On How Zonal Fields Suppress Reversed Shear Alfvén Eigenmode in Tokamak Plasmas: Simulation and Theory

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This work is supported by 2022YFE03040002, 2024YFE03170000, CNNC-JCKY-202640, the Strategic Priority Research Program of the Chinese Academy of Sciences (CAS) (XDB0500302), and the Italian Ministry of Foreign Affairs (CN23GR02).

16th International West Lake Symposium, 2026.4.9-4.12, Hangzhou

Outline

- Background and Motivation
- GTC Simulations
- Theoretical Analyses
  - ‘RSAE-ZF-MHD’ model + variational Analysis
  - ‘RSAE-ZF-KAW’ model
- Numerical Validation
- Conclusion

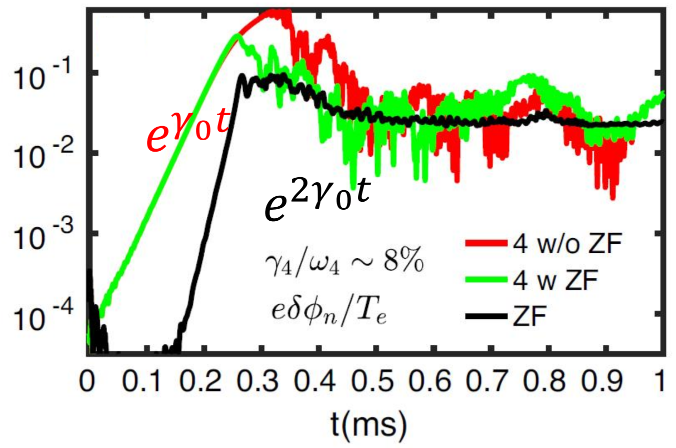
Background and Motivation

- Energetic particles (EPs) can drive Alfvén eigenmodes (AEs), impacting plasma stability and confinement [L. Chen & F. Zonca 2016RMP].

- Zonal electromagnetic Fields (ZFs, zonal flow + zonal current) regulate AEs and suppress instabilities [L. Chen et al 2001NF, 2012PRL, Y. Todo et al 2010NF, Y. Chen et al. 2018POP, Z. Qiu et al 2016NF].

- Recent RSAE simulations [Y. Chen et al. 2018POP , P. Liu, et al 2023RMPP]:
  - Beat-driven ZFs (grow at $\sim 2\gamma_{RSAE}$) reduce saturation amplitude by ~30%
  - But E×B shearing rate is negligible

GTC Simulations of an $n = 4$ RSAE [P. Liu, et al 2023RMPP]

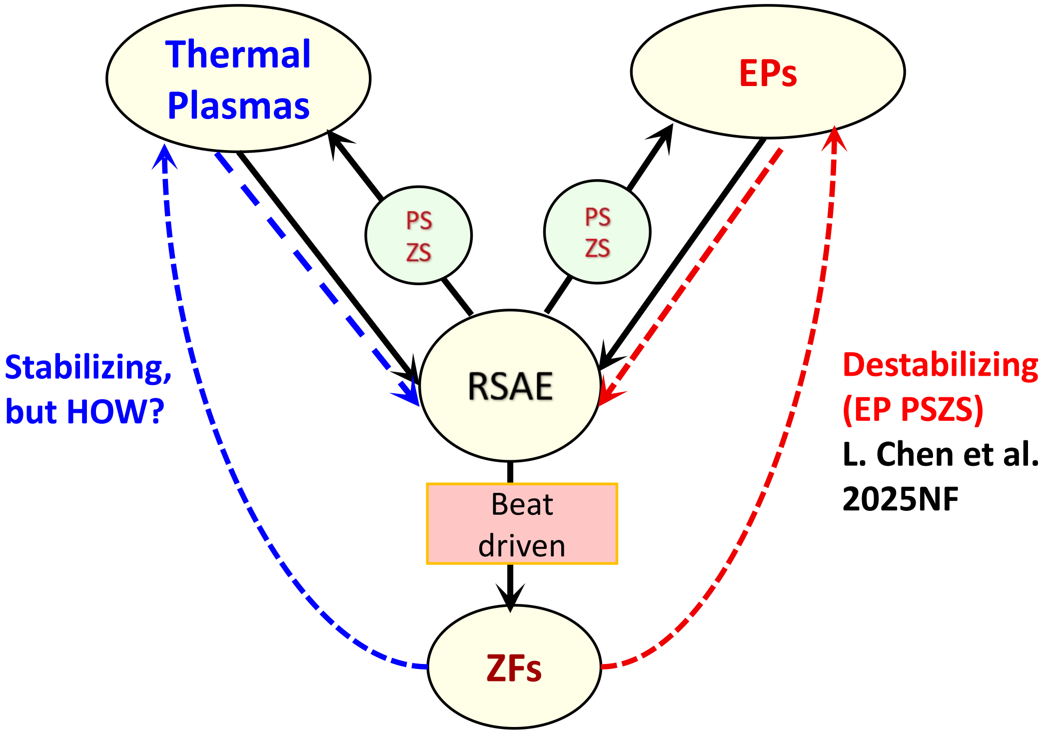


GTC: Time history of perturbed $e\delta\phi/T_e$ on q_{min} surface.

Background and Motivation

- Energetic particles (EPs) can drive Alfvén eigenmodes (AEs), impacting plasma stability and confinement [L. Chen & F. Zonca 2016RMP].
- Zonal electromagnetic Fields (ZFs, zonal flow + zonal current) regulate AEs and suppress instabilities [L. Chen et al 2001NF, 2012PRL, Y. Todo et al 2010NF, Y. Chen et al. 2018POP, Z. Qiu et al 2016NF].
- Recent RSAE simulations [Y. Chen et al. 2018POP , P. Liu, et al 2023RMPP]:
  - Beat-driven ZFs (grow at ~ $2\gamma_{RSAE}$) reduce saturation amplitude by ~30%
  - But E×B shearing rate is negligible
- Two main routes of ZF effects on AE Saturation [L. Chen & F. Zonca, 2013POP]:
  - ☐ nonlinear EP dynamics
    ZFs modify EP phase-space zonal structure [F. Zonca et al. 2015NJP; 2021JPCS; M. Falessi, et al. 2023NJP], affecting wave-particle resonance and mode drive. ⇒ **Destabilizing!** [L. Chen et al 2025NF]
  - ☐ nonlinear thermal plasma dynamics
    ZFs alter the mode through nonlinear frequency shift or q-profile modification, leading to enhanced continuum damping.
- Suppression must come from the thermal plasma route

Nonlinear Diagram [Courtesy of Prof. L. Chen, private notes]



But underlying mechanism unknown → **Motivation**

GTC Simulations

GTC [Z. Lin et al. 1998 Science] simulaton setup

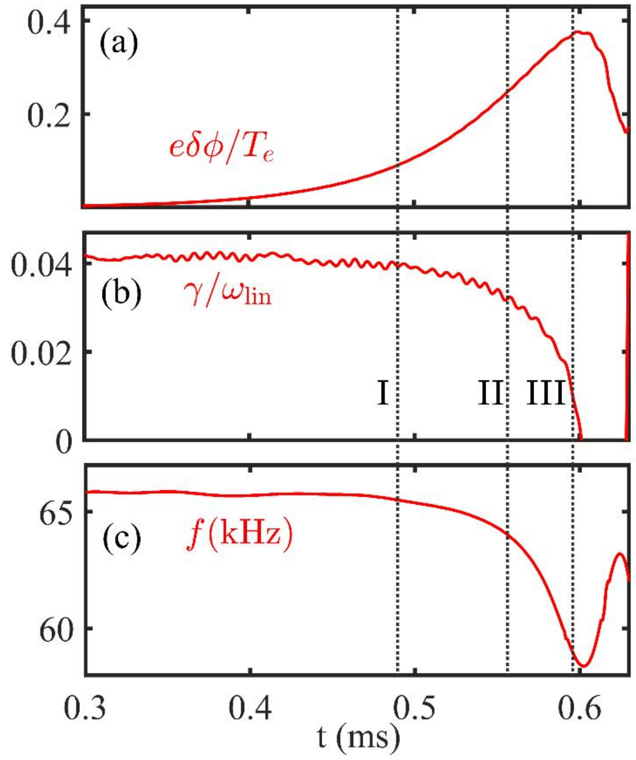
- **Equilibrium:**
  DIII-D discharge #159243 at 805 ms [Collins et al 2016PRL];
  Reversed magnetic shear, $q_m = 2.9747$
  $n = 4$ RSAE simulated

- **Physics models:**
  EP & thermal ions: gyrokinetic;
  Electrons: fluid-kinetic hybrid [Z. Lin & L. Chen, 2001POP].

- **Key setup:**
  Thermal plasma: fully nonlinear
  EP dynamics: linear

  ⇒ to isolate the thermal plasma nonlinearity route

GTC simulation results (RSAE, n =4) [Courtesy of P.F. Liu]



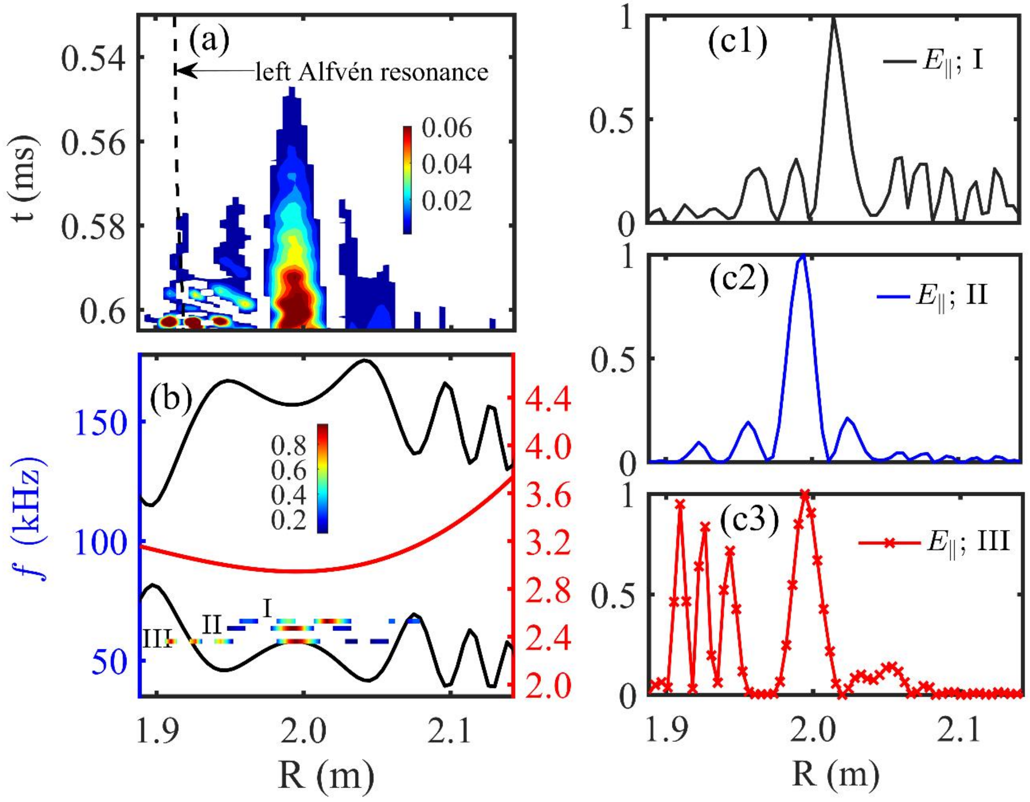
Even with linear EP drive

✓ mode saturates at finite amplitude;
✓ suppression of the nonlinear growth rate;
✓ pronounced downward frequency chirping

GTC Simulations

KAW excitation & RSAE frequency evolution [Courtesy of P.F. Liu]

$$E_{\parallel} = -(\nabla_{\parallel}\delta\phi + \partial\delta A_{\parallel}/c\partial t)$$



- (a) Time evolution of $E_{\parallel}$ radial profile;
  Radial spreading of the mode converted KAW.
- (b) RSAE frequency vs. Alfvén continua (black) from MAS [Bao2023 et al. 2023NF]; q profile (red, right axis). Horizontal lines mark phases I-III;
  Frequency chirps downward toward continuum
- (c) $E_{\parallel}$ radial profiles at phases I-III,
  Increasing k_r & propagating radially inward with downward chirping until mode saturation.

⇒ First-principles theoretical analyses **are needed** to quantify the dominant damping mechanism and distinguish the respective roles of zonal current vs. zonal flow.

Theoretical Analyses — 'RSAE-ZF-MHD' model

Given ZFs produced by beat-driven RSAE —
How do $\delta\phi_z$ and $\delta A_{\parallel z}$ affect the saturation of RSAE via thermal plasmas?

- **The governing equations for beat-driven ZFs on RSAE-MHD** [Frieman & Chen, 1982Phys. Fluids B; Chen & Zonca, 2016RMP]

- NL gyrokinetic equation:

$$(\partial_t + v_\parallel b_0 \cdot \nabla + v_d \cdot \nabla)\delta g_{jk} = i\left(\frac{e}{m}\right)_j QF_{Mj}\langle \delta L_k \rangle - \left(\frac{c}{B_0}\right)\sum_{k=k'+k''} b_0(k'' \times k')\langle \delta L'_k \rangle \delta g''_{jk} \quad (3)$$

- Quasi-neutrality condition:

$$\frac{N_0 e^2}{T_e}\left(1 + \frac{T_e}{T_i}\right)\delta\phi_k = \sum_j \left\langle e_j J_k \delta g_{jk} \right\rangle_v, \quad (4)$$

- NLGK vorticity equation:

$$\frac{c^2 B}{4\pi\omega^2}\frac{\partial}{\partial l}\frac{k_\perp^2}{B}\frac{\partial}{\partial l}\delta\psi_k + \frac{e^2}{T_i}\langle (1 - J_k^2)F_0 \rangle \delta\phi_k - \sum_j \left\langle \frac{q}{\omega} J_k \omega_d \delta g_j \right\rangle =$$

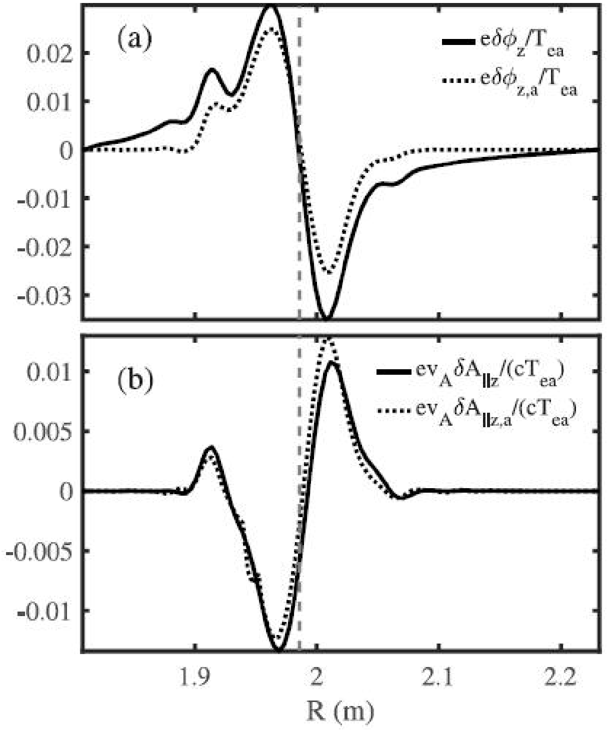
$$- i\frac{c}{B\omega}\sum_{k=k'+k''} \boldsymbol{b} \cdot \boldsymbol{k}'' \times \boldsymbol{k}' \left[\langle (J_k J_{k'} - J_{k''})\delta L_{k'} \delta g_{k''} \rangle + \frac{k_\perp''^2 c^2}{4\pi} \frac{1}{\omega_{k'}\omega_{k''}} \frac{\partial \delta\psi_{k'}}{\partial l} \frac{\partial \delta\psi_{k''}}{\partial l} \right] \quad (5)$$

LHS: field line bending, inertia, ballooning-interchange
RHS: gyrokinetic Reynolds stress, Maxwell stress

Beat-driven ZFs by RSAE [**L. Chen et al 2025NF**].

$$\delta\phi_z \simeq \frac{c}{B_0\omega_0^2} k_{\theta 0}\omega_{*pi}\frac{\partial}{\partial r}(|\delta\phi_0|^2) \quad (1)$$

$$\delta A_{\parallel z} \simeq \frac{c^2}{B_0\omega_0^2} k_{\theta 0}\frac{\partial}{\partial r}(k_{\parallel 0}|\delta\phi_0|^2) \quad (2)$$



Theoretical Analyses — 'RSAE-ZF-MHD' model

◻ The particle non-adiabatic responses $\boldsymbol{\delta g_j = \delta g_j^{(1)} + \delta g_j^{(2)}}$

- For electrons, $|\omega_0|, |\omega_{de}|_0 \ll |k_\parallel v_{te}|_0$ and $|k_\perp \rho_e|_0^2 \ll 1$

$$\delta g_{e0}^{(1)} \simeq -\left(\frac{e}{T_e}\right) F_{Me}\left(1-\frac{\omega_{*ne}}{\omega}\right)_0 \delta\psi_0$$

$$\delta g_{e0}^{(2)} \simeq -i\left(\frac{c}{B_0}\right)\frac{k_{\theta 0}k_z}{\omega_0}(\delta\psi_0\delta g_z - \delta\psi_z\delta g_0)_e \qquad (6)$$

- For ions, $|\omega_0| \gg |k_\parallel v_{ti}|_0$

$$\delta g_{i0}^{(1)} \simeq \frac{e}{T_i} F_{Mi}\left(1-\frac{\omega_{*i}}{\omega}\right)_0 J_0\delta\phi_0\left(1+\frac{\omega_{di}}{\omega}\right)_0$$

$$\delta g_{i0}^{(2)} \simeq i\left(\frac{c}{B_0}\right)\frac{k_{\theta 0}k_z}{\omega_0}(J_z\delta\phi_z\delta g_0 - J_0\delta\phi_0\delta g_z)_i \qquad (7)$$

Eqs. (6) & (7) ⇒ Q-N condition, Eq. (4),

NL Ohm's law:

$$\delta\phi_0 - \delta\psi_0 \simeq i\frac{c}{B_0}\frac{k_{\theta 0}k_z}{\omega_0}\delta\phi_0\left(\delta\phi_z - \frac{\omega_0}{c}\frac{\delta A_{\parallel z}}{k_{\parallel 0}}\right) + O\left(k_\perp^2\rho_i^2\right) \qquad (8)$$

- ZF-induced finite $E_\parallel$
- The FILR corrections are neglected.

◻ NLGK vorticity Eq. (5), for RSAE, $\boldsymbol{\nabla\cdot\delta J_0^{(1)} + \nabla\cdot\delta J_0^{(2)} = 0}$

$$\nabla\cdot\delta J_0^{(1)} = \frac{\delta B_{\perp 0}}{B_0}\cdot\nabla J_{\parallel 0} - \frac{ik_{\parallel 0}c}{4\pi}\nabla_\perp^2\delta A_{\parallel 0} + \frac{ic^2}{4\pi}\nabla_\perp\cdot\frac{\omega_0-\omega_{*pi}}{v_A^2}\nabla_\perp\delta\phi_0 - \frac{ic^2}{4\pi}\nabla_\perp\cdot\frac{\left(1-\frac{\omega_{*pi}}{\omega_0}\right)}{v_A^2}\frac{\omega_{gam}^2}{\omega_0}\nabla_\perp\delta\phi_0 + \lambda_a \qquad (9)$$

$$\lambda_a = -\frac{c}{B_0}b_0\times\kappa\cdot\nabla\left[\sum_j\left\langle\left(\frac{em}{T}F_M\right)_j\frac{\omega_{*j}}{\omega_0}\left(\frac{v_\perp^2}{2}+v_\parallel^2\right)_j\right\rangle_v\right]_s\delta\phi_0$$

$$\nabla\cdot\delta J_0^{(2)} = \frac{c^3k_{\theta 0}k_z}{4\pi B_0}v_A^2(k_z^2-k_{\perp 0}^2)\left[\left(\frac{k_{\parallel 0}v_A}{\omega_0}\right)\left(\frac{v_A}{c}\right)\delta A_{\parallel z} - \left(1-\frac{\omega_{*pi}}{\omega_0}\right)\delta\phi_z\right]\delta\phi_0 \qquad (10)$$

Theoretical Analyses — 'RSAE-ZF-MHD' model

Eqs. (8)-(10) ⇒ 'RSAE-ZF-MHD' model

$$\nabla_\perp \cdot (\epsilon_A \nabla_\perp \delta\phi_0) + \Lambda\delta\phi_0 + \nabla\cdot[(\nabla\alpha_{\phi 2})\delta\phi_0] = 0, \quad (11)$$

$$\epsilon_A = \epsilon_{A0} + \alpha_{\phi 1} + \alpha_A,$$

$$\epsilon_{A0} = \left[\left(1 - \frac{\omega_{*pi}}{\omega_0}\right)(\omega_0^2 - \omega_{\rm gam}^2)\right] - k_{\parallel 0}^2 v_A^2,$$

$$\alpha_{\phi 1} = \frac{c}{B_0}\omega_0 k_{\theta 0}\left(\frac{k_{\parallel 0}^2 v_A^2}{\omega_0^2} + 1 - \frac{\omega_{*pi}}{\omega_0}\right)\frac{\partial\delta\phi_z}{\partial r},$$

$$\alpha_A = -2\frac{c}{B_0}\omega_0 k_{\theta 0}\left(\frac{k_{\parallel 0} v_A}{\omega_0}\right)\left(\frac{v_A}{c}\right)\frac{\partial\delta A_{\parallel z}}{\partial r},$$

$$\alpha_{\phi 2} = \frac{c}{B_0}\omega_0 k_{\theta 0}\left(\frac{k_{\parallel 0}^2 v_A^2}{\omega_0^2} - 1 + \frac{\omega_{*pi}}{\omega_0}\right)\frac{\partial\delta\phi_z}{\partial r}$$

Λ: favorable average curvature and toroidal couplings [F. Zonca 2000&2002 POP].

Variational Analysis [courtesy of Prof. L. Chen's private notes]

- For an RSAE localized near $r = r_m$, introducing $\zeta \equiv k_{\theta 0}(r - r_m)$ and $\psi = \epsilon_A^{1/2}\delta\phi_0$, Eq. (11) ⇒ the variational form:

$$\mathcal{L}[\psi] = \frac{1}{2}\int_{-\infty}^{+\infty} d\zeta\left\{-\left|\frac{d\psi}{d\zeta}\right|^2 + |\psi|^2\left[-1 + \frac{1}{4}\left(\frac{\hat\epsilon_A'}{\hat\epsilon_A}\right)^2 - \frac{1}{2}\left(\frac{\hat\epsilon_A''}{\hat\epsilon_A}\right) + \frac{\hat\Lambda}{\hat\epsilon_A} + \frac{1}{2}\frac{\hat\alpha_{\phi 2}''}{\hat\epsilon_A}\right]\right\} \quad (12)$$

$$\hat\epsilon_A = \hat\epsilon_{A0} + \hat\alpha_{\phi 1} + \hat\alpha_A,\ \hat\Lambda = \frac{\Lambda}{k_{\theta 0}^2\omega_{A0}^2},\ \hat\epsilon_{A0} = \left(1 - \frac{\Omega_{*pi}}{\Omega_0}\right)(\Omega_0^2 - \Omega_{\rm gam}^2) - K_\parallel^2,$$

$$\Omega_0 = \frac{\omega_0}{\omega_{A0}},\ \Omega_{*pi} = \frac{\omega_{*pi}}{\omega_{A0}},\ K_\parallel = (n_0 q - m_0) = K_m + \frac{1}{2}\frac{S^2}{n_0 q_m}\zeta^2,\ K_{\parallel m} = n_0 q_0 - m_0 < 0,$$

$$S^2 = r_0^2 q''/q_0,\ q_0 = q(r_m)$$

Three scale lengths:
$\zeta \sim \zeta_0 \ll 1$: potential well region (near $q(r_m)$)
$\zeta \sim \zeta_1 \sim \mathcal{O}(1)$: potential barrier region
$\zeta \sim \zeta_2 \gg 1$: Alfvén resonance layer, $\hat\epsilon_A(\zeta_2) = 0$

Theoretical Analyses — Variational Analysis

1) Variational D.R. without ZFs.

Case1: Around $\zeta = 0$, $\hat{\epsilon}_{A0} \simeq \varDelta(\zeta_0^2 + \zeta^2)$, with $\varDelta = \frac{S^2|K_m|}{n_0 q_m}$, $\zeta_0^2 = \left(\frac{\hat{\epsilon}_{Am}}{\varDelta}\right) \ll 1$, $\hat{\epsilon}_{Am} = \left(1 - \frac{\Omega_{*pi}}{\Omega_0}\right)\left(\Omega_0^2 - \Omega_{\mathrm{gam}}^2\right) - K_m^2$,

For $|\zeta| \sim \mathcal{O}(1)$ outer region, $\left(\frac{d^2}{d\zeta^2} - 1\right)\psi_O = 0 \quad \Rightarrow \psi_O = e^{-|\zeta|}$ exponential decay solution

For $|\zeta| \sim |\zeta_0| \ll 1$ inner layer, $\left[\frac{d^2}{d\zeta^2} - \frac{1}{\zeta_0^2+\zeta^2} + \frac{\zeta^2}{(\zeta_0^2+\zeta^2)^2} + \frac{\hat{\Lambda}}{\varDelta(\zeta_0^2+\zeta^2)}\right]\psi_I = 0$

Matching $\lim_{|\zeta|\to\infty}\psi_I = \lim_{|\zeta|\to 0^+}\psi_O = 1$ and $\lim_{|\zeta|\to\infty}\frac{d\psi_I}{d\zeta} = \lim_{|\zeta|\to 0^+}\frac{d\psi_O}{d\zeta} = -1$

$$\Rightarrow \zeta_0 = \frac{\pi}{2}\left(\frac{\hat{\Lambda}}{\varDelta} - \frac{1}{2}\right)$$

(13); in agreement with [H.L. Berk, et al, 1993Phys. Fluids B; F. Zonca, et al. 2002POP]

Case2: Including Alfvén resonance absorption

Letting $\zeta_2 > \zeta_* > 1$, then $\mathcal{L}_0[\psi] = \frac{1}{2}\int_{-\zeta_*}^{\zeta_*} d\zeta\,[\ldots]$, $\mathcal{L}_1[\psi] = \int_{\zeta_*}^{\infty} d\zeta\,[\ldots]$, $\psi = \psi_t = e^{-|\zeta|}$, $\Rightarrow D_0 + D_1 = 0$,

The lowest-order eigenmode D.R.: $D_0 = \frac{\pi}{2\zeta_0}\left(\frac{\hat{\Lambda}}{\varDelta} - \frac{1}{2}\right) - 1$

Dominant contribution from the Alfvén resonance: $D_1 \simeq \frac{1}{4}\int_{\zeta_*}^{\infty} d\zeta\,\psi_t^2\frac{1}{(\zeta-\zeta_2)^2} \simeq -\frac{1}{2}i\pi e^{-2\zeta_2} = -i\delta(\Omega)$

The complete RSAE linear D.R.:

$$\zeta_0(1 + \mathrm{i}\delta_c) = \frac{\pi}{2}\left(\frac{\hat{\Lambda}}{\varDelta} - \frac{1}{2}\right)$$

(14); recovers analytic theory [F. Zonca, et al. 2002POP]

$\delta_c = \frac{\pi}{2}e^{-2\zeta_2}$: continuum resonant-absorption damping; exponential small.

Theoretical Analyses — Variational Analysis

2) Variational D.R. with ZFs.

Case3: $\boldsymbol{\alpha_z} = \boldsymbol{\widehat{\alpha}_{\phi 1}} + \boldsymbol{\widehat{\alpha}_A}$ and expanding around $\zeta = 0$, $\Rightarrow \hat{\epsilon}_A \simeq \varDelta_z(\zeta_{0z}^2 + \zeta^2)$,

$$\varDelta_z = \frac{1}{(n_0 q_m)^2}\left[|K_m| n_0 \left(\frac{r^2 d^2 q}{dr^2}\right)_m + \frac{1}{2}\left(\frac{r^2 d^2 \alpha_z}{dr^2}\right)_m\right]$$

$$\zeta_{0z}^2 = \frac{1}{\varDelta_z}\left[\left(1 - \frac{\Omega_{*pi}}{\Omega_0}\right)\left(\Omega_0^2 - \Omega_{\mathrm{gam}}^2\right) - K_m^2 + \alpha_{zm}\right]$$

$\psi = \psi_t = e^{-|\zeta|}$, $\Rightarrow D_z = D_{0z} + D_{1z} = 0$, $D_{0z} = \frac{\pi}{2\zeta_{0z}}\left(\frac{\widehat{\Lambda}_z}{\varDelta_z} - \frac{1}{2}\right) - 1$, $D_{1z} \simeq -\frac{1}{2} i\pi e^{-2\zeta_2} = -i\delta(\Omega)$

$$\zeta_{0z}(1 + i\delta) = \frac{\pi}{2}\left(\frac{\widehat{\Lambda}_z}{\varDelta_z} - \frac{1}{2}\right) \qquad (15)$$

$$\varDelta_z = \varDelta + \frac{1}{2}\alpha_{zm}'', \ \widehat{\Lambda}_z = \widehat{\Lambda} + \frac{1}{2}\widehat{\alpha}_{\phi 2m}'',$$

$$\alpha_{zm} = -\frac{2c^2}{B_0^2 \omega_A^2}\left(\frac{n_0 q}{r}\right)_m^4 \left\{\left(\frac{K_{\parallel 0}}{\Omega_0}\right)^2 - \frac{1}{2}\frac{\Omega_{*pi}}{\Omega_0}\left[\left(\frac{K_{\parallel 0}}{\Omega_0}\right)^2 + 1 - \frac{\Omega_{*pi}}{\Omega_0}\right]\right\}_m \frac{\partial^2 |\delta\phi_0|^2}{\partial \zeta^2}\Big|_m$$

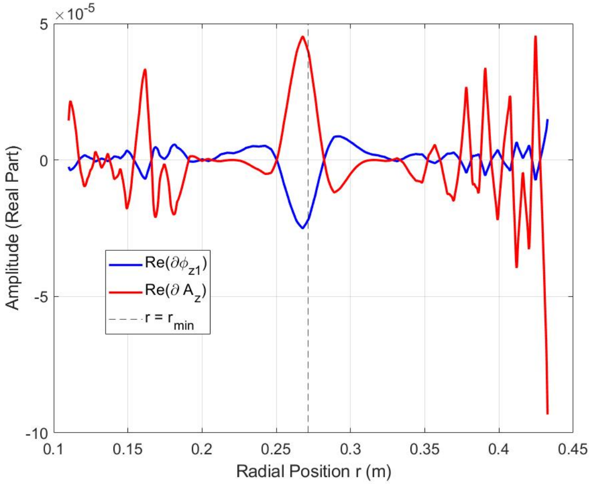
$$\left(1 - \frac{\Omega_{*pi}}{\Omega_0}\right)\left(\Omega^2 - \Omega_{gam}^2\right) = K_m^2 - \alpha_{zm}$$

For a mode peaking at $\zeta = 0$, $\Rightarrow \frac{\partial^2 |\delta\phi_0|^2}{\partial \zeta^2}\Big|_m < 0$

Under the typical ordering $|\Omega_0| \approx |K_m| > |\Omega_{*pi}|$, $\Rightarrow$

$\partial_{zm} > 0$ dominated by the zonal current ($\propto K_{\parallel 0}$) and leads to a net downward frequency shift. Zonal flow ($\propto \Omega_{*pi}$) alone would give an opposite minor upward frequency shift.

$$\alpha_{zm} = (\widehat{\alpha}_{\phi 1} + \widehat{\alpha}_A)|_{r_m}$$



Time 0.30 ms, at q_m (r=0.2704 m):

$\mathrm{Re}(\widehat{\alpha}_{\phi 1})$= -2.2858e-05

$\mathrm{Re}(\widehat{\alpha}_A)$= 4.1560e-05

Zonal current

⇩

Downward chirp

⇩

Frequency →continuum point

⇩

$k_r^2 \propto \frac{1}{\omega_0^2 - \omega_{\mathrm{cont}}^2} \uparrow$

⇩

Mode conversion to KAW ↑

KAW physics is needed!

Theoretical Analyses — 'RSAE-ZF-KAW' model

For the coupled KAW-RSAE system

- Electron Landau damping becomes relevant ($|\omega_0| \sim |k_{\parallel 0} v_{te}|$),

  $\delta g_e \simeq -\frac{e}{T_e} F_{Me} \left(1 - \frac{\omega_{*ne}}{\omega_0}\right) \left(\delta\psi_0 - \frac{\omega_0}{k_{\parallel 0} v_{te} - \omega_0} \delta\phi_\parallel\right)$, (16)

- For ions,

  $\delta g_i \simeq \frac{e}{T_i} F_{Mi} \left(1 - \frac{\omega_{*i}}{\omega}\right)_0 J_0 \delta\phi_0 \left(1 + \frac{\omega_{di}}{\omega}\right)_0$, (7)

  $\delta\phi_\parallel \equiv \delta\phi_0 - \delta\psi_0$.

Substituting $\delta g_{e,i}$ into the Q-N condition, ⇒ $\delta\psi_0 = \sigma_k \delta\phi_0$,

$$\sigma_k = 1 + \tau(1 - \Gamma_0)(1 - \omega_{*pi}/\omega_0)/[(1 - \frac{\omega_{*ne}}{\omega_0})(1 + \zeta_e Z(\zeta_e))]$$

incorporates FILR effects ($\Gamma_0 \equiv I_0(b_i) e^{-b_i}$, $b_i = k_\perp^2 \rho_i^2$) and electron kinetics ($Z(\xi_e) = \pi^{-\frac{1}{2}} \int_{-\infty}^{\infty} \frac{e^{-t^2}}{t - \xi_e} dt$, $\xi_e = \frac{\omega}{|k_\parallel| v_e}$).

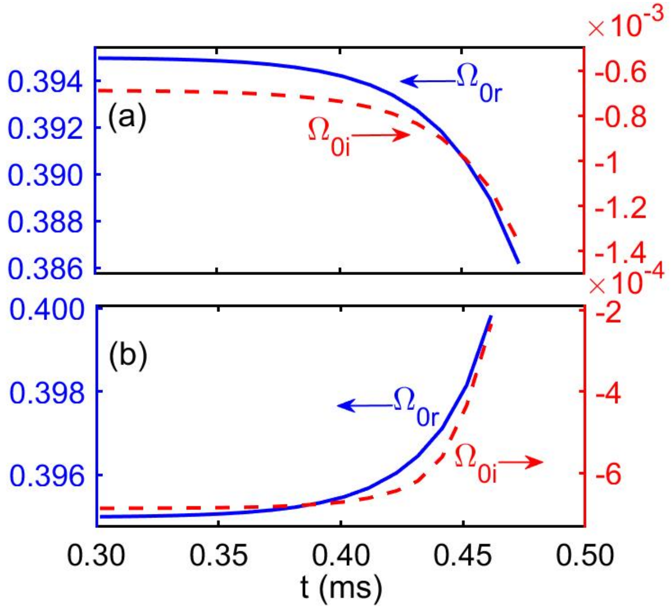
- NLGY vorticity eqn ⇒ the 4th –order eigenmode eqn for 'RSAE-ZF-KAW' model:

$$\rho_i^2 \nabla_\perp^2 \left(1 - \frac{\omega_{*pi}}{\omega_0}\right) \left[\frac{3}{4}(\omega_0^2 - \omega_{gam}^2) + \frac{\tau k_\parallel^2 v_A^2}{\left(1 - \frac{\omega_{*ne}}{\omega}\right)\left(1 + \xi_e Z(\xi_e)\right)}\right] \nabla_\perp^2 \delta\phi_0 + \nabla_\perp \cdot (\epsilon_A \nabla_\perp \delta\phi_0) + \Lambda \delta\phi_0 + \nabla \cdot [(\nabla \alpha_{\phi 2}) \delta\phi_0] = 0 \quad (17)$$

- ✓ Self-consistently describes the coupling between RSAE, ZFs and KAWs
- ✓ **Electron Landau damping** → radiative damping → responsible for the NL suppression and eventual saturation of RSAE

Numerical Validation

'RSAE-ZF-MHD' Model (Eq. (11))



$\Omega_{0r} \equiv \frac{\mathrm{Re}(\omega_0)}{\omega_A}$, $\Omega_{0i} \equiv \frac{\mathrm{Im}(\omega_0)}{\omega_A}$

(a) With zonal flow and zonal current.

- clear downward frequency chirping

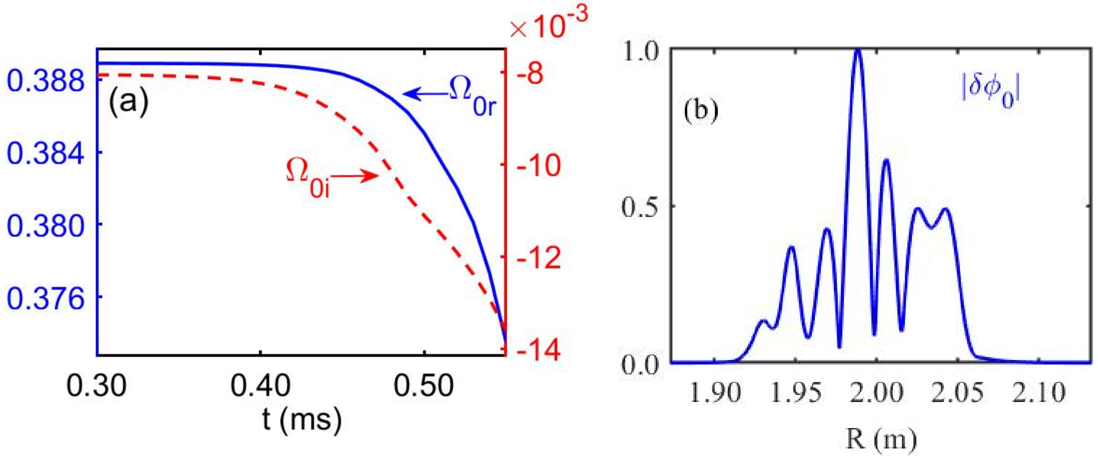
(b) With zonal flow only.

- minor upward frequency chirping

⇒ Confirming the theoretical prediction of zonal current dominance

⇒ Continuum damping remains negligible, $\mathcal{O}(10^{-4} - 10^{-3})$, in agreement with theory.

'RSAE-ZF-KAW' Model (Eq. (17))



(a) With zonal flow and zonal current.

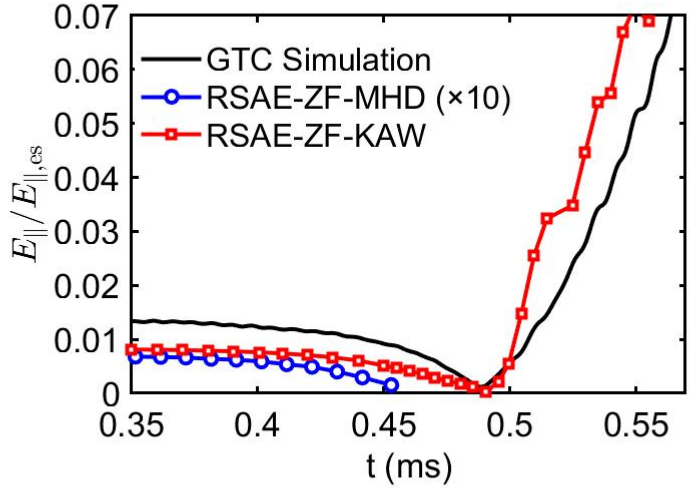
- downward frequency chirping;
- radiative damping nearly 2 orders stronger than MHD continuum damping; balancing the EP drive;

(b) Radial profile of normalized $|\delta\phi_0|$ at $t = 0.55$ ms (phase II).

- fine-scale radial structures;
- confirms the increase in k_r;
- in agreement with theory and GTC simulations

Numerical Validation

Quantitative comparison of $E_\parallel / E_{\parallel,es}$ evolution



$$E_\parallel = -\left(\nabla_\parallel \delta\phi + \partial \delta A_\parallel / c\partial t\right)$$

$$E_{\parallel,es} = -\nabla_\parallel \delta\phi$$

NL Ohm's law:

$$\delta\phi_0 - \delta\psi_0 \simeq i \frac{c}{B_0} \frac{k_{\theta 0} k_z}{\omega_0} \delta\phi_0 \left(\delta\phi_z - \frac{\omega_0}{c} \frac{\delta A_{\parallel z}}{k_{\parallel 0}}\right) + O\left(k_\perp^2 \rho_i^2\right) \quad (8)$$

- Similar trends as the freq. approaches the accumulation point of the continuum spectrum; the system recovers the ideal MHD limit and $E_\parallel$ vanishes.
- **RSAE-ZF-KAW**: order‑ of‑ magnitude increase after $t = 0.48$ ms; agrees with **GTC**
- **RSAE-ZF-MHD:** fails as RSAE merges into Alfvén continuous spectrum;
- The sharp rise in $E_\parallel$ is dominated by KAW excitation; the ZF contribution remains >10× smaller

⇒ Numerically validating that KAW generation is the primary mechanism for $E_\parallel$ enhancement and RSAE suppression.

Conclusion

The effects of ZFs on saturation of RSAE via thermal plasma route:

1. Key findings from GTC simulations

Single-n RSAE interacting with beat-driven ZFs exhibits:

- Downward frequency chirping and suppression of the growth rate
- KAW excitation (enhanced $E_\parallel$ near q_m surface, outward propagation) and mode saturation

2. Theoretical insights

- Zonal current, not zonal flow, dominates downward frequency chirping;
- Radiative damping, not continuum damping, governs energy dissipation via mode-converted KAWs, balancing EP drive

3. Mechanism: thermal plasma nonlinearity route

- ZFs suppress RSAE primarily through thermal plasma route, not direct EP modification
- Zonal current → frequency down-chirping → KAW mode conversion → radiative damping

4. Future perspective

- Opens new avenues for alpha-particle transport research in burning plasmas

Acknowledgements

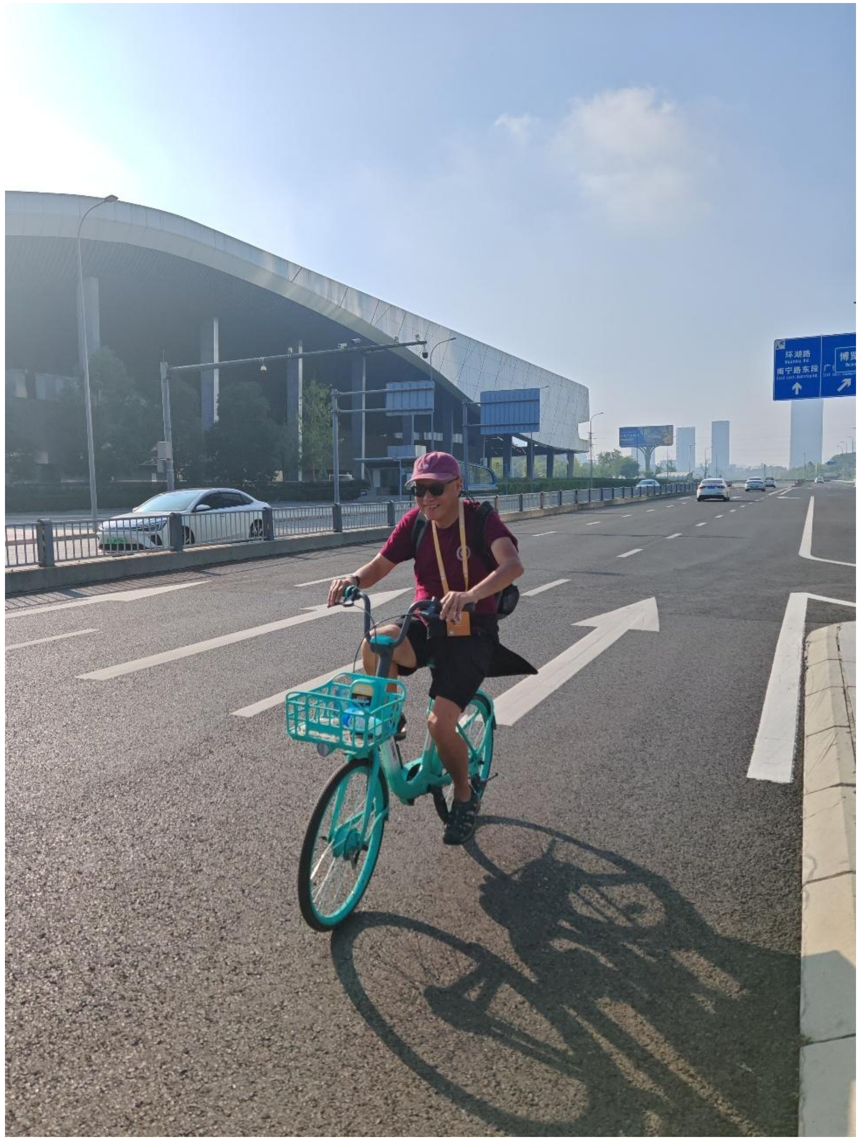
On this special occasion,
I would like to express my deepest gratitude to Prof. Chen —
for his foundational theories and ideas that have illuminated our path.

As my mentor,
he guided me step by step —
from linear to nonlinear physics.

He taught me, above all,
to bravely and honestly say, “I don’t know.”

Without his insight, patience, and generosity,
I would not be who I am today.

Thank you, Prof. Chen!



2025.10.15 IAEA-FEC, Chengdu

Thanks for your attention!

Ruirui Ma

16th International West Lake Symposium, 2026.4.9-4.12, Hangzhou

Numerical Validation

Numerical Approach

GTC: $\Omega_0 = \Omega_{0r} + i\Omega_{0i}$ (EP-driven), equilibrium parameters, $q(r)$, $n_i(r)$, etc

w/o ZFs ⇩

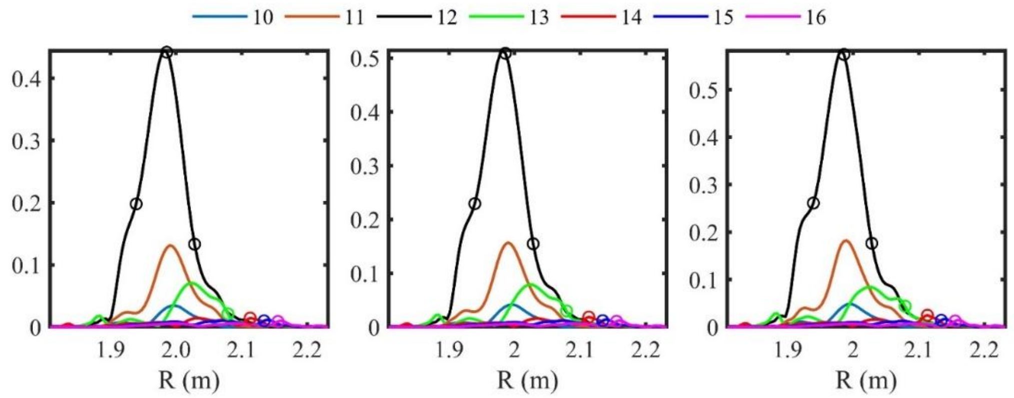
$\hat{\Lambda}$ (complex, $\hat{\Lambda}_r$ is positive and $\geq \hat{\Lambda}_{crit}$)

⇩

Fixing $\hat{\Lambda}_r$ and setting $\hat{\Lambda}_i = 0$ (w/o EP drive)

w/ ZFs ⇩ ZFs evolve with time via $\delta\phi_0(r,t)$ given by GTC

Numerically solving two models



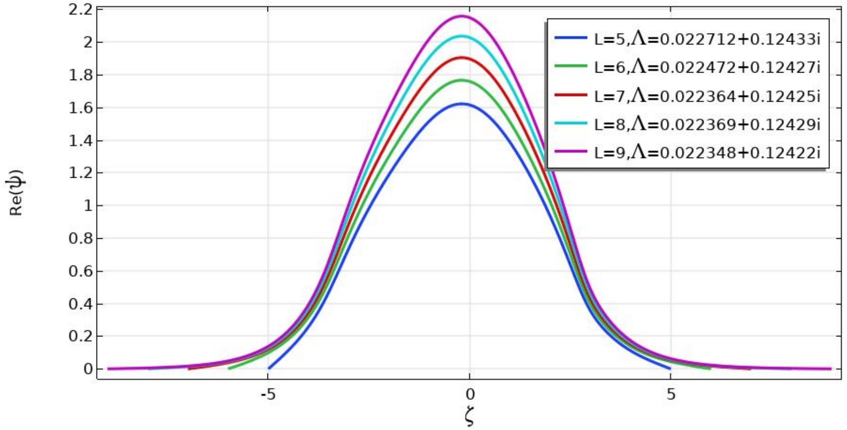
R=[1.9, 2.1] m

The radial position of $q_{\min}$: $R_m = 1.9938$ m

$\zeta = k_{\theta 0}(r - r_m) \simeq [-5,\ 5]$

$$\mathrm{ZFs} = 0, \left(\frac{\omega_0}{\omega_A}\right)_{gtc} = 0.47065 + 0.12716i,$$

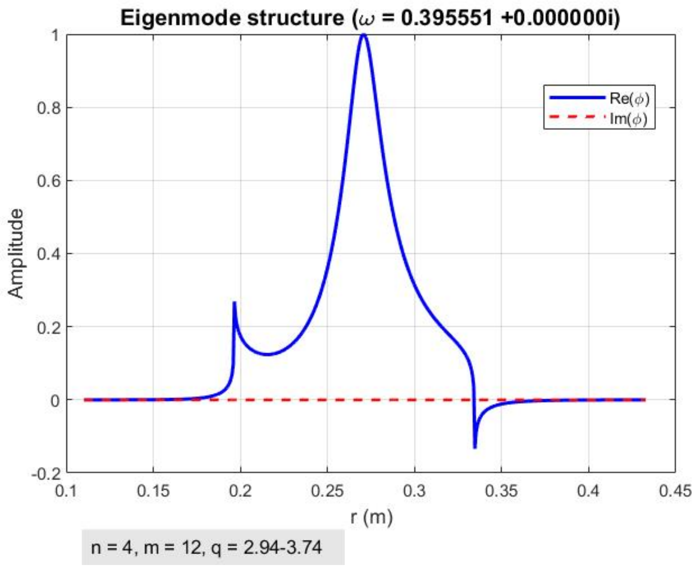
$$\Rightarrow \hat{\Lambda} = \Lambda/(\omega_A^2 k_{\theta 0}^2) = 0.022364 + 0.12425i$$



Numerical Validation

'RSAE-ZF-MHD': $\nabla_{\perp} \cdot (\epsilon_A \nabla_{\perp} \delta\phi_0) + \Lambda \delta\phi_0 + \nabla \cdot [(\nabla \alpha_{\phi 2}) \delta\phi_0] = 0,$ (11)

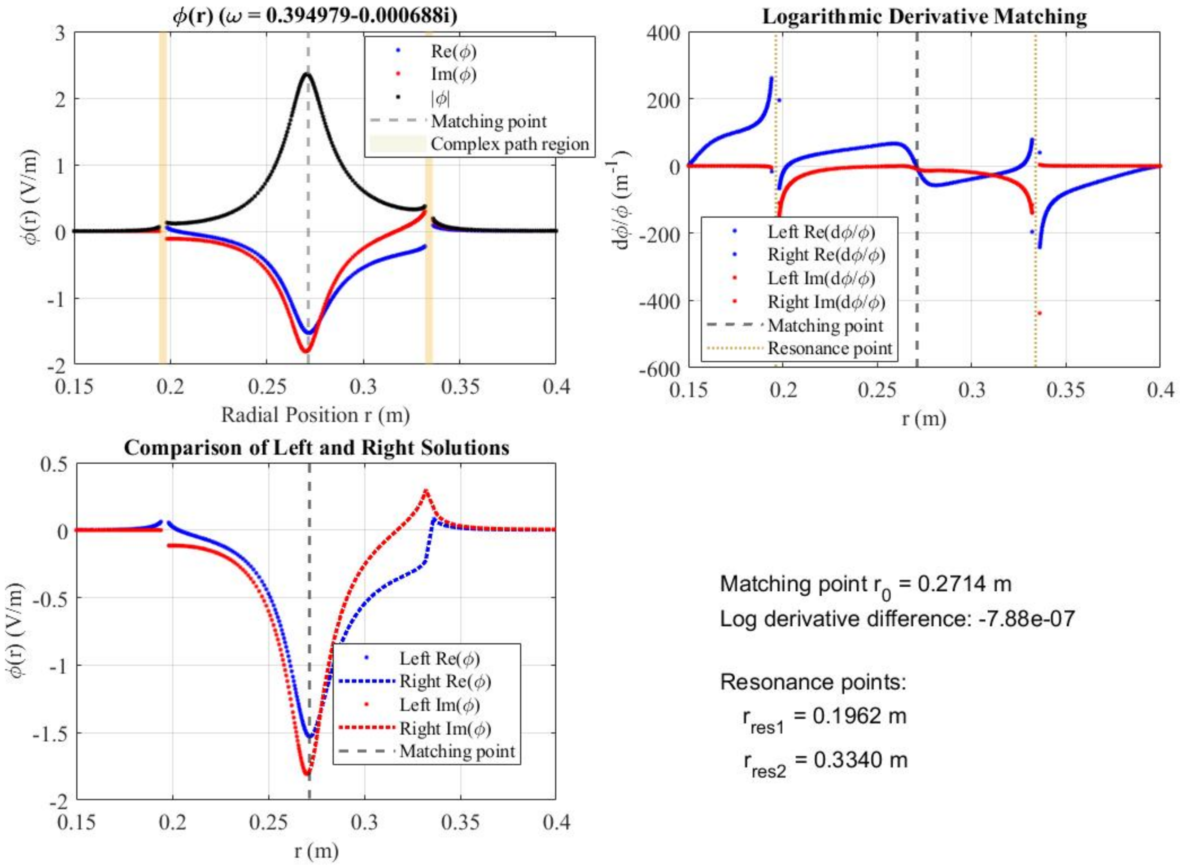
Matrix solver:



Cannot handle Alfvén resonance points

Shooting method:

Extended to the complex plane → handles Alfvén resonance



Matching point r_0 = 0.2714 m

Log derivative difference: -7.88e-07

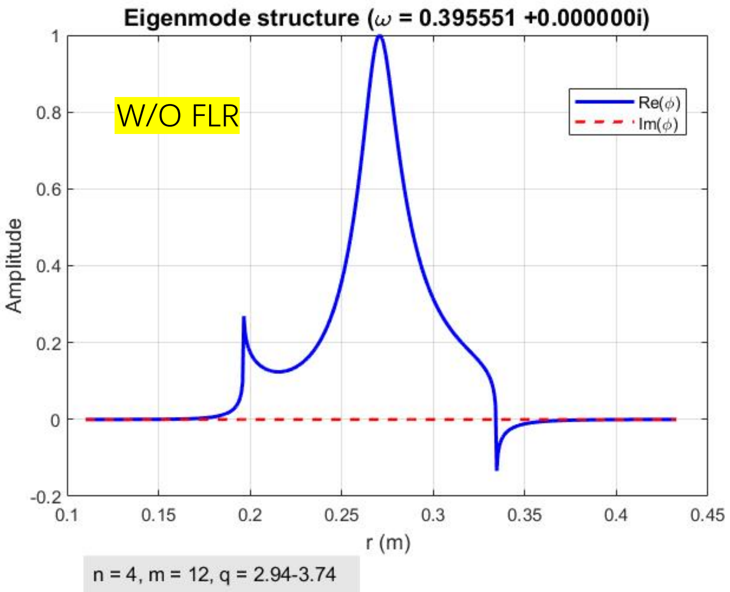
Resonance points:

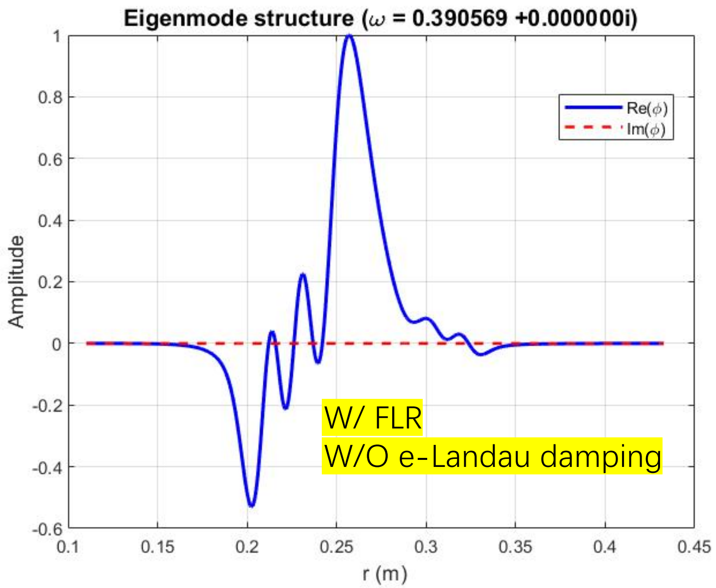
r_{res1} = 0.1962 m

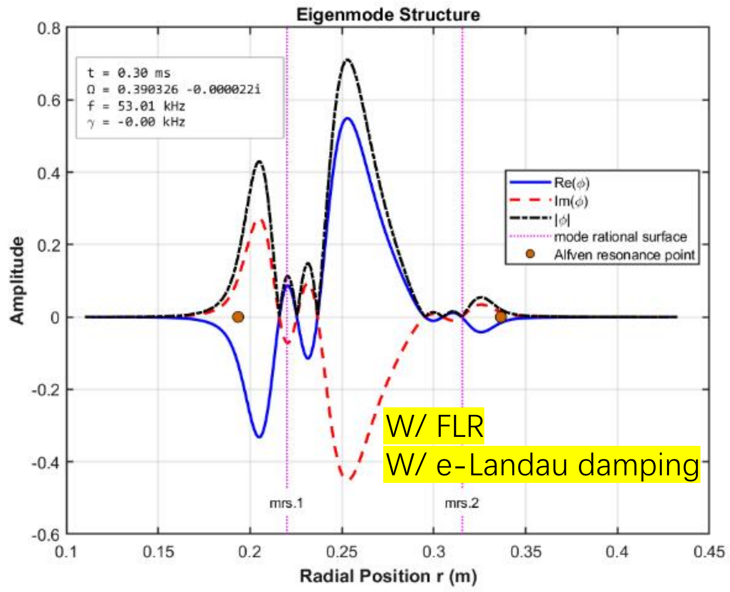
r_{res2} = 0.3340 m

Numerical Validation

'RSAE-ZF-KAW': $\rho_i^2 \nabla_\perp^2 \left(1 - \frac{\omega_{*pi}}{\omega_0}\right) \left[\frac{3}{4}(\omega_0^2 - \omega_{gam}^2) + \frac{\tau k_\parallel^2 v_A^2}{\left(1 - \frac{\omega_{*ne}}{\omega}\right)(1 + \xi_e Z(\xi_e))}\right] \nabla_\perp^2 \delta\phi_0 + \nabla_\perp \cdot (\epsilon_A \nabla_\perp \delta\phi_0) + \Lambda\delta\phi_0 + \nabla \cdot [(\nabla\alpha_{\phi 2})\delta\phi_0] = 0$ (17)







- FILR removes the Alfvén singularity.
- Electron-Landau damping captures radiative (convective) damping.