Drift wave soliton and zonal flow generation: implications for “stair-case” formation

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*In collaboration with **Ningfei Chen**, Liu Chen, and Fulvio Zonca

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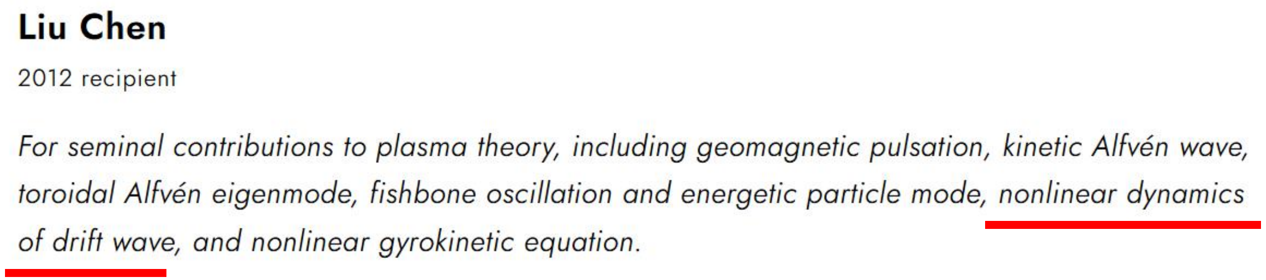
Outline

- ☐ **Zonal flow on drift wave regulation and spreading**
  - –spontaneous excitation v.s. beat-driven
  - –DW spreading as soliton
- ☐ Theoretical framework of DW-ZF interaction
- ☐ DW soliton due to beat-driven ZF
  - –threshold on DW soliton formation
  - –soliton trapped inside potential well: little effects on turbulence spreading
- ☐ DW soliton due to beat-driven and spontaneous excitation ZF
  - –SZF creates micro-barrier and potential-well simultaneously
  - –implications on “stair-case” formation?
- ☐Summary

Zonal flow on drift wave turbulence regulation

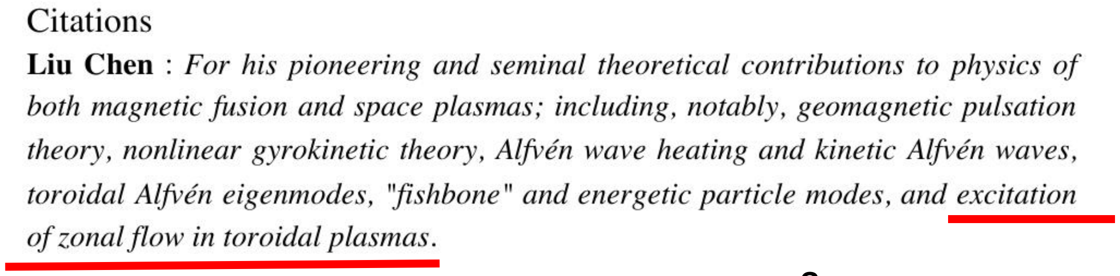
- **Drift waves** (DW) turbulence considered as one of the major candidates for inducing anomalous plasma transport [Chen PoF80, PoFB91ab]
- Zonal flows expected to regulate DW turbulence [Chen PoP2000, PoP2024]
    - –meso-scale radial corrugations with toroidally symmetric structures, linearly stable to ∇P
    - –nonlinearly driven by DWs via nonlinear wave-wave interactions [Lin Science1998]
    - –regulation achieved by scattering DWs into the linearly stable short wave-length regime
    - –two mechanisms for ZF excitation: spontaneous excitation [PoP00] v.s. beat-driven [PoP24]
- Zonal flow may also contribute to turbulence spreading
- A topic Prof. Liu Chen continuously contribute ~ 50 years



Liu Chen

2012 recipient

For seminal contributions to plasma theory, including geomagnetic pulsation, kinetic Alfvén wave, toroidal Alfvén eigenmode, fishbone oscillation and energetic particle mode, nonlinear dynamics of drift wave, and nonlinear gyrokinetic equation.

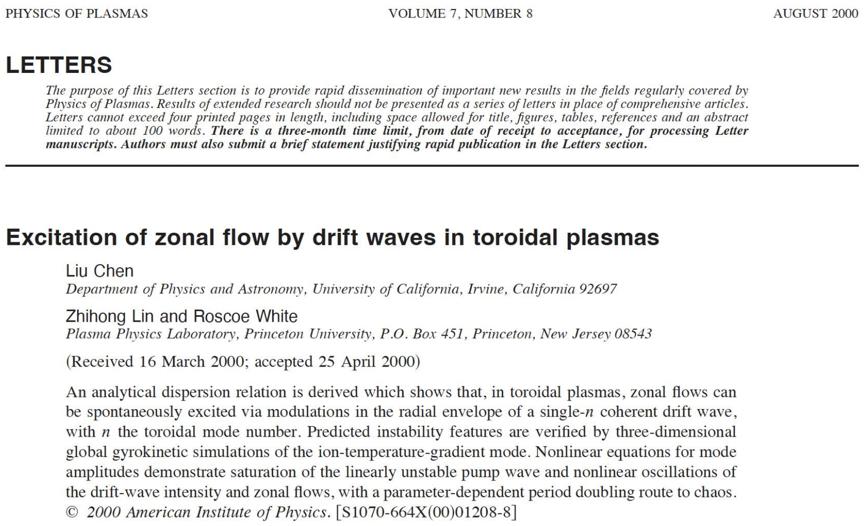


Citations

Liu Chen : *For his pioneering and seminal theoretical contributions to physics of both magnetic fusion and space plasmas; including, notably, geomagnetic pulsation theory, nonlinear gyrokinetic theory, Alfvén wave heating and kinetic Alfvén waves, toroidal Alfvén eigenmodes, "fishbone" and energetic particle modes, and excitation of zonal flow in toroidal plasmas.*

“Spontaneous excitation” of ZF by DW

- Spontaneous excitation via modulational instability [Chen PoP2000]
  - –pump DW decay into ZF and lower/upper sidebands
  - –ZF growth rate approximately proportional to pump wave amplitude
  - –threshold on pump DW amplitude to overcome frequency mismatch



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LETTERS

The purpose of this Letters section is to provide rapid dissemination of important new results in the fields regularly covered by Physics of Plasmas. Results of extended research should not be presented as a series of letters in place of comprehensive articles. Letters cannot exceed four printed pages in length, including space allowed for title, figures, tables, references and an abstract limited to about 100 words. ***There is a three-month time limit, from date of receipt to acceptance, for processing Letter manuscripts. Authors must also submit a brief statement justifying rapid publication in the Letters section.***

Excitation of zonal flow by drift waves in toroidal plasmas

Liu Chen
Department of Physics and Astronomy, University of California, Irvine, California 92697

Zhihong Lin and Roscoe White
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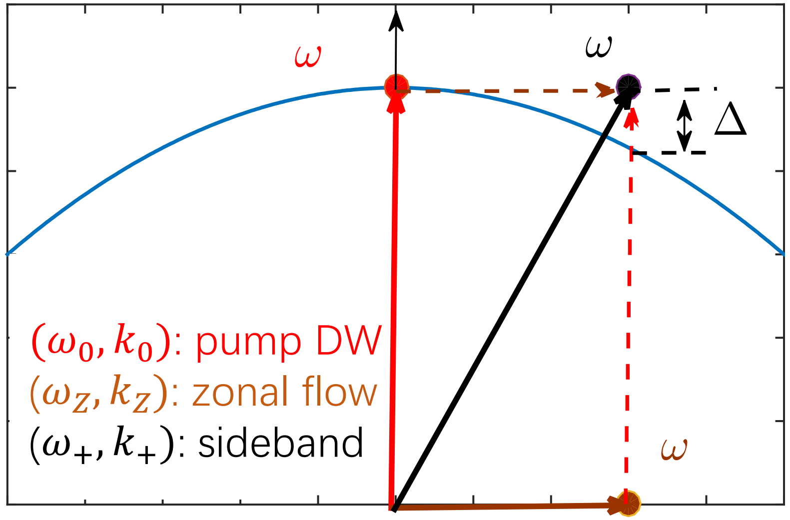
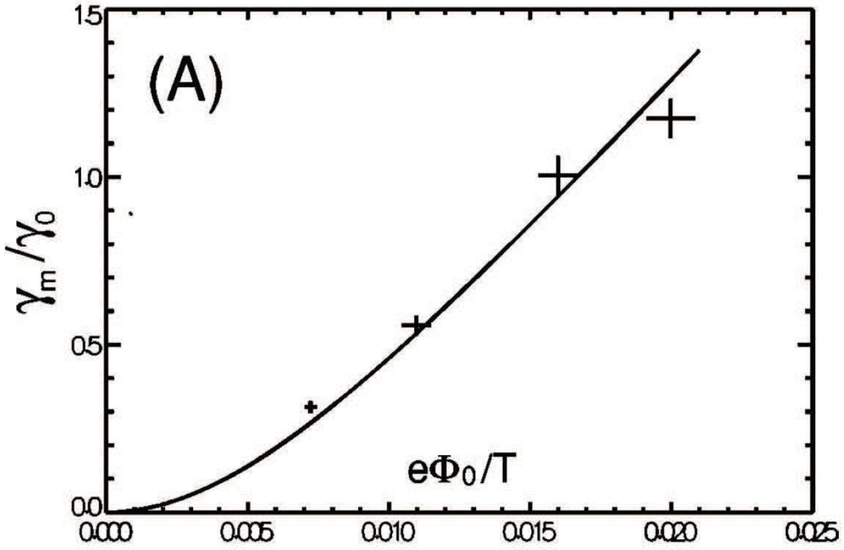
(Received 16 March 2000; accepted 25 April 2000)

An analytical dispersion relation is derived which shows that, in toroidal plasmas, zonal flows can be spontaneously excited via modulations in the radial envelope of a single-*n* coherent drift wave, with *n* the toroidal mode number. Predicted instability features are verified by three-dimensional global gyrokinetic simulations of the ion-temperature-gradient mode. Nonlinear equations for mode amplitudes demonstrate saturation of the linearly unstable pump wave and nonlinear oscillations of the drift-wave intensity and zonal flows, with a parameter-dependent period doubling route to chaos. © *2000 American Institute of Physics.* [S1070-664X(00)01208-8]

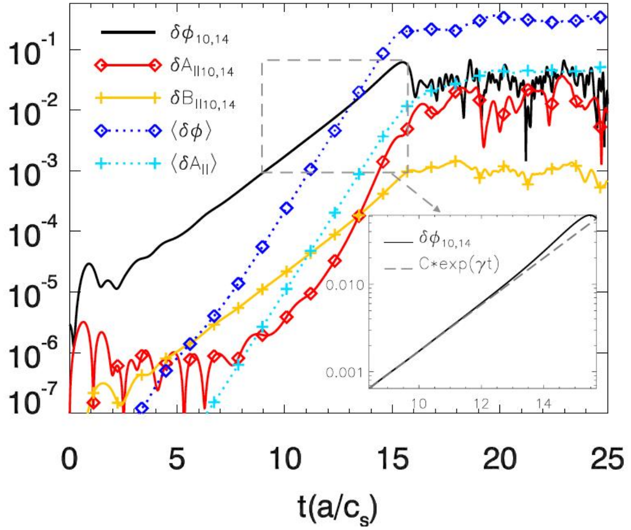




[Chen, Lin & White, Phys. Plasmas **7**, 3129 (2000)]

"Beat-driven" ZF by DW self-beating

- ☐ Beat-driven extensively observed in numerical simulations, e.g., [Dong PoP19]
  - –ZF growth rate roughly equals twice instantaneous DW growth rate
  - –threshold-less: ZF observed when DW amplitude is very small
  - –explained in TAE cases as resonant EP induced symmetry-breaking in ω [Qiu PoP2016]
- ☐ Theory with ZF spontaneous excitation and beat-driven on the same footing [Chen PoP2024]




Physics of Plasmas LETTER pubs.aip.org/aip/pop

On beat-driven and spontaneous excitations of zonal flows by drift waves

Cite as: Phys. Plasmas **31**, 040701 (2024); doi: 10.1063/5.0203053
Submitted: 9 February 2024 · Accepted: 18 March 2024 ·

Liu Chen,[1,2,3] Zhiyong Qiu,[1,3,a)] and Fulvio Zonca[1,3]

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ABSTRACT

Using the slab plasma as a paradigm model, we have derived analytically equations for the nonlinear generation of zero-frequency zonal flows by electron drift waves including, on the same footing, both the beat-driven and spontaneous excitations. It is found that the beat-driven zonal flow tends to reduce the frequency mismatch between the electron drift waves and, thereby, contributes to a significant $O(1)$ enhancement of the modulational instability drive and lowering its threshold. Implications to tokamaks plasmas as well as drift-wave soliton formation are also discussed.

$$\frac{\partial^2}{\partial x^2}\phi_{Zb} = -\frac{c}{B_0}k_y\frac{\omega_{*i}}{\omega_{dr}^2\rho_i^2}\frac{\partial}{\partial x}\left|\phi_d\right|^2$$

$$\frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial x^2}\phi_{Zs}\right) \simeq i\frac{c}{B_0}k_y\alpha_i\frac{\partial^2}{\partial x^2}\left(\phi_d\frac{\partial}{\partial x}\phi_d^* - \phi_d^*\frac{\partial}{\partial x}\phi_d\right)$$

$$\epsilon_d\phi_d = \frac{c}{B_0}\frac{k_y}{\omega_{dr}}\phi_d\frac{\partial}{\partial x}\left(\phi_{Zb} + \phi_{Zs}\right)$$

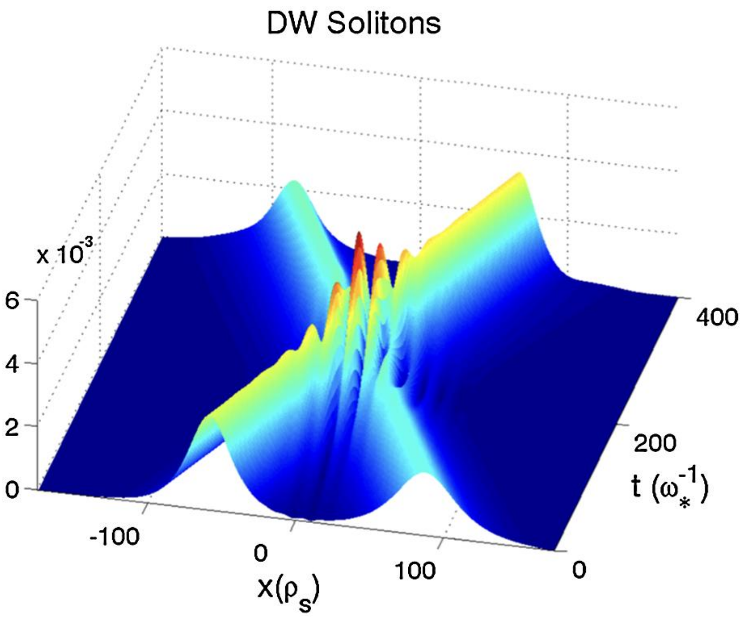
[Chen, Qiu and Zonca, Phys. Plasmas **31**, 040701 (2024)]

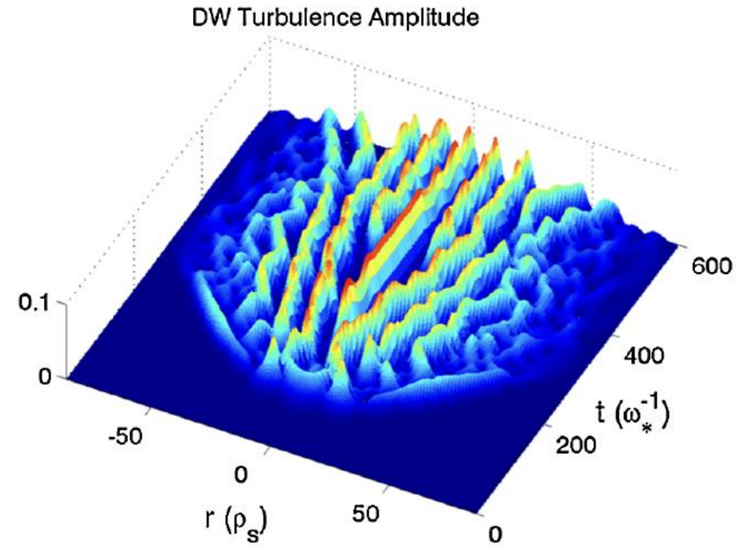
Zonal flow on turbulence spreading

- ☐ Turbulence spreading: redistribution of turbulence from the linearly unstable to stable region
- ☐ ZF as potential mediator of turbulence spreading [Zonca PoP2004, Guo PRL2009]
  - –Fully nonlinear two-field ZF-DW equations
  - –Dispersive-less DW propagation as solitons: enhanced turbulence spreading
  - –DW soliton generation as the nonlinear trapping and kinetic dispersiveness balance each other

$$(1 + k_y^2 - \partial_x^2)\partial_t \phi_d + i\frac{\omega_*(x)}{\omega_*(0)}\phi_d = -iC\phi_d\partial_x\phi_z,$$

$$\partial_t \phi_z = iC\langle \phi_d \partial_x \phi_d^* - \text{c.c.}\rangle;$$

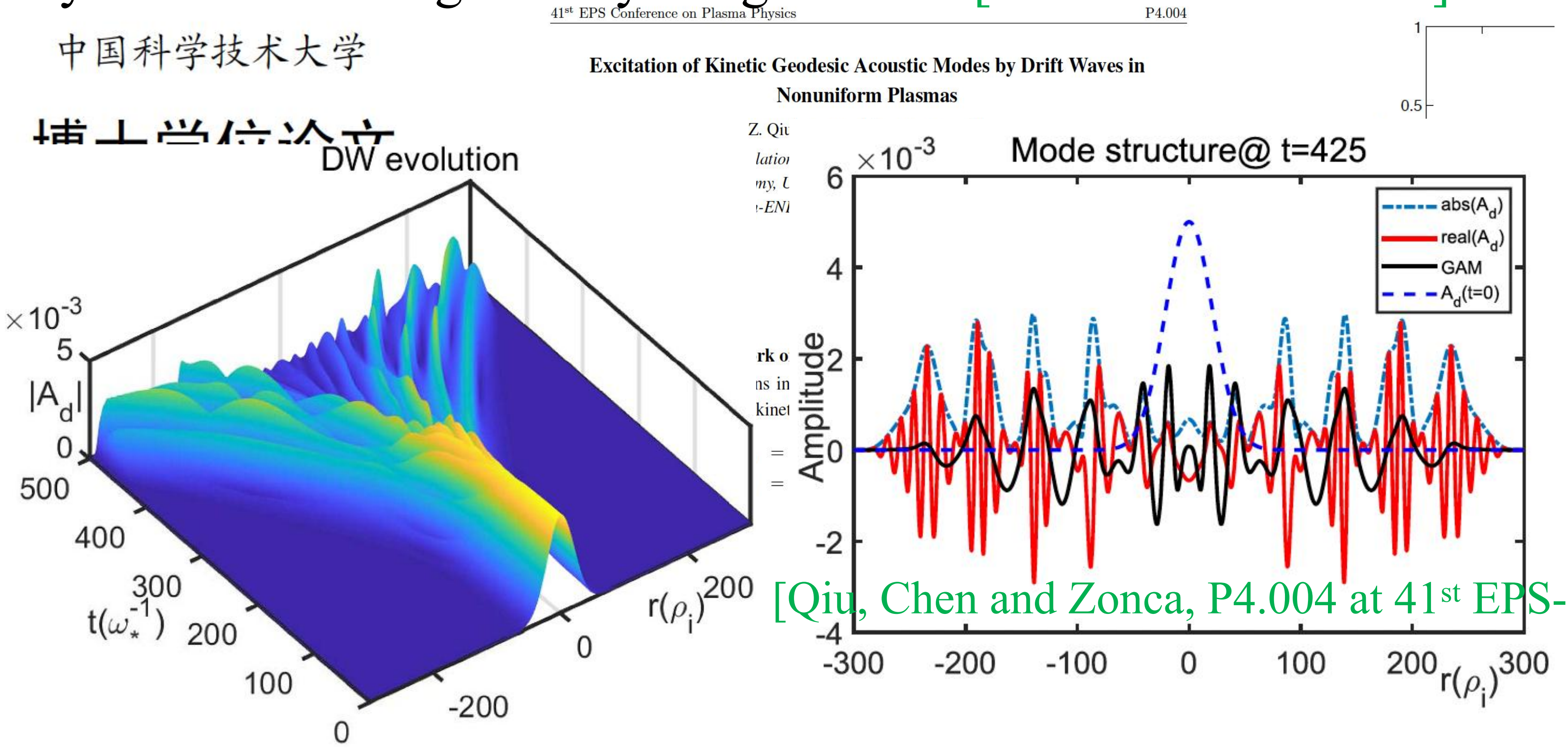




[Guo, Chen and Zonca, Phy. Rev. Lett. **103**, 055002 (2009)]

Extension to DW-GAM solitons

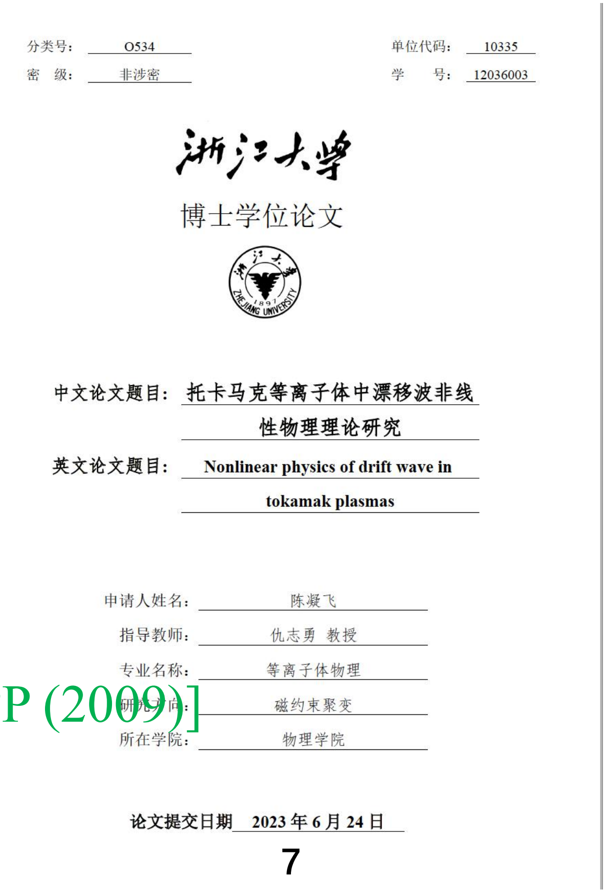
- ☐ Mar. 2007, discussion of my thesis plan during 2nd Westlake symposium: “Theories of geodesic acoustic mode in tokamak plasmas”
- ☐ Fully nonlinear DW-EGAM equations: hint of DW-GAM solitons formation [Qiu EPS 2014]
- ☐ Systematic investigation by Ningfei Chen [N. Chen PPCF2022]


中国科学技术大学

41st EPS Conference on Plasma Physics P4.004

Excitation of Kinetic Geodesic Acoustic Modes by Drift Waves in Nonuniform Plasmas

[Qiu, Chen and Zonca, P4.004 at 41st EPS-DPP (2009)]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浙江大学
博士学位论文

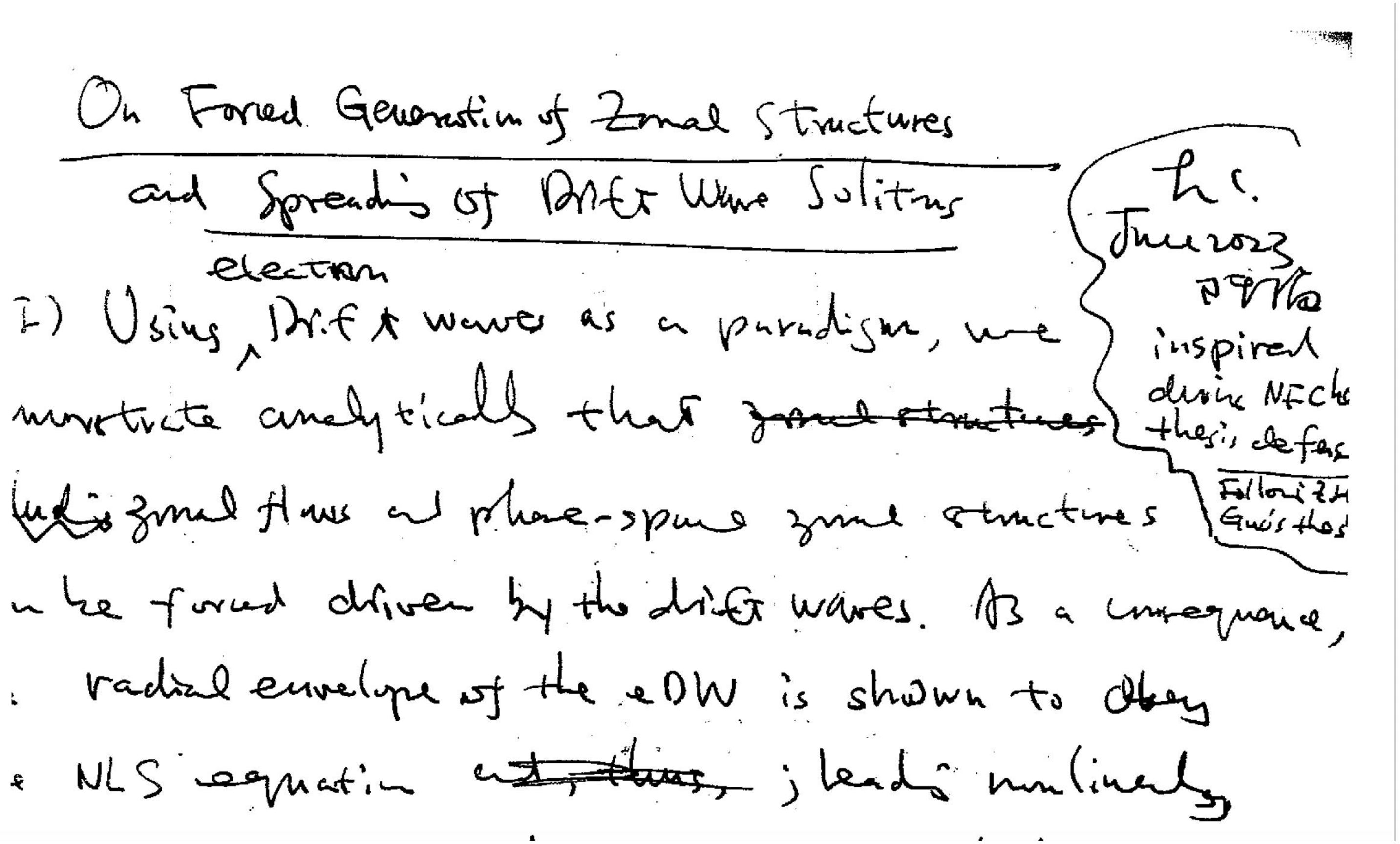
中文论文题目: 托卡马克等离子体中漂移波非线性物理理论研究
英文论文题目: Nonlinear physics of drift wave in tokamak plasmas

申请人姓名: 陈凝飞
指导教师: 仇志勇 教授
专业名称: 等离子体物理
研究方向: 磁约束聚变
所在学院: 物理学院

论文提交日期 2023 年 6 月 24 日

Ningfei Chen's project during the gap year

- ☐ How about beat-driven ZF on DW soliton generation? Initiative by Prof. Chen and investigated later in [N. Chen PoP2024]



On Forced Generation of Zonal Structures and Spreading of Drift Wave Solitons

I) Using electron Drift waves as a paradigm, we demonstrate analytically that ~~zonal structures~~ ~~including~~ zonal flows and phase-space zonal structures n be forced driven by the drift waves. As a consequence, radial envelope of the eDW is shown to obey NLS equation ~~and, thus,~~ ; leading nonlinearly

L.C.
June 2023
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Followi ZH
Guo's thes

- ☐ Beat-driven & spontaneous excitation on the same footing [N. Chen NF2025]
- ☐ Micro-barrier formation in the presence of multiple DWs: missed in [Guo PRL2009]

Theoretical framework of DW-ZF interactions

☐ Coupled ZF-DW equations with spontaneous-excitation and beat-driven ZF treated on the same footing [Chen, Qiu, Zonca, PoP2024]

$$\omega_n \mathscr{E}_{\mathrm{d}} \delta\phi_n = -\frac{c\tau}{B_0} k_{\theta n} \frac{\omega_{*\mathrm{i},n}}{\omega_n} \delta\phi_n \left(\partial_r \delta\phi_{\mathrm{Z,S}} + \partial_r \delta\phi_{\mathrm{Z,B}}\right), \tag{1}$$

$$\partial_t \chi_{\mathrm{iZ}} \delta\phi_{\mathrm{Z,S}} = -\frac{c}{B_0} \alpha_{\mathrm{i}} k_{\theta n} \rho_{\mathrm{ti}}^2 \partial_r \left(\delta\phi_n \partial_r^2 \delta\phi_n^* - \delta\phi_n^* \partial_r^2 \delta\phi_n\right), \tag{2}$$

$$\chi_{\mathrm{iZ}} \delta\phi_{\mathrm{Z,B}} = \frac{c}{B_0} k_{\theta n} \frac{\omega_{*\mathrm{i},n}}{\omega_n^2} \left|\partial_r \delta\phi_n\right|^2 . \tag{3}$$

–(1): modulation of DW $\delta\phi_n$ by ZF $(\delta\phi_{Z,S} + \delta\phi_{Z,B})$

–(2): finite ZF $\delta\phi_{Z,S}$ excitation due to DW radial envelope modulation

⇒ spontaneous excitation ⇒ $\gamma_Z \propto \delta\phi_n$

–(3): "passive"-excitation of $\delta\phi_{Z,B} \propto |\delta\phi_n|^2$: beat-driven with $\gamma_Z \sim 2\gamma_n$

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  - –threshold on DW soliton formation
  - –soliton trapped inside potential well: little effects on turbulence spreading
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- ☐Summary

First step: DW soliton due to beat-driven ZF

- Coupled ZF-DW equations with only beat-driven ZF $\delta\phi_{Z,B}$

$$\omega_n \mathscr{E}_{\mathrm{d}} \delta\phi_n = -\frac{c\tau}{B_0} k_{\theta n} \frac{\omega_{*\mathrm{i},n}}{\omega_n} \delta\phi_n \left(\partial_r \cancel{\delta\phi_{\mathrm{Z,S}}} + \partial_r \delta\phi_{\mathrm{Z,B}}\right),$$

$$\chi_{\mathrm{iZ}} \delta\phi_{\mathrm{Z,B}} = \frac{c}{B_0} k_{\theta n} \frac{\omega_{*\mathrm{i},n}}{\omega_n^2} \left|\partial_r \delta\phi_n\right|^2 .$$

- Equation for DW nonlinear evolution due to beat-driven ZF

$$\left(\partial_t - \mathrm{i}\tau\Omega(r) - \gamma_{\mathrm{L}} + \mathrm{i}\tau^2\partial_r^2 - \mathrm{i}\alpha|A|^2\right) A = 0$$

–Space and time normalized to ρ_{ti} and ω_{*i}; $A \equiv eA_n/T_i$

–Nonlinear Schrodinger equation with potential well ($\propto -|A|^2$) due to DW self-trapping through beat-driven ZF

–$\Omega(r) \equiv \omega_{*i}(r)/\omega_{*i}(0) - 1$: plasma nonuniformity induced by diamagnetic well

–$\Omega(r) = 0$: uniform; $\Omega(r) = \exp(-r^2/L_p^2) - 1$: nonuniform

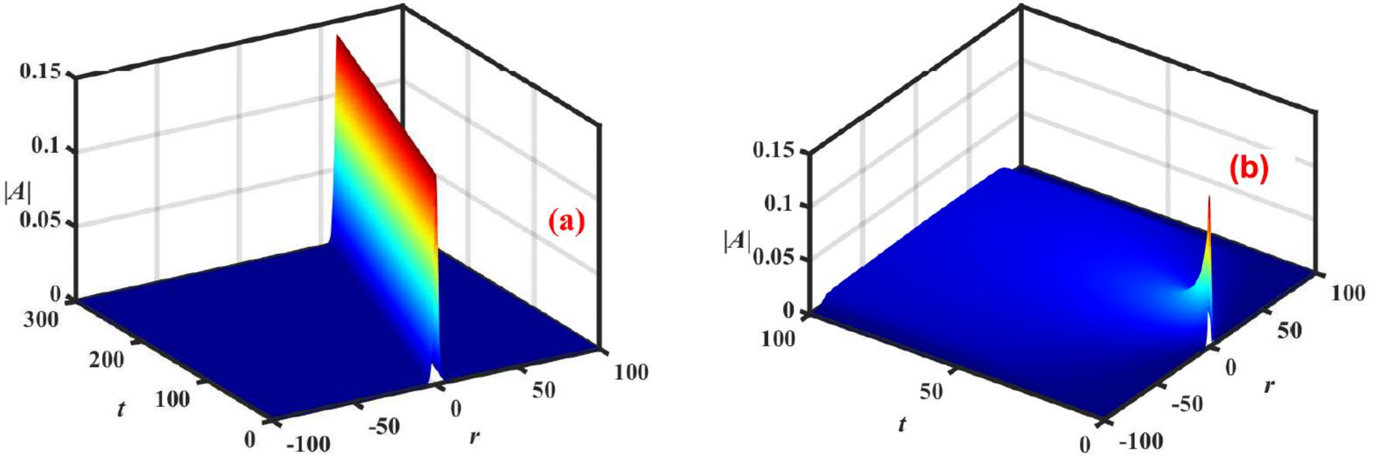
DW soliton generation in uniform plasmas

☐ One-soliton solution using travelling wave transformation: $A = \widehat{A}(T, \xi)\exp(-i\omega t + ik_r r)$; with $\xi = r - v_g t$, and $v_g = 2\tau^2 k_r$

$$\left(\tau^2\partial_\xi^2 - \tau^2 k_r^2 + \omega + \alpha\hat{A}^2\right)\hat{A} \; = \; 0$$

$\Rightarrow$ soliton solutions: $A = \sqrt{2/\alpha}\,\mathrm{sech}(\mathrm{r} - \mathrm{v_g t})\exp(-\mathrm{i}\omega t + ik_r r)$

☐ For DW amplitude above threshold, soliton form and propagate with its shape and amplitude unchanged



☐ Threshold for soliton generation $e\delta\phi/T_e \simeq 0.02$: well within relevant parameter range of fusion plasmas. Simulationwith ORB5 ongoing [N. Chen]

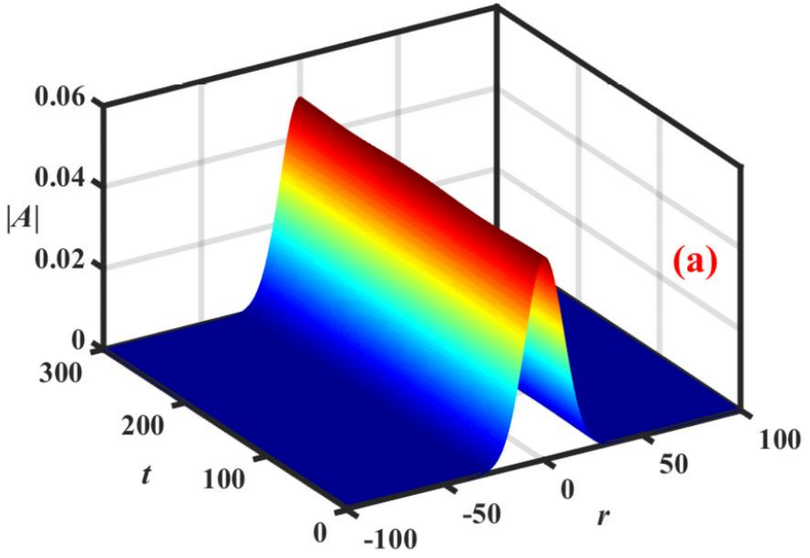
DW soliton generation and propagation in nonuniform plasmas

- ☐ Nonuniform plasma with $\Omega(r) = \exp\left(r^2/L_p^2\right) - 1 \Rightarrow$

$$\left(\tau^2\partial_r^2 + \omega + \tau\Omega(r) + \alpha|\hat{A}|^2\right)\hat{A} = 0$$

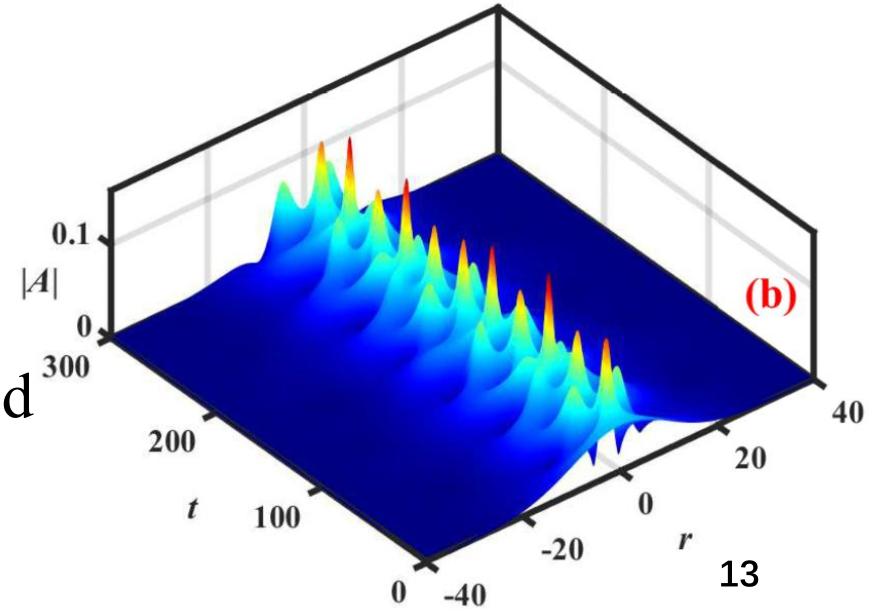
– Linear limit: expanding $\Omega(r)$ around $r = 0 \Rightarrow$ Weber equation $\Rightarrow$ eigen-function $A = A_0 \exp\left(-r^2/2L_d^2\right)H_l(r/L_d)$, with $L_d = \left(\tau L_p^2\right)^{1/4}$

–Nonlinear: soliton generation and sloshing in the nonlinear well



⇐ Linear eigenmode remain stationary

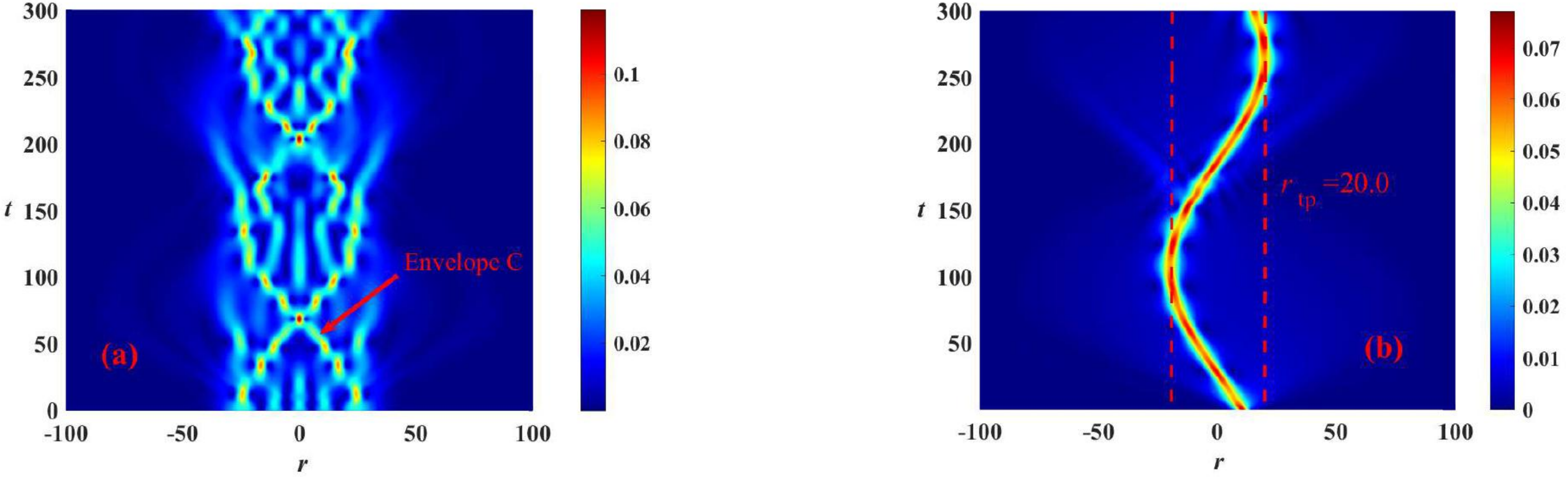
⇒ Nonlinear: soliton formation and trapped by nonlinear well



Soliton propagation in nonuniform plasmas

☐ Isolation of soliton: a filter $\exp\left(-(r-r_0)^2/L_f^4\right)$ applied to radial mode structure

☐ Envelope “C” is isolated and evolve: bounce back and forth at the turning points $r = r_\pm$



☐ Radial extent for DW solitons propagation is not sensitive to either DW strength or existence of nonlinearity, with reasonable DW amplitude

☐ Contribute little to turbulence spreading?

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- ☐ Summary

Beat-driven and spontaneous excitation ZF considered simultaneously

☐ Include spontaneous-excitation and beat-driven ZF by DW on the same footing

$$\left(\partial_t - i\tau\Omega(r) - i\tau C_{\mathrm{d}}\partial_r^2\right) A_n = \mathrm{i}\beta A_n \delta E_{\mathrm{Z,S}} + \mathrm{i}\frac{\sqrt{\epsilon}}{1.6q^2}\beta^2 |A_n|^2 A_n,$$

$$\partial_t \delta E_{\mathrm{Z,S}} = -\mathrm{i}\frac{\sqrt{\epsilon}}{1.6q^2}\beta\alpha_{\mathrm{i}}\partial_r\left(A_n\partial_r A_n^* - A_n^*\partial_r A_n\right),$$

–$\delta E_{Z,S}$: spontaneously excited ZF (SZF) electric field;

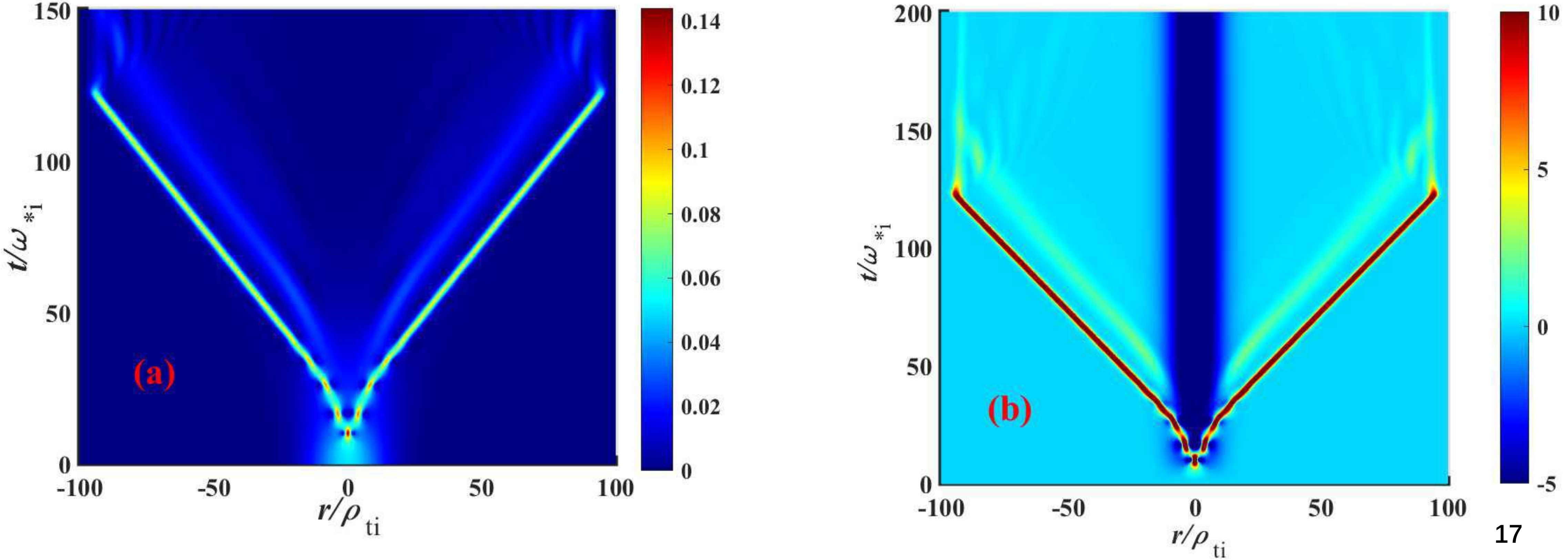
–SZF equation similar to [Guo PRL 103, 055002 (2009)], with the additional $\sqrt{\epsilon}/(1.6\,q^2)$ factor due to neoclassical inertia enhancement

–Nonlinear potential: $-\beta\delta E_{Z,S} - \sqrt{\epsilon}/(1.6q^2)\beta^2|A_n|^2$

– BZF is always a potential well, while SZF can be potential well/barrier depending on the sign of $\delta E_{Z,S}$

Uniform: soliton evolution and micro-barrier formation

- With uniform $\Omega(r)$, and single initial DW pulse, besides soliton formation, formation of micro-barrier at the origin can also be observed;
- Micro-barrier is not found in previous study [Guo PRL2009]: micro-barrier doesn't affect soliton propagation in uniform plasmas with single DW envelope



Simultaneous excitation of soliton and micro-barrier

$$\partial_t \chi_{\mathrm{iZ}} \delta\phi_{\mathrm{Z,S}} = -\frac{c}{B_0} \alpha_{\mathrm{i}} k_{\theta n} \rho_{\mathrm{ti}}^2 \partial_r \left(\delta\phi_n \partial_r^2 \delta\phi_n^* - \delta\phi_n^* \partial_r^2 \delta\phi_n \right)$$

☐ Simultaneous generation of potential-well and potential-barrier:

–Integrating the SZF equation over the radial domain $\Rightarrow \partial_t \int_{-\infty}^{+\infty} \delta E_{Z,S} dr = 0$

$\Rightarrow \; W \equiv \int_{-\infty}^{+\infty} \delta E_{Z,S} dr$ is a conserved quantity

–SZF grows from noise-level: $W = 0$

–Simultaneous generation of potential well ($\delta E_{Z,S} > 0$) and potential barrier ($\delta E_{Z,S} < 0$): **co-existence** of solitons and micro transport-barriers

☐ Determined by structure of Reynolds stress and zero-frequency nature of ZF

–not sensitive to DW types, i.e., ITG, CTEM

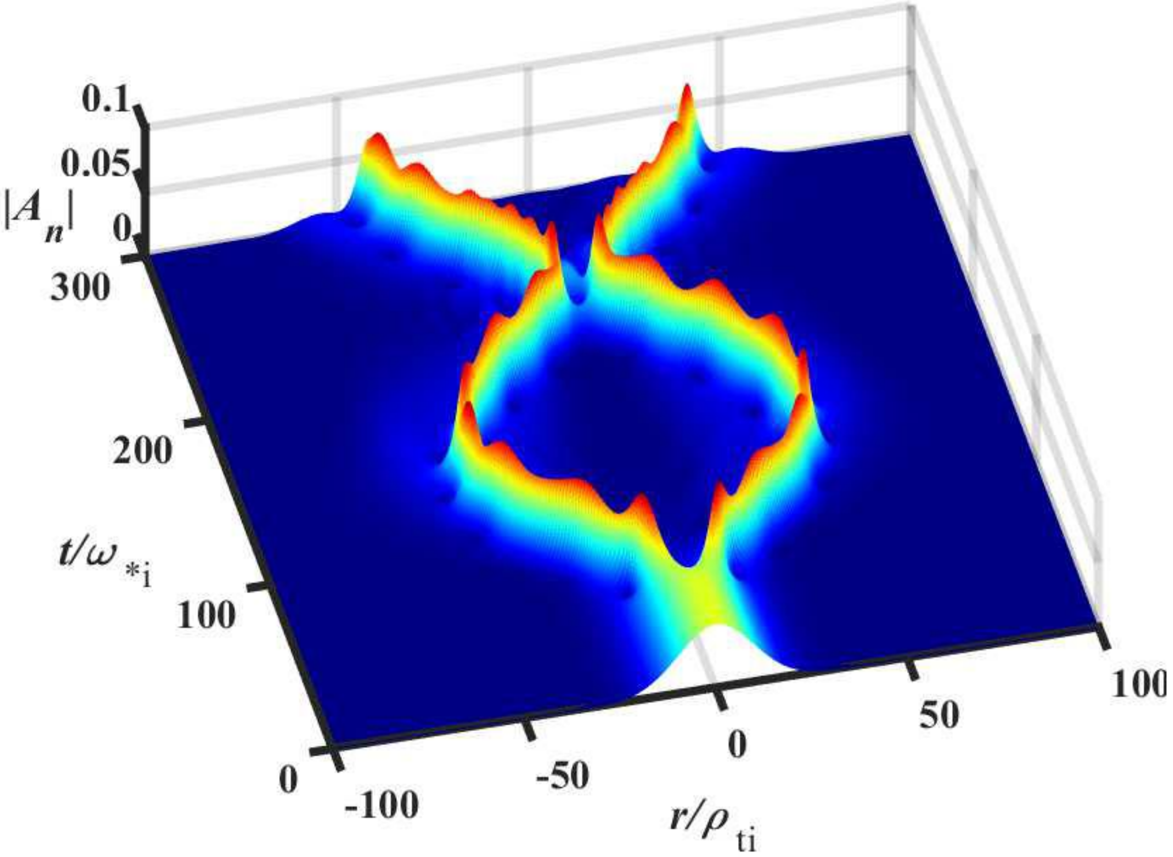
Nonuniform: single soliton confined within diamagnetic well

- ☐ Initial condition: DW as the lowest order eigenmode with diamagnetic well;
- ☐ DW solitons are reflected by micro-barrier around the origin and diamagnetic well induced turning points

  −outward propagation of solitons is reflected by diamagnetic well;

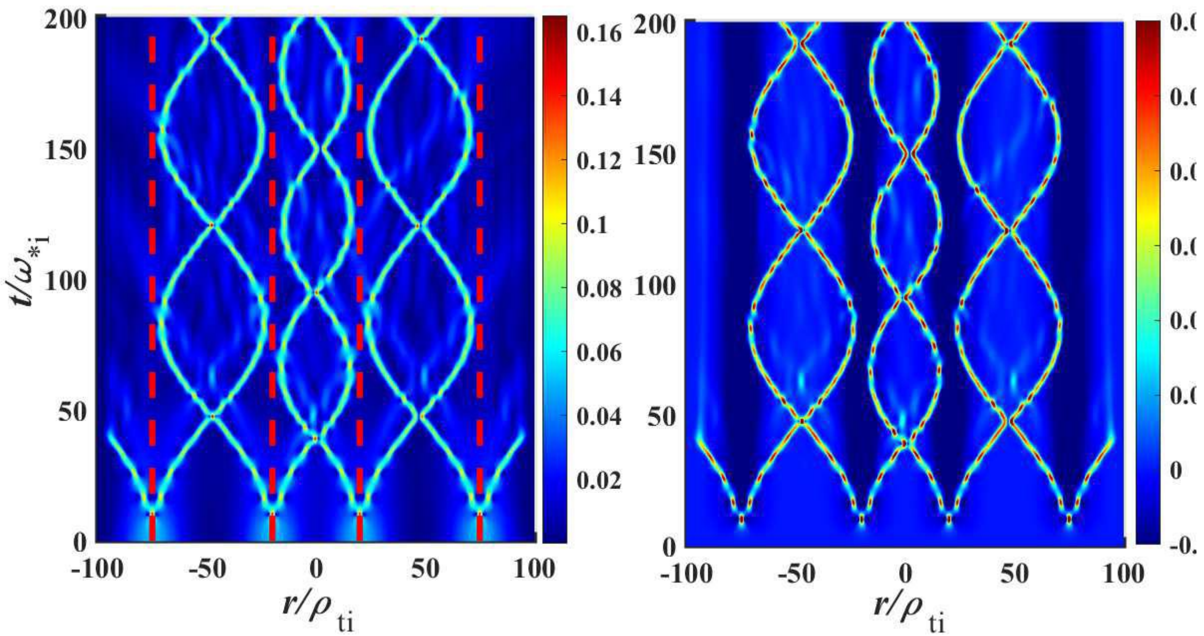
  −inward propagation is stopped by micro-barrier located near origin;

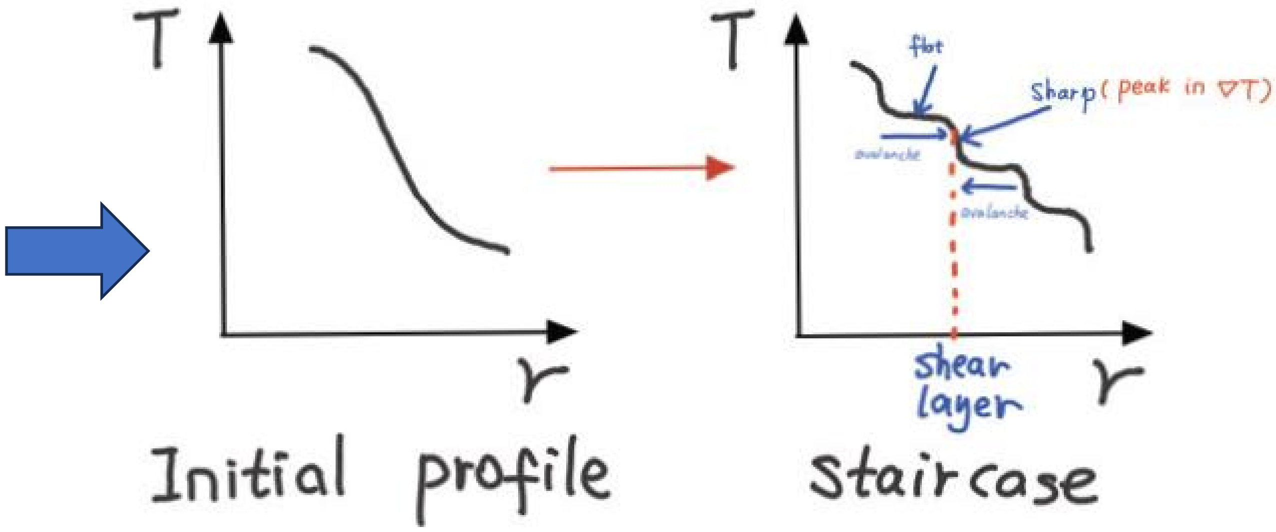
  ⇒ in nonuniform case, single DW envelope can itself manifest the effect of micro-barrier induced by SZF



Multi-DWs: staircase-like structure formation

- ☐ Multiple DW envelopes with random initial phase
  - –solitons confined within neighboring micro-barriers
  - –micro-barriers are generated by SZF; since potential given by BZF is always a potential well
  - –potential first-principle explanation of $E \times B$ staircase pattern formation





Summary

- □ Continuous contributions of Prof. Chen on DW dynamics: linear instability, zonal flow generation, and turbulence spreading. Selected publications:
  - drift wave stability [PoF80, PoFB91ab,… NF25]
  - zonal flow generation [PoP00, NF07, PRL12, PRL22, PoP24ab…]
  - turbulence spreading [PRL04, PoP04, PRL09, PoP14,PoP15, PoP24, NF25…]

- □ Wish Prof. Chen good health, and enduring academic excellence!

身体健康，学术长青！