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April 10th, 2026

Physics of Alfvén wave and energetic particles in fusion plasmas*

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*In collaboration with L. Chen, M. V. Falessi, Z. Qiu, X. Tao and G. Wei

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Ministero degli Affari Esteri e della Cooperazione Internazionale

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Institute for Fusion Theory and Simulation, Zhejiang University

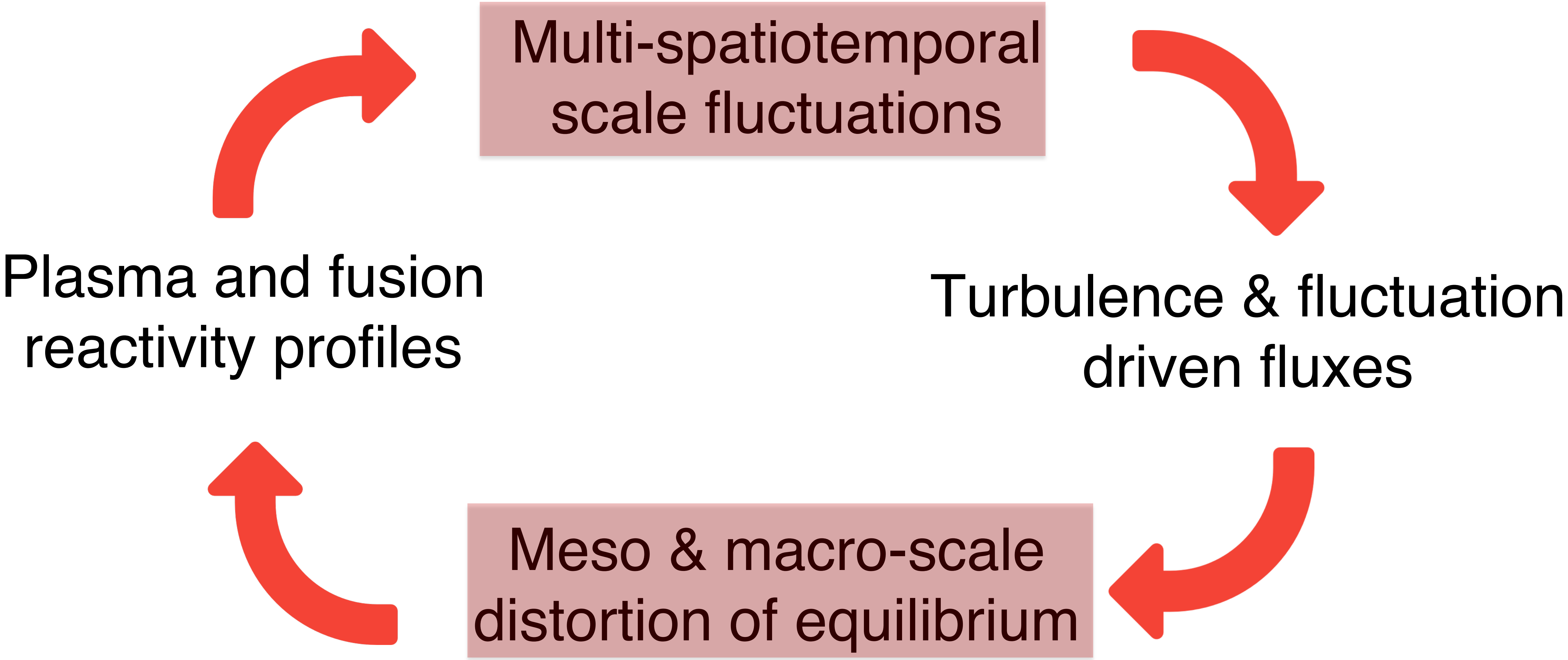
ENEA ITALIAN NATIONAL AGENCY FOR NEW TECHNOLOGIES, ENERGY AND SUSTAINABLE ECONOMIC DEVELOPMENT

Center for Nonlinear Plasma Science CNPS - Since 2020 -

Shear Alfvén waves and the physics of burning plasmas

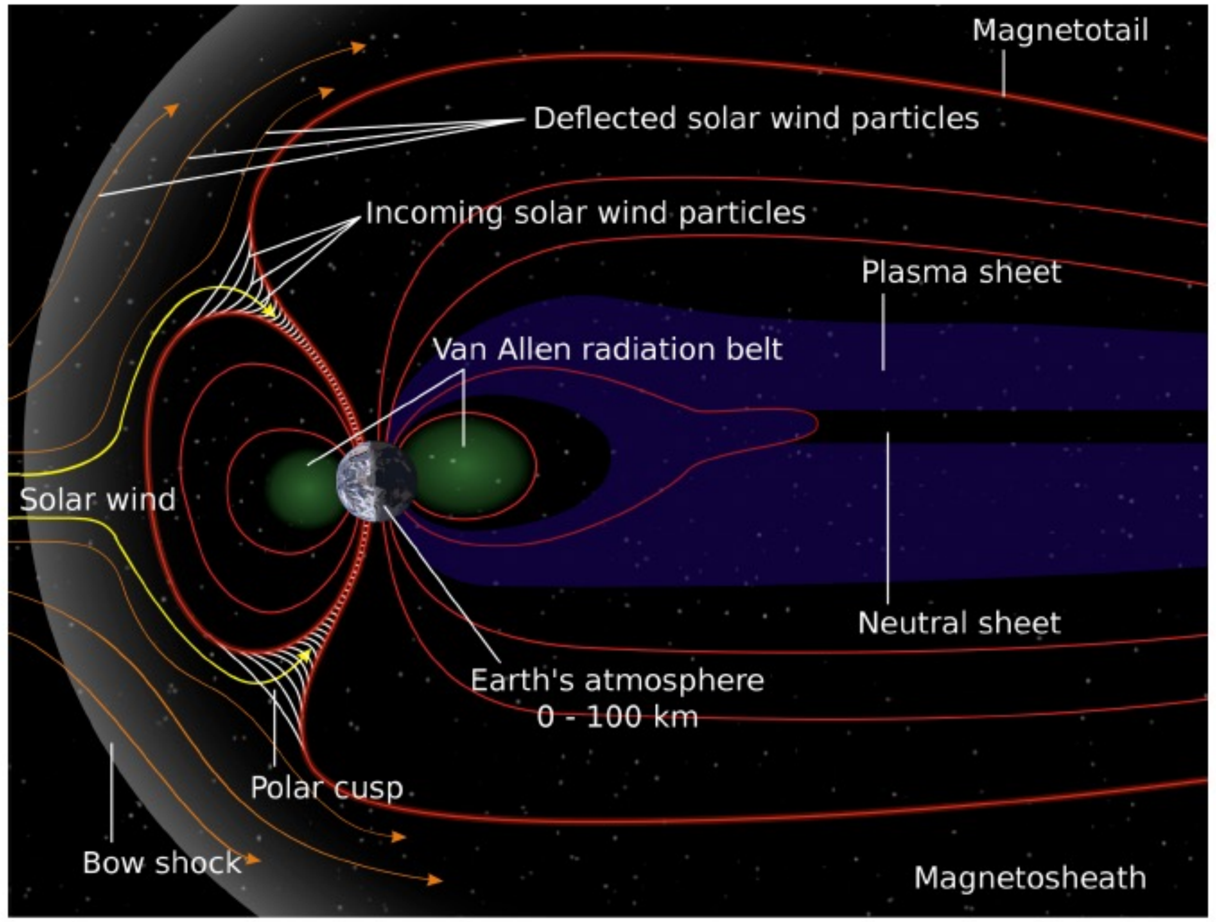
- Burning plasmas as complex self-organized systems:

[C&Z Rev. Mod. Phys. Review article]: Nonlinear gyrokinetic description]



Broader relevance: Earth's radiation belts

Diagram of Earth's magnetosphere



Source: https://en.wikipedia.org/wiki/Magnetosphere
Original: NASA Vector: Aaron Kaase, Medium69 - Structure of the magnetosphere numbered.svg

- ❑ Significant MeV electron population
- ❑ Formation mechanism?

Earth's radiation belts

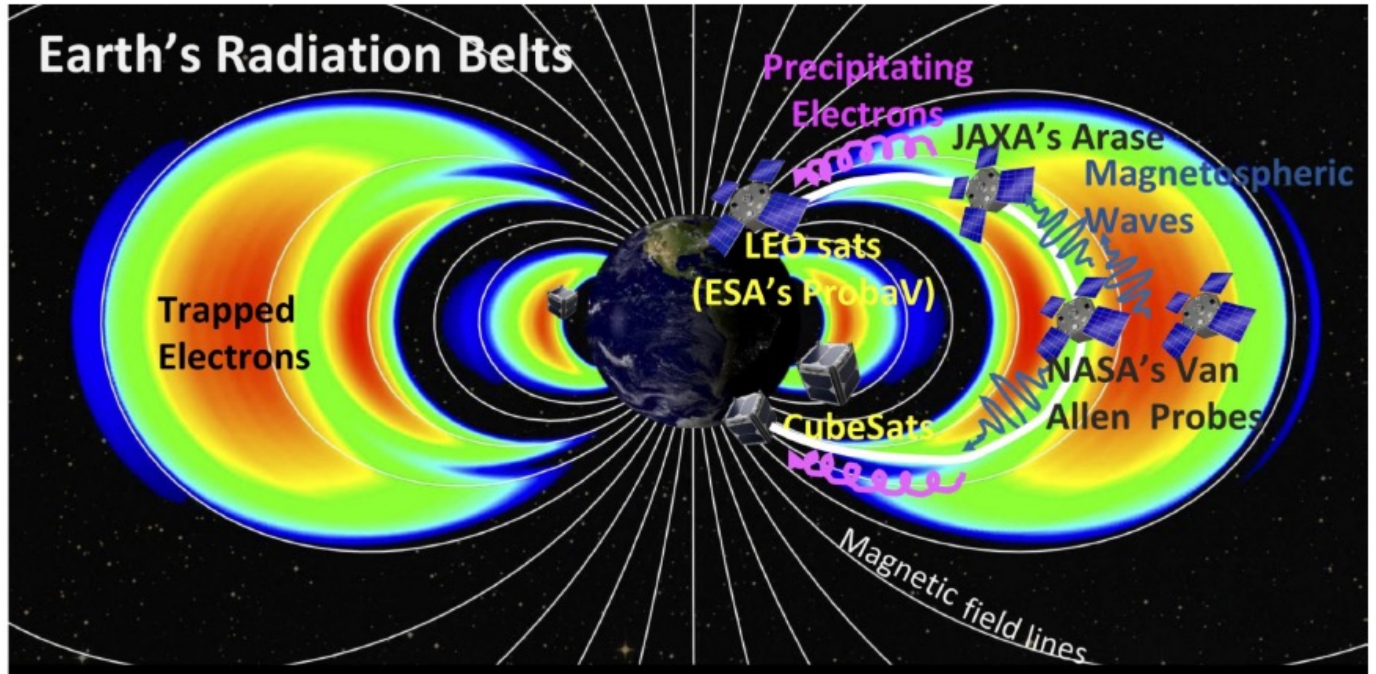


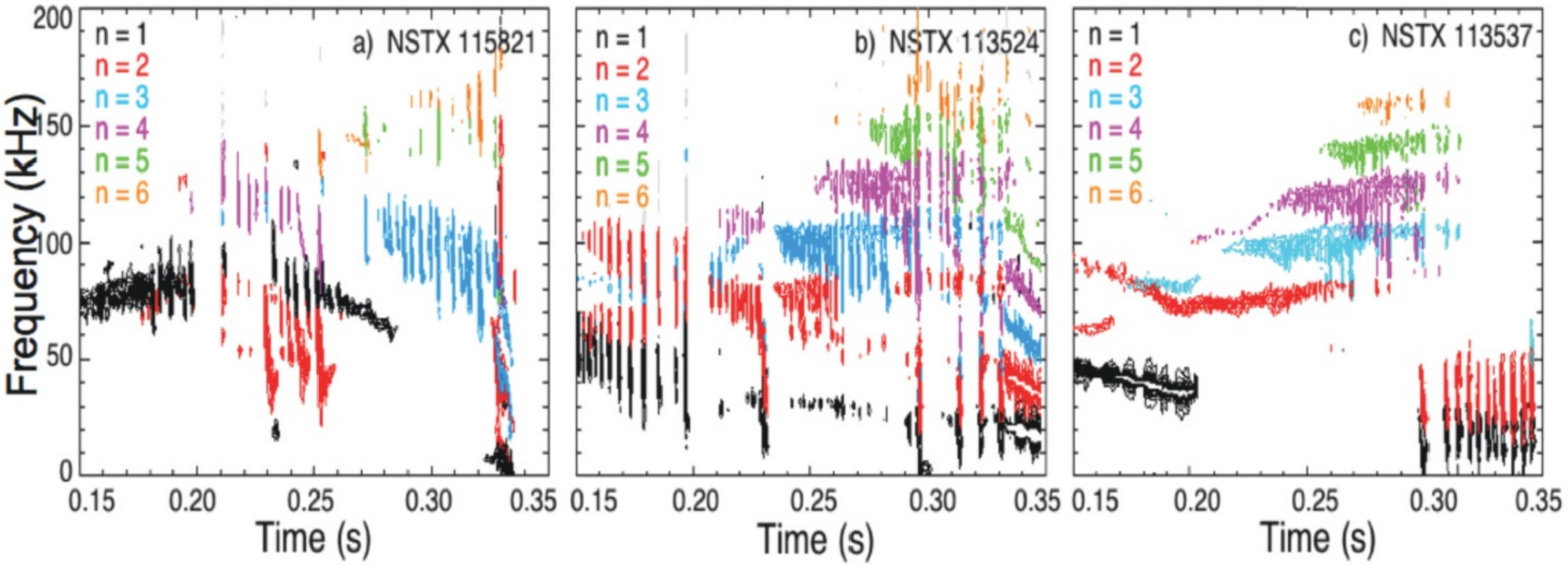
Figure: Latidunal repartition of the Arase, Proba V (LEO), Van Allen Probes satellites within the Earth's inner magnetosphere and radiation belts. Here belts are illustrated with a newly found and unusual 3 Belt structure.

Source: https://www.issibern.ch/teams/radbeltphysics/

- ❑ Whistler mode chorus is responsible for electron acceleration to MeV energy in Earth's radiation belts (R. B. Horne, Nature 2005)

Experimental observations

- Rich spectrum of Alfvénic fluctuations is observed experimentally (E. D. Fredrickson et al., Phys. Plasmas 2006)



- Resonant excitation of Alfvén eigenmodes, EPM and fishbones by energetic particles and ensuing effects on transport

Unified theoretical approach

- The general fishbone like dispersion relation (GFLDR) provides the unified description of spectral properties, mode structure, and resonant excitation by energetic particles in realistic nonuniform equilibria (Z&C Phys. Plasmas 2014)

$$
\begin{aligned}
\mathcal{F}[\delta\phi, \delta A_\parallel] \;=\;& (2\pi)^3 \sum_{j\in\mathbb{Z}} \int_0^a dr \frac{d\psi/dr}{2} e^{i2\pi nqj} \int_{-\infty}^{\infty} \mathcal{J} d\vartheta \\
& \left[-\boldsymbol{\nabla} \times \left(\boldsymbol{b}_0 c \partial_t^{-1} \nabla_\parallel \delta\hat{\phi}^\dagger(r, \vartheta + 2\pi j) \right) \cdot \frac{\delta \hat{\boldsymbol{B}}_\perp(r,\vartheta)}{4\pi} \right. \\
& \left. + \partial_t^{-1} \delta\hat{\phi}^\dagger(r, \vartheta + 2\pi j) \boldsymbol{\nabla} \cdot \delta \hat{\boldsymbol{J}}_\perp(r,\vartheta) \right] = \delta W - \delta I \; .
\end{aligned}
$$

- Constructed from the GK energy functional (Hasegawa and Chen 1991)

The General fishbone like dispersion relation

- The GFLDR can be cast as (Z&C Phys. Plasmas 2014)

"fluid" potential energy/MHD

$$i|s|\Lambda = \delta\hat{W}_f + \delta\hat{W}_k$$

generalized GK inertia

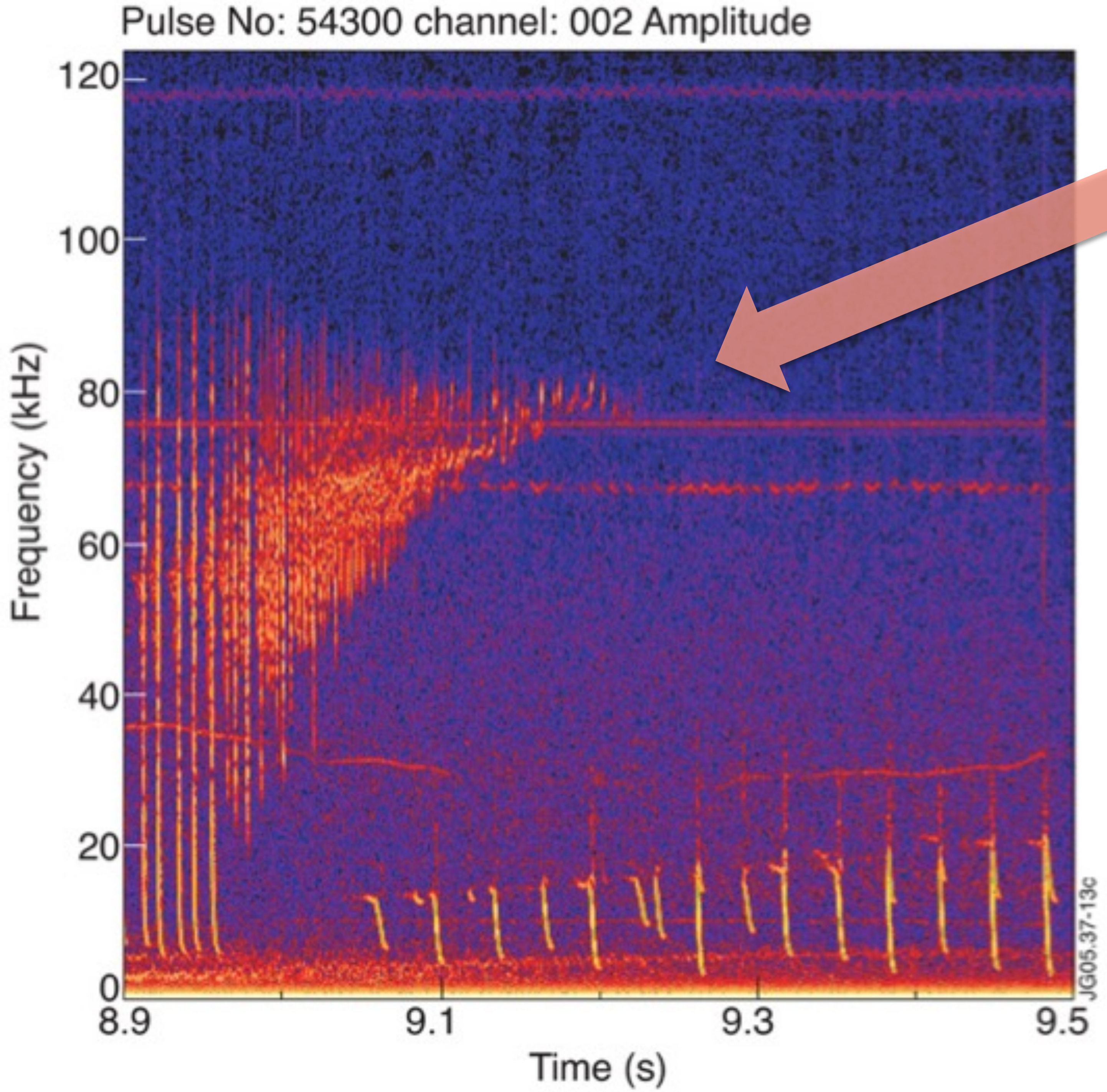
GK potential energy/compression

- The GFLDR is a prediction first approach, which readily offers a falsification test to experimental observations

- The GFLDR identifies spatiotemporal scales underlying relevant physics: it elevates interpretation of experimental observations and numerical simulation results

High frequency fishbones at JET

- ❑ The GFLDR has successfully been applied to AUG, DIII-D, FTU, HL-2A, HL-3, JET, NSTX, DTT, …



$$i|s|\Lambda = \delta\hat{W}_f + \delta\hat{W}_k$$

Z&C et al 2009 Nuc. Fus. ***49*** *085009*

- ❑ Shear Alfvén continuum accumulation point is degenerate with GAM frequency
- ❑ Thermal plasma kinetic effects set onset threshold for high frequency fishbones

Courtesy of F. Nabais et al. 2005 Phys. Plasmas ***12*** *102509*

High frequency fishbones at JET

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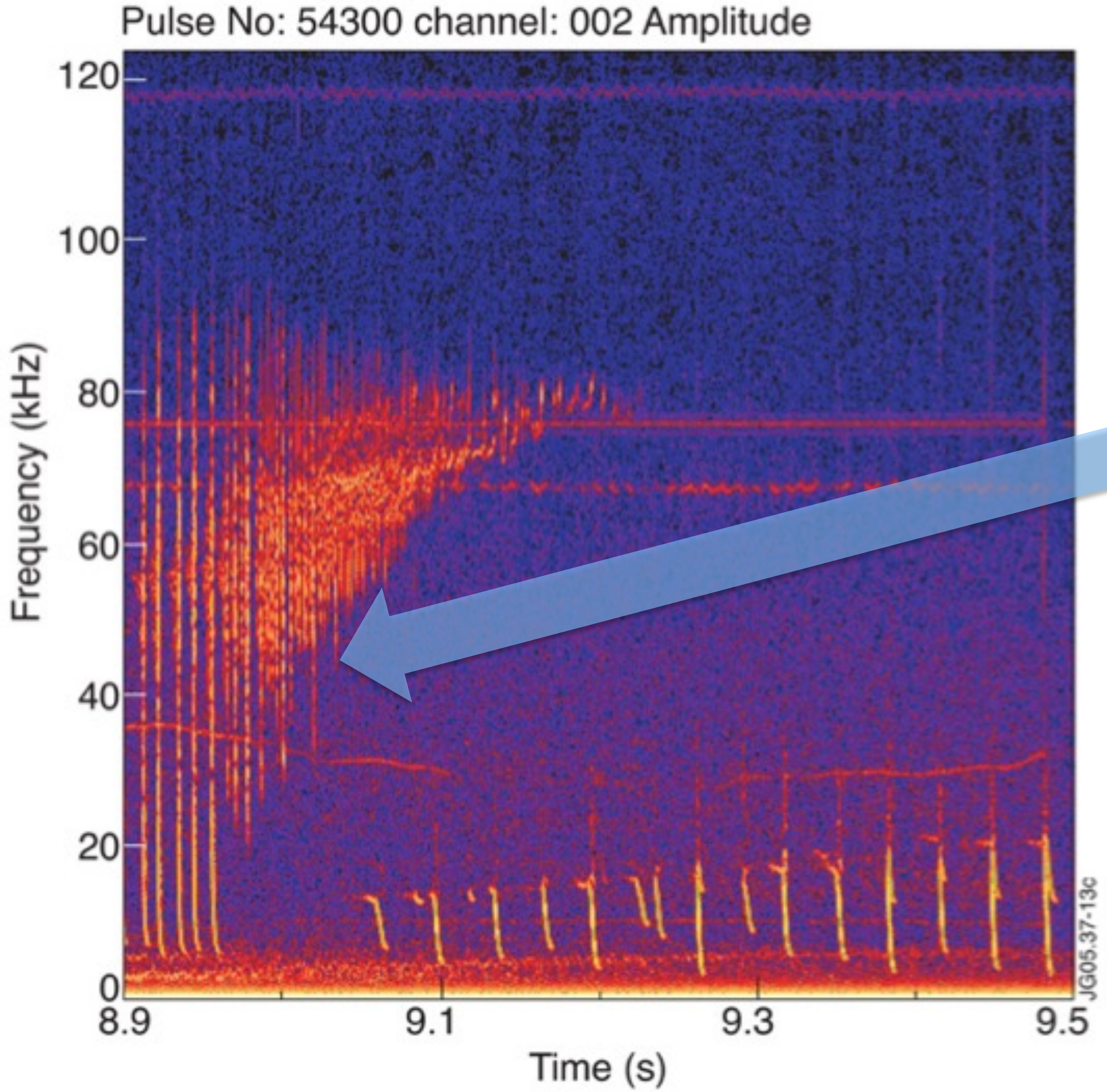


$$i|s|\Lambda = \delta\hat{W}_f + \delta\hat{W}_k$$

Z&C et al 2009 Nuc. Fus. ***49*** *085009*

- Energetic particles provide effective drive with strong ICRH in low-density plasma

- Nonlinear chirping dynamics is consistent with predicted evolution due to precession resonance of EP tail

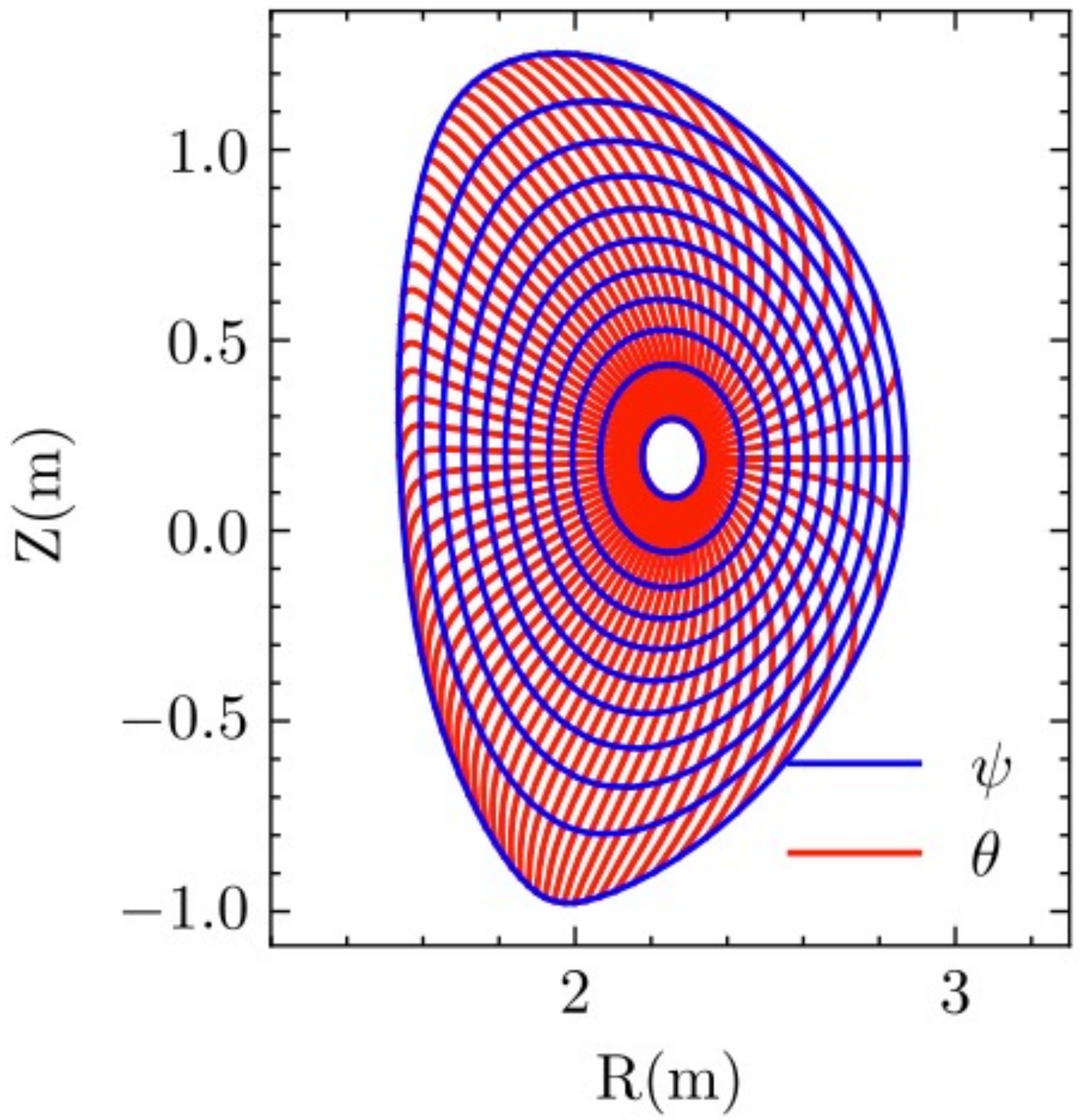
Courtesy of F. Nabais et al. 2005 Phys. Plasmas ***12*** *102509*

Importance of polarization

- ❑ SAW-ISW (Shear Alfvén-Ion Sound) continuum computed by GFLDR for realistic DTT equilibrium

G. Wei et al. 2024 Phys. Plasmas ***31*** *072505*

Divertor Tokamak Test



$$g_1(\vartheta) = \frac{\hat{\kappa}_\perp \hat{\Phi}_s(\vartheta)}{(\beta q^2)^{1/2}} \frac{ck_\vartheta}{B_0 R_0}, \quad g_2(\vartheta) = \frac{i\delta \hat{P}_{\rm comp}(\vartheta)}{(2\Gamma)^{1/2} P_0},$$

$$\delta\boldsymbol{\xi}_\perp = (c/B_0)\, \boldsymbol{b}_0 \times \nabla\Phi_s \qquad \delta P_{\rm comp} = -\Gamma P_0 \boldsymbol{\nabla}\cdot\delta\boldsymbol{\xi}$$

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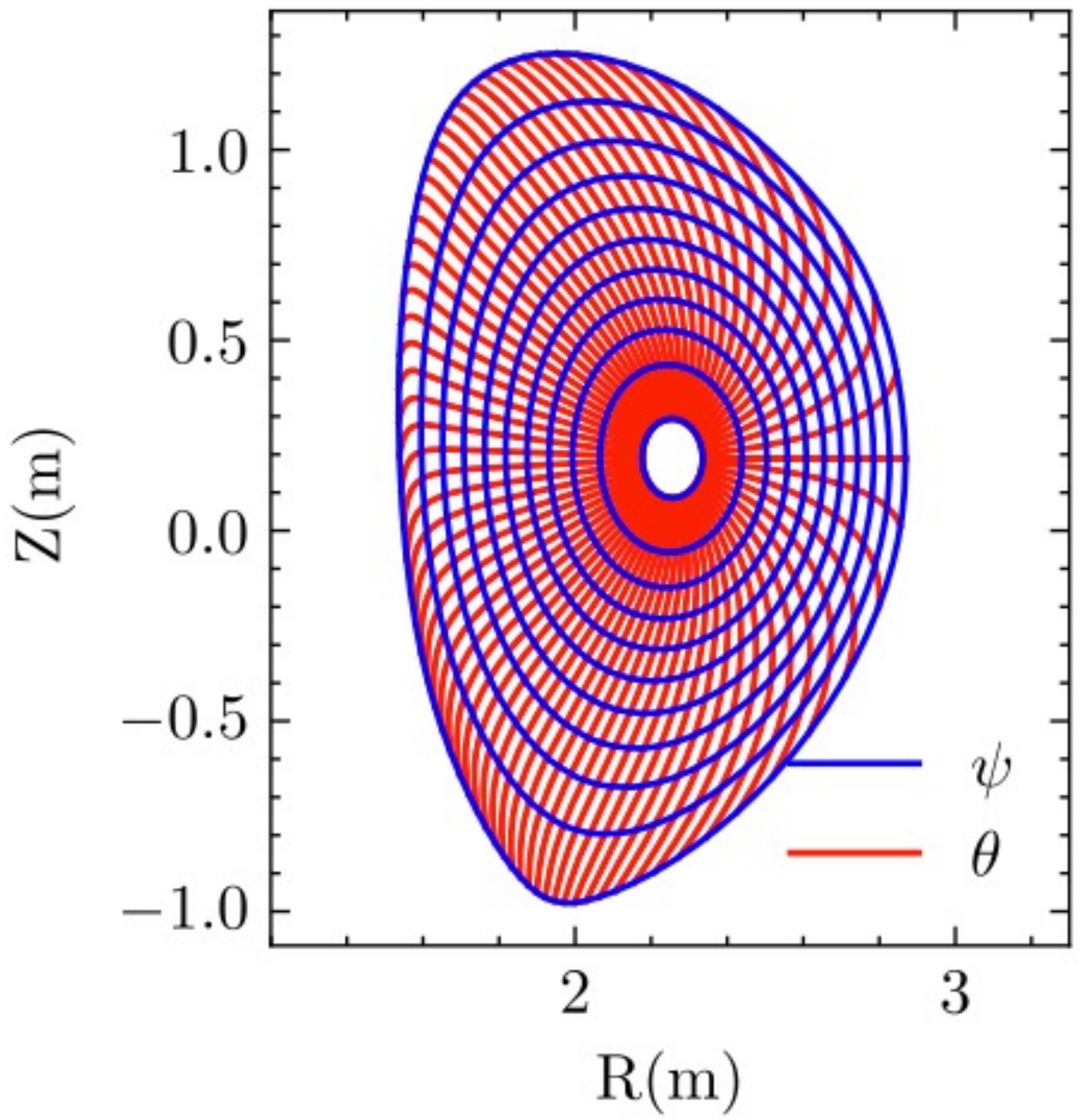

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Alfvénicity

$$A = \frac{\int_0^{2\pi} g_1^2(\vartheta; \nu, r) d\vartheta}{\int_0^{2\pi} [g_1^2(\vartheta; \nu, r) + g_2^2(\vartheta; \nu, r)] d\vartheta},$$

M.V. Falessi et al. 2019 Phys. Plasmas ***26*** *082502*

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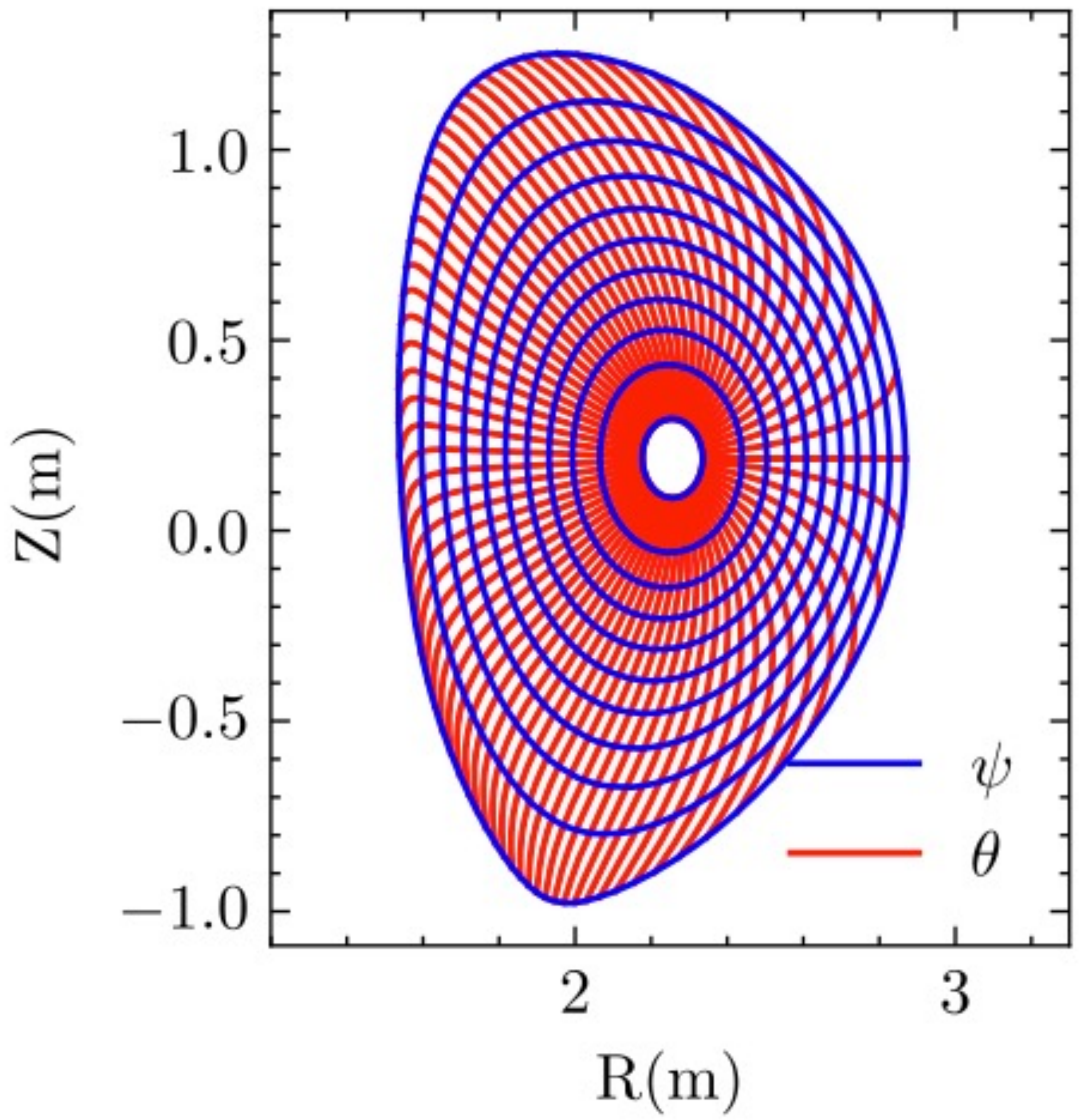
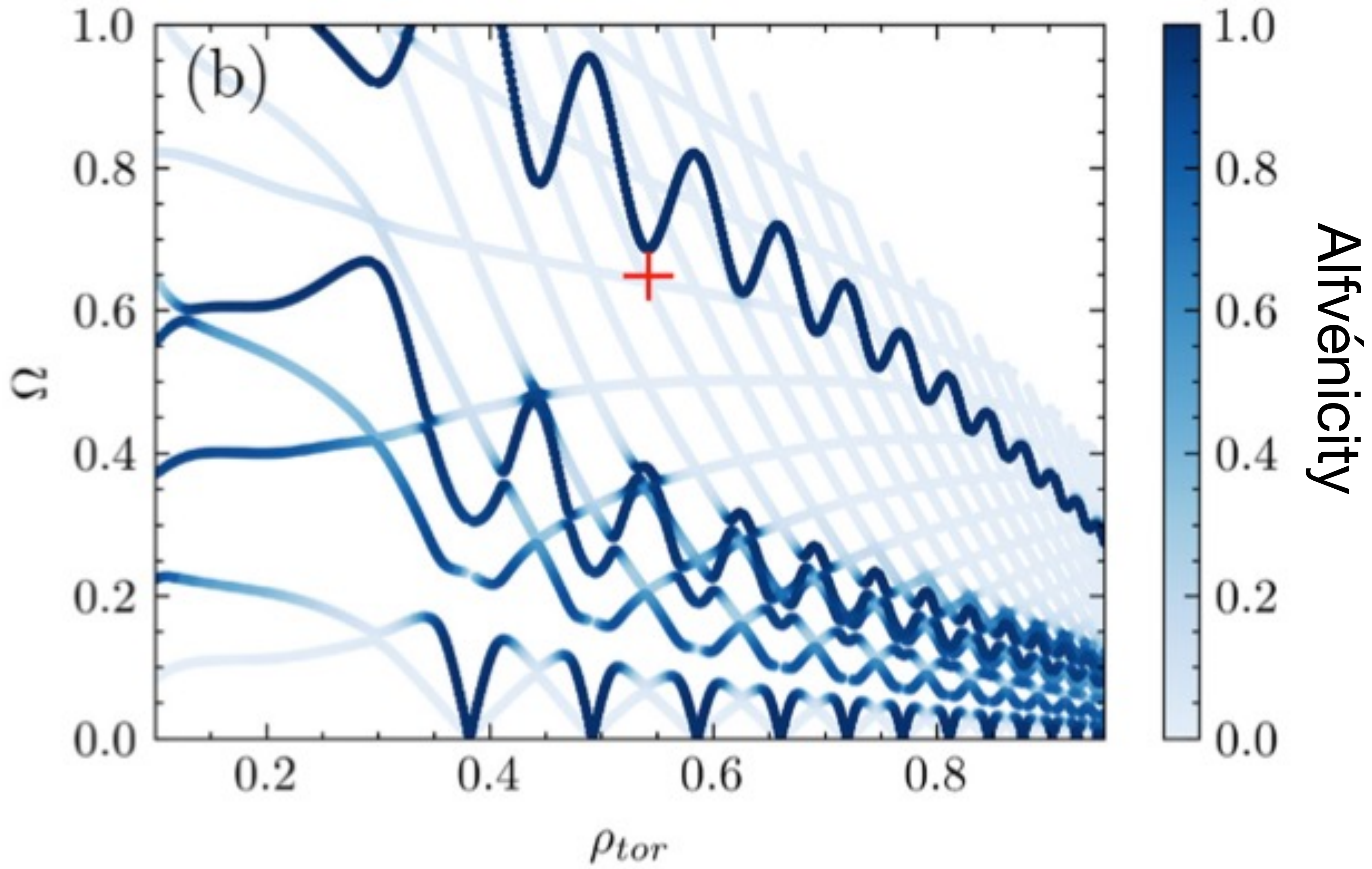
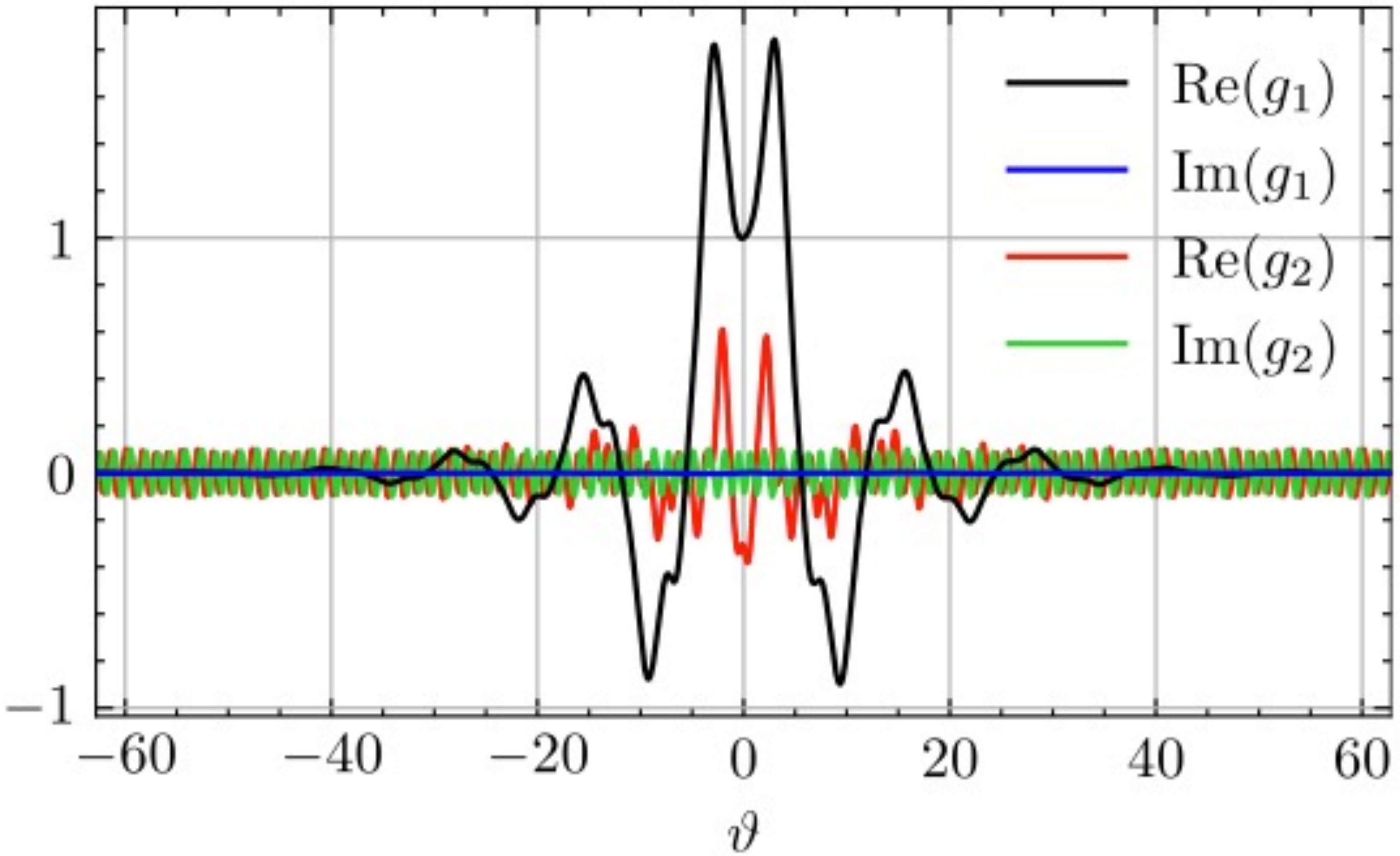


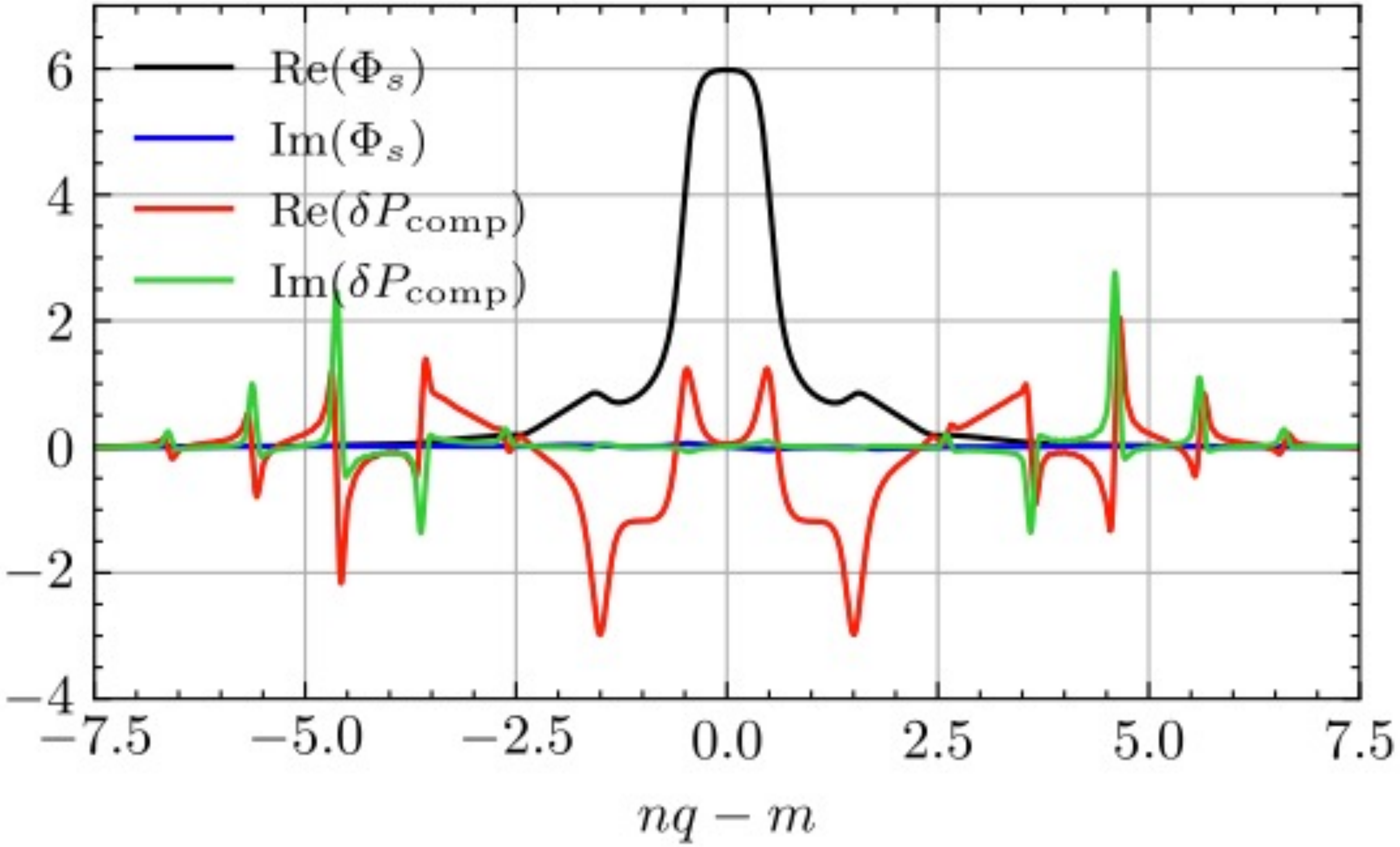


Crucial role of mode structures

- Interaction of TAE with SAW-ISW continuum leads to resonant absorption and damping

G. Wei et al. 2024 Phys. Plasmas ***31*** *072505*





- Mode structures in ballooning and real space are crucial for correct description of damping and drive

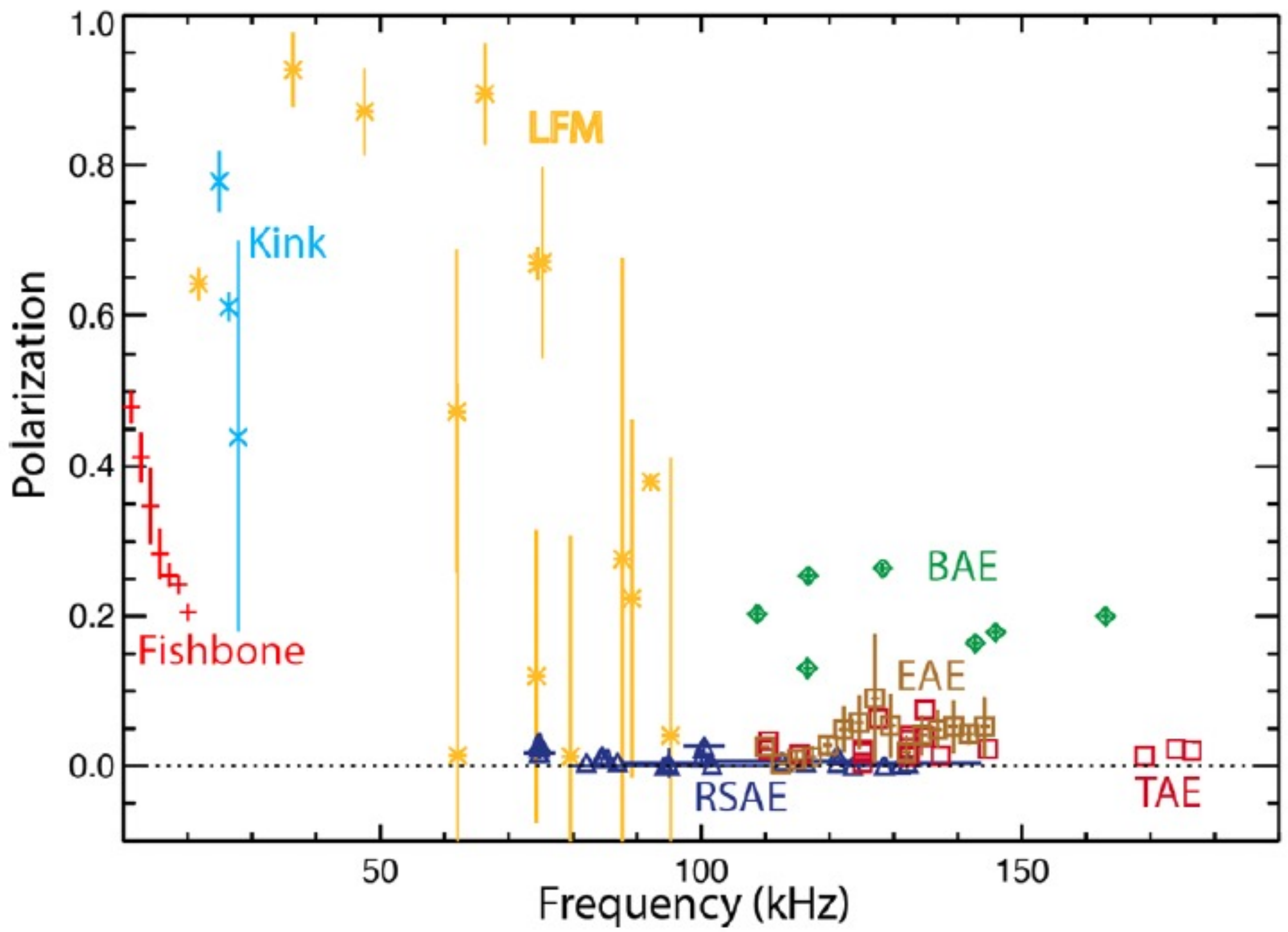
Recent experimental observations

- Recent experimental observations from DIII-D capture mode polarization effects

W.W Heidbrink et al. 2025 Nucl. Fusion ***65*** *112002*

Polarization analogous to **Alfvénicity** defined in

M.V. Falessi et al. 2019 Phys. Plasmas ***26*** *082502*



- GFLDR provides precise prediction of experimental measurements and awaits for falsification test

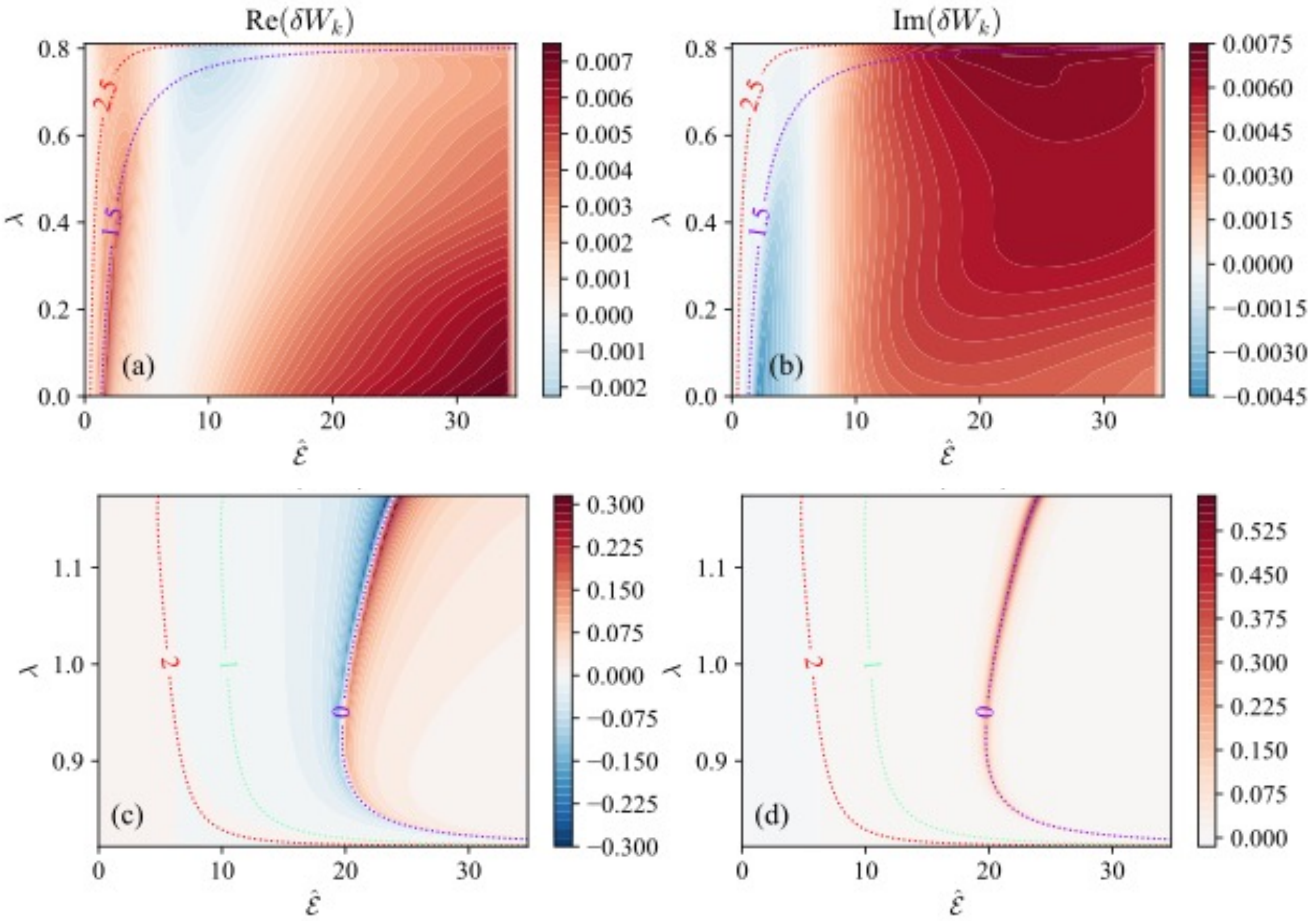
Crucial role of wave particle resonances

- Mode structures in ballooning and real space also affect the wave-particle resonances and power exchange

G. Wei et al. 2025 Nucl. Fusion ***65*** *106035*

EP: isotropic slowing down

GFLDR synthetic diagnostics



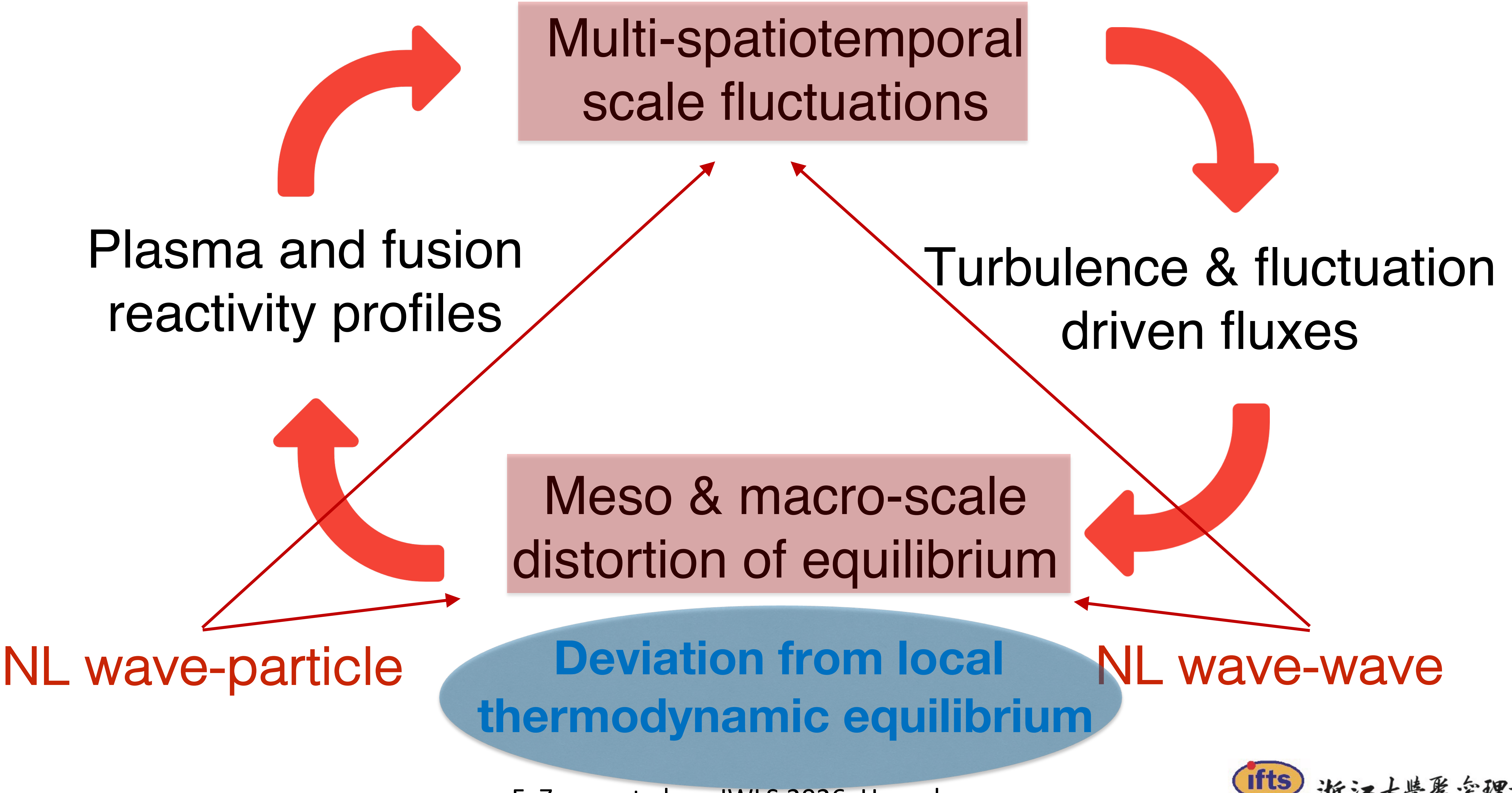
Circulating

Trapped

Nonlinear dynamics and transport

- ❑ Burning plasmas as complex self-organized systems:

[C&Z Rev. Mod. Phys. Review article]: Nonlinear gyrokinetic description]



Phase space transport - I

- A **first-principle-based**, **self-consistent approach** is crucial for developing a predictive capability of plasma behaviors on long time scales
- **Proper nonlinear equilibrium**
  - Long lived (undamped by collision) zonal e.m. fields and corresponding phase space zonal structures: **Zonal State**
  - Must self-consistently compute Zonal State in the presence of finite level of fluctuations

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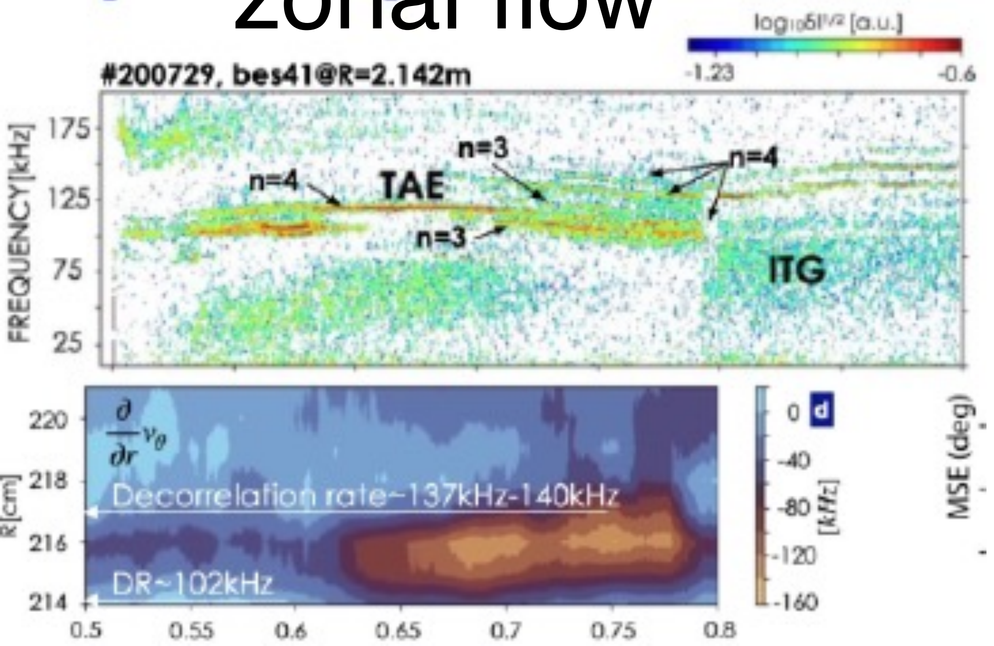
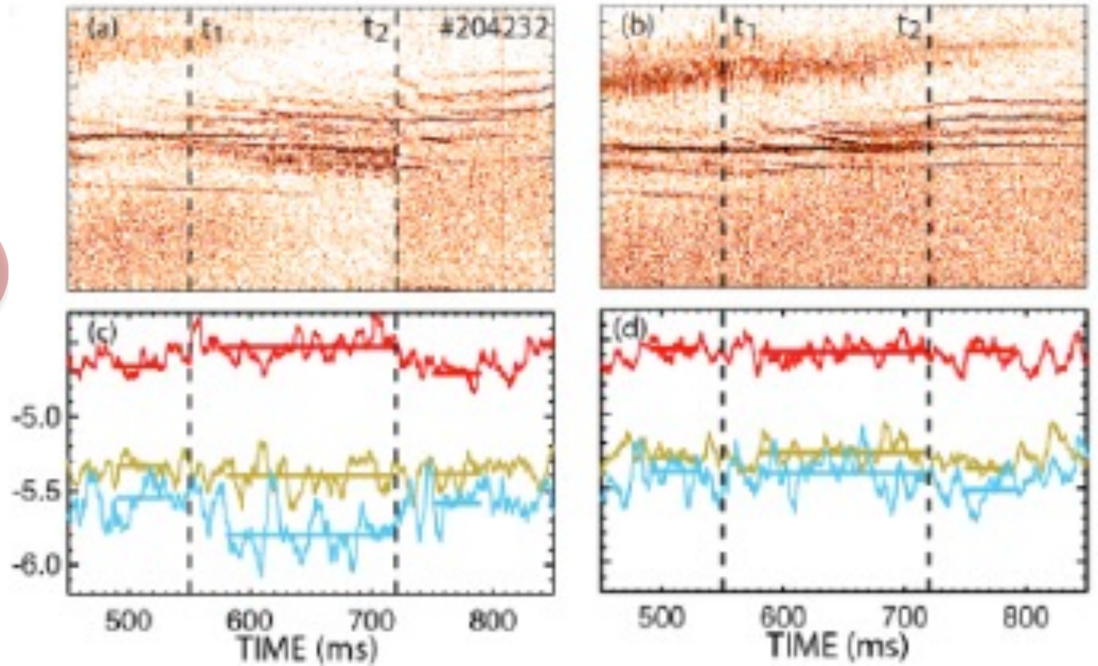
  - Routinely observed experimentally [zonal flows – X. Du et al. PRL 2025; zonal currents – W.W. Heidbrink et al. ITPEA EP 2026]

zonal flow



Presentations by Z. Qiu and M.V. Falessi

zonal current



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  - Must self-consistently compute Zonal State in the presence of finite level of fluctuations
- Separate macro-meso spatiotemporal scales from micro-meso scales
  - Renormalized equilibrium accounting for self-interaction

$$F_{0*} \equiv \bar{F}_0 + \left. e^{-iQ_z} \overline{e^{iQ_z} \delta F_z} \right|_F .$$

$\overline{\qquad\qquad}$ (orbit averaging)

meso-macro

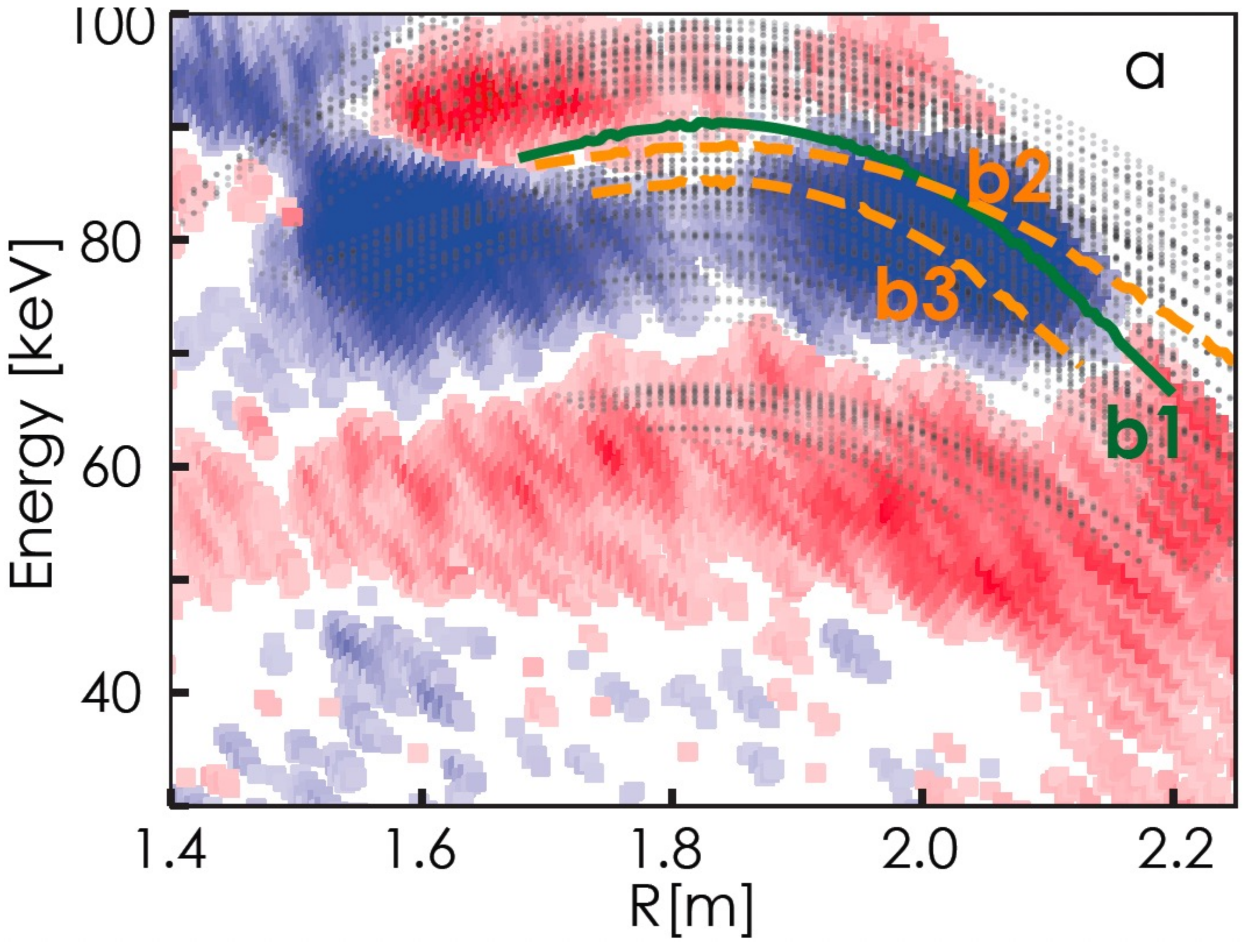
micro-meso

Phase space transport - II

❑ **Phase space zonal structure (PSZS)** evolution equation (nearly integrable system) [Falessi et al NJP 2023]

$$\frac{\partial}{\partial t}\overline{F_{z0}} + \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b \delta \dot{P}_\phi \delta F\right)_z} + \frac{\partial}{\partial \mathcal{E}}\overline{\left(\tau_b \delta \dot{\mathcal{E}} \delta F\right)_z}\right]_S = \overline{\left(\sum_b C_b^g\left[F, F_b\right] + \mathcal{S}\right)_{z\,S}}$$

Measured RSAE induced flow images along with the streamlines; at INPA measured pitch of 0:78 in DIII-D



X. Du et al. 2021 Phys. Rev. Lett. ***26*** *082502*

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❑ Intrinsically full-F [Falessi et al NJP 2023] but reduced to the 3D action space (contants of motion) ➔ verified numerically with HMGC+HYMAGYC codes [S. Briguglio and G. Vlad, 2020]

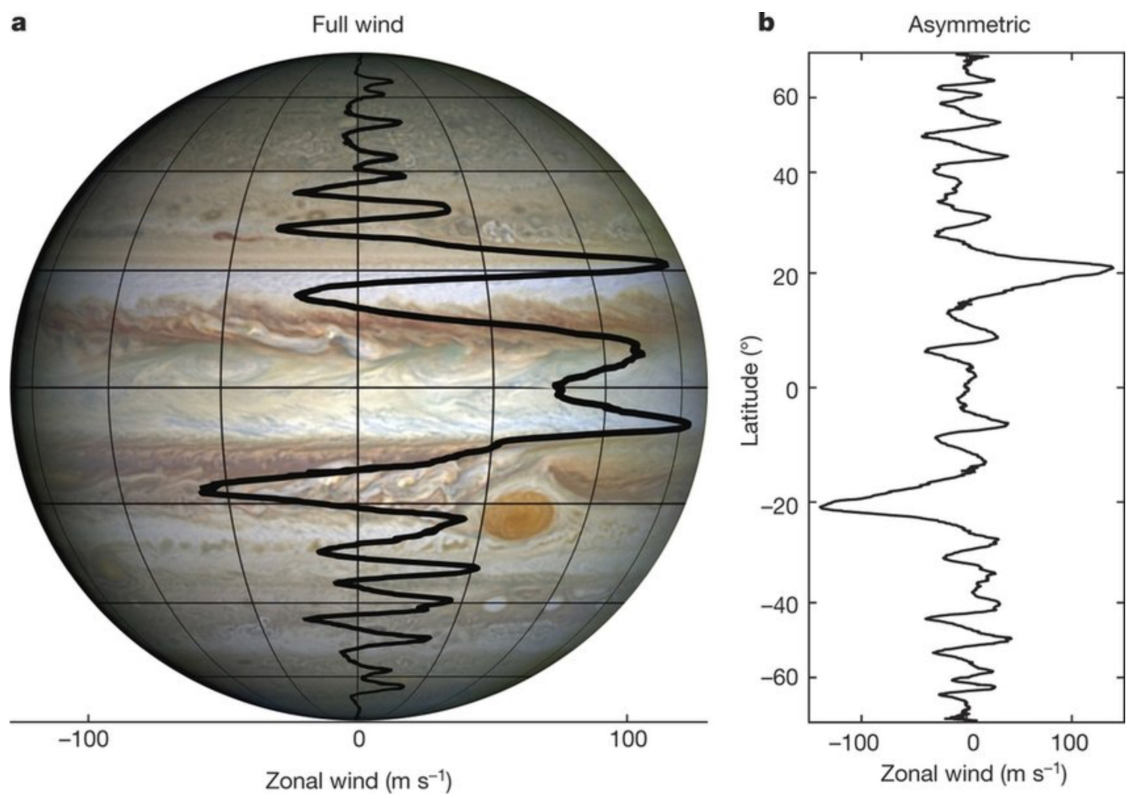
❑ Ready for implementation in non-linear GK codes, including thermal component if **whole transport description is lifted to the phase space** [S.J. Wang et al. PRL 2024, ITB by ITG] ➔ Presentation by S.J. Wang

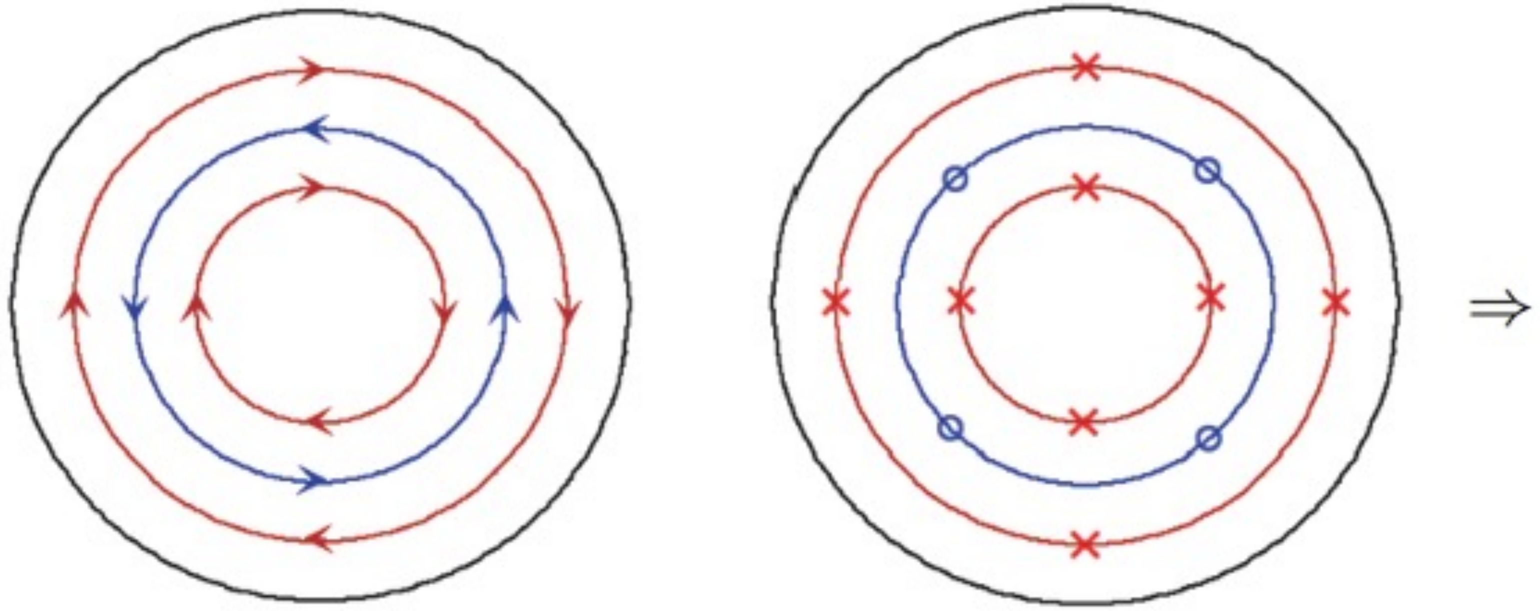
Zonal electromagnetic fields

❑ Zonal flows and currents are equally important in NL dynamic evolution

*C&Z. 2012 Phys. Rev. Lett.. **109** 145002*
*Z. Qiu et al. 2013 Europhys. Lett.. **101** 35001*

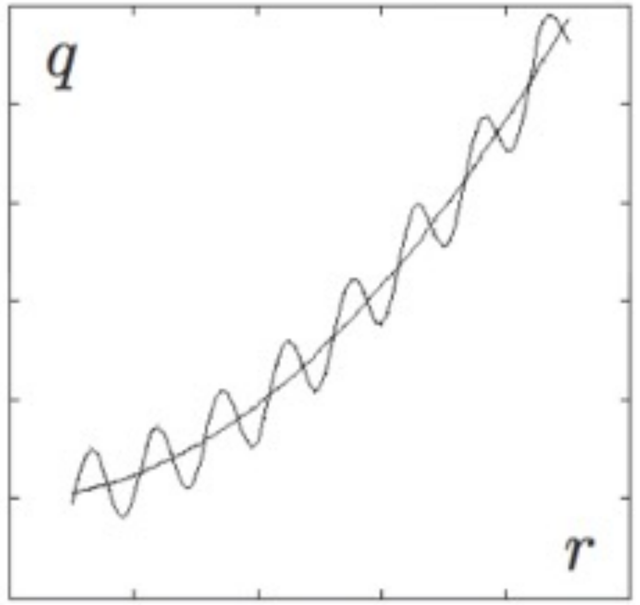
Jupiter's wind





zonal flow

zonal current



[Y. Kaspi et al 2018 Nature 555(7695):223-226]

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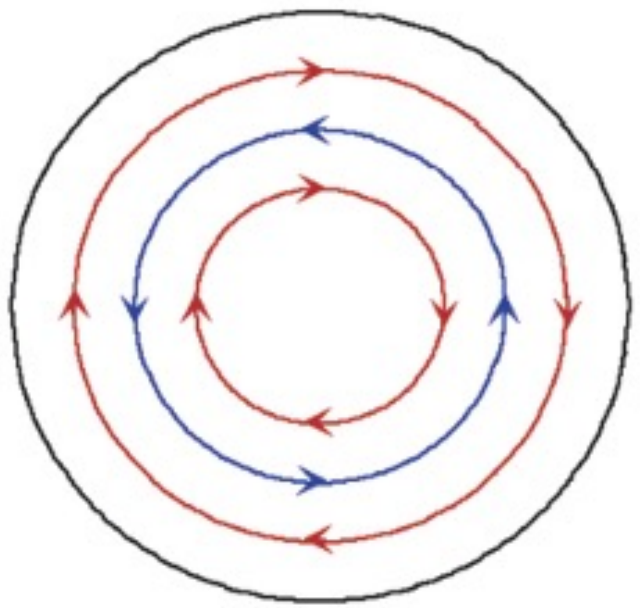
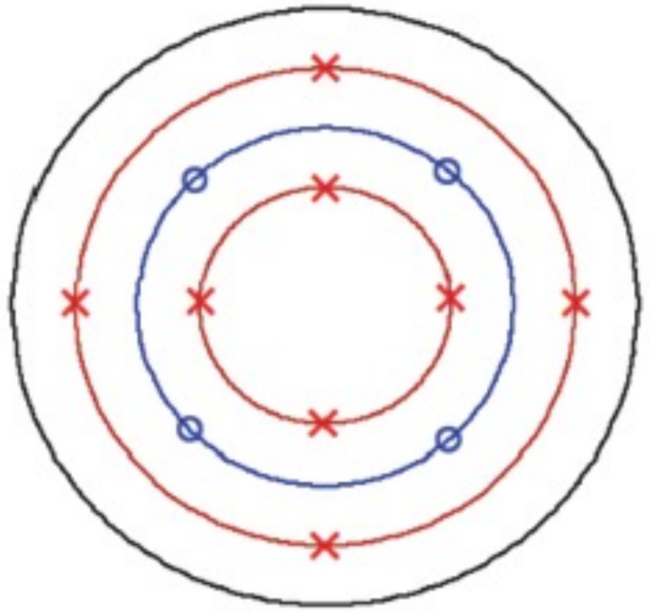
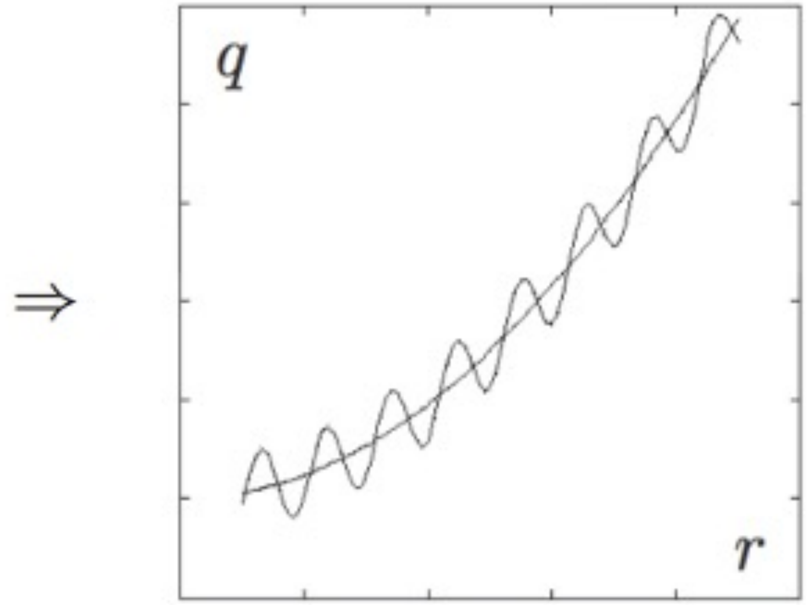
Presentations by L. Chen and R. Ma



zonal flow



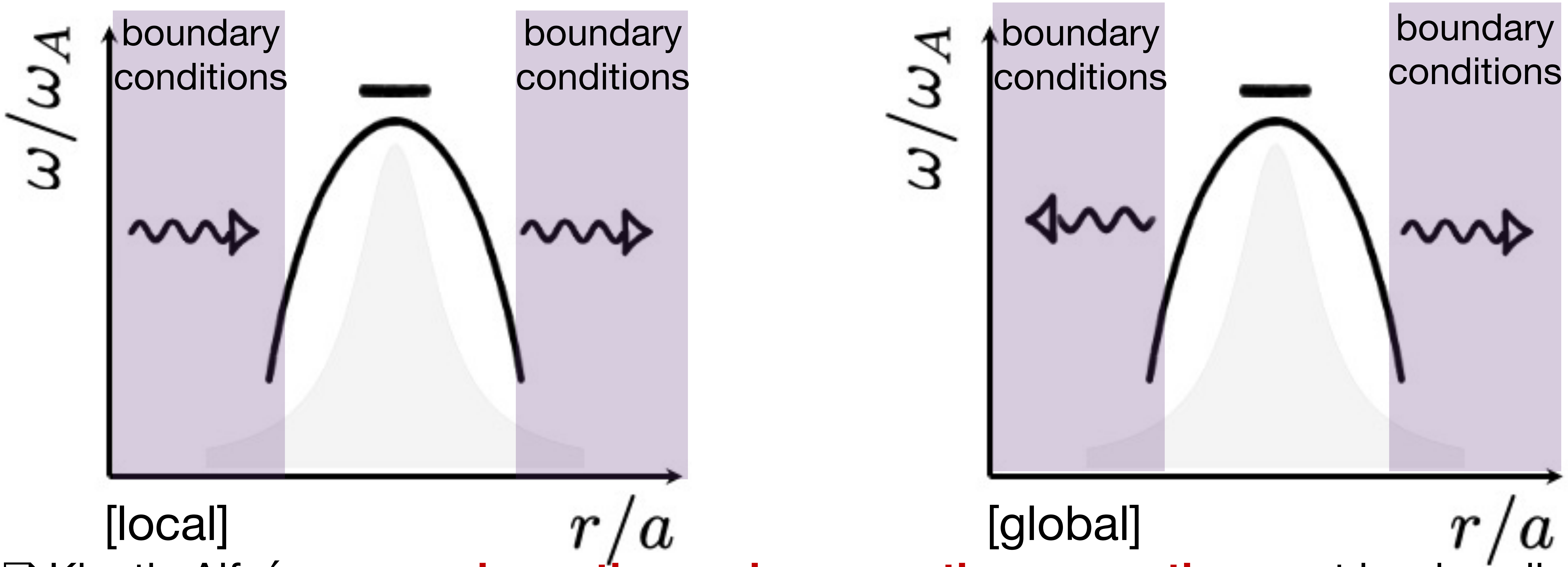
zonal current



- ❑ Core plasma also plays a crucial role in determining the properties of the zonal state
- ❑ Reversed shear Alfvén eigenmodes (RSAE): paradigm for Alfvénic fluctuation dominating the core of reactor relevant plasmas

Importance of global kinetic approaches

- ❑ Generation of **zonal electromagnetic fields** strongly depends on the **spectral features** of the underlying fluctuations [F. Zonca et al., AAPPS-DPP 2025]



- ❑ Kinetic Alfvén waves **absorption and propagation properties** must be described by **proper global kinetic approaches.** ➔ Presentation by A. Mishchenko

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Extending the GFLDR to short wavelength

- Separate linear and nonlinear responses and project on linear parallel mode structures to derive nonlinear envelope equations [C&Z Rev. Mod. Phys. Review article]

$$\hat{e}_n^+ \cdot \boldsymbol{D}\left(r,t,k_{nr},\omega_n\right) \cdot \boldsymbol{A}_n(r,t) e^{iS_n(r,t)} = \hat{e}_n^+ \cdot \boldsymbol{F}(r,t)$$

linear response nonlinear response

- Introduced envelope $\boldsymbol{A}_n(r,t) = \hat{\boldsymbol{e}}_n A_n(r,t)$, with polarization vector $\hat{\boldsymbol{e}}_n$, eikonal representation and parallel mode structure, $y_{1,2}(r,\vartheta)$

$$\begin{pmatrix} e\delta\hat{\psi}_n(r,\vartheta;t)/T_{0i} \\ e\delta\hat{\phi}_{\parallel n}(r,\vartheta;t)/T_{0i} \end{pmatrix} \equiv A_n(r,t) e^{iS_n(r,t)} \begin{pmatrix} e_1(r,t) y_1(r,\vartheta) \\ e_2(r,t) y_2(r,\vartheta) \end{pmatrix}$$

$$\delta\hat{\phi}_{\parallel n} \equiv \delta\hat{\phi}_n - \delta\hat{\psi}_n, \qquad \int_{-\infty}^{\infty} |y_{1,2}(r,\vartheta)|^2 d\vartheta = 1 \,. \qquad \hat{e}_n^+ \cdot \hat{e}_n = 1 \,.$$

Extending the GFLDR to short wavelength

- Nonlinear evolution equation has NLSE-like form

propagation
freq.shift/growth
dispersiveness
nonlinear/forcing

$$\frac{\partial}{\partial t}\left(\frac{\partial D^0_{Rn}}{\partial \omega_n}A_n^2\right) - \frac{\partial}{\partial r}\left(\frac{\partial D^0_{Rn}}{\partial k_{nr}}A_n^2\right) + 2D^1_{An}A_n^2 - 2iD^1_{Rn}A_n^2$$

$$+iA_n\left(\frac{\partial^2 D^0_{Rn}}{\partial k_{nr}^2} + 2\frac{\partial \hat{e}_n^+}{\partial k_{nr}}\cdot \boldsymbol{D}^0_{Rn}\cdot\frac{\partial \hat{e}_n}{\partial k_{nr}}\right)\frac{\partial^2 A_n}{\partial r^2} = -2ie^{-iS_n}A_n\hat{e}_n^+\cdot\boldsymbol{F}$$

$$-\left(\hat{e}_n^+\cdot\frac{d}{dt}\hat{e}_n - \frac{d}{dt}\hat{e}_n^+\cdot\hat{e}_n\right)\frac{\partial D^0_{Rn}}{\partial \omega_n}A_n^2 + \left(\frac{\partial \hat{e}_n^+}{\partial \omega_n}\cdot\boldsymbol{D}^0_{Rn}\cdot\frac{\partial \hat{e}_n}{\partial t} - \frac{\partial \hat{e}_n^+}{\partial t}\cdot\boldsymbol{D}^0_{Rn}\cdot\frac{\partial \hat{e}_n}{\partial \omega_n}\right)A_n^2$$

$$-\left(\frac{\partial \hat{e}_n^+}{\partial k_{nr}}\cdot\boldsymbol{D}^0_{Rn}\cdot\frac{\partial \hat{e}_n}{\partial r} - \frac{\partial \hat{e}_n^+}{\partial r}\cdot\boldsymbol{D}^0_{Rn}\cdot\frac{\partial \hat{e}_n}{\partial k_{nr}}\right)A_n^2\,.$$

From local dispersion rel.

[FZ etal JPCS 2021]

- By formal manipulation of spectral evolution and assumption on NL term it is possible to reduce this equation to a Stochastic Differential Equation (SDE)

➔ Presentation by H. Jhang

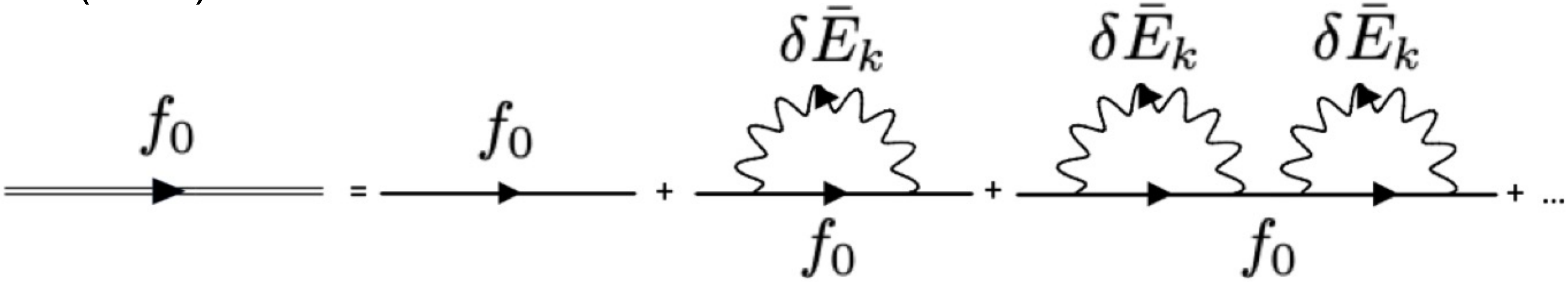
The Dyson-like equation

- Evolution of PSZS are dominated by emission and absorption of same k

$$\partial_t\left(\overline{e^{iQ_z}\bar{F}_0} + \overline{e^{iQ_z}\delta F_z}\right) = -\frac{1}{\tau_b}\frac{\partial}{\partial\psi}\left[\tau_b\overline{e^{iQ_z}\delta\dot{\psi}\delta F}\right]_z - \frac{1}{\tau_b}\frac{\partial}{\partial\mathcal{E}}\left[\tau_b\overline{e^{iQ_z}\delta\dot{\mathcal{E}}\delta F}\right]_z$$

Van Hove L (1955)
PrigogineI(1962)
BalescuR(1963)

- **Slow terms generate "secularities"**
- **Renormalization & Dyson like equation**



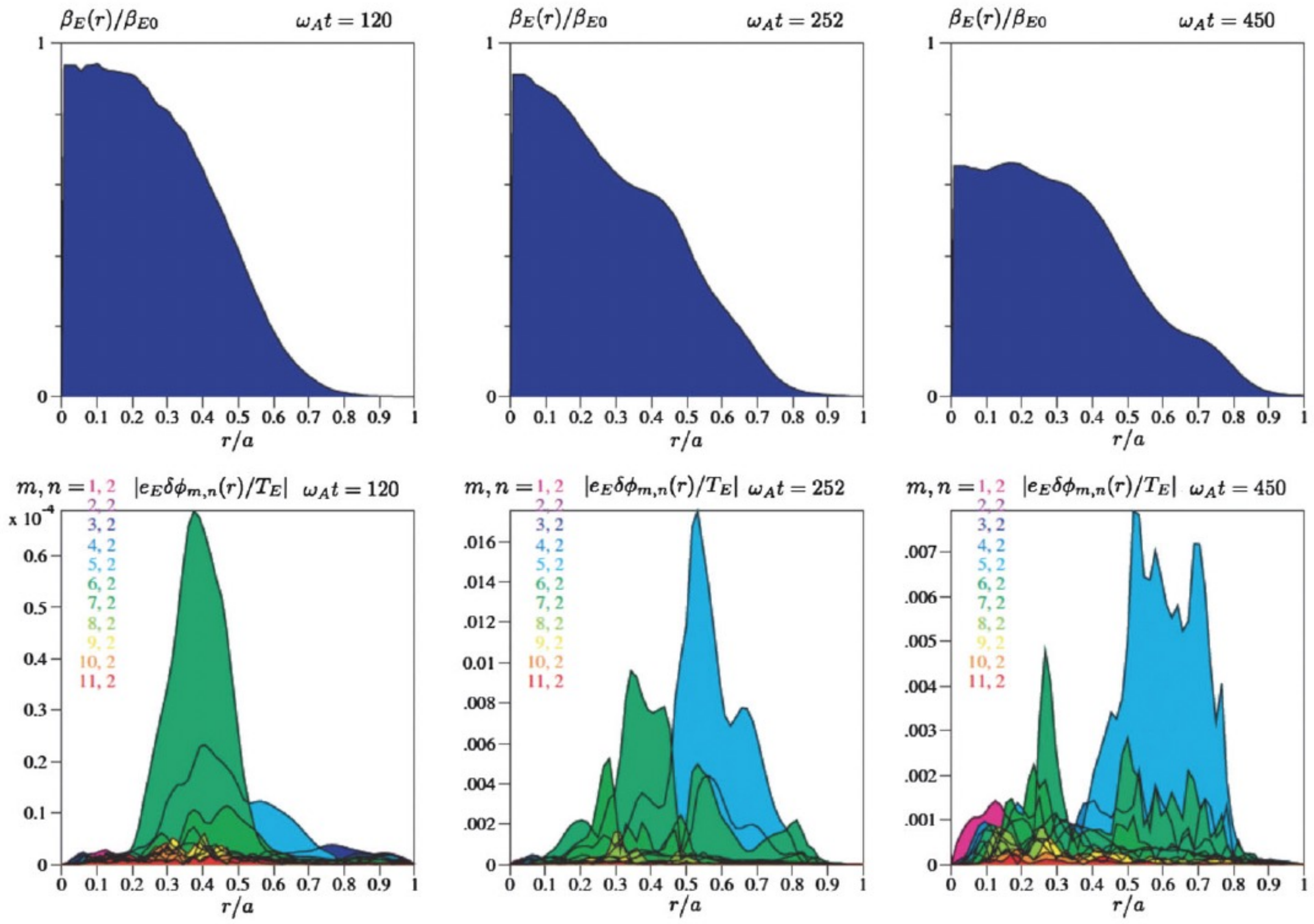
- Principal series of secular terms

The Dyson Schrödinger Model

- Combining the NLSE description of radial envelope and DSE for PSZS one can obtain description of EP avalanches and due to EPM evolution

Z&C equation

[C&Z Rev. Mod. Phys. Review article]



From Vlad et al PPCF 2004

$$\partial_\xi^2 U = \lambda U - 2iU|U|^2$$

$$U(\xi) \equiv e^{i\varphi(\xi)} W(\xi) \begin{cases} W(\xi) = \mathrm{sech}\left[(\sqrt{2}/3)^{1/2}\xi\right] \,, \\ \varphi(\xi) = -2\ln\cosh\left[(\sqrt{2}/3)^{1/2}\xi\right] \,, \\ \lambda = -\sqrt{2}/3 + i4/3 \end{cases}$$

$$\xi - \xi_0 \equiv \frac{k_{n0}}{|sk_\vartheta|}(x - x_0)$$

$$\equiv \frac{k_{n0}}{|sk_\vartheta|}\left(x - |sk_\vartheta| \int_0^t v_g(t')dt'\right), \qquad x = |sk_\vartheta|(r - r_0)$$

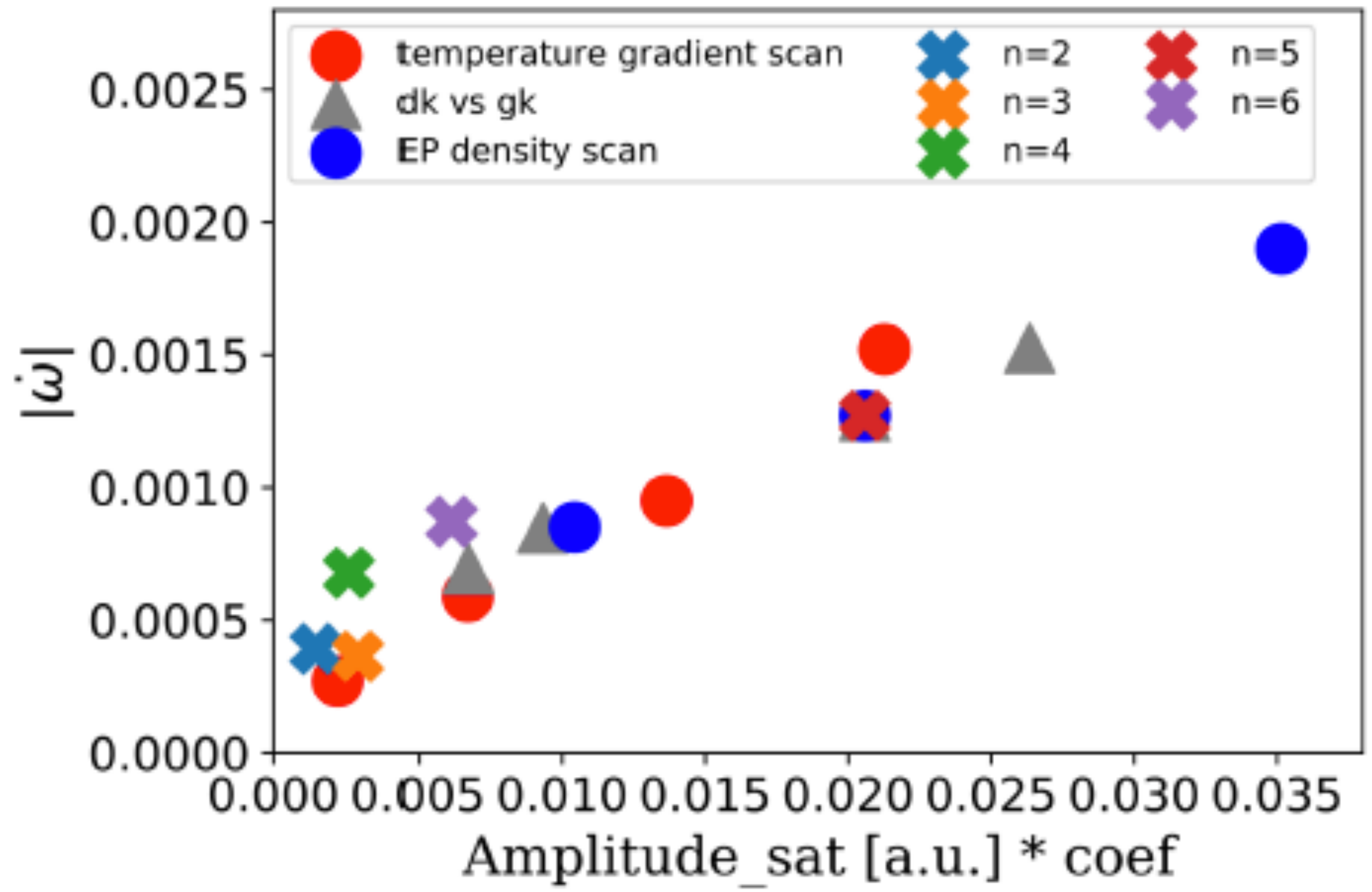
- Radial group velocity of wave-packet maximizes wave-particle power transfer for chirping rate

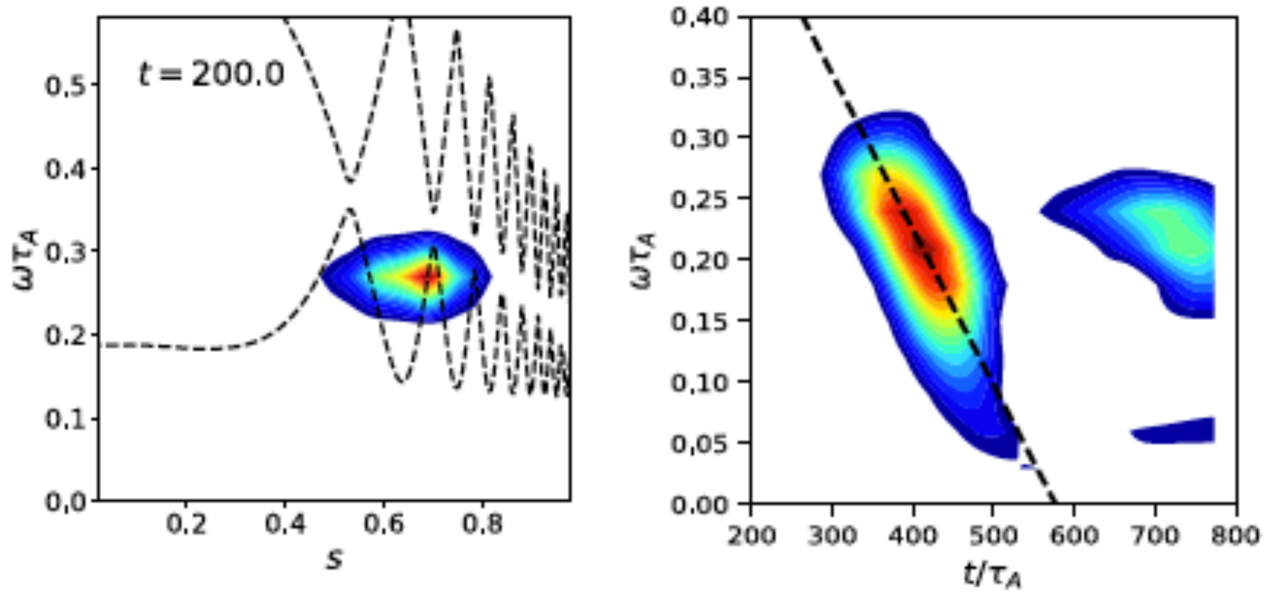
$$\dot{\omega} = R\omega_{tr}^2 \qquad R \simeq 1/2$$

Numerical simulation of EPM chirping

- PIC simulations in tokamaks show linear scaling of chirping rate with amplitude [X. Wang et al, EPS-DPP invited 2023]

◉ Chirping rate vs. Saturation amplitude





intensity contour plot

$$\dot{\omega} \simeq \delta \dot{X}_{\perp} \cdot \nabla \omega_{res}$$

← theory prediction

Earth's chorus chirping

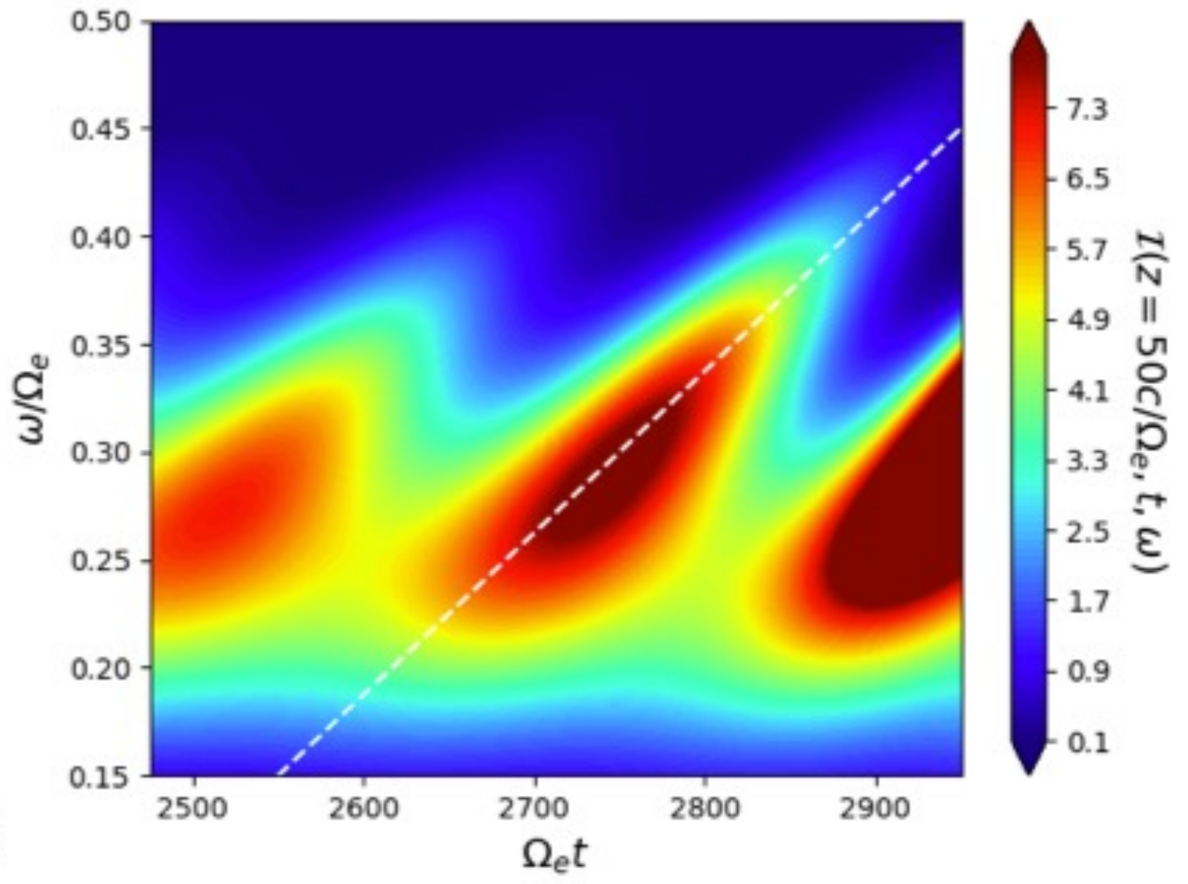
❑ Adopt the general approach to construct the nonlinear growth rate and frequency shift

$$\frac{\partial \omega}{\partial t} = \pm \frac{1}{2} \frac{\langle\langle \omega_{\mathrm{trk}}^4 \rangle\rangle^{1/2}}{(1 - v_{r\omega}/v_{g\omega})^2}$$

[Vomvoridis et al 1982]

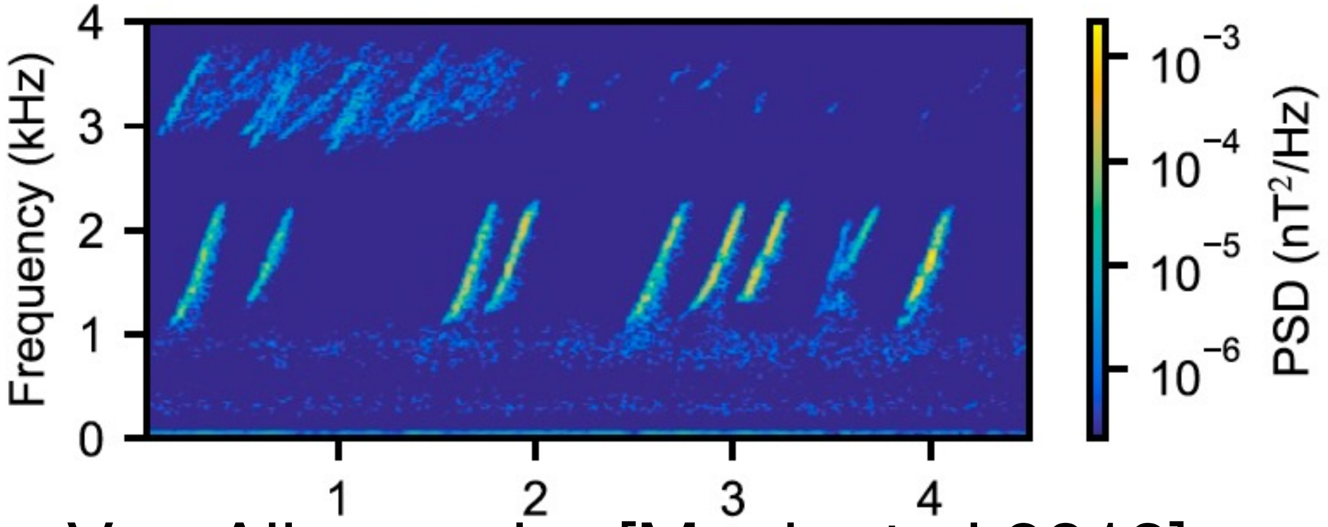
X. Tao, F. Zonca, L. Chen, A "Trap-Release-Amplify" model of Chorus Waves, JGR: Space Physics, 126, e2021JA029585

F. Zonca, X. Tao, L. Chen, Nonlinear dynamics and phase space transport by chorus emission RMPP 5, 8; A theoretical framework of chorus wave excitation, JGR 127, e2021JA029760



(b)

Chorus Power Spectral Densities (PSD)



Van Allen probe [Mauk et al 2013]

ENEA ITALIAN NATIONAL AGENCY FOR NEW TECHNOLOGIES, ENERGY AND SUSTAINABLE ECONOMIC DEVELOPMENT

ifts 浙江大學聚變理論與模擬中心

Institute for Fusion Theory and Simulation, Zhejiang University

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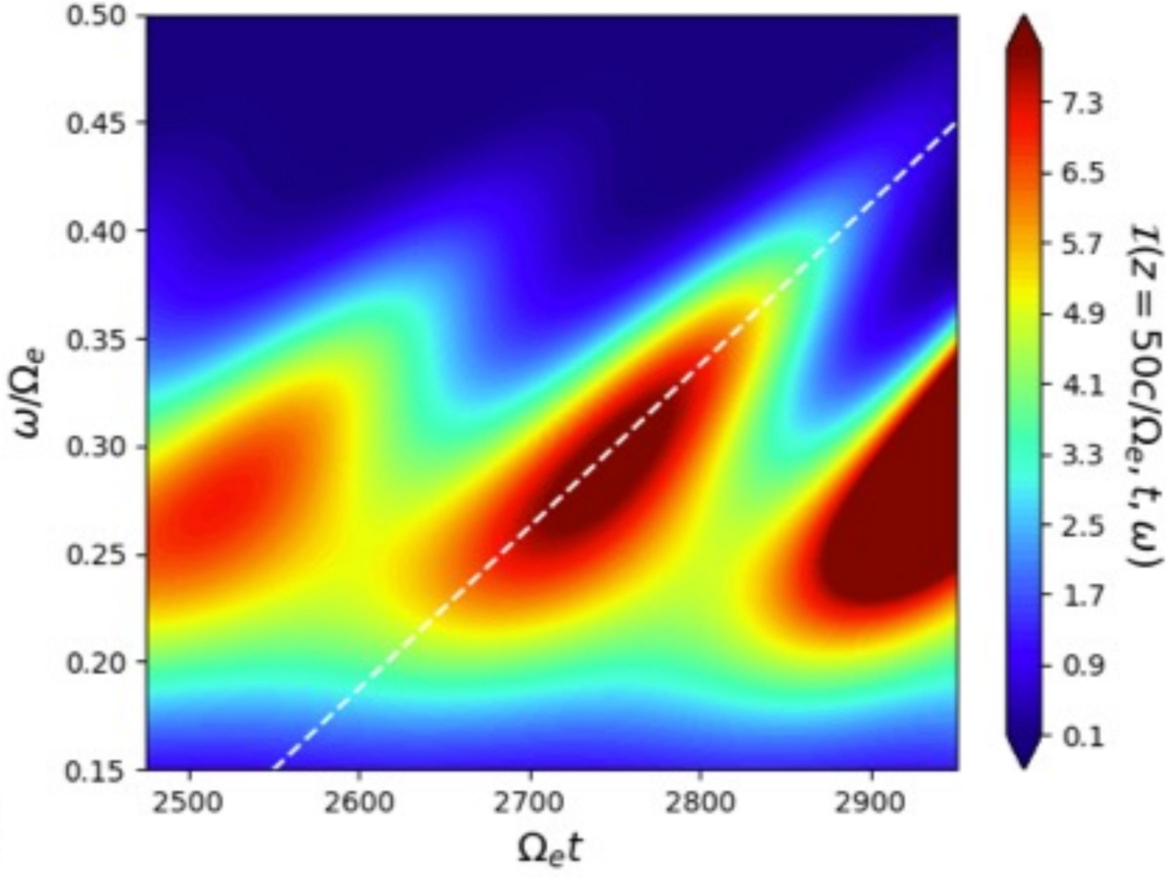

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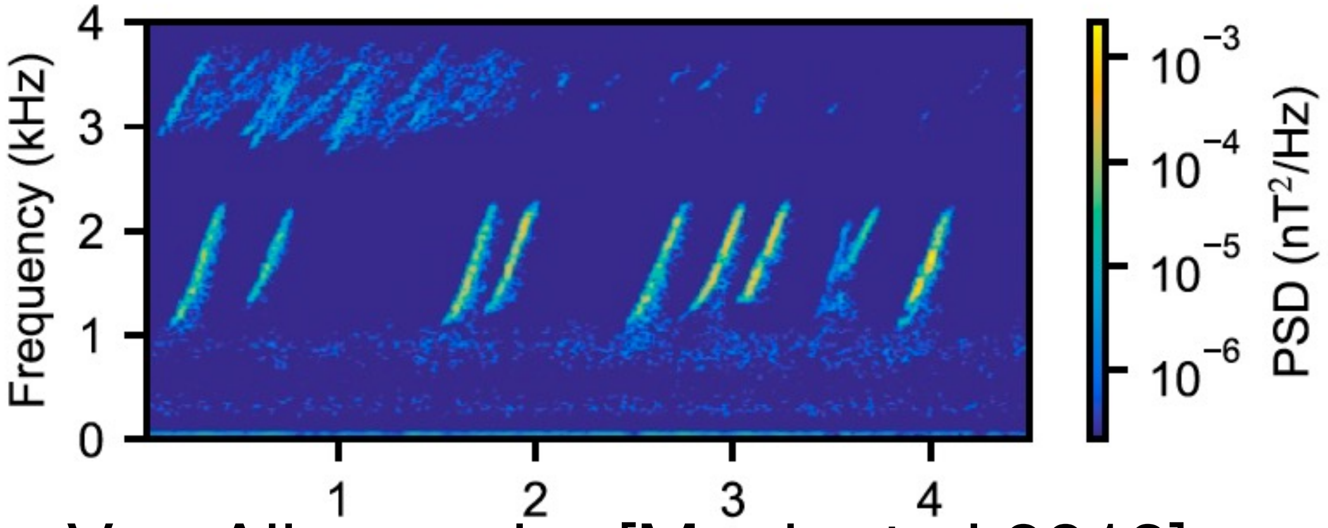

Van Allen probe [Mauk et al 2013]

Chorus Power Spectral Densities (PSD)

Universality of chirping process

← Presentation by X. Tao

Some memories from Irvine 1994