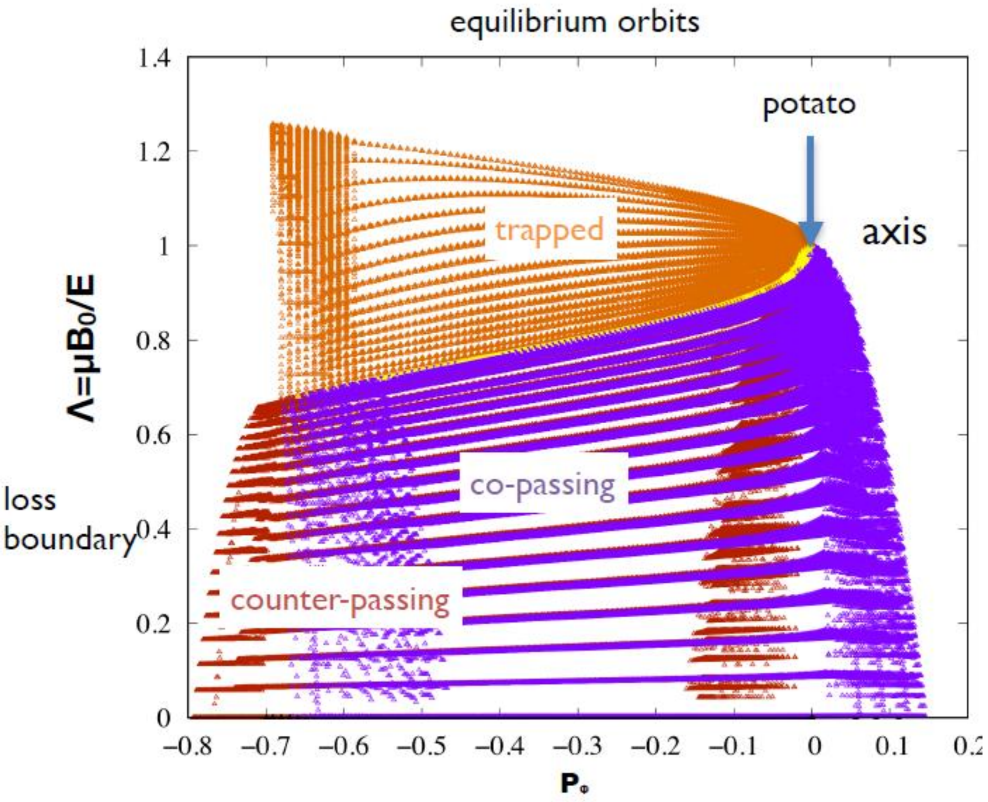
DAS (G. Wei et al 2025)

- Ballooning decomposition for fluctuations;
- Based on fish-bone like dispersion relation;
- Mode structure decomposition, separation of radial envelope and parallel mode structure;
- Calculate nonlinear fluxes by the DSM model or a saturation rule.

LIGKA-HAGIS (Lauber et al 2007)

- Fourier decomposition for fluctuations;
- Solve linear gyrokinetic equation;
- Assume fluctuations amplitude, e.g., kick-model or quasilinear;
- Use IMAS-coupled EP stability WF (HAGIS/LIGKA) to calculate Phase Space Zonal Structures fluxes.

Phase Space fluxes



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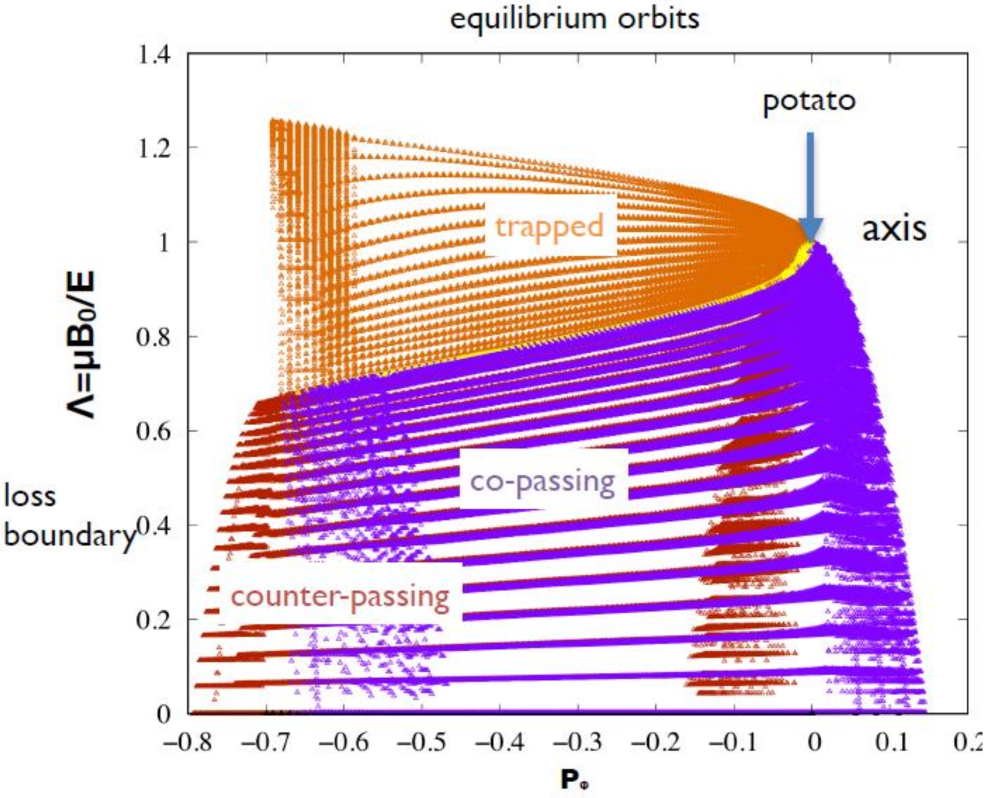
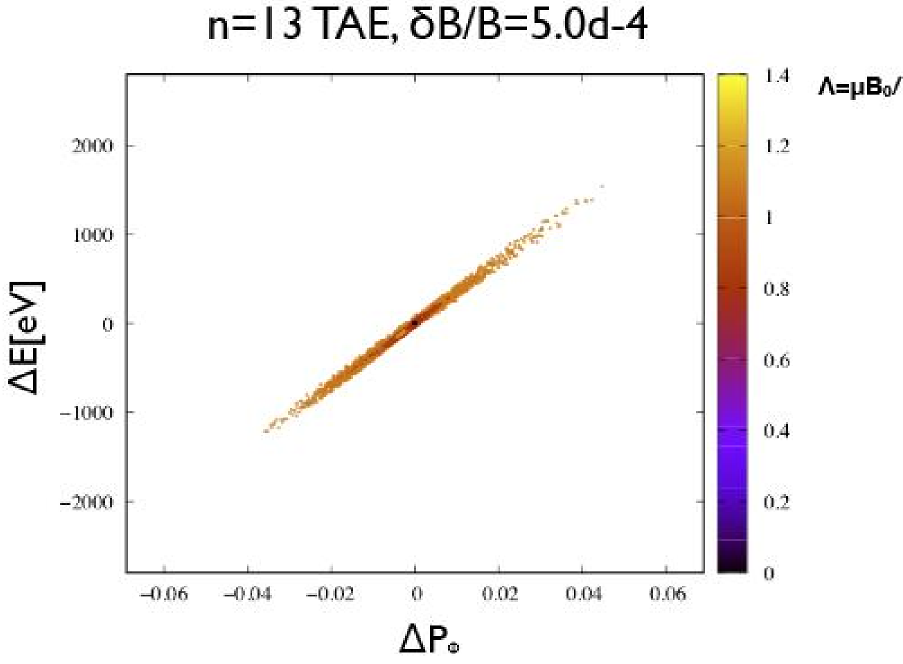
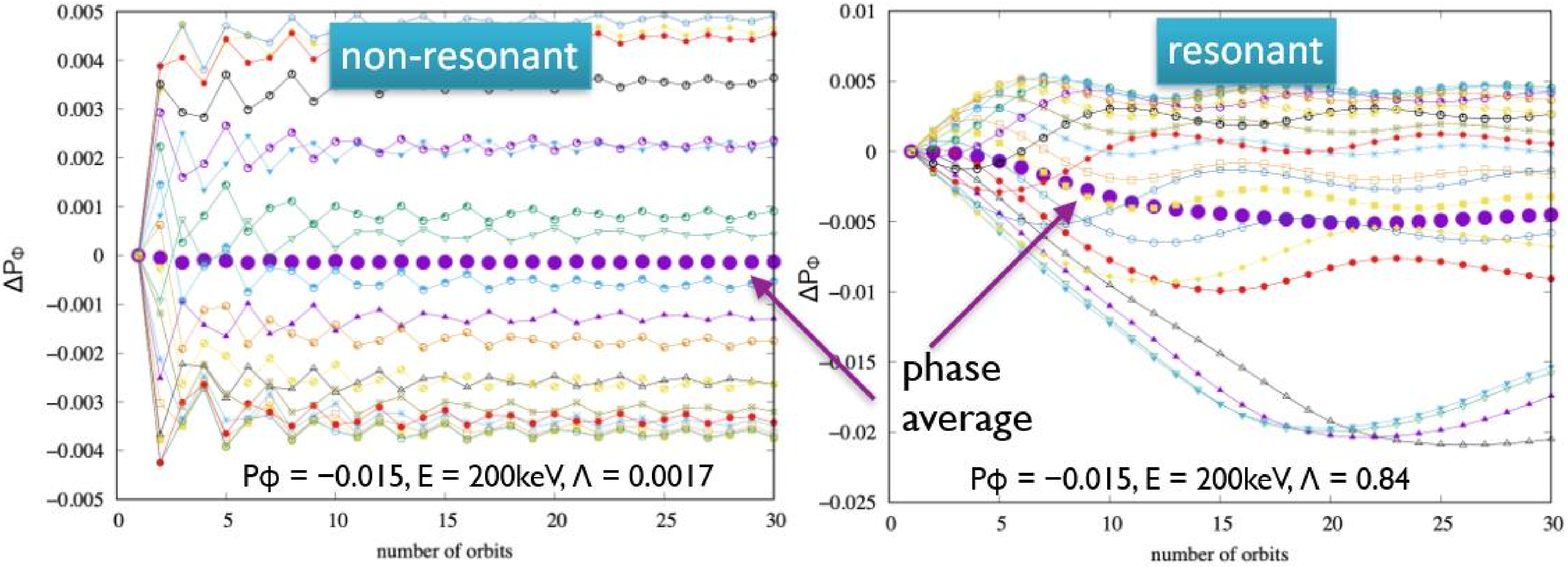




Zonal and orbit averaging



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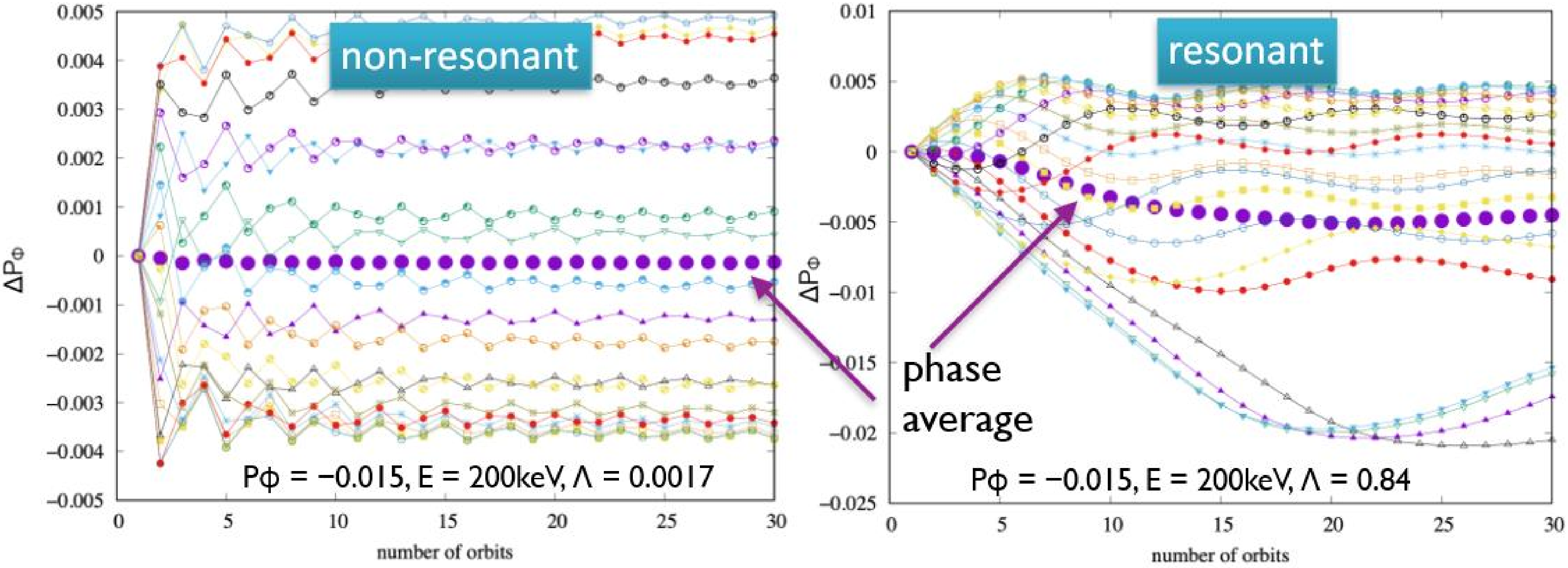

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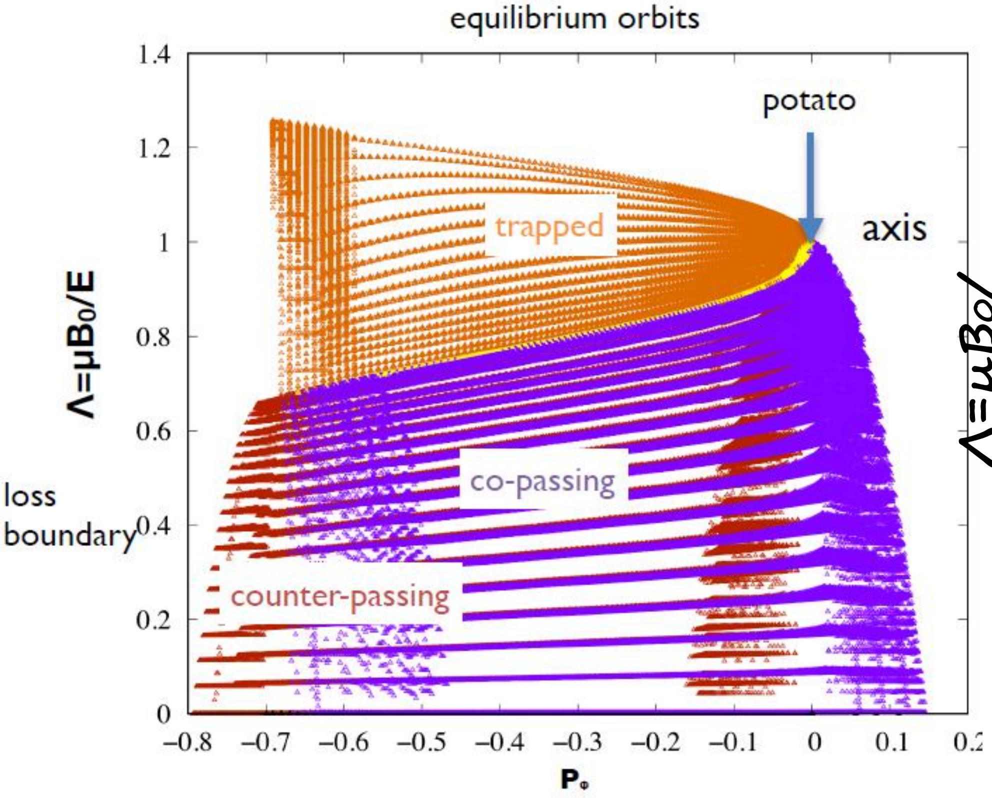

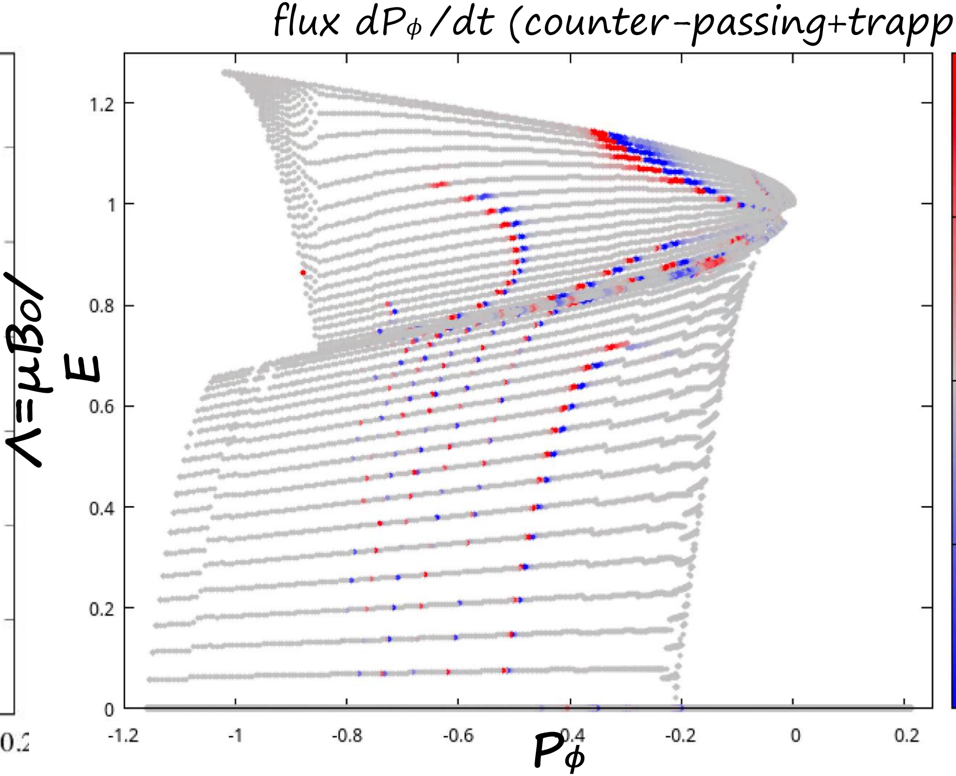

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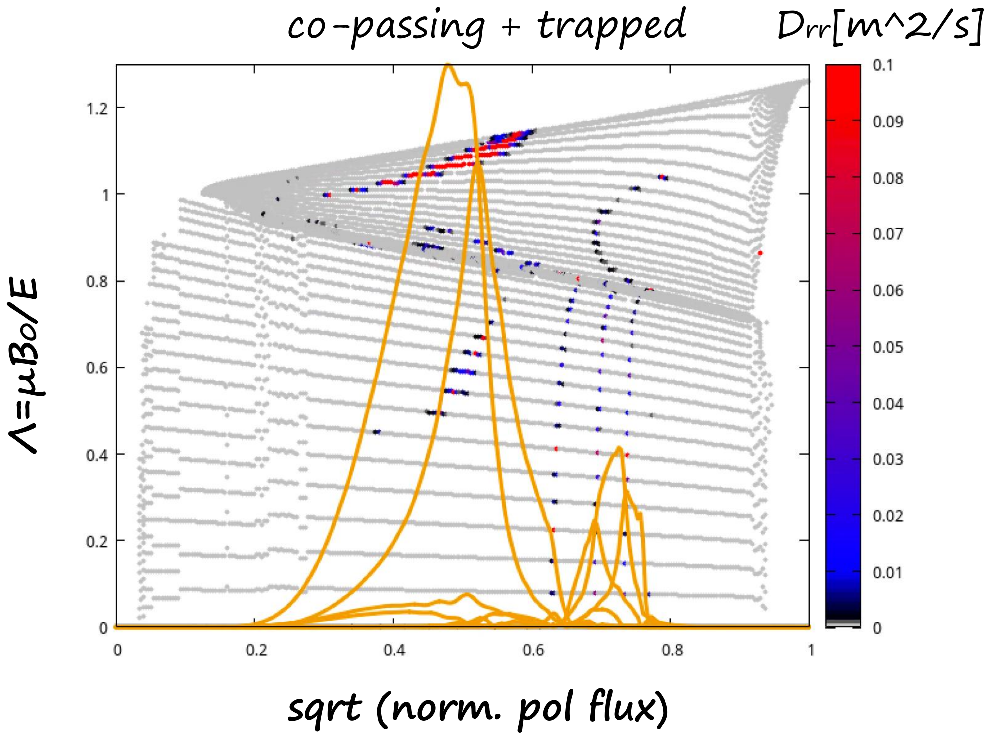
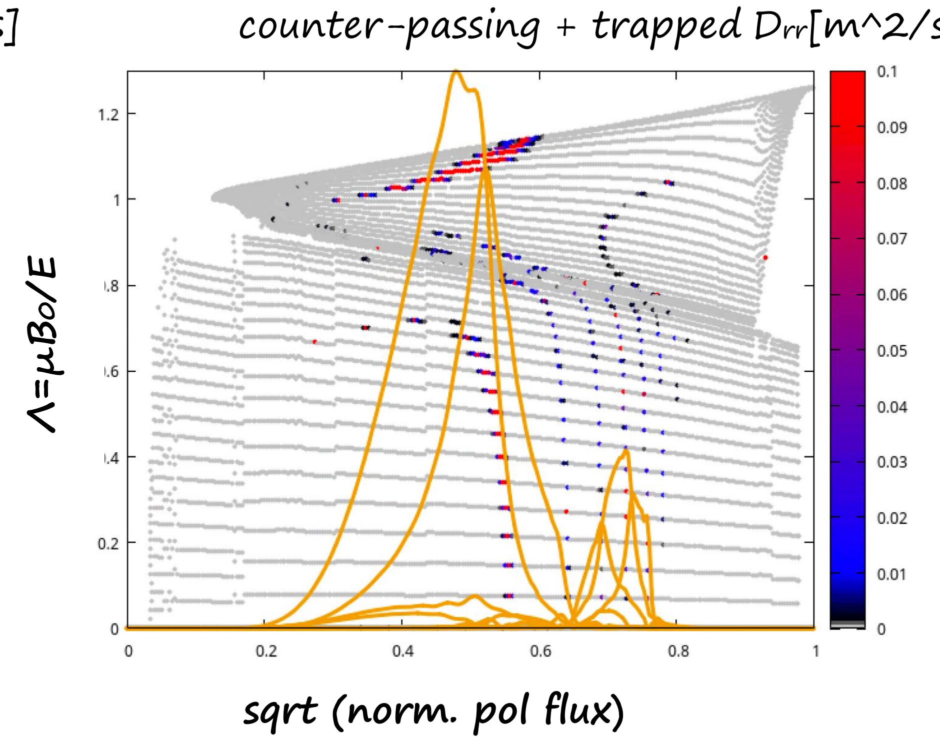
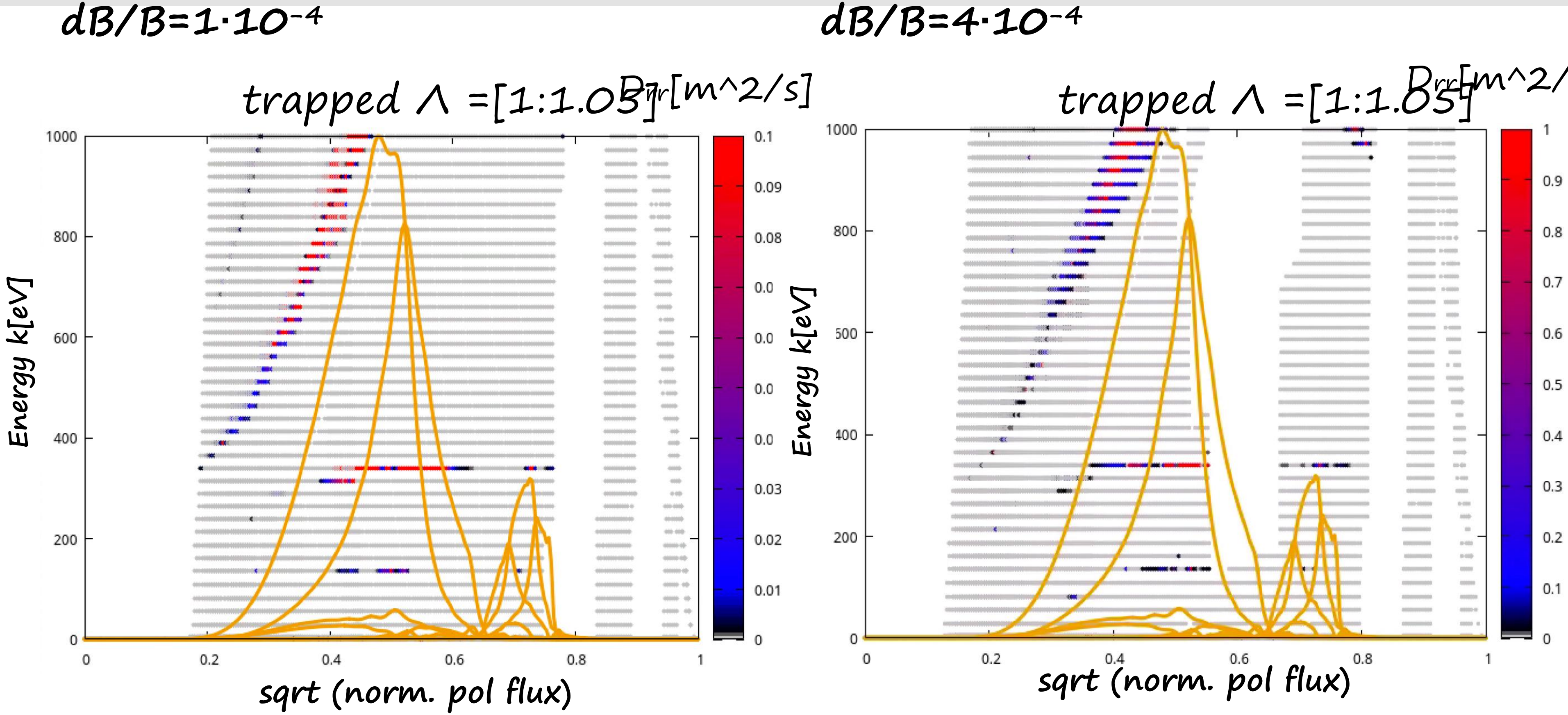




Courtesy of Ph. Lauber

Mode amplitude scaling

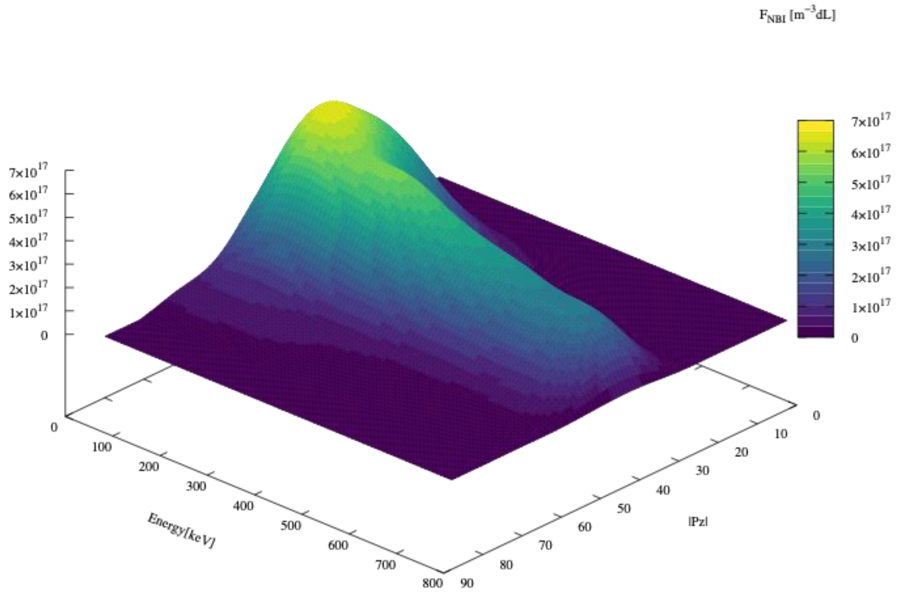


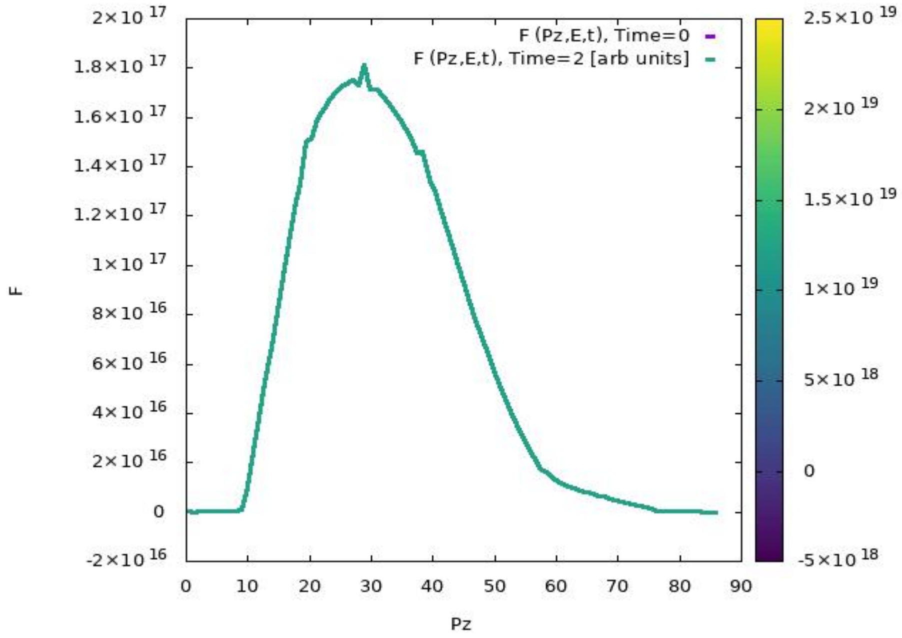
Courtesy of Ph. Lauber

PSZS evolution

ATEP code: solve transport equation for PSZS with sources and collisions, Lauber 2022

$$\frac{\partial}{\partial t}\overline{F_{z0}} + \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b \delta \dot{P}_\phi \delta F\right)}_z + \frac{\partial}{\partial \mathcal{E}}\overline{\left(\tau_b \delta \dot{\mathcal{E}} \delta F\right)}_z\right]_S = \overline{\left(\sum_b C_b^g\left[F, F_b\right] + \mathcal{S}\right)}_{zS}$$

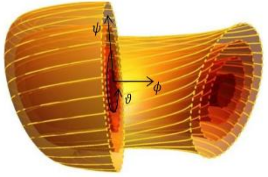


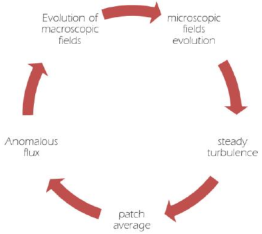


ITER plasma from H&CD WF by M. Schneider

Summary & Conclusions

- Need of predictive transport models on timescales comparable with magnetic fusion experiments, i.e., ITER, properly describing EPs;





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- Need of predictive transport models on timescales comparable with magnetic fusion experiments, i.e., ITER, properly describing EPs;

- we have shown how to describe meso-scales and non-Maxwellian distribution functions using appropriate phase transport equations;

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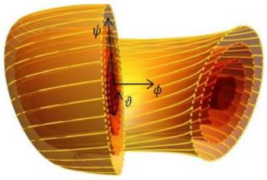

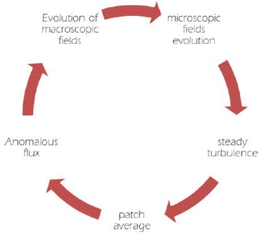

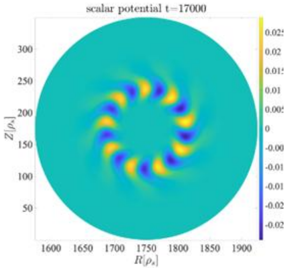

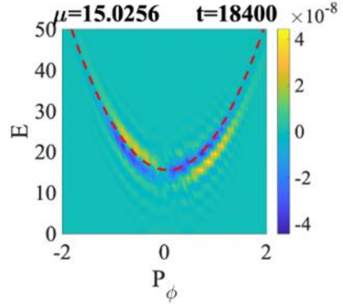

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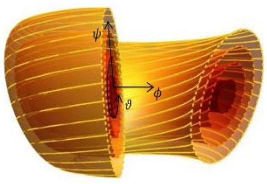
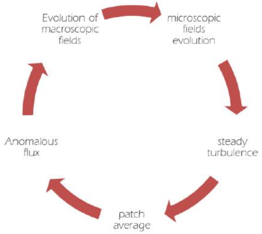
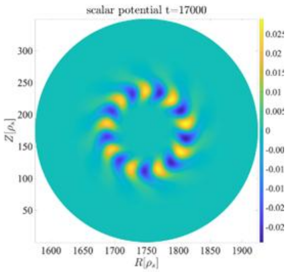
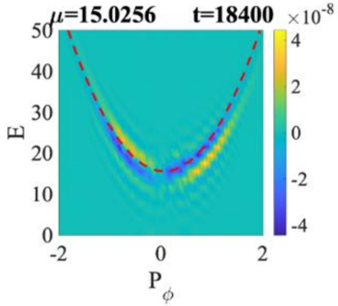
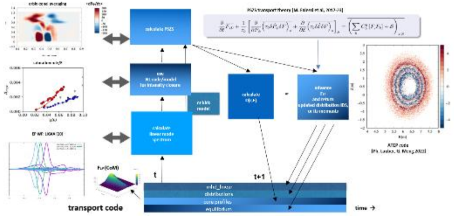
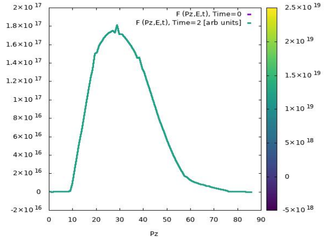




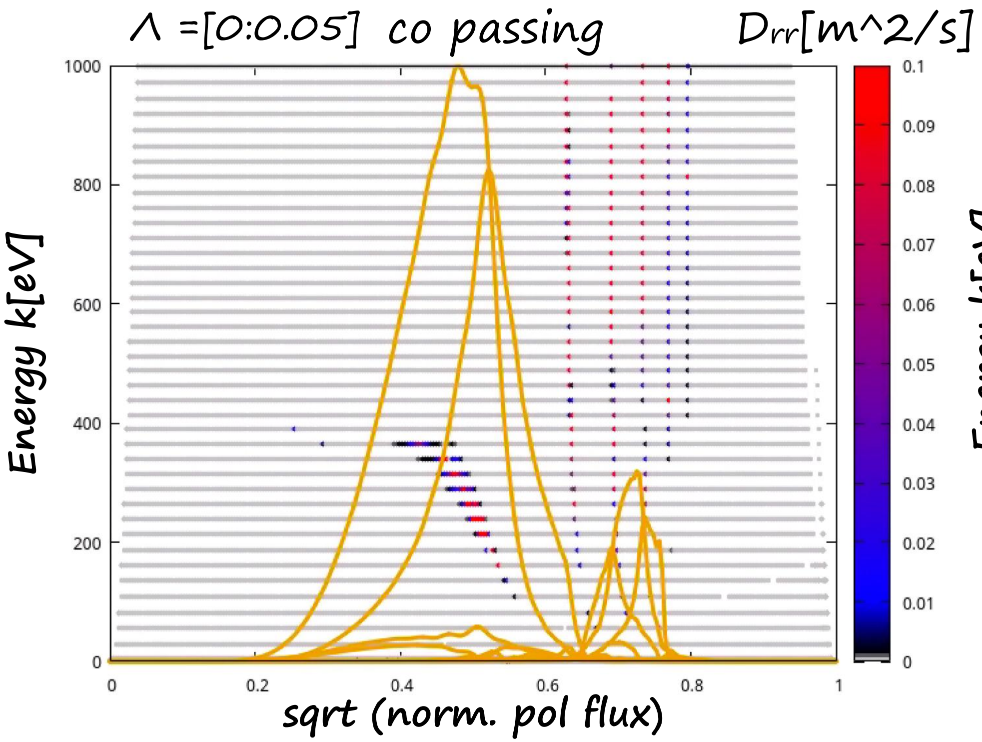


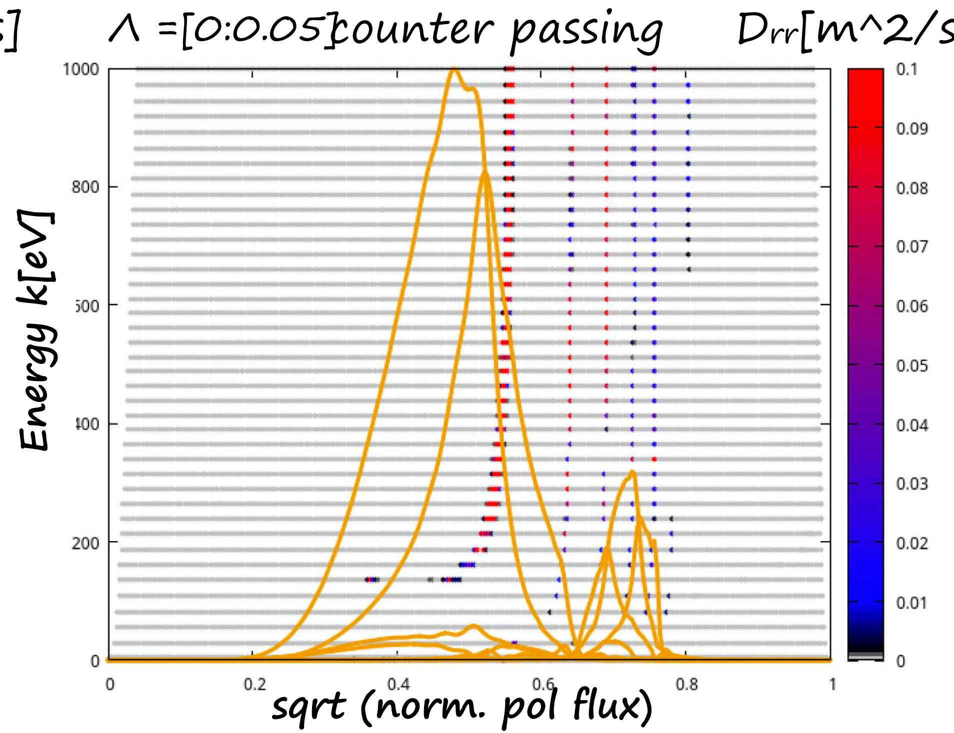






Phase Space fluxes



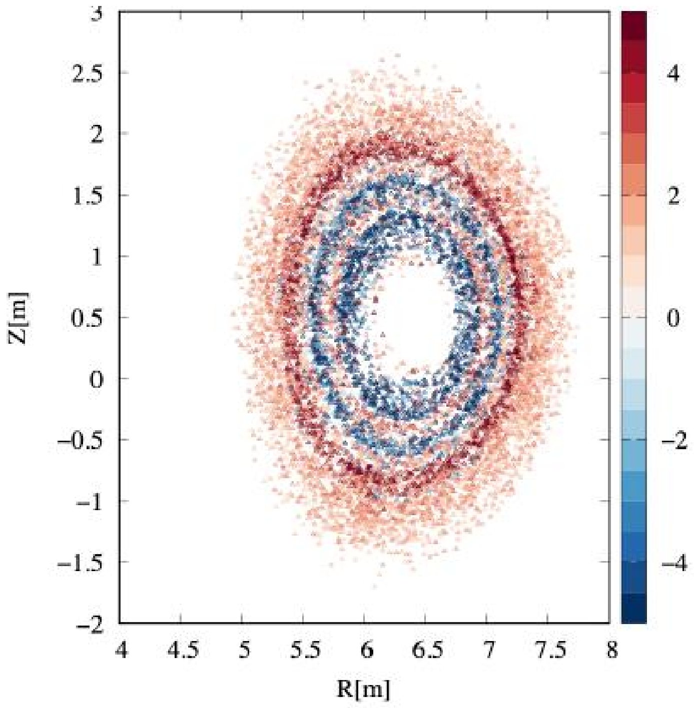


Courtesy of Ph. Lauber

PSZS moments

- Mapping to 1d profile of PSZS;
- The CGL equilibrium can be readily ;
- Next step is the calculation of the corresponding magnetic equilibrium;

$$\sigma \frac{\mathbf{J}_z \times \mathbf{B}}{c} = \nabla P_{\parallel} + (\sigma - 1)\nabla\left(\frac{B^2}{8\pi}\right) + \frac{B^2}{4\pi}\nabla_{\perp}\sigma$$

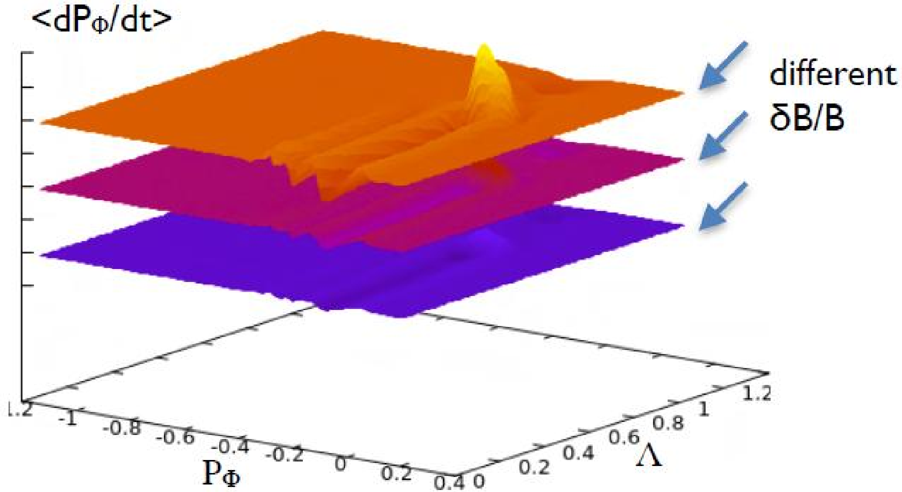


quasi-linear model: add global energy conservation equation

$$\frac{d}{dt}\left(\mathscr{E}+\sum_k W_k\right) = -2\sum_k \gamma_{d,k} W_k$$

$$\mathscr{E}(t) = \int dv_{P_\phi,E,\Lambda} E \cdot F_{EP}(t)$$

amplitude dependent <dP$_\Phi$/dt>, <dE/dt> needed!



- pre-calculate <dP$_\Phi$/dt> for different amplitudes δB/B e.g. $=5 \cdot 10^{-6}, 5 \cdot 10^{-5}, 5 \cdot 10^{-4}, 5 \cdot 10^{-3}$
- interpolate in CoM space, then construct 4D object
- use E-conservation of PSZS transport equation to determine energy transfer to mode and change mode amplitude(s) accordingly
- it includes resonance broadening and transitions from isolated to overlapping modes
- it is NOT yet fully self-consistent, i.e. ratio of mode amplitudes is fixed according to linear growth rates i.e. no radial envelope evolution (but in line with QL assumptions) - DAEPS/DSM model will include this
- in preparation: use information on individual modes' power exchange in HAGIS to construct individual energy balance equation

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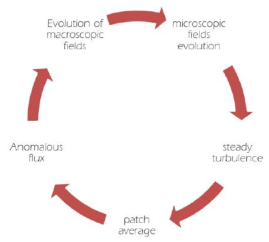

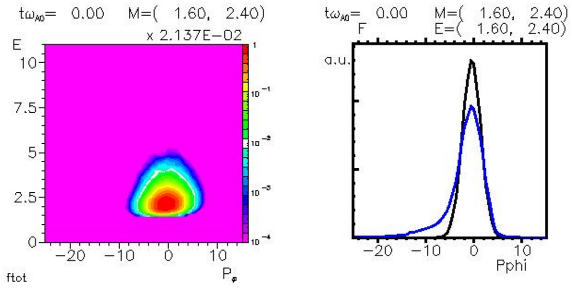

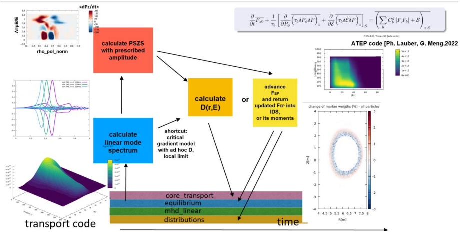

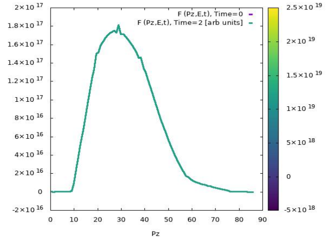

Phase Space Zonal Structures equation

- We want to explicit identify the part of the toroidally symmetric distribution function unaffected by rapid collisionless dissipation that is the PSZS Falessi et al 2023 sub.;
- we start by writing the nonlinear Gyrokinetic equation:

$$\partial_t(DF) + \nabla \cdot \left(D\dot{\mathbf{X}}F\right) + \partial_{\mathcal{E}}\left(D\delta\dot{\mathcal{E}}F\right) = 0$$

D is the Jacobian of the velocity space and $\dot{\mathbf{X}} = \dot{\mathbf{X}}_0 + \delta\dot{\mathbf{X}}$ is the gyrocenter velocity due to magnetic equilibrium and to fluctuating fields;

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- we decompose F as $F_0 + \delta F$ where F_0 is the steady state solution of the lowest order gyrokinetic equation, i.e.:

$$F_0(\psi, \theta, \mathcal{E}, \mu) = e^{-iQ_z} F_{B0}(\psi, \theta, \mathcal{E}, \mu) = F_{B0}\left(\psi - \frac{F v_\parallel}{\Omega}, \mathcal{E}, \mu\right) = F_{B0}\left(\bar{\psi}(\psi, \theta, \mathcal{E}, \mu), \mathcal{E}, \mu\right)$$

where the drift/banana center shift operator e^{iQ_z} with $Q_z \equiv F(v_\parallel/\Omega)k_z/(d\,\psi\,/\,d\,r)$

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where the drift/banana center shift operator e^{iQ_z} with $Q_z \equiv F(v_\parallel/\Omega)k_z/(d\psi/dr)$

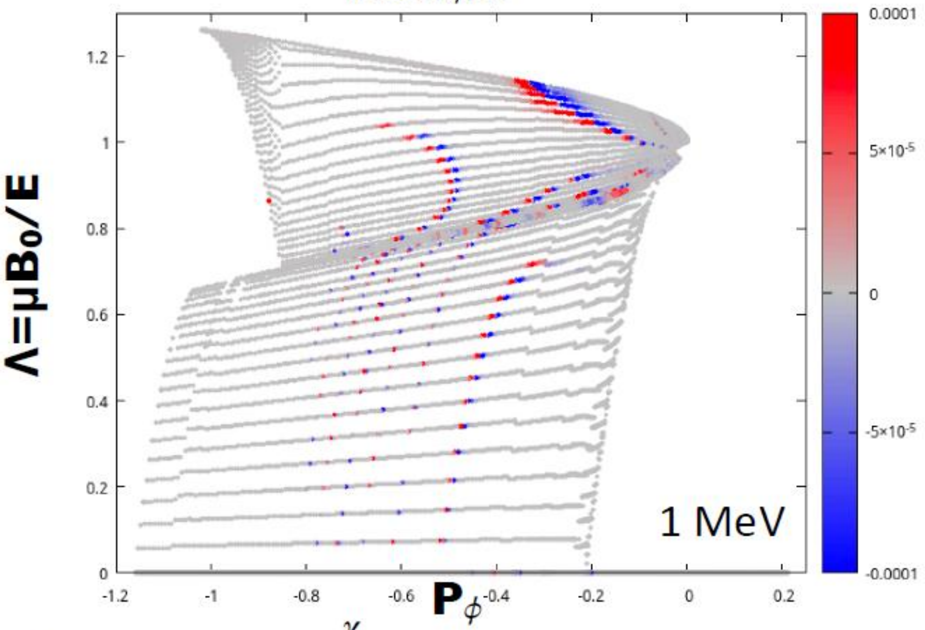
- we can calculate $D\left(\delta\dot{\mathbf{X}} \cdot \nabla + \delta\dot{\mathcal{E}}\partial_{\bar{\mathcal{E}}}\right)F_{B0}(\bar{\psi}(\psi, \theta, \mathcal{E}, \mu), \mathcal{E}, \mu)$…

from fluxes to diffusion coefficients

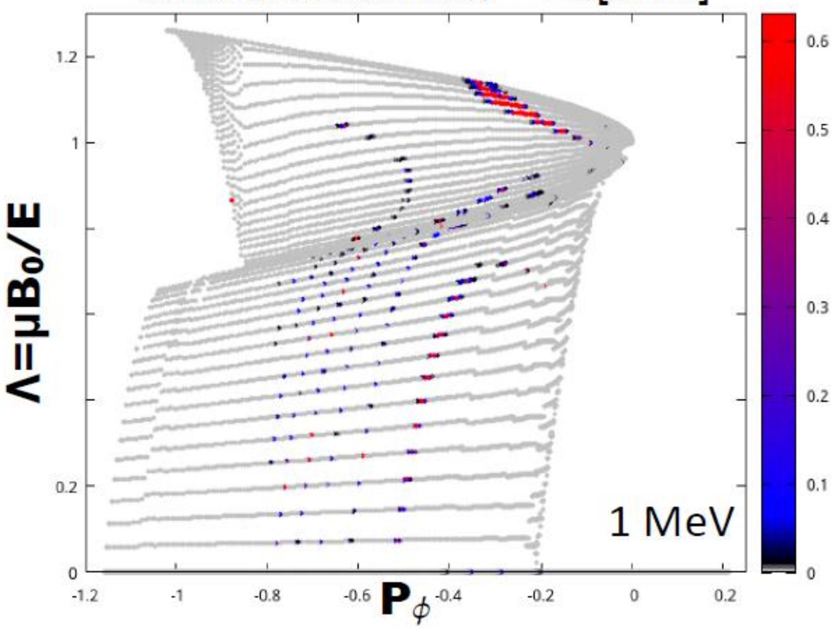
[L Chen, JGR 104, 1999]

$$D_{P,P} = < dP_\phi/dt >^2 \cdot \tau_{ac}$$

flux dP_ϕ/dt



diffusion coefficient $D_{PP} \rightarrow D_{rr}$[m^2/s]



$$\tau_{ac} = \frac{\gamma}{[\omega_{TAE} - \omega_{prec} - p\omega_{t,b}]^2 + \gamma^2}$$

τ_{ac} calculated numerically: include natural line broadening, finite island width; turbulent broadening can be added [Berk, 1995/96, Gorelenkov 2018]

similarly for: $$D_{E,E} = < dE/dt >^2 \cdot \tau_{ac}$$

$$D_{P,E} = < dP_\phi/dt > < dE/dt > \cdot \tau_{ac}$$

Collisional fluxes

$$\frac{\partial}{\partial t}\overline{F}_{z0} + \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b \delta \dot{P}_\phi \delta F\right)_z} + \frac{\partial}{\partial \mathcal{E}}\overline{\left(\tau_b \delta \dot{\mathcal{E}} \delta F\right)_z}\right]_S = \overline{C^g_{z0}} + [\bar{S}]_S$$

- expanding the collision operator we obtain:

$$\bar{C}^g_{z0} = \overline{C^g_z\left[\overline{F}_{z0}, \overline{F}_{z0}\right]} + \left[\overline{C^g_z\left[\overline{F}_{z0}, \delta F_z\right]} + \overline{C^g_z[\delta F, \delta F]}\right]_S$$

- the second term, in the presence of a Maxwellian reference state, describes neoclassical transport;
- corrections to neoclassical transport are given by the first and the third terms;
- PSZS evolution with collisions and sources have been studied in G. Meng et al. 2023

Phase Space Zonal Structures equation

- … and re-write the toroidally symmetric component of the Gyrokinetic equation:

$$D\left(\partial_t + \dot{\mathbf{X}}_0 \cdot \boldsymbol{\nabla}\right)\left(F_z - \frac{e}{m}\langle \delta L_g \rangle_z \frac{\partial}{\partial \mathcal{E}}|_{\bar{\psi}} F_0 + \frac{F}{B_0}\frac{\partial F_0}{\partial \bar{\psi}}\langle \delta A_{\parallel g}\rangle_z\right) + D\frac{e}{m}\frac{\partial}{\partial \mathcal{E}}|_{\bar{\psi}} F_0 \partial_t \langle \delta L_g\rangle_z$$
$$+ \frac{1}{J}\frac{\partial}{\partial \theta}\left(JD\delta\dot{\theta}\delta F\right) + \frac{1}{J}\frac{\partial}{\partial \psi}\left(JD\delta\dot{\psi}\delta F\right) + \frac{\partial}{\partial \mathcal{E}}\left(JD\delta\dot{\mathcal{E}}\delta F\right) = 0$$

where $\langle \delta L_g \rangle_z = J_0\left(\delta\phi_z - \frac{v_\parallel}{c}\delta A_{\parallel z}\right) + 2\frac{m}{\lambda e}\mu J_1 \delta B_{\parallel z}$ and J is the Jacobian of the curvilinear coordinate system;

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$$G_z = F_z - \frac{e}{m}\langle\delta L_g\rangle_z \frac{\partial}{\partial\mathcal{E}}|_{\bar{\psi}} F_0 + \frac{F}{B_0}\langle\delta A_{\parallel g}\rangle_z \frac{\partial}{\partial\bar{\psi}} F_0$$

- We now apply e^{iQ_z} and write the governing equation for $G_B = e^{iQ_z} G_z$

Phase Space Zonal Structures equation

- … we obtain the following equation:

$$\partial_t(JDG_B) + \partial_\theta G_B$$
$$= e^{iQ_z}\left[-\frac{e}{m}\frac{\partial}{\partial\mathcal{E}}\Big|_{\bar{\psi}} F_0 \partial_t\left(JD\langle\delta L_g\rangle_z\right) - \frac{1}{J}\frac{\partial}{\partial\theta}\left(JD\delta\dot{\theta}\delta F\right) - \frac{1}{J}\frac{\partial}{\partial\psi}\left(JD\delta\dot{\psi}\delta F\right) - \frac{\partial}{\partial\mathcal{E}}\left(JD\delta\dot{\mathcal{E}}\delta F\right)\right]$$

It can be shown that e^{iQ_z}, up to the required order, commute with partial derivatives;

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It can be shown that e^{iQ_z}, up to the required order, commute with partial derivatives;

- finally, integrating over θ and introducing the bounce/transit average $\overline{[\ldots]} = \tau_b^{-1}\oint\frac{d\ell}{v_\parallel}[\ldots]$, we obtain:

$$\partial_t\overline{G}_B = -\overline{e^{iQ_z}\frac{e}{m}\frac{\partial}{\partial\mathcal{E}}|_{\bar{\psi}}F_0\partial_t\langle\delta L_g\rangle_z} - \frac{1}{\tau_b}\frac{\partial}{\partial\psi}\left(\tau_b\overline{e^{iQ_z}\delta\dot{\psi}\delta F}\right) - \frac{1}{\tau_b}\frac{\partial}{\partial\mathcal{E}}\left(\tau_b\overline{e^{iQ_z}\delta\dot{\mathcal{E}}\delta F}\right)$$

Particle transport

- We can re-write the low frequency (transport) component of δf_z in term of only $\delta \overline{G}_B$;
- taking the time derivative of the surface averaged velocity integral we obtain:

$$\partial_t \langle\langle \delta f_z \rangle_v \rangle_\psi = \frac{e}{m} \left\langle \left[1 - \overline{\left(e^{-iQ_z} J_0\right)} \, \overline{\left(e^{iQ_z} J_0\right)} \right] \frac{\partial F_0}{\partial \mathcal{E}} \partial_t \delta \phi_z \right\rangle_v +$$

$$\frac{1}{V'} \frac{\partial}{\partial \psi} \left\langle \left\langle V' \overline{\left(e^{-iQ_z} J_0\right) \left[c e^{iQ_z} R^2 \boldsymbol{\nabla} \phi \cdot \boldsymbol{\nabla} \langle \delta L_g \rangle \delta G \right]} \right\rangle_v \right\rangle_\psi$$

- This equation describes the radial oscillations on any length-scale of the density profile

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- This equation describes the radial oscillations on any length-scale of the density profile in the absence of collisions and assuming GK ordering Falessi and Zonca 2019;
- mesoscales are spontaneously created by the turbulence;
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Equilibrium renormalization

- This naturally leads to define a reference distribution function F_0 at each instant of time. Going back to the previous derivation and rewriting terms such as:

$$\delta\dot{\mathbf{X}}\big|_z \cdot \nabla\delta F_z = \delta\dot{\mathbf{X}}\big|_z \cdot \nabla e^{-iQ_z}\left(\delta\bar{F}_{Bz} + \delta\widetilde{F}_{Bz}\right)$$

- $e^{-iQ_z}\delta\bar{F}_{Bz}$ analogously to F_0, is a function of the constants of motion;

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- $e^{-iQ_z}\delta\bar{F}_{Bz}$ analogously to F_0, is a function of the constants of motion;
- we obtain a newly defined G_z:

$$G_z = F_z - \frac{e}{m}\langle\delta L_g\rangle_z \frac{\partial}{\partial\mathcal{E}}|_{\bar{\psi}} F_{0*} + \frac{F}{B_0}\langle\delta A_{\parallel g}\rangle_z \frac{\partial}{\partial\bar{\psi}} F_{0*}$$

- Where $F_{0*} = F_0 + e^{-iQ_z}\delta\bar{F}_{Bz}$ is the renormalized F_0;
- Phase Space Zonal Structures are the macro-/meso- scopic component of F_{0*}

Orbit averaging

- Particle motion in the reference magnetic field is characterized by three integrals of motion, i.e. $P_\phi, \mu, \mathcal{E}$;
- Phase Space Zonal Structures equation is connected with the macro-/meso- scopic component, i.e. $[...]_S$, unperturbed orbit averaged distribution function (Falessi and Zonca 2019);

$$\partial_t \overline{F_{z0}} + \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b \delta\dot{P}_\phi \delta F\right)_z} + \frac{\partial}{\partial \mathcal{E}}\overline{\left(\tau_b \delta\dot{\mathcal{E}} \delta F\right)_z}\right]_S = \overline{\left(\sum_b C_b^g\left[F, F_b\right] + \mathcal{S}\right)_{zS}}$$

where $\overline{(\ldots)} = \tau_b^{-1}\oint d\theta/\dot{\theta}\,(\ldots) = \oint d\theta/\dot{\theta}\, e^{iQ_z}(\ldots)(\bar{\psi},\theta)$, with $\tau_b = \oint d\theta/\dot{\theta}$ and $\psi = \overline{\psi} + \delta\widetilde{\psi}(\theta)$;

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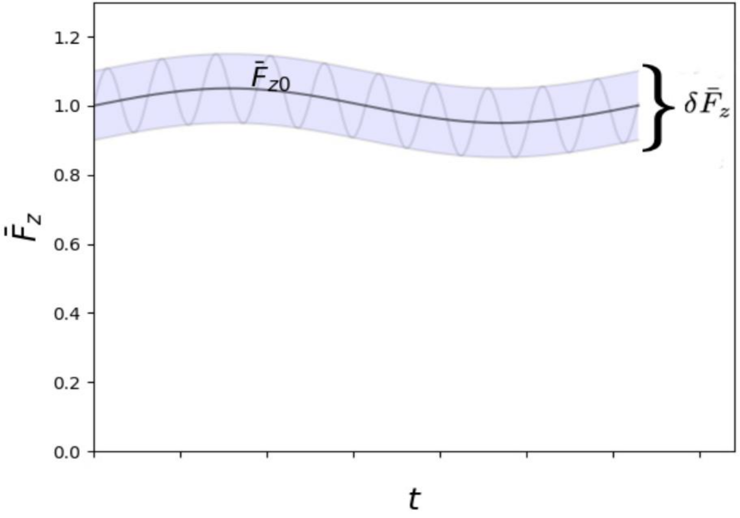
- equivalent to bounce/transit averaging a quantity shifted with the e^{iQ_z} operator;
- This expression describe transport processes in the phase space due to fluctuations, collisions and sources.

Neighboring nonlinear equilibria

- Having defined Phase Space Zonal Structures, we can decompose the toroidally symmetric distribution function;
- $\overline{F_{z0}}$ describe macro- & meso-scales;
- micro-scales are accounted by $\delta\overline{F}_z$;

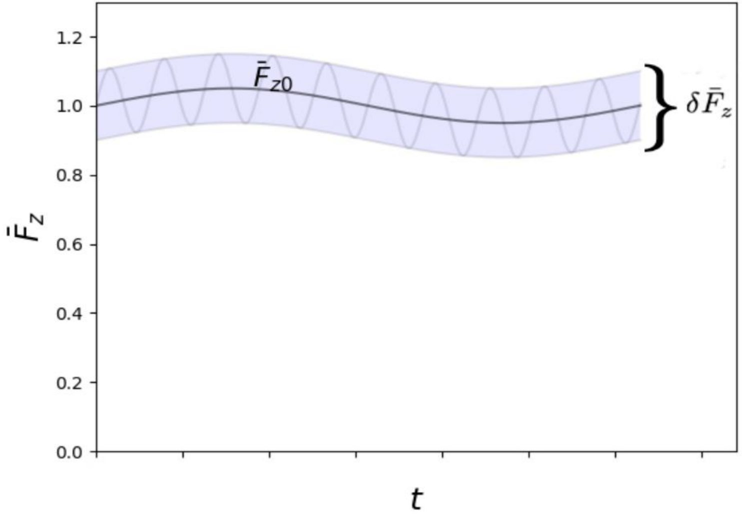
$$F_z = \overline{F_{z0}} + \delta\bar{F}_z + \delta\tilde{F}_z$$



Neighboring nonlinear equilibria

- Having defined Phase Space Zonal Structures, we can decompose the toroidally symmetric distribution function;
- $\overline{F_{z0}}$ describe macro- & meso-scales;
- micro-scales are accounted by $\delta\overline{F}_z$;
- they describe system transitions between neighboring nonlinear equilibria, see Chen and Zonca 2007; Falessi and Zonca 2019;
- nonlinear equilibria, together with zonal fields, form a zonal state.

$$F_z = \overline{F_{z0}} + \delta\bar{F}_z + \delta\tilde{F}_z$$



Collisional fluxes

$$\frac{\partial}{\partial t}\overline{F}_{z0} + \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b \delta \dot{P}_\phi \delta F\right)_z} + \frac{\partial}{\partial \mathcal{E}}\overline{\left(\tau_b \delta \dot{\mathcal{E}} \delta F\right)_z}\right]_S = \overline{C^g_{z0}} + [\bar{S}]_S$$

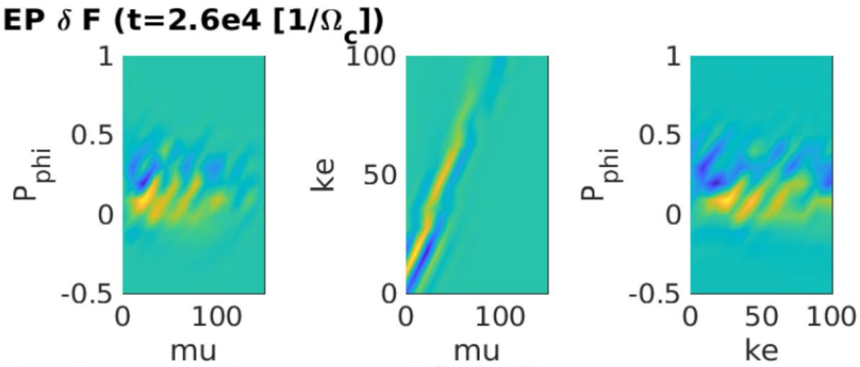
- expanding the collision operator we obtain:

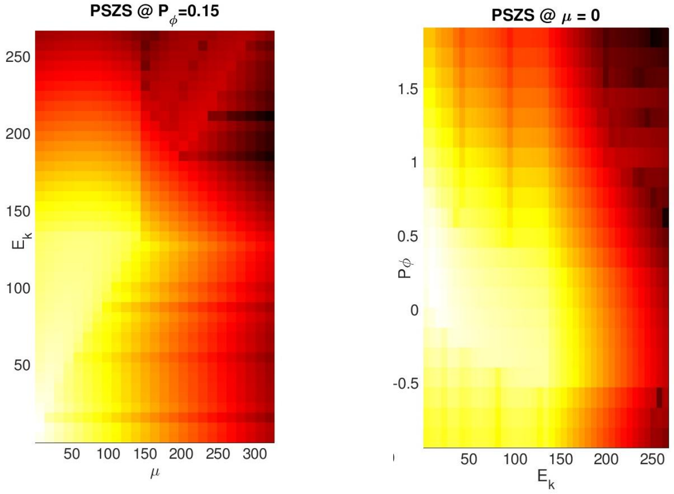
$$\bar{C}^g_{z0} = \overline{C^g_z[\overline{F}_{z0}, \overline{F}_{z0}]} + \left[\overline{C^g_z[\overline{F}_{z0}, \delta F_z]} + \overline{C^g_z[\delta F, \delta F]}\right]_S$$

- the second term, in the presence of a Maxwellian reference state, describes neoclassical transport;
- corrections to neoclassical transport are given by the first and the third terms;
- PSZS evolution with collisions and sources have been studied in G. Meng et al. 2023

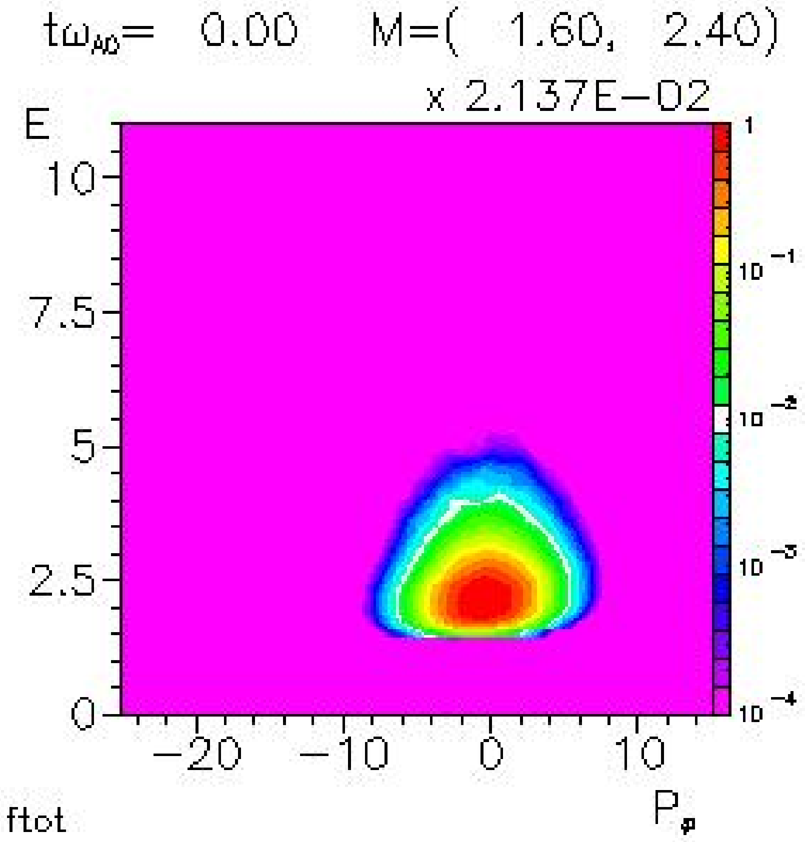
Phase Space Zonal Structures diagnostic in ORB5

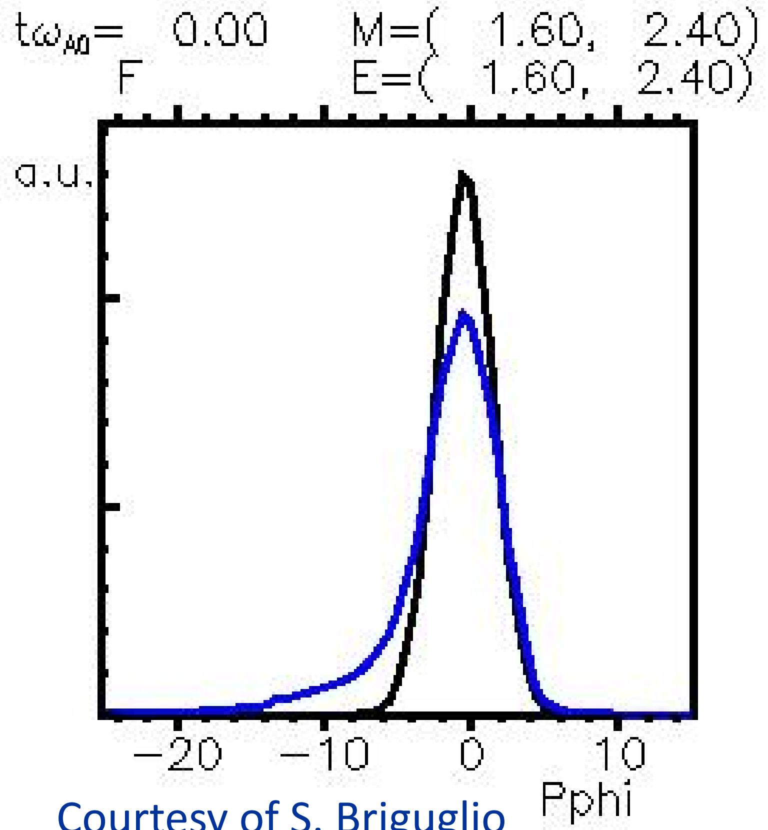
- an ORB5 diagnostic for PSZS has been developed, i.e., see Bottino et al (2022);
- PSZS can accumulate over time…
- A restart of ORB5 from PSZS data is the next step, see the contribution by A. Bottino at this conference;
- Illustration of ORB5 application to frequency chirping modes (Invited Talk by X. Wang)





PSZS diagnostic in HMGC





Courtesy of S. Briguglio

Zonal fields

- We have shown how to calculate an evolving renormalized distribution function consistent with the finite level of fluctuations; this is connected to the evolution of a macro-/meso- scopic corresponding CGL equilibrium …

- Following Cary & Brizard 2009 , we write every moment of the zonal distribution function:

$$
\begin{aligned}
\mathbf{J}_z \quad &= e\int d\mathcal{E}d\mu d\alpha d^3\mathbf{X}D(T^{-1}\mathbf{v})\delta(\mathbf{X}+\boldsymbol{\rho}-\mathbf{r})\left[F_0+\delta F_z-\frac{e}{m}\langle\delta L_g\rangle_z\frac{\partial}{\partial\mathcal{E}}F_0-\frac{e}{m}\frac{\partial}{\partial\mu}F_0\langle\delta L_g\rangle_z\right]\\
&+\frac{e^2}{m}\int d\mathcal{E}d\mu d\alpha d^3\mathbf{X}D\left[\frac{\partial}{\partial\mathcal{E}}F_0\delta\phi_z+\frac{1}{B_0}\frac{\partial}{\partial\mu}F_0\delta L_z\right]
\end{aligned}
$$

- we can substitute F_z for the Phase Space Zonal Structures $\bar{F}_{z0}=\left[F_0+e^{-iQ_z}\delta\bar{F}_{Bz}\right]_S$;

- we obtain, from a multipole expansion, an equilibrium that is consistent with an

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$$+\frac{e^2}{m}\int d\mathcal{E}d\mu d\alpha d^3\mathbf{X}D\left[\frac{\partial}{\partial\mathcal{E}}F_0\delta\phi_z+\frac{1}{B_0}\frac{\partial}{\partial\mu}F_0\delta L_z\right]$$

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- we obtain, from a multipole expansion, an equilibrium consistent with a CGL pressure tensor:

$$\sigma\frac{\mathbf{J}_z\times\mathbf{B}}{c}=\nabla P_\parallel+(\sigma-1)\nabla\left(\frac{B^2}{8\pi}\right)+\frac{B^2}{4\pi}\nabla_\perp\sigma$$

consistent with the finite level of fluctuations; this is connected to the evolution of a macro-/meso- scopic corresponding CGL equilibrium …
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$= e\int d\mathcal{E}d\mu d\alpha d^3\mathbf{X}D(T^{-1}\mathbf{v})\delta(\mathbf{X}+\boldsymbol{\rho}-\mathbf{r})\left[F_0+\delta F_z-\frac{e}{m}\langle\delta L_g\rangle_z\frac{\partial}{\partial\mathcal{E}}F_0-\frac{e}{m}\frac{\partial}{\partial\mu}F_0\langle\delta L_g\rangle_z\right]$
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we obtain, from a multipole expansion, an equilibrium consistent with a CGL pressure

Reduced transport models

- PSZS transport is studied by means of a hierarchy of verified and validated reduced models within an EUROfusion Enabling Research Project with P. Lauber as P.I.;
- explicit expression of EP fluxes in PSZS equations have been calculated within the following hierarchy of simplifying assumptions:

- the zeroth level of simplification consist in the gyrokinetics description of plasma dynamics;
- the first level of simplification consist in assuming $|\omega| \gg \tau_{NL}^{-1} \sim \gamma_L$, Zonca et al 2021;
- the second and final level of simplification is the quasilinear model

Reduced transport models

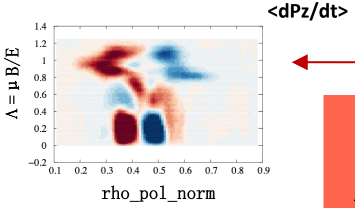
DAEPS (Y. Li et al 2020)

- Ballooning decomposition for fluctuations;
- Based on fish-bone like dispersion relation;
- Mode structure decomposition, separation of radial envelope and parallel mode structure;
- Calculate nonlinear fluxes by the DSM model or a saturation rule.

LIGKA-HAGIS (Lauber et al 2007)

- Fourier decomposition for fluctuations;
- Solve linear gyrokinetic equation;
- Assume fluctuations amplitude, e.g., kick-model or quasilinear;
- Use IMAS-coupled EP stability WF (HAGIS/LIGKA) to calculate Phase Space Zonal Structures fluxes.

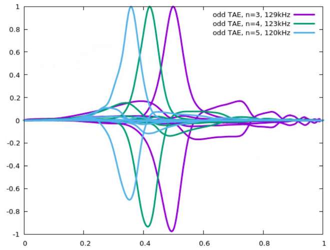
ATEP: kick model limit



ATEP code: solve transport equation for PSZS with sources and collisions, Lauber 2022

$$\frac{\partial}{\partial t}\overline{F_{z0}} + \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b \delta\dot{P}_\phi \delta F\right)}_z + \frac{\partial}{\partial \mathcal{E}}\overline{\left(\tau_b\delta\dot{\mathcal{E}}\delta F\right)}_z\right]_S = \overline{\left(\sum_b C_b^g\left[F,F_b\right] + \mathcal{S}\right)}_{z\,S}$$

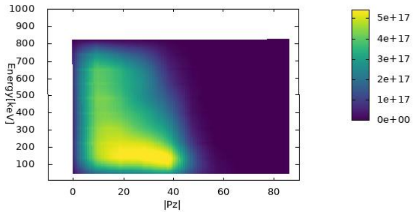
calculate PSZS fluxes with prescribed amplitude

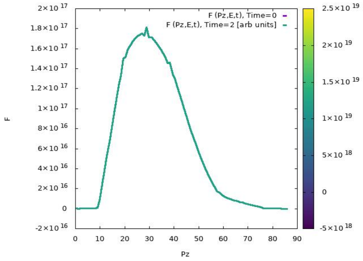


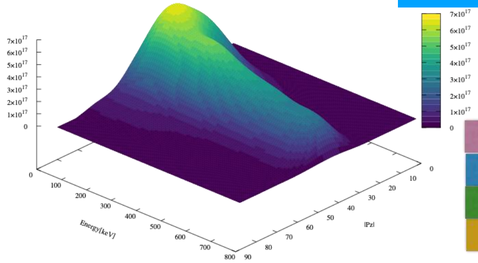
calculate linear mode spectrum

DAEPS and LIGKA Are interchangeable Thanks to IMAS

advance F_{EP} and return updated F_{EP} into IDS, or its moments





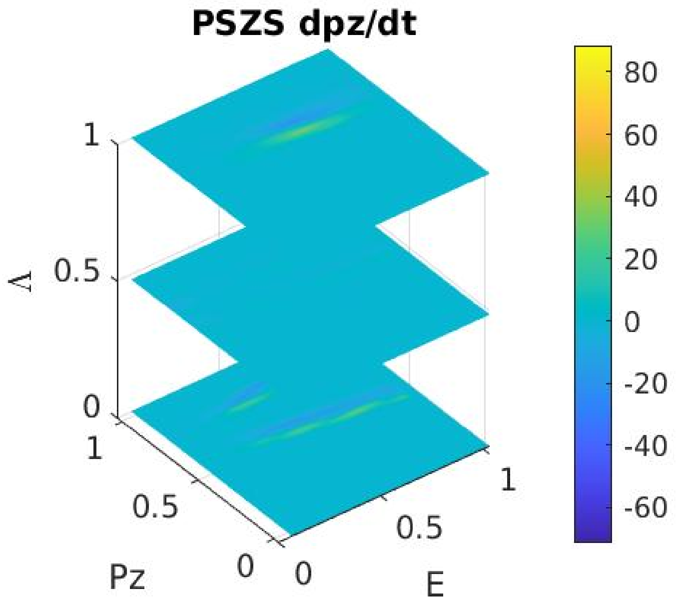


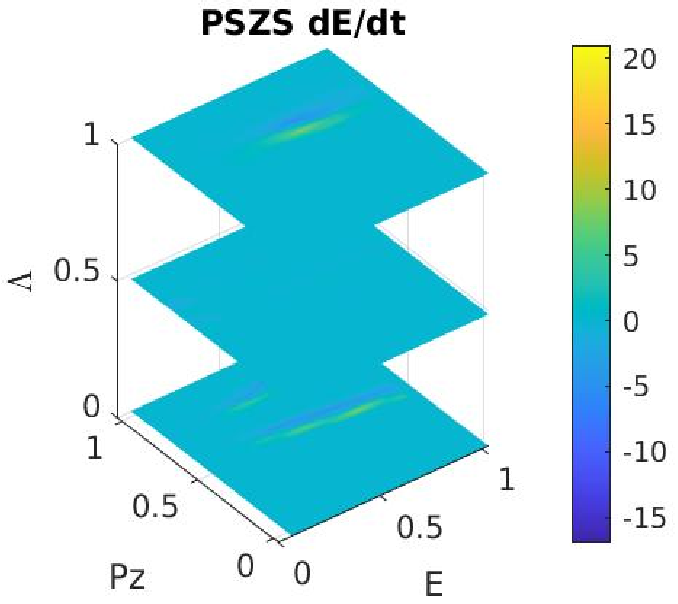
core_transport
equilibrium
mhd_linear
distributions

time

Phase Space fluxes

$$\frac{\partial}{\partial t}\overline{F_{z0}} + \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b \delta \dot{P}_\phi \delta F\right)_z} + \frac{\partial}{\partial \mathcal{E}}\overline{\left(\tau_b \delta \dot{\mathcal{E}} \delta F\right)_z}\right]_S = \overline{\left(\sum_b C_b^g\left[F, F_b\right] + \mathcal{S}\right)_{z\,S}}$$



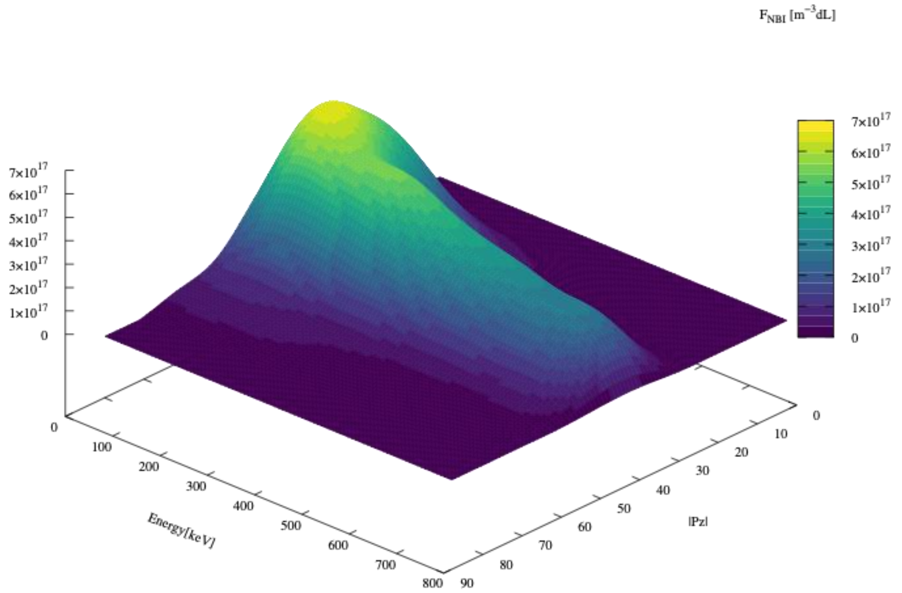


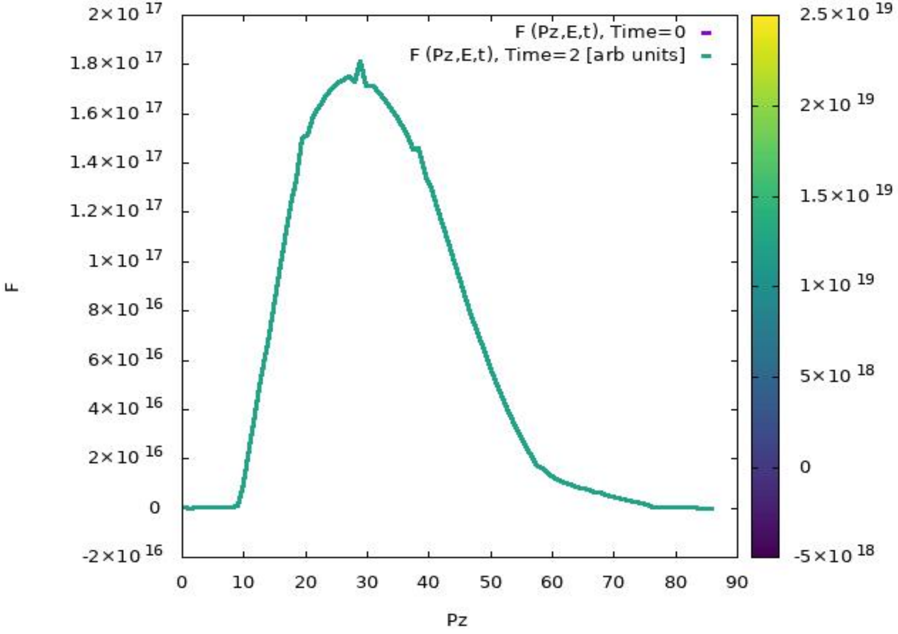
G. Meng et al. 14th International West Lake Symposium

PSZS evolution

ATEP code: solve transport equation for PSZS with sources and collisions, Lauber 2022

$$\frac{\partial}{\partial t}\overline{F_{z0}} + \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b \delta \dot{P}_\phi \delta F\right)}_z + \frac{\partial}{\partial \mathcal{E}}\overline{\left(\tau_b \delta \dot{\mathcal{E}} \delta F\right)}_z\right]_S = \overline{\left(\sum_b C_b^g\left[F, F_b\right] + \mathcal{S}\right)}_{zS}$$

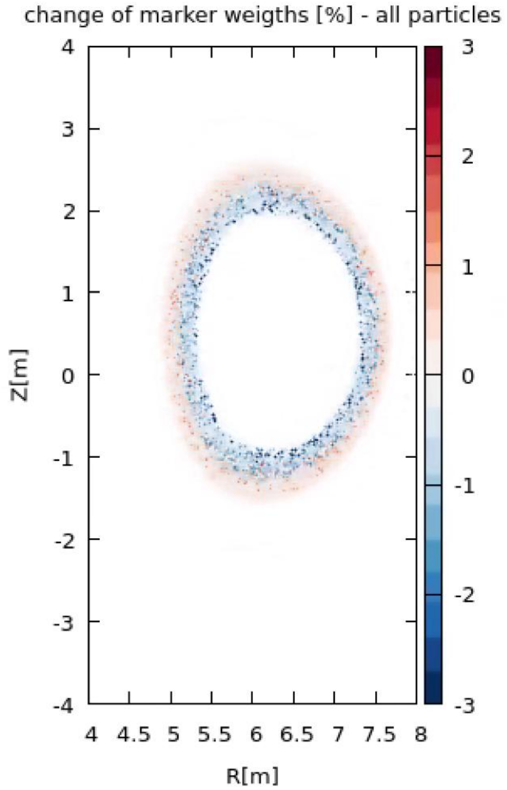


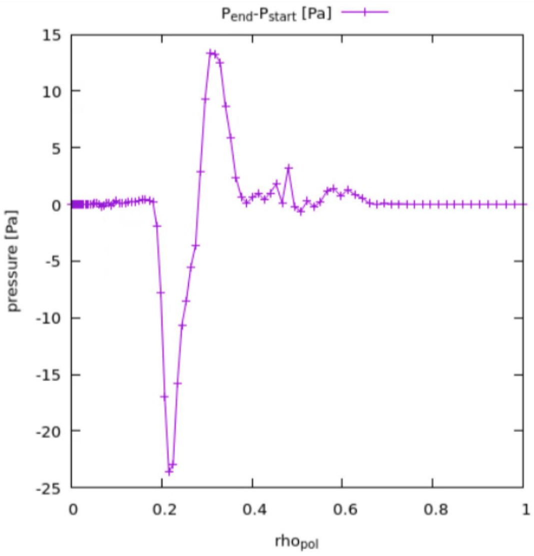


ITER plasma from H&CD WF by M. Schneider

PSZS moments

- Mapping to 1d profile of PSZS;
- The CGL equilibrium can be readily constructed Falessi et al 2023 sub;
- transport is zonal by construction;
- Next step is the calculation of the corresponding magnetic equilibrium;

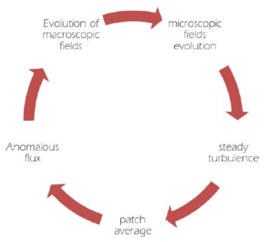




ATEP code: solve transport equation for PSZS with sources and collisions, Lauber 2022

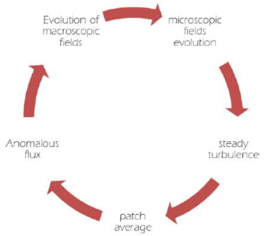
Summary & Conclusions

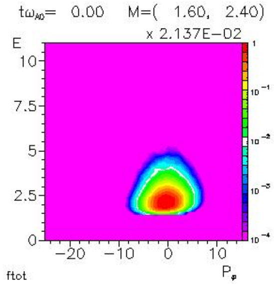
- Need of predictive transport models on timescales comparable with magnetic fusion experiments, i.e., ITER, properly describing EPs;

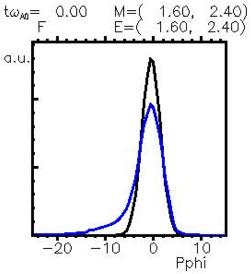


Summary & Conclusions

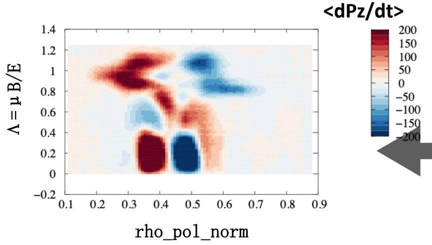
- Need of predictive transport models on timescales comparable with magnetic fusion experiments, i.e., ITER, properly describing EPs;
- we have shown how to describe meso-scales and non-Maxwellian distribution functions using appropriate phase transport equations;
- we have introduced the concept of zonal state to describe the evolution of the plasma between neighboring nonlinear equilibria;



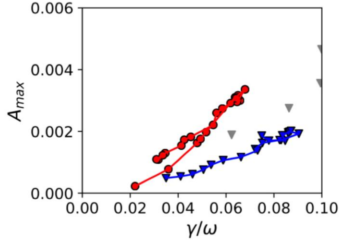




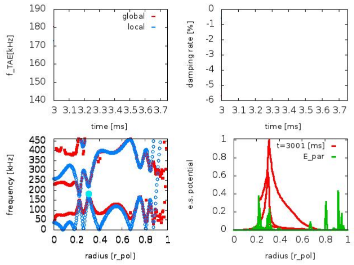
ATEP: Quasilinear model



saturation rule: HAGIS or ORB5



EP WF (LIGKA)



calculate PSZS

use NL code/model for intensity closure

calculate linear mode spectrum

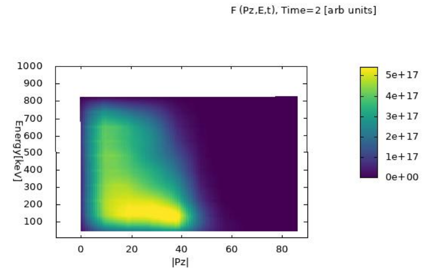
ATEP code: solve transport equation for PSZS with sources and collisions Lauber 2022

$$\frac{\partial}{\partial t}\overline{F_{z0}} + \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b \delta\dot{P}_\phi \delta F\right)}_z + \frac{\partial}{\partial \mathcal{E}}\overline{\left(\tau_b \delta\dot{\mathcal{E}} \delta F\right)}_z\right]_S = \overline{\left(\sum_b C_b^g\,[F,F_b] + \mathcal{S}\right)}_{z\,S}$$

calculate D(r, E)

or

advance F_{EP} and return updated distribution IDS, or its moments



t

t+1

transport code

core_transport

equilibrium

mhd_linear

distributions

time

Zonal fields: macro- /meso- scopic component

- the current $\mathbf{J}_z$ and the pressure tensor $\mathbf{P}$ satisfy the following force balance equation:

$$\sigma \frac{\mathbf{J}_z \times \mathbf{B}}{c} = \nabla P_\parallel + (\sigma - 1)\nabla\left(\frac{B^2}{8\pi}\right) + \frac{B^2}{4\pi}\nabla_\perp \sigma$$

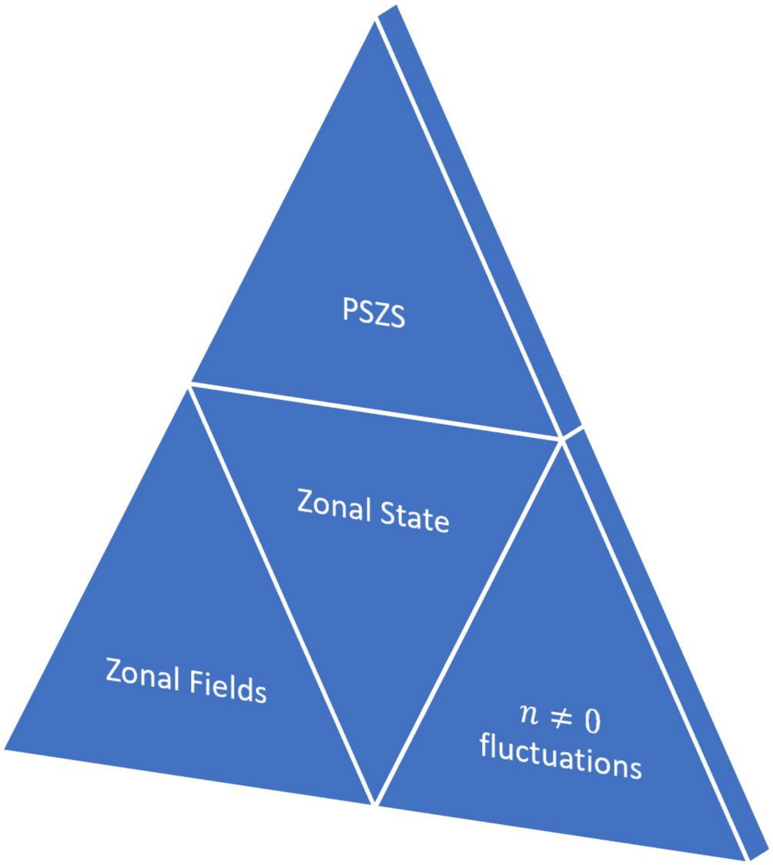
where $\sigma = 1 + \frac{4\pi}{B^2}(P_\perp - P_\parallel)$;

- the self-consistent modification of the equilibrium magnetic field due to PSZS can be calculated solving this equation, e.g.:

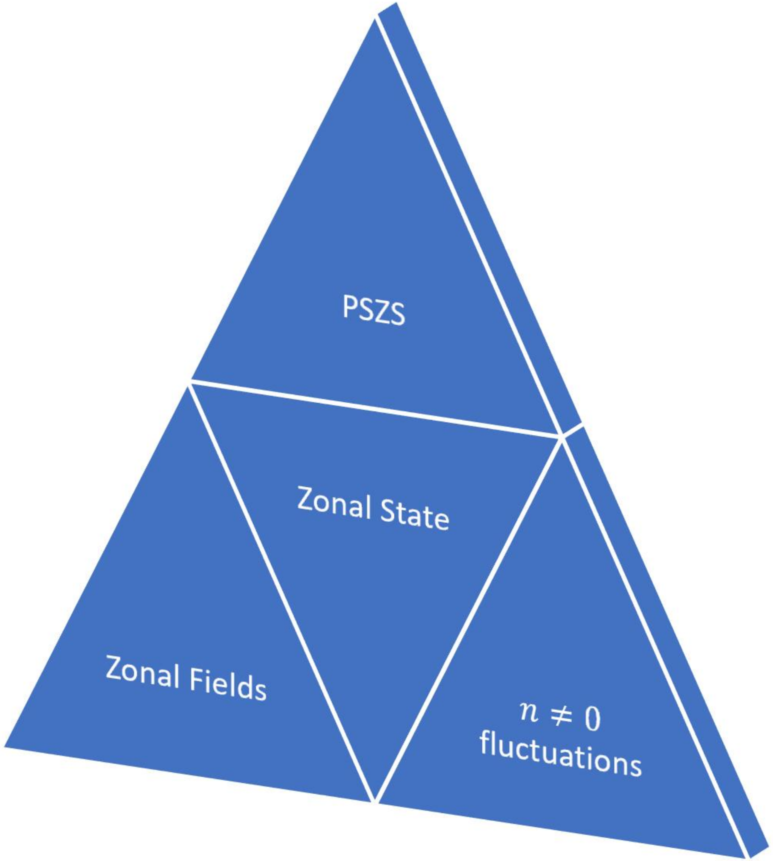
$$\Delta^*\psi + \nabla \ln\sigma \cdot \nabla\psi = -\frac{4\pi R^2}{\sigma} - \frac{1}{\sigma^2}\frac{\partial G}{\partial \psi}$$

- where $G \equiv \sigma F^2/2$ is a flux function;

Zonal state



Zonal state



Transport processes in burning plasmas

- EPs are generally non Maxwellian, and transport processes take place in the phase space!
- Separation of scales is questionable…
- Particularly important for ITER physics;
- interplay of meso-scale structures with micro-scales generated by EPs, i.e., $\rho_{LE} \sim (\rho_L L)^{1/2}$, Zonca et al 2015;

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- interplay of meso-scale structures with micro-scales generated by EPs, i.e., $\rho_{LE} \sim (\rho_L L)^{1/2}$, Zonca et al 2015;
- interaction with thermal plasma over long timescales can modify bulk transport processes;
- the derivation, see Falessi 2017; Falessi and Zonca 2019, is based on the theory of Phase space zonal structures (PSZS), see Chen and Zonca 2016.