# A Scheme for Reconstructing Diagnostic Data Based on Compressed Sensing Theory[1]

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[1] a tribute to Prof. Liu Chen.

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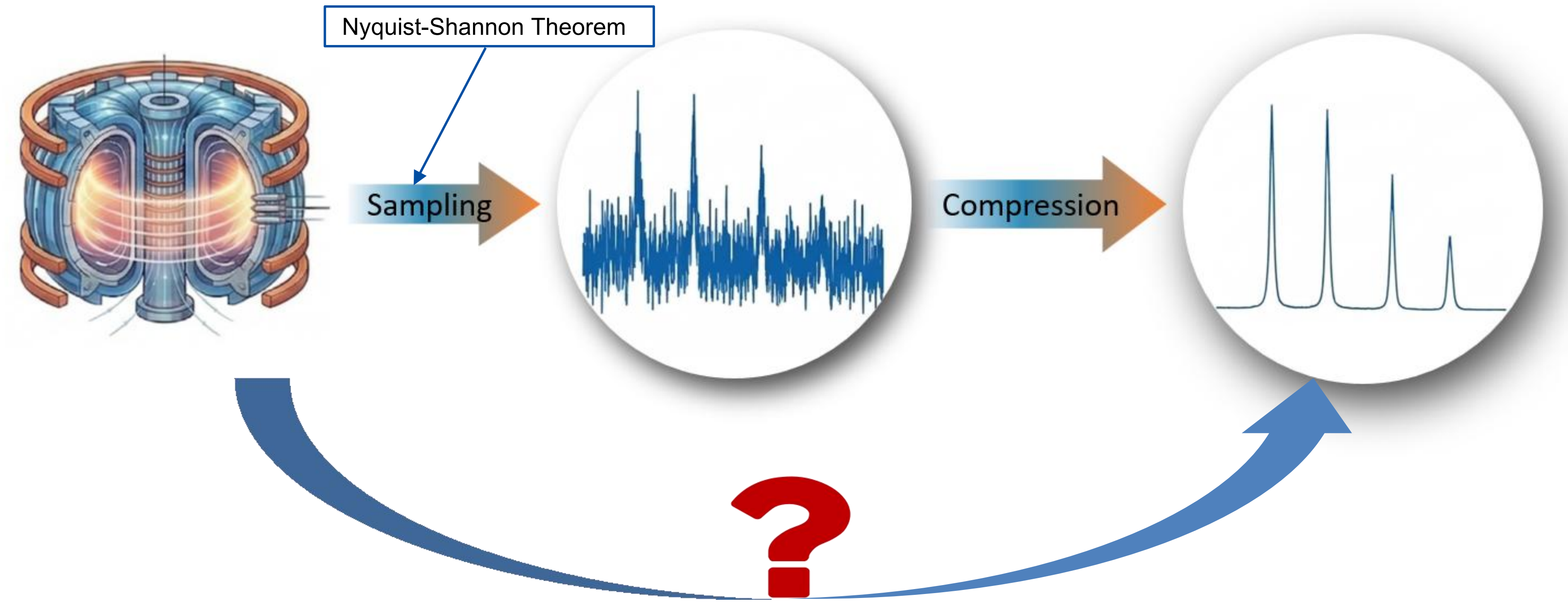
# 01 Background

# Background

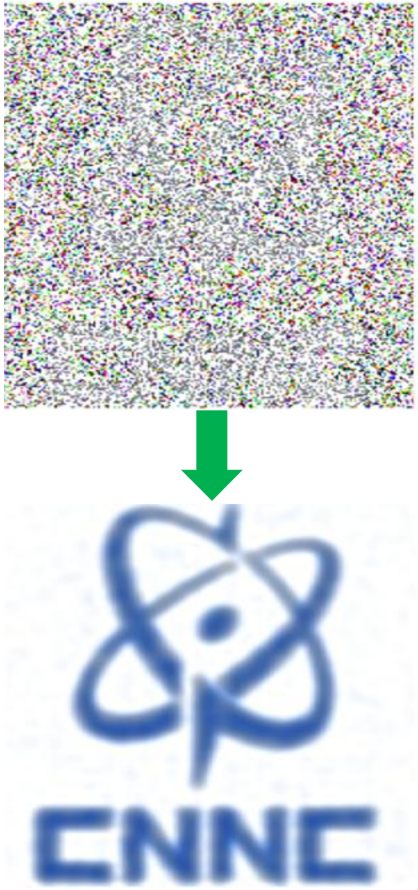
## Signal processing




**Could we combine the two steps into one?**

# Background › Compressed sensing theory

Compressed Sensing is a signal processing theory that breaks the Nyquist-Shannon theorem by leveraging sparsity to enable perfect reconstruction of the original signal [Donoho, IEEE Trans. IT, 2006; Candes, Romberg & Tao, IEEE Trans. IT, 2006]. It is interpretable.




**Sparsity:**

- Signal is sparse in a transform domain;
- Only a few coefficients are non-zero.

**Incoherent measurements:**

- Measurement matrix is incoherent with the sparse basis;
- Each measurement captures global information;
- Example: random Gaussian/Fourier matrix.

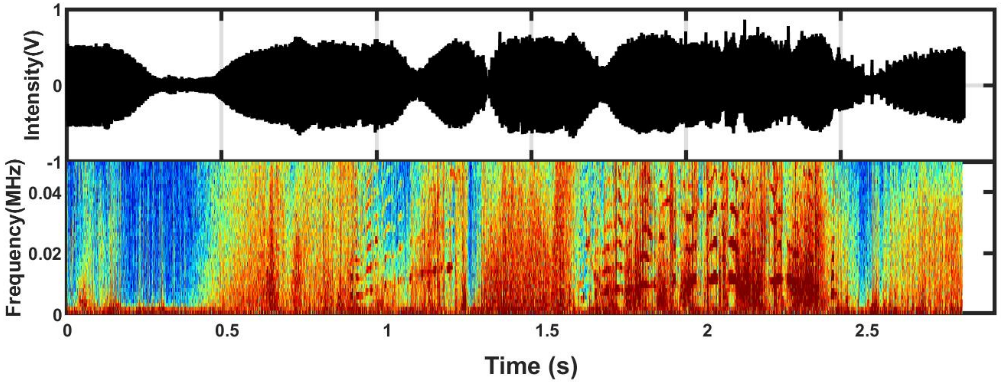
**Optimization-based reconstruction:**

- Sparest solution is recovered from an optimization problem;
- $L_0$-norm minimization: nonconvex optimization problem, NP-hard;
- $L_1$-norm minimization: convex optimization problem, solvable, but high per-iteration cost and numerous iterations emerge for high-dimensional signals.

# 02 Method

**Plasma signals are heterogeneous.**

- Localized gradients
- Smooth background
- Non-differentiable perturbations




Single transform basis is insufficient. Different signal structures demand distinct representation domains.

- To address this, we propose an **adaptive, data-driven** multi-domain sparse representation:

$$x = \sum_{k=1}^{K} D_k \alpha_k$$

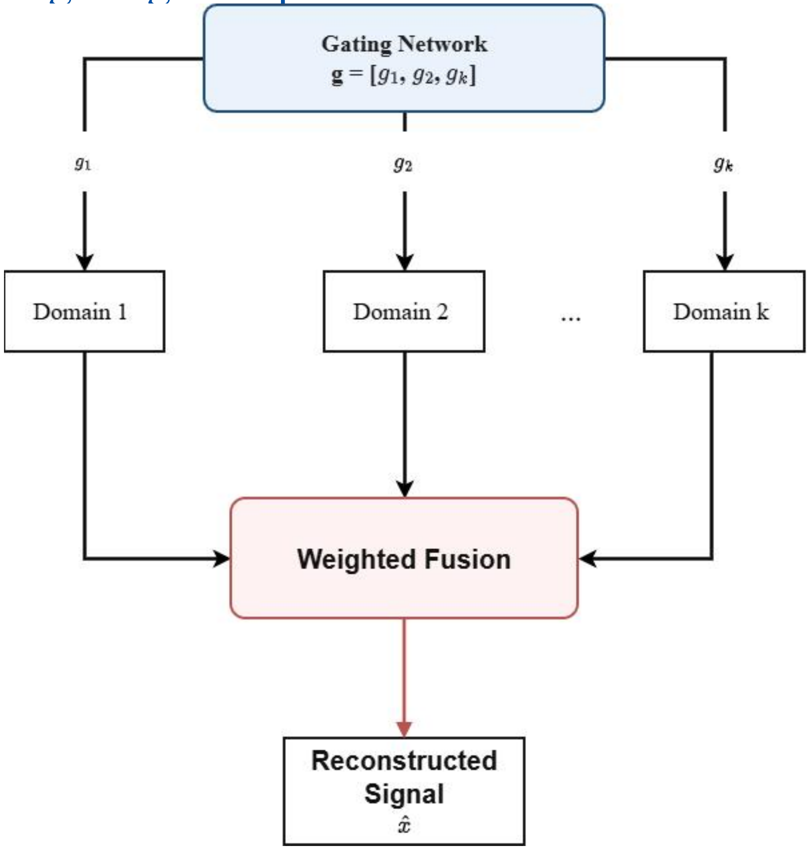
# Gated multi-domain sparse representation

Different domains contribute differently → adaptive weighting is required

Adaptive, data driven weighted combination of multiple domains [Wu, et al, ICLR, 2020]:

$$x = \sum_{k=1}^{K} g_k D_k \alpha_k$$

- Different domains capture different structures
- Dynamic combination
- Gating network



## Method — L1-norm minimization

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Classical compressed sensing:

- Model-driven algorithm
- Convex optimization problem

$$\min_{\alpha} \|y - A\Psi\alpha\|_2^2 + \lambda\|\alpha\|_1$$

sparsity

- Solved by iterative algorithms

$$x^{k+1} = S_{\lambda}(x^k - \eta A^T(Ax^k - y))$$

fidelity

- Numerous iterations
- High computational cost
- Slow convergence

map iterations to layers

Deep Unfolding

Layer 1

Layer 2

Layer 3

…

Layer k

We introduce the deep unfolding algorithm [Gregor & LeCun, ICML, 2010] that bridges model-based and data-driven methods.

# Method — Deep unfolding reconstruction

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- The optimization problem is formulated as:

$$\min_{\alpha} \frac{1}{2}||y - A\Psi_{eff}\alpha||_2^2 + \lambda||\alpha||_1$$

- y: observed signal
- A: measurement matrix
- $\Psi_{eff}$: adaptive multi-domain dictionary
- $\alpha$: sparse coefficients

- This problem is typically solved using iterative shrinkage algorithms.

$$\alpha^{(t+1)} = S_\lambda(\alpha^{(t)} + \eta\Psi_{eff}^T A^T(y - A\Psi_{eff}\alpha^{(t)}))$$

- Gradient step: reduces data fidelity error
- Shrinkage operator $S_\lambda$: enforces sparsity

Each iteration = Linear transform + Nonlinear shrinkage

# Deep unfolding reconstruction

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Classical iteration:

$$\alpha^{(t+1)} = S_{\lambda}((I - \eta\Psi^T A^T A\Psi)\alpha^{(t)} + \eta\Psi^T A^T y)$$

- Repeated matrix multiplications
- Slow convergence
- Fixed operator

- Deep unfolding:

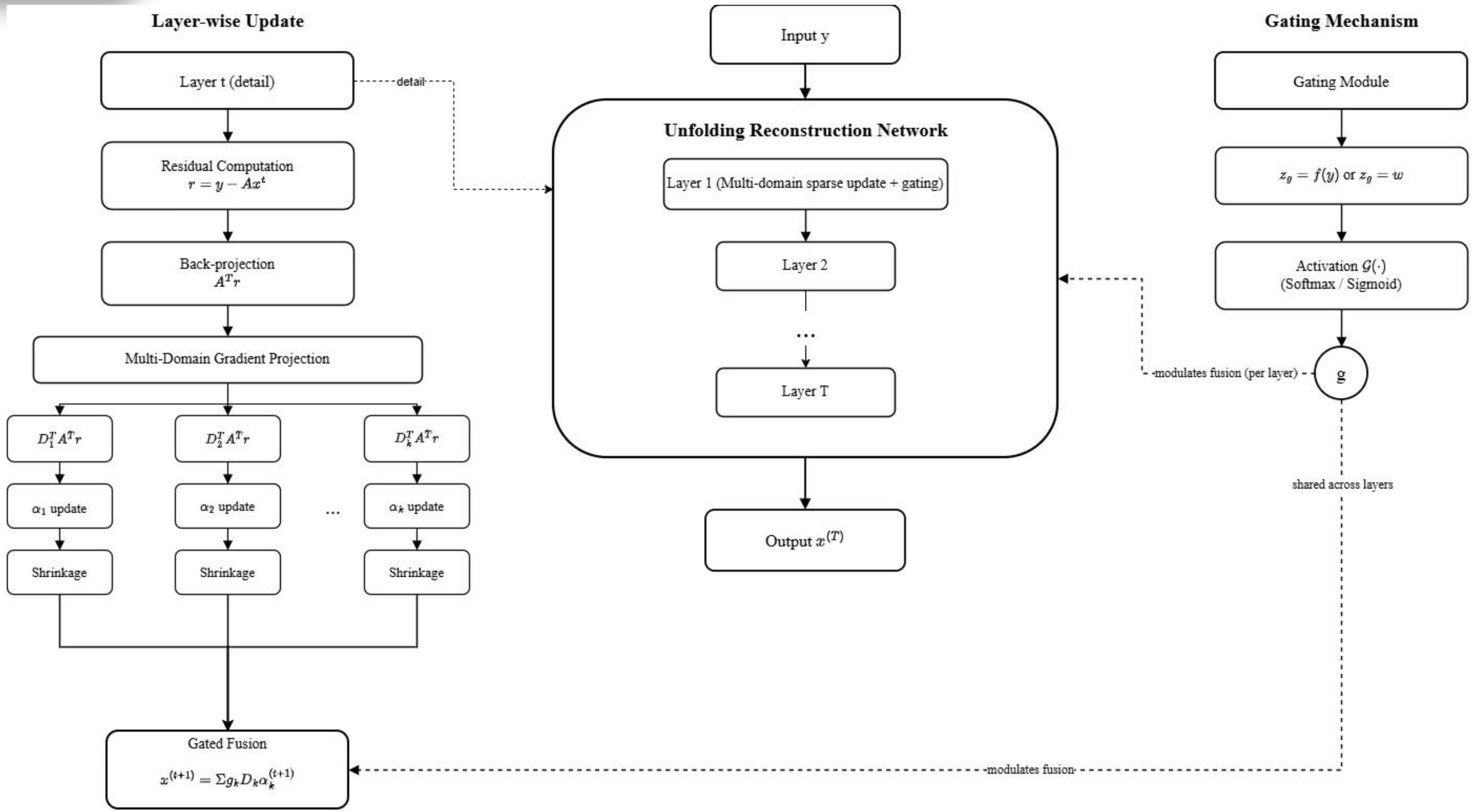
$$\alpha^{(t+1)} = S_{\theta(t)}(W^{(t)}\alpha^{(t)} + b^{(t)})$$

- No explicit matrix construction
- Fixed-depth feed-forward
- Learnable operators

➢ Replace iterative matrix operations with learnable operators
➢ Efficient inference while preserving optimization structure

**interpretability of compressed sensing theory + computational efficiency of deep unfolding**

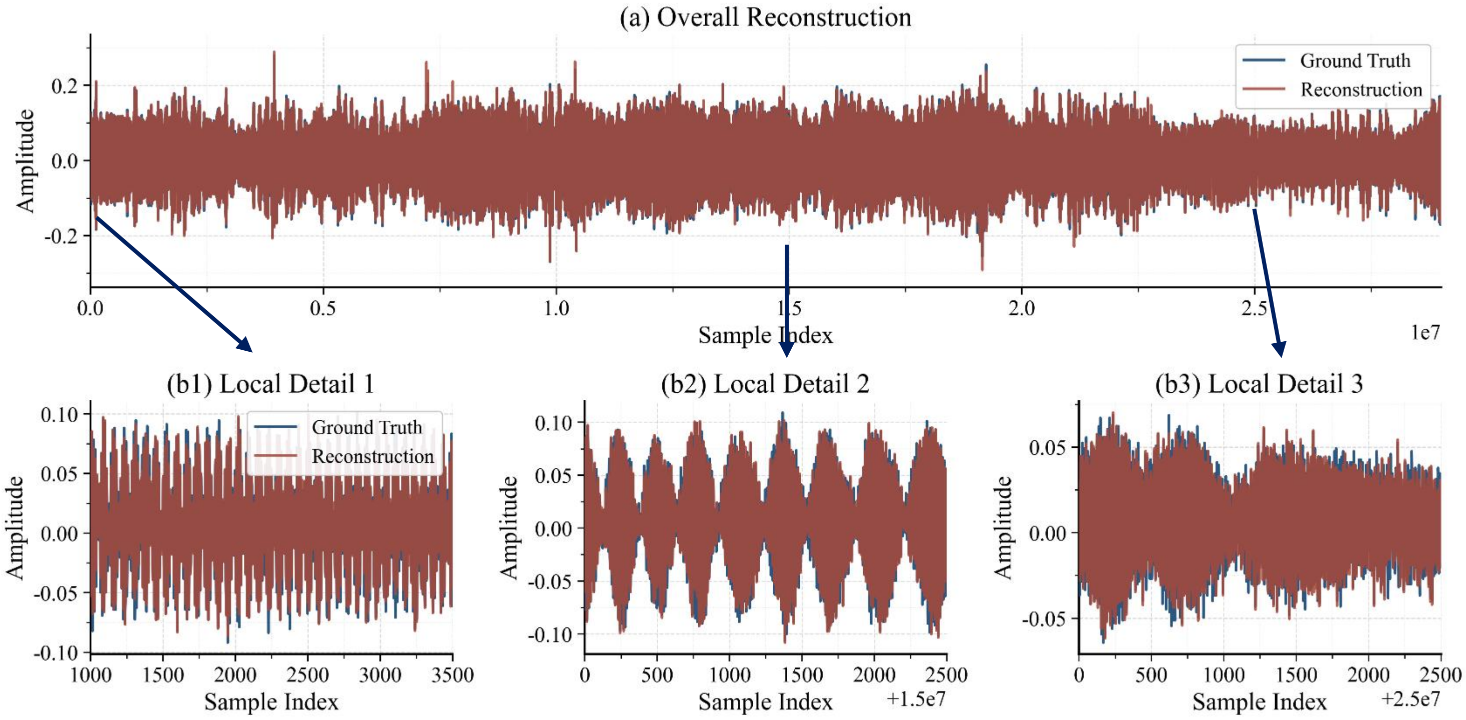
# Network architecture




**Multi-domain representation + adaptive gating + deep unfolding**

# 03 Results

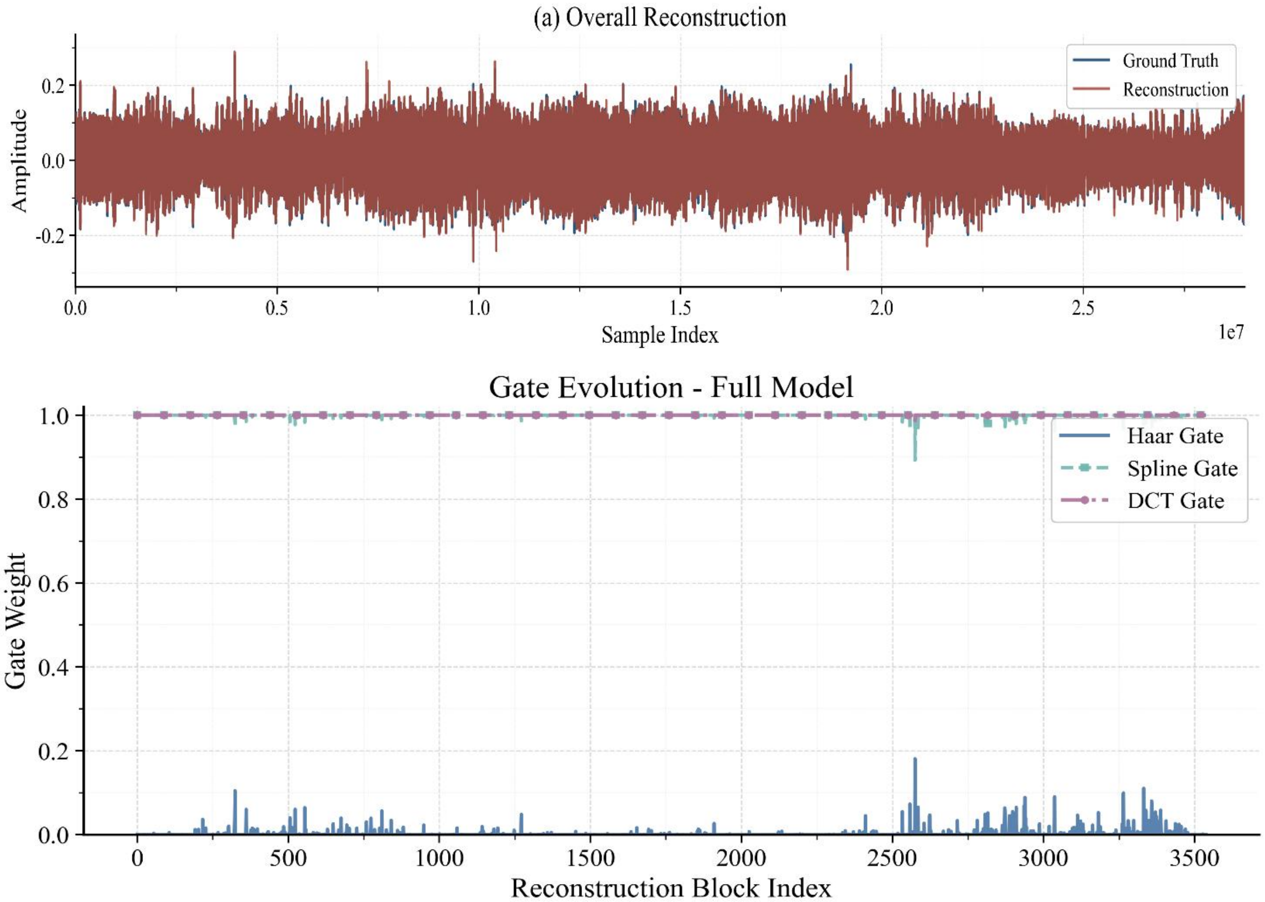
# Results — Coherent scattering data completion




- High-fidelity reconstruction under severe undersampling (40% completeness)
- Global consistency: Reconstruction are consistent with ground truth

# Results — Adaptive reconstruction and gate evolution

- Adaptive domain selection via gating mechanism
- Dominant contribution from smooth basis (DCT)
- Sparse activation of Haar/Spline for local structures
- Enables multi-scale representation of heterogeneous signals



# Results

## Comparison with Baseline Methods

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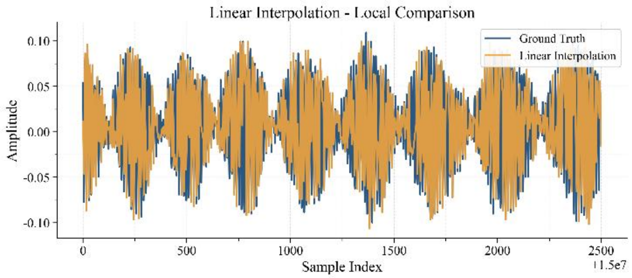
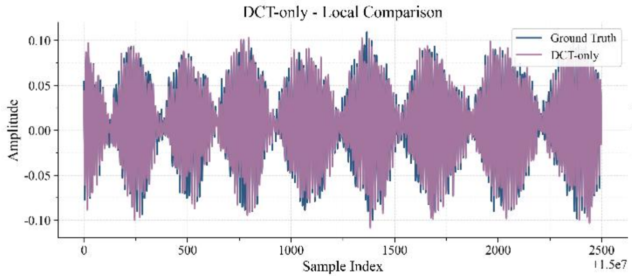
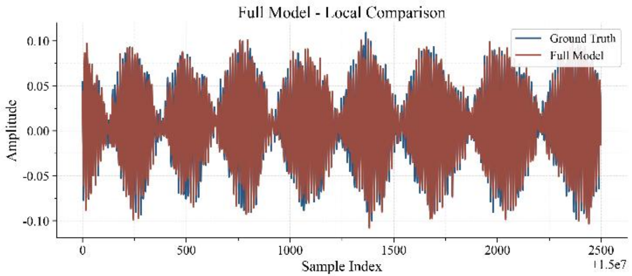



Linear Interpolation




DCT-Only




Full Model

Comparison for data with only 40% completeness

**RMSE: Root Mean Square Error**
**NMSE: Normalized Mean Square Error**
**SSIM: Structural Similarity Index Measure**

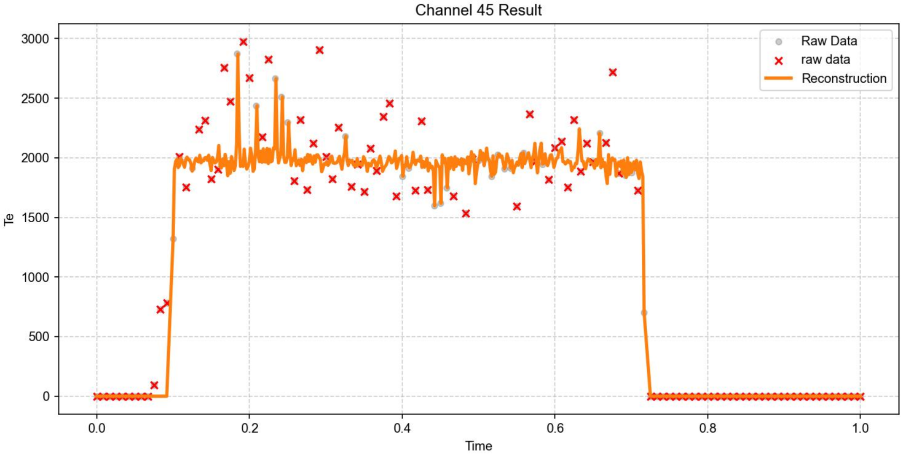
| Model | RMSE↓ | NMSE↓ | SSIM↑ |
|---|---|---|---|
| Linear Interpolation | 0.022 | 0.285 | 69.7% |
| DCT-only (No Gate) | 0.011 | 0.066 | 92.8% |
| Full Model | 0.006 | 0.023 | 93.4% |

Compared with linear interpolation: RMSE reduced by ~73% SSIM improved by ~23%

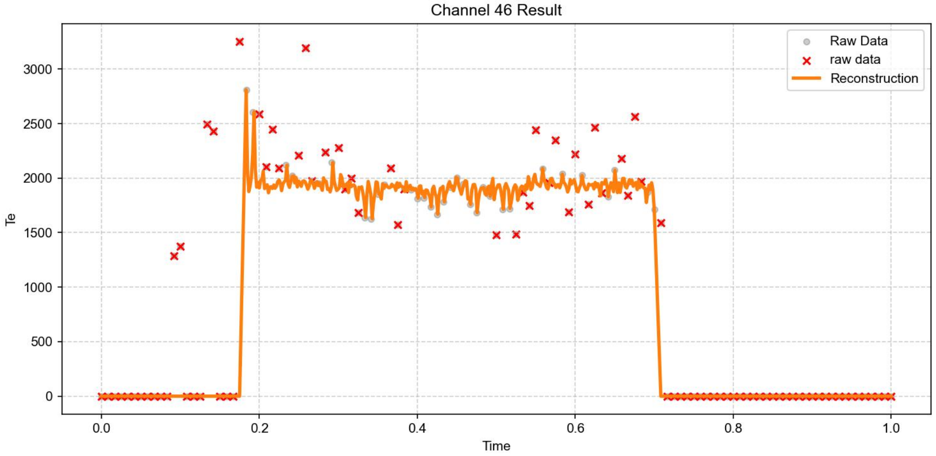
# Results

## Super-resolution Thomson scattering

Building on the signal reconstruction framework, we further apply it to enhance the temporal resolution of Thomson Scattering.




Channel 45



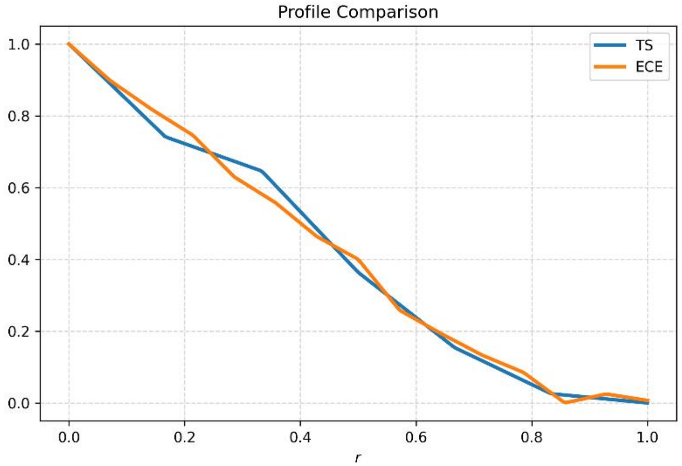

Channel 46

Temporal resolution enhanced from 121 to 633 sampling points, enabling reconstruction of fine dynamic features.

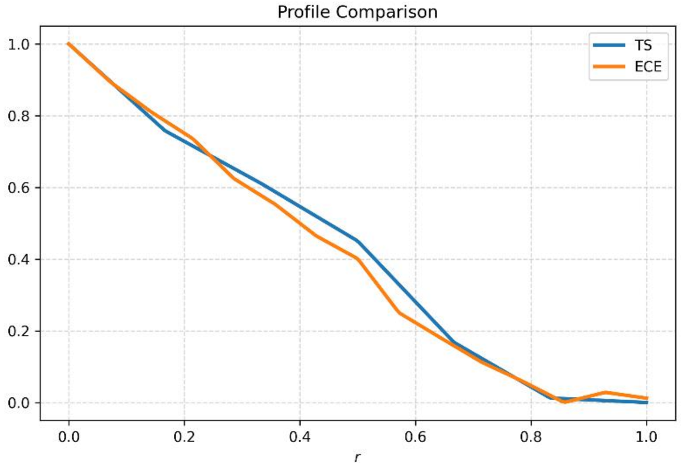
## Results

# Super-resolution Thomson scattering

To verify the validity of Thomson scattering temporal super-resolution, the reconstructed results were compared with ECE cross-sectional images.




1114ms




785ms

Reconstructed TS profiles show good agreement with ECE measurements, verifying the physical consistency.

# 04 Discussions

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We propose an effective data reconstruction framework based on the compressed sensing theory.

- For the first time, we integrate compressed sensing theory with a deep unfolding–based gated multi-domain sparse representation framework, establishing a structured and interpretable reconstruction paradigm.
- A joint dictionary is introduced to capture heterogeneous structures in plasma diagnostic signals, including smooth backgrounds and localized variations.
- The model effectively combines interpretability (model-driven) with flexibility (data-driven), achieving a balance between physical consistency and learning capability.
- It exhibits strong performance in reconstructing incomplete data under realistic acquisition constraints.

# Loss functions

- **Overall Loss:**

$$L = \|x_{rec} - x_{true}\|_2^2 + \lambda_1 L_{SSIM} + \lambda_2 \|z\|_1 + \lambda_3 \|g\|_1$$

- **Each Term Explanation**

$\|x_{rec} - x_{true}\|_2^2$ → reconstruction accuracy

$L_{SSIM}$ → structure preservation

$z$ sparsity → multi-domain prior

$g$ regularization → stabilize gating

**Evaluation Metrics**:

- RMSE:

$$RMSE = \sqrt{\frac{1}{N}\sum(x_{rec} - x_{true})^2}$$

sensitive to large deviations

- NMSE:

$$NMSE = \frac{\sum(x_{rec} - x_{true})^2}{\sum x_{true}^2}$$

removes scale dependency

- SSIM:

$$SSIM(x, y) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)}$$

evaluates structural similarity;
sensitive to local patterns and edges

**RMSE/NMSE → error， SSIM → structure**