NLT simulation of the Internal Transport Barrier Formation

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1. Introduction

- The anomalous transport in a tokamak fusion plasma is believed to be caused by the turbulence driven by drift-waves, such as the Ion Temperature Gradient (ITG) mode.
- When the turbulence is reduced/suppressed across a confinement transition in the core region, the Internal Transport Barrier (ITB) has been observed in the reversed shear configuration and the weak shear configuration (the hybrid scenario) for more than 40 years.

J. Strachan et al. Phys. Rev. Lett. **58**, 1004 (1987).
TFTR
Koide, Y. et al. Phys. Rev. Lett. **72**, 3662 (1994).
JT-60U
C. Rettig et al. Phys. Plasmas **5**, 1727 (1998).
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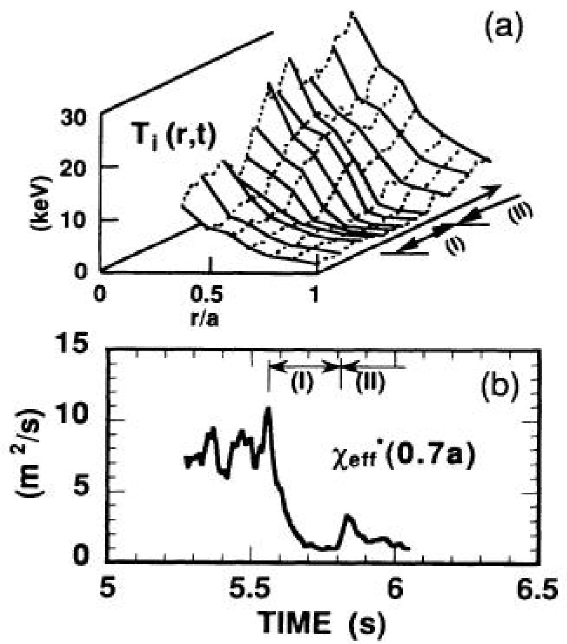
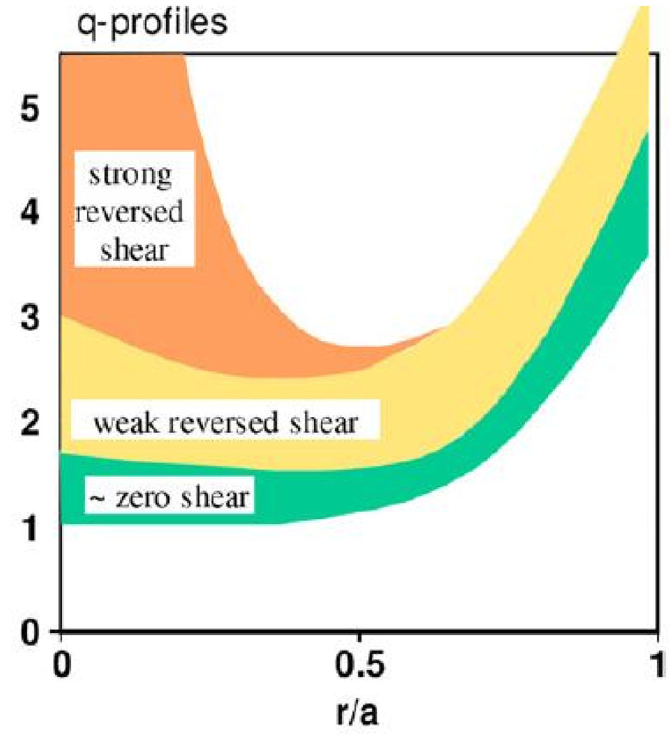


ITB, JT-60U

Why the ion ITB in the hybrid scenario?

- The hybrid scenario is the most promising one for the International Thermonuclear Experimental Reactor (ITER), since it is relatively easy to be realized and controlled.



E. Doyle et al, Nucl. Fusion **47**, S18 (2007).
C. Gormezano et al, Nucl. Fusion **47,** S285 (2007)

Nonlinear GK simulation of the ITB formation by the GYSELA code

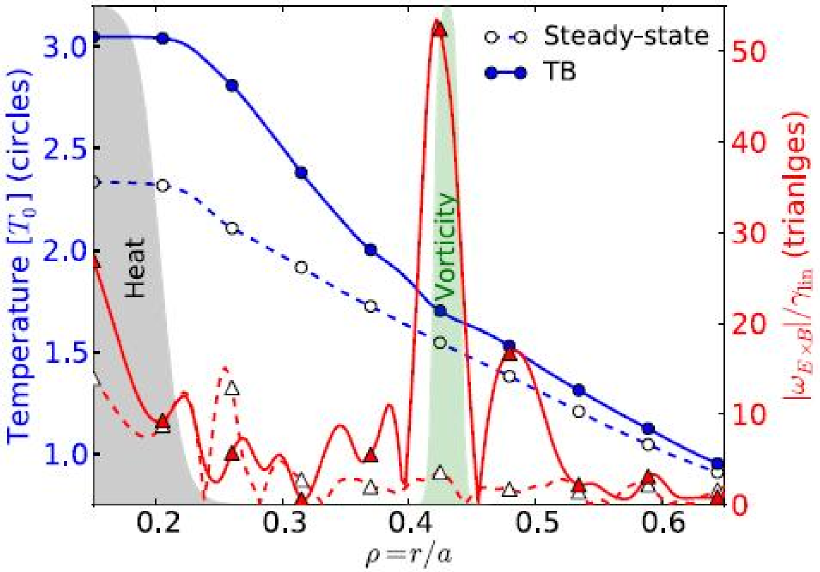


FIG. 1 (color online). Temperature (blue circles) and $E \times B$ shear (red triangles) during steady-state (open) and a TB phase (solid). The kinetic sources are labeled by the black and green areas.

Turbulence suppressed by the shearing of E_r generated by externally injected very localized vorticity (charge).

A. Strugarek et al, Phys. Rev. Lett. **111**, 145001 (2013).

Nonlinear GK simulation of the ITB formation by the GKNET code

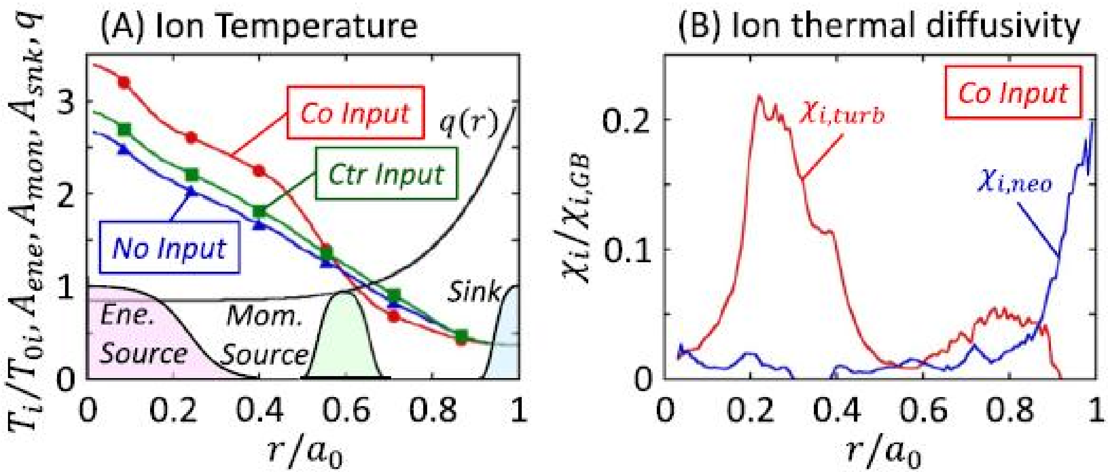


Figure 1. (A) Time-averaged radial ion temperature profile in the case without momentum input (blue, triangle), with co-input (red, circle) and counter-input (green, square) at $r = 0.6a_0$. Safety factor profile (black) and the deposition profiles of energy/momentum source and sink operators are also shown. (B) Time-averaged radial profiles of turbulent (red) and neoclassical (blue) ion thermal diffusivity in the case with co-input, normalized by the Gyro-Bohm one.

Turbulence suppressed by the shearing of E_r
generated by externally injected very localized toroidal momentum.

K. Imadera and Y. Kishimoto, Plasma Phys. Control. Fusion **65**, 024003 (2023).

2. 1 Neighboring Equilibrium Update in the long-time perturbative simulation

- The full-f equation

$$\partial_t f + \{H,\, f\} = S + C(f).$$

- The δf equation

$$H = H_0 + \delta H,\ f = f_0 + \delta f.$$

$$\{H_0,\, f_0\} = 0.$$ **Self-consistent equilibrium!**

$$\partial_t \delta f + \{H_0,\, \delta f\} = -\{\delta H, f_0\} - \{\delta H,\, \delta f\} + S + C.$$

It has higher precision, but involves the secularity problem when the perturbation becomes large in a long-time simulation.

Usually, f_0 is taken approximately as the local Maxwellian, although it should be a Constant Of Motion!

S. Wang, Phys. Rev. E **64**, 056404 (2001).
Y. Idomura et al, Nucl. Fusion 43, 234 (2003).

Neighboring Equilibrium Update (NEU)

- The system consists of **the field & the distribution function.**
- When the perturbation becomes large, the equilibrium is updated by repartitioning the system into an updated combination of the perturbation and **the self-consistent equil. (field & distribution)** .

$$H = \overline{H_0} + \overline{\delta H},\ f = \overline{f_0} + \overline{\delta f}.$$

$$\{\overline{H_0}, \overline{f_0}\} = 0.$$

- The updated δf equation is formally unchanged,

$$\partial_t \overline{\delta f} + \{\overline{H_0}, \overline{\delta f}\} = -\{\overline{\delta H}, \overline{f_0}\} - \{\overline{\delta H}, \overline{\delta f}\} + S + C.$$

S. Wang, Z. Wang, T. Wu, Phys. Rev. Lett. **132**, 065106 (2024).

Reconstruction of the equil.

- Define the ensemble average as the toroidal and the time average,

$$f_{en}(\mathbf{Z}, t_0) = \frac{1}{2\pi\,\tau_{en}} \int_{t_0-\tau_{en}}^{t_0} dt \oint d\alpha\, f(\mathbf{z}, t).$$

$$\mathbf{Z} = (\psi, \theta, v_{||}, \mu),\ \mathbf{z} = (\mathbf{Z},\ \alpha).$$

- **Updated equil. Hamiltonian,**

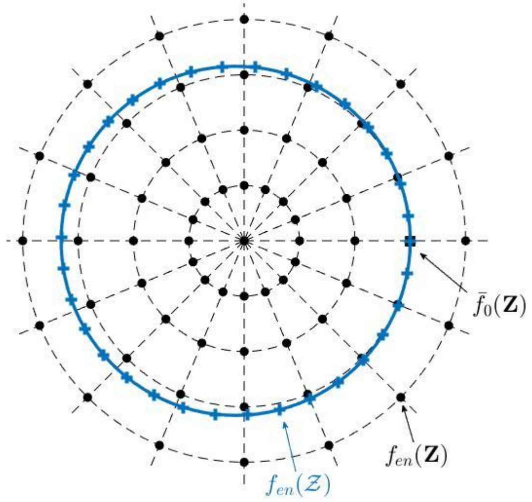
$$\overline{H_0}(\mathbf{Z}) = H_{en}(\mathbf{Z}).$$

- **Updated equil. Distribution,**
  COM in the updated equil. field,

$$\overline{f_0}(\mathbf{Z}) = \frac{1}{\tau_b} \oint d_{\overline{H_0}}\tau\, f_{en}[\mathcal{Z}(\tau; \mathbf{Z}, \tau_0)].$$

$\mathcal{Z}(\tau; \mathbf{Z}, \tau_0)$ is the phase point at τ, for a particle launched from **Z** at τ_0.

Average along the orbit given by the UPDATED equil. field!



Notes on NEU

- The concept of the nonlinear Neighboring Equilibrium was introduced to discuss the linear instability under **the nonlinearly evolved equilibrium (zonal E_r and the temperature gradient)**.

L. Chen and F. Zonca, Nucl. Fusion **47**, 886 (2007).
S. Sun et al, Nucl. Fusion **62**, 126005 (2022).

- Equil. distribution as a COM has been used in the long-time evolution theories on the averaged distribution or the Phase-Space-Zonal-State, and in the neoclassical theory.

$$\overline{f_0}(P, W, \mu) = \frac{1}{\tau_b}\oint d_{H_0}\theta \frac{1}{\dot{\theta}} f_{en}(P,\ W, \mu, \theta).$$ **Previous.** COM **in the old field.**

$$\overline{f_0}(\boldsymbol{Z}) = \frac{1}{\tau_b}\oint d_{\overline{H_0}}\tau\, f_{en}[\boldsymbol{\mathcal{Z}}(\tau; \boldsymbol{Z}, \tau_0)].$$ **NEU.** COM **in the updated field.**

S. Wang, Sci. Reports 10:6986 (2020).
A. J. Bottino et al, Phys. Conf. Ser. **2397**,012019 (2022).
M. V. Falessi et al, New J. Phys. **25**,123035 (2023).
F. S. Zaitsev, et al, Phys. Fluids B **5**, 509 (1993).
S. Wang, Phys. Plasmas **6**, 1393 (1999). of thermal ions)

$(\psi, \theta, v_{||}, \mu) \rightarrow (P, W, \mu, \theta).$
P, the toroidal canonical angular momentum.
W, the energy.

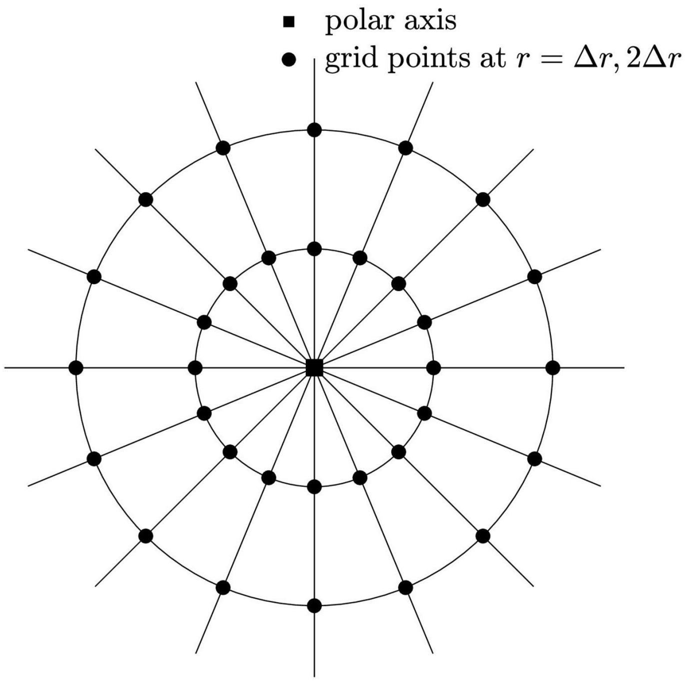
- **The previous equil. distribution is not a COM in the updated field; they are not the NEU!**

2.2 Pole problem in simulation including the magnetic axis

1/r singularity in solving the Vlasov equation.
Inner boundary condition in solving the Poisson equation.

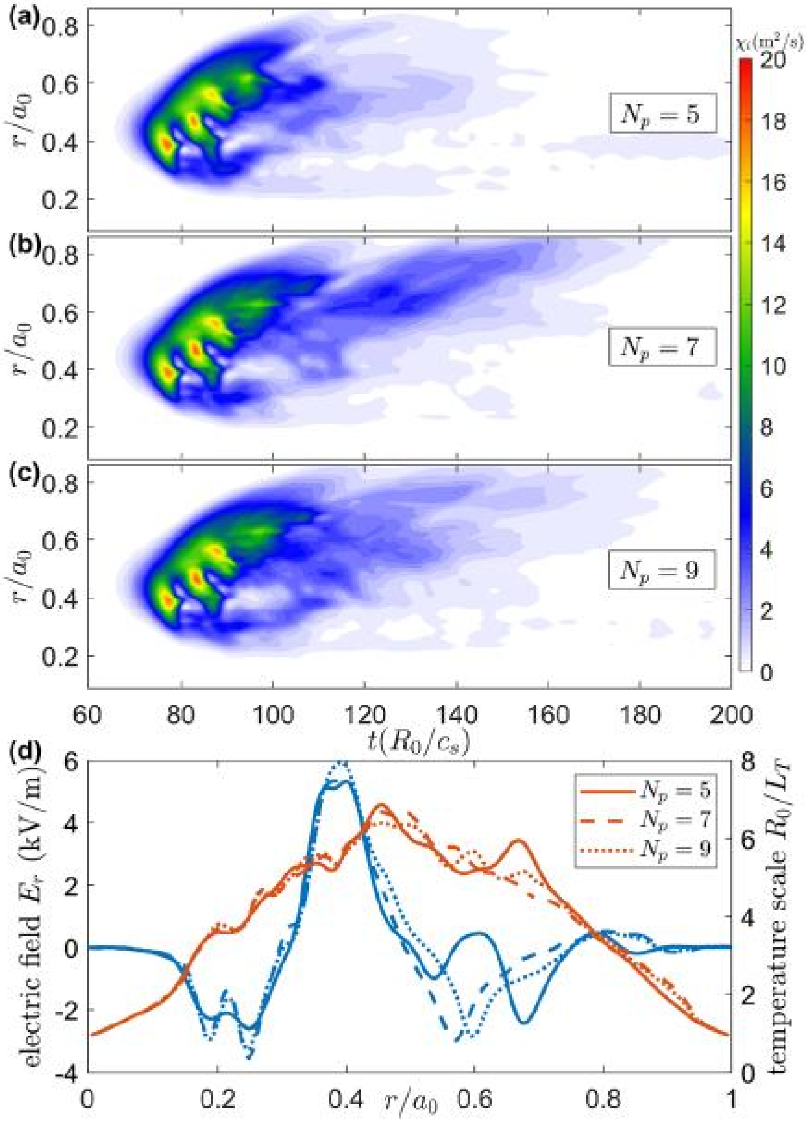
$$\hat{g}(0,\theta) = \frac{4}{3N_\theta}\sum_{k=1}^{N_\theta}\hat{g}(\Delta r,\theta_k) - \frac{1}{3N_\theta}\sum_{k=1}^{N_\theta}\hat{g}(2\Delta r,\theta_k).$$

Mean value theorem,
derived from the continuity condition



T. Wu, Z. Wang, S. Wang,
Plasma Sci. technol. 28, 035101 (2026)

2.3 Numerical dissipation control in the radial low-pass filtering



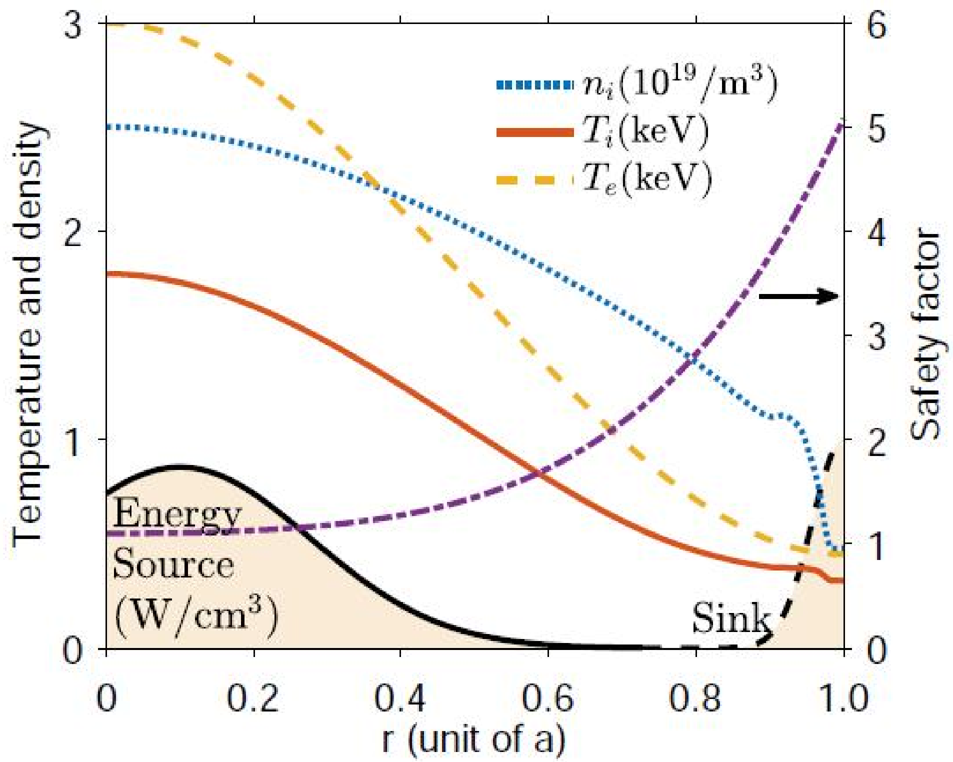
$$\chi(k) \approx \epsilon N_{f,0} \cdot \Delta^{N_p-1} \cdot k^{N_p-3} \propto k^{N_p-3}.$$

Dissipation of different k_r component, with the Np-point low-pass filtering

Dissipation of the fine zonal structure can be reduced to the sub-neoclassical level,
by choosing Np>7.

Z. Wang, S. Wang, Phys. Plasmas 31, 083903 (2024)

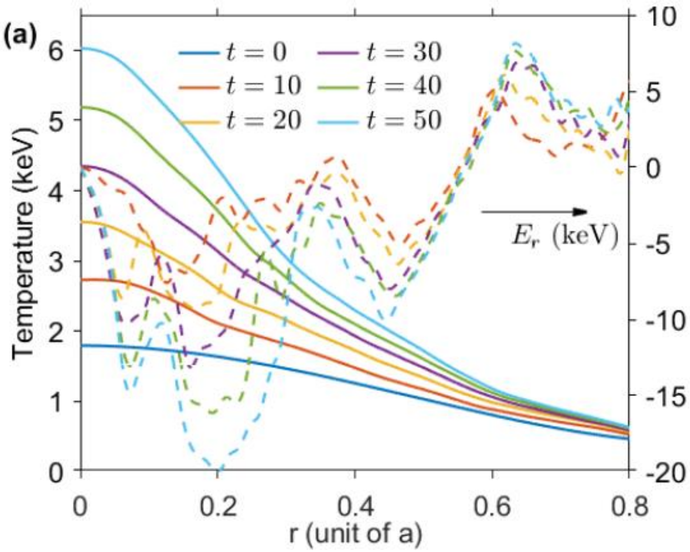
3. Self-organization dynamics of the ITB

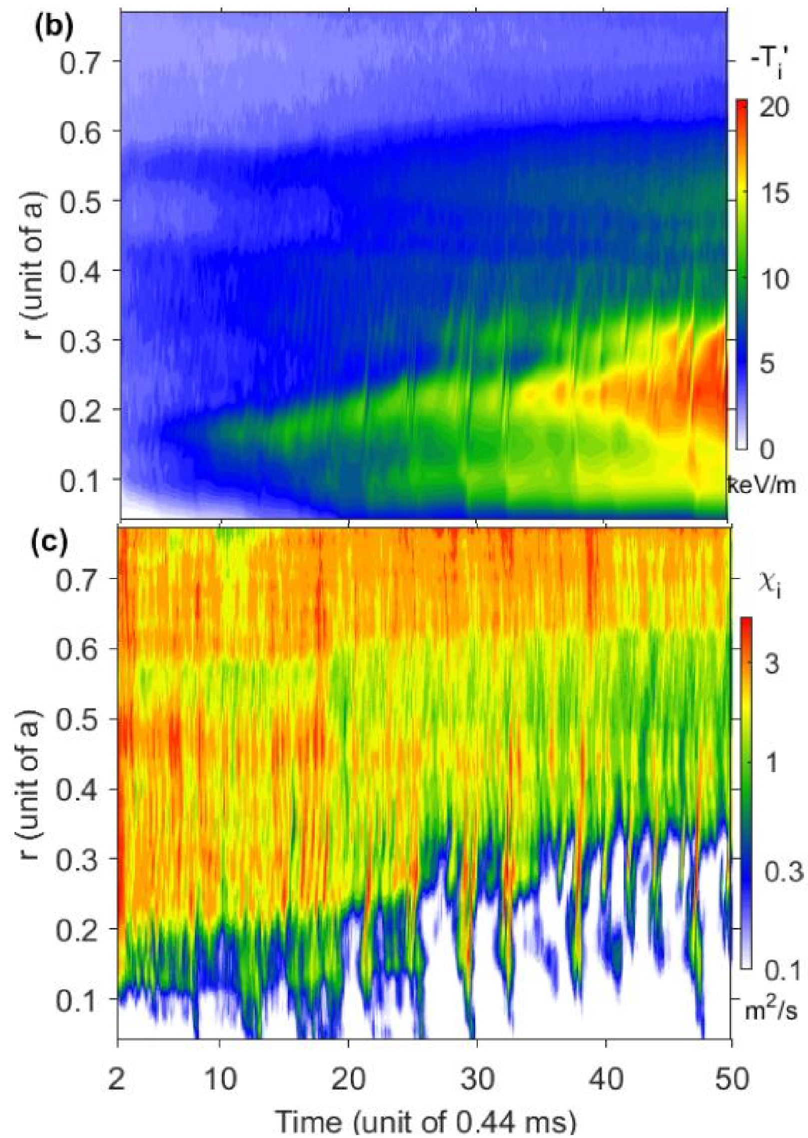


Set-up of the initial equilibrium.
Heating power 2.5MW. Typical DIII-D parameters.

S. Wang, Z. Wang, T. Wu,
Phys. Rev. Lett. **132**, 065106 (2024)

General results of the spontaneously formed ITB

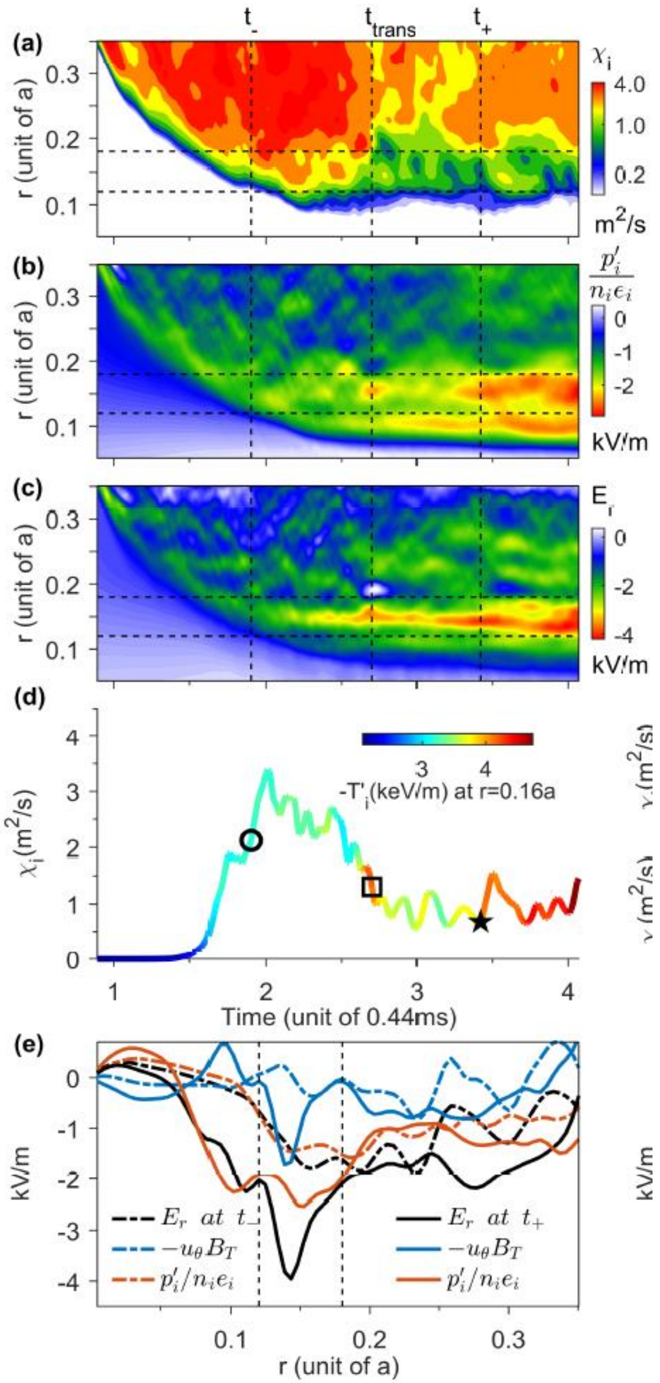




Agreements with experiments:

- emergence near the magnetic axis.
- Outward expansion with ~3m/s.
- E_r shear at the ITB.
- Intermittent burst on both sides.
- Power threshold behavior.

F. Crisanti et al, Phys. Rev. Lett. **88**, 145004 (2002).
P. Mantica et al, Sci. Technol. **53**, 1152 (2008).
K. Ida and F. Fujita, Plasma Phys. Control. Fusion **60**, 033001 (2018).



(a) ITB is initially induced by the inwardly propagated avalanche, **near the magnetic axis where the magnetic shear is weak.**
[F. Romanelli, F. Zonca, Phys. Plasmas **5**, 4081 (1993)]

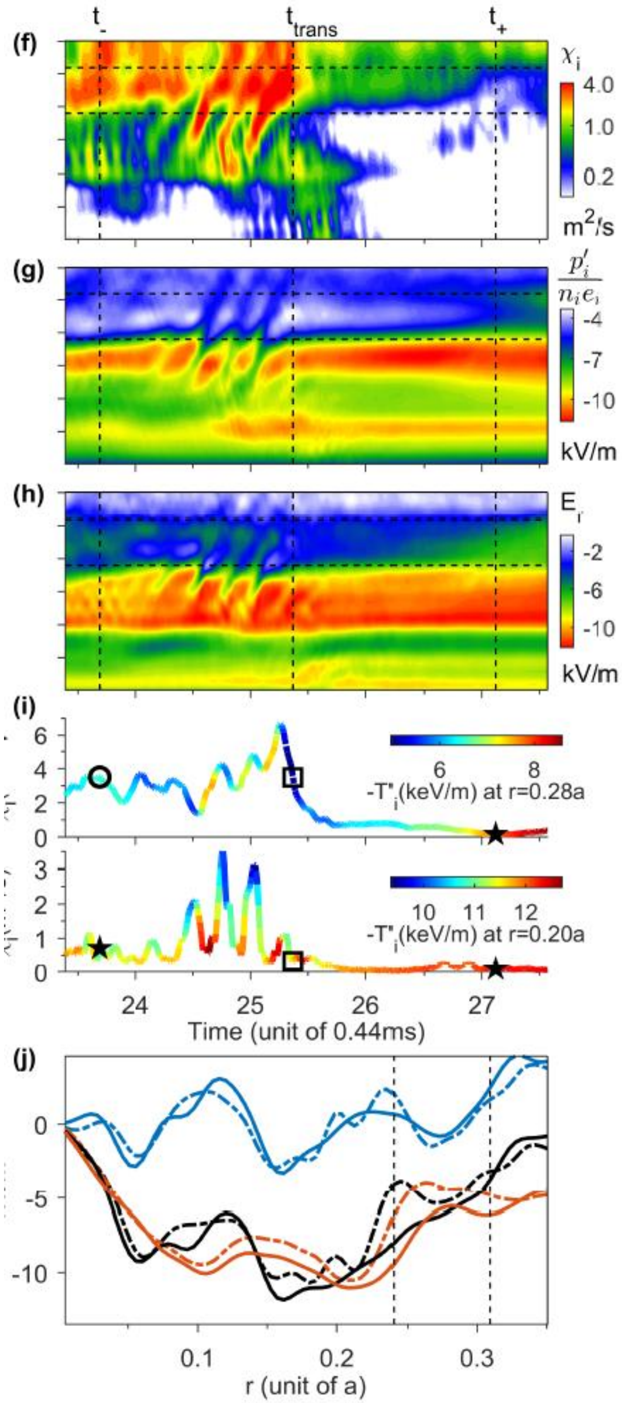
(b) Evolution of the temperature gradient.

(c) E_r changes significantly before the ITB emergence.
[H. Biglary et al, Phys. Fluids B **2**, 1 (1990).]

(d) Confinement transition when the ITB emerges.

(e) E_r change across the transition is induced by the turbulent energy flux and Reynold stress.
[S. Wang, Phys. Plasmas **24**, 102508 (2017).
P. Diamond et al, Phys. Rev. Lett. **72**, 2565 (1994)]

Initial formation of ITB



(f) **ITB expansion induced by outwardly propagated avalanche.**

(g) **Increase of T_i gradient inside ITB destabilizes ITG there; the avalanche leads to the destabilization of the outer region.**

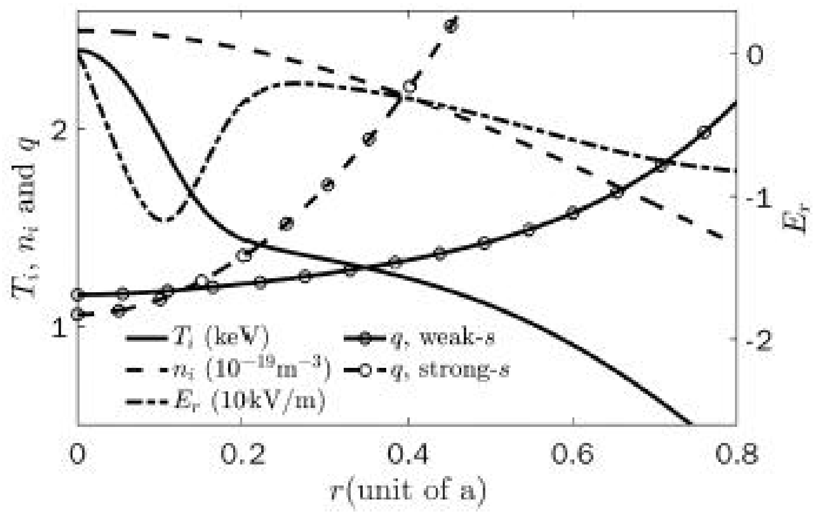
(h) Significant change of E_r before the ITB expansion.

(i) L-H transition in the outer region. **H-L-H transition inside the ITB.** (robustness, resilience, **Critical** structure.) (insensitive to the initial perturbation, **Self-Organized**.)
[P. Bak, et al, Phys. Rev. Lett. **59**, 381 (1987); Phys. Rev. A **38**, 364 (1988). Y. Kishimoto, et al, PoP 3, 1289 (1996)]

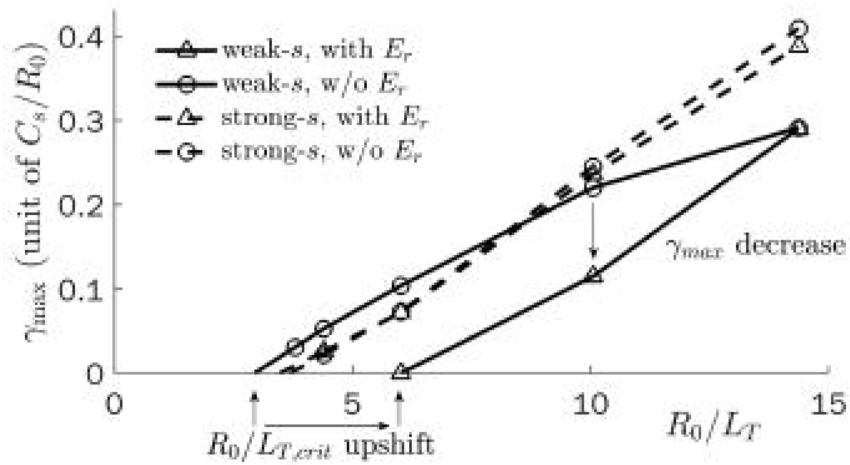
(j) **Expansion of E_r structure is due to the turbulent energy flux and Reynold stress.**
[Wang, PoP17; Diamond, PRL94]

ITB expansion

4. Geometric curvature and weak magnetic effects on the near axis ITB



Equilibrium profiles
（strong/weak s）



Effects of Er-well on the linear instability of ITG mode in the near axis region (strong/weak s)

T. Wu and S. Wang, PRL 136, 015102 (2026)

ITG is stabilized when s<0.1

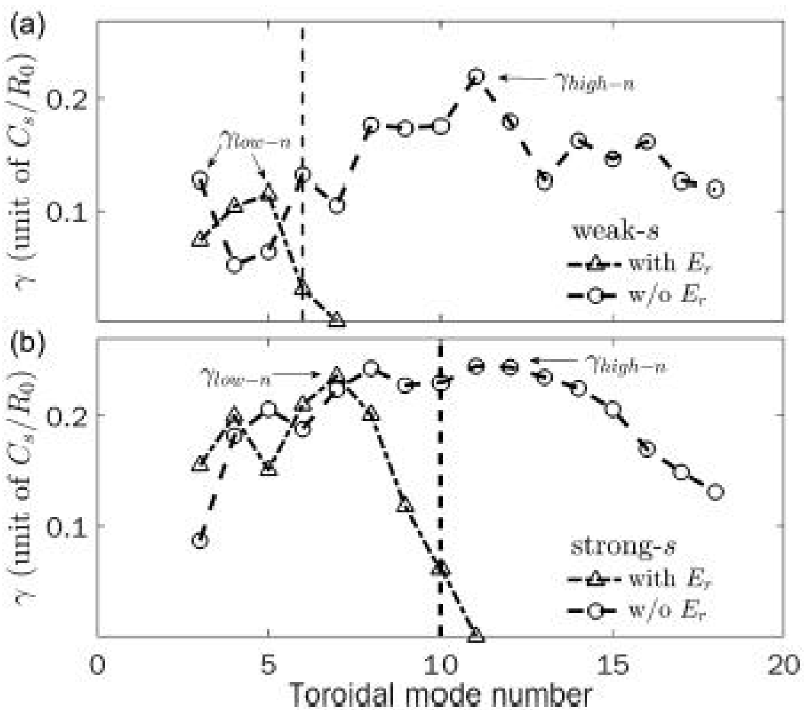


FIG. 3. Effects of E_r on the growth rates of ITG modes for different toroidal mode numbers, with $R_0/L_T = 10$ fixed. (a) the weak and (b) the strong s configurations. The high-n ($n > n_c$) modes are stabilized by E_r. $n_c = 6, 10$ for the weak and strong s configurations, respectively..

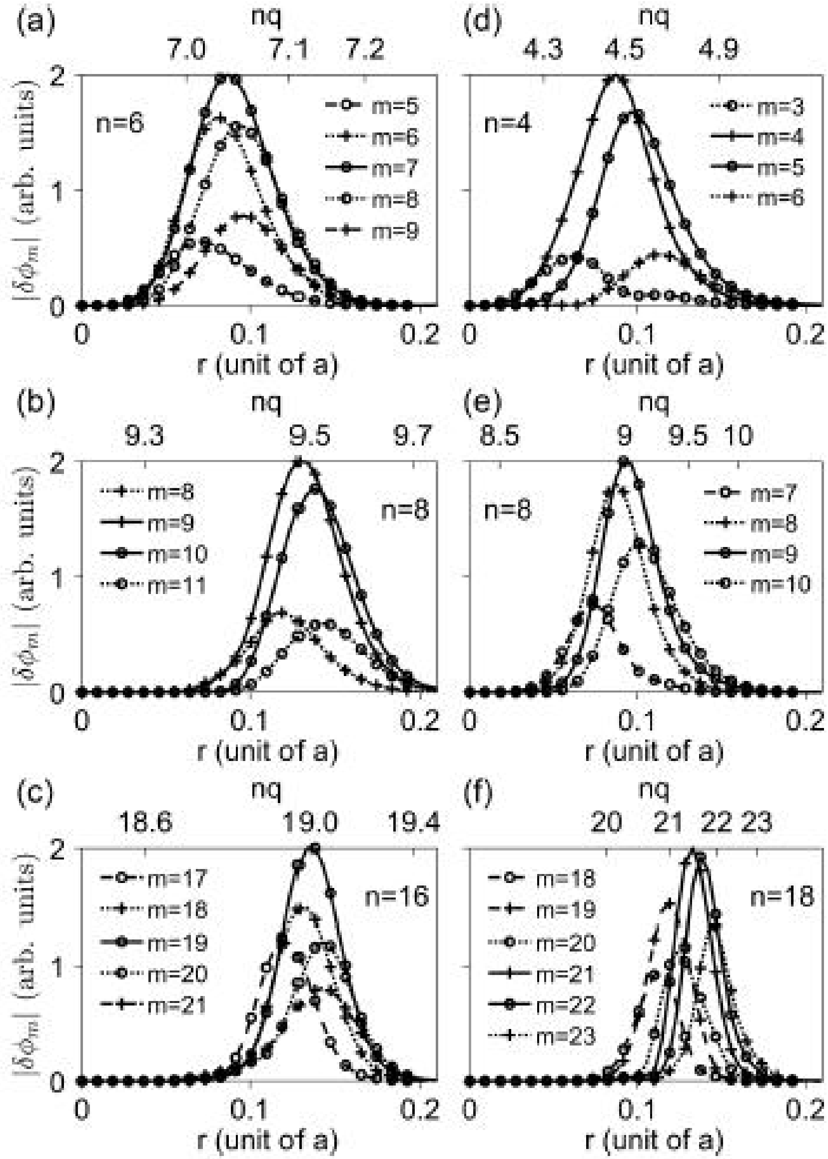
Er stabilizes high-n modes, but has little effect on the low-n modes?

Theory of the ITG in the weak s configuration

J . Q. Dong, et al, PFB92 （local theory, s_c=0.5）

F. Romanelli and F. Zonca, PFB93

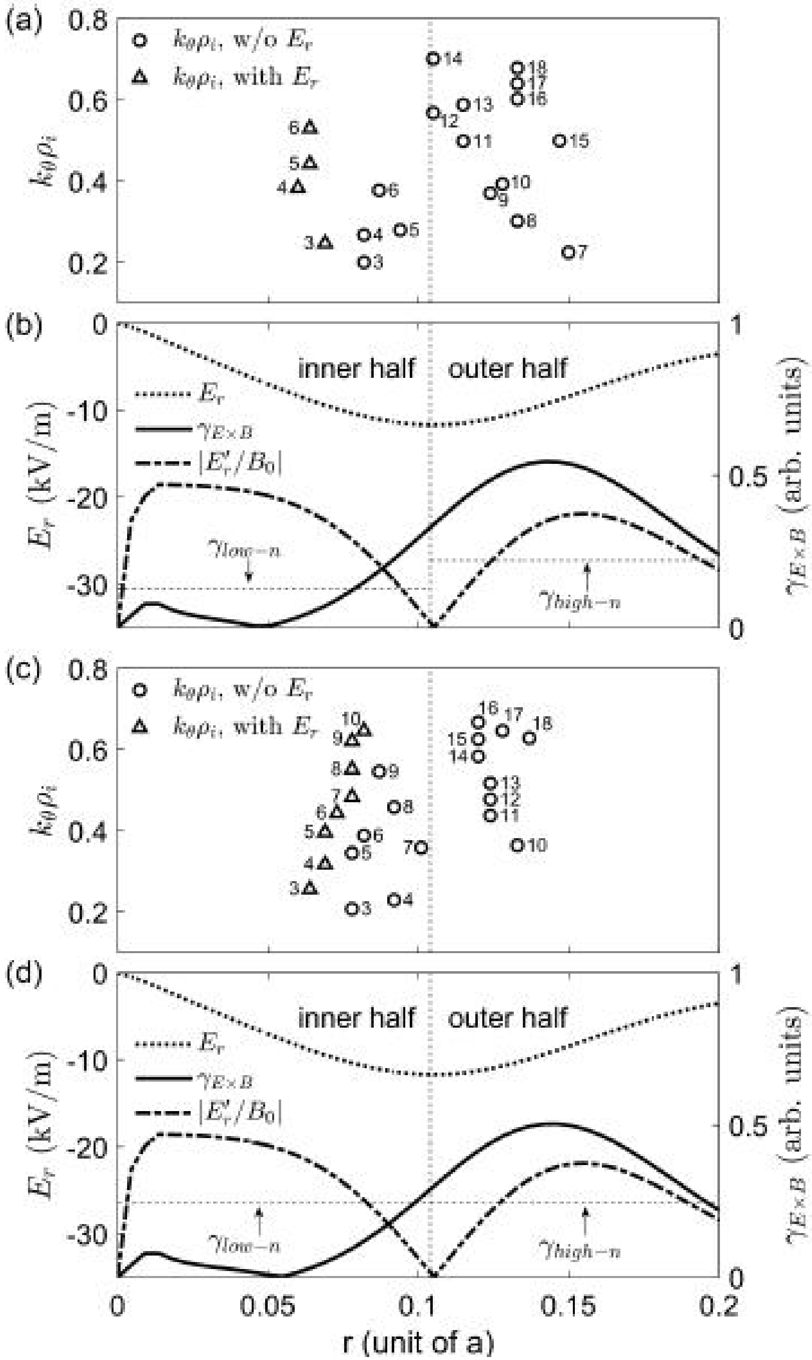
J. W. Connor and R. J. Hastie, PPCF04



Mode structure,
Type I ballooning mode

weak s

strong s



The ExB shear of the near-axis Er-well, is cancelled/enhanced by the geometric curvature effect in the inner/outer half.

T. Wu and S. Wang, PRL26

$$\gamma_{E\times B} = |E_r' - E_r(1-s)/r|/B_0,$$

R. Waltz, R. Miller, PoP99

5. Conclusions

- **The NEU method has been developed, which solved the secularity problem in a long-time nonlinear GK perturbative simulation,** and significantly speeded up the computation in the NLT code.
- **Spontaneous formation of the ion ITB with the external heating in the hybrid scenario has been demonstrated,** with the simulation results well consistent with experiments in various aspects.
- **Both the turbulent energy flux and the turbulent Reynolds stress are important in the nonlinear excitation of zonal flows during the ITB formation.**

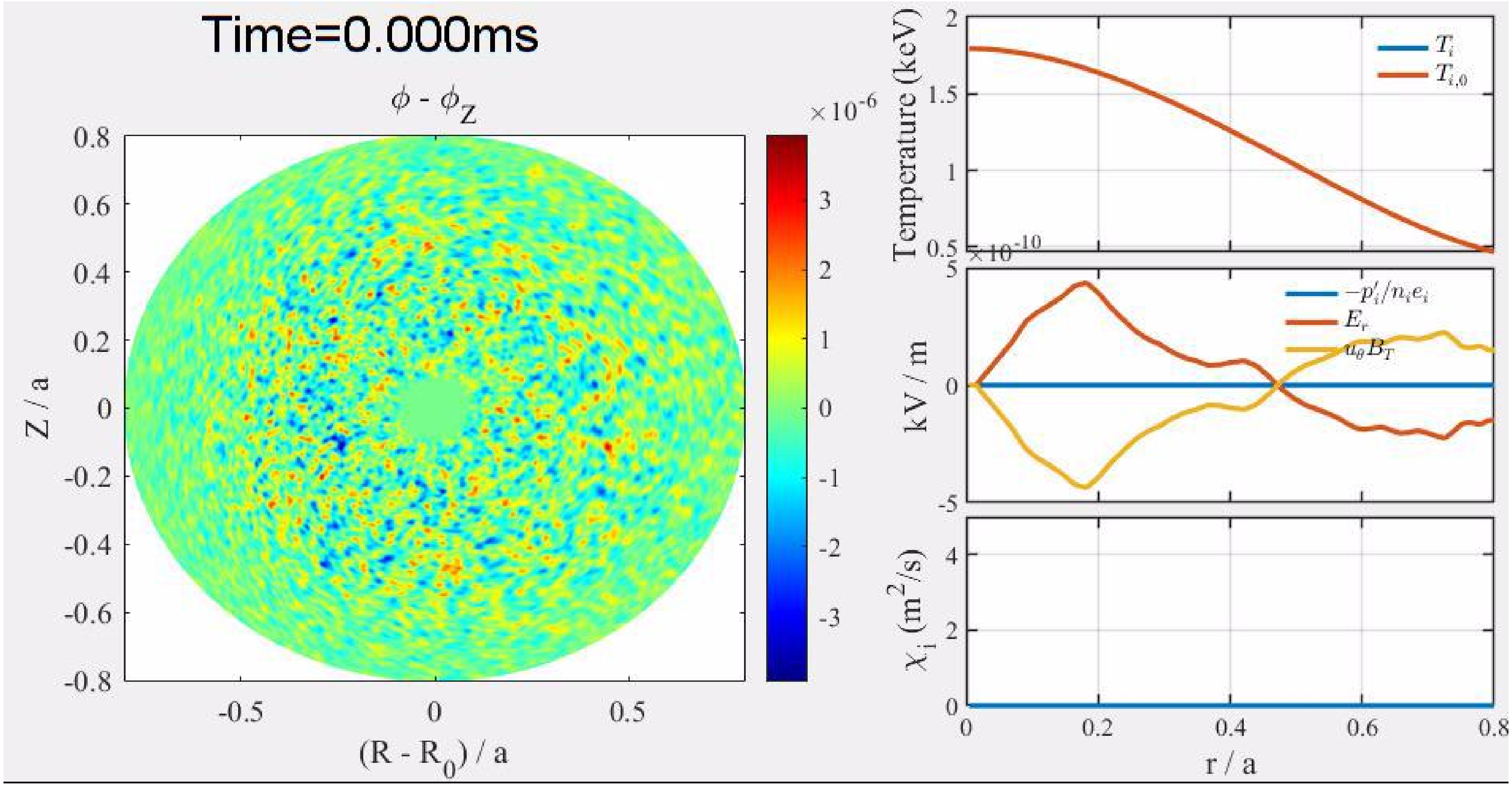
Physical picture of the ITB expansion

- **The ITB is a Self-Organized Critical structure**; its expansion is an catastrophe induced by the outward propagated avalanche.
- **The ion temperature gradient is the driving force of the ITG mode, while the shearing E_r structure is the stabilizing force.**
- **The burst of ITG mode driven by heating inside the ITB induces an outward avalanche and destabilizes the ITG mode outside the ITB, which expands the shearing E_r structure through zonal flows, therefore the ITB is expanded.**

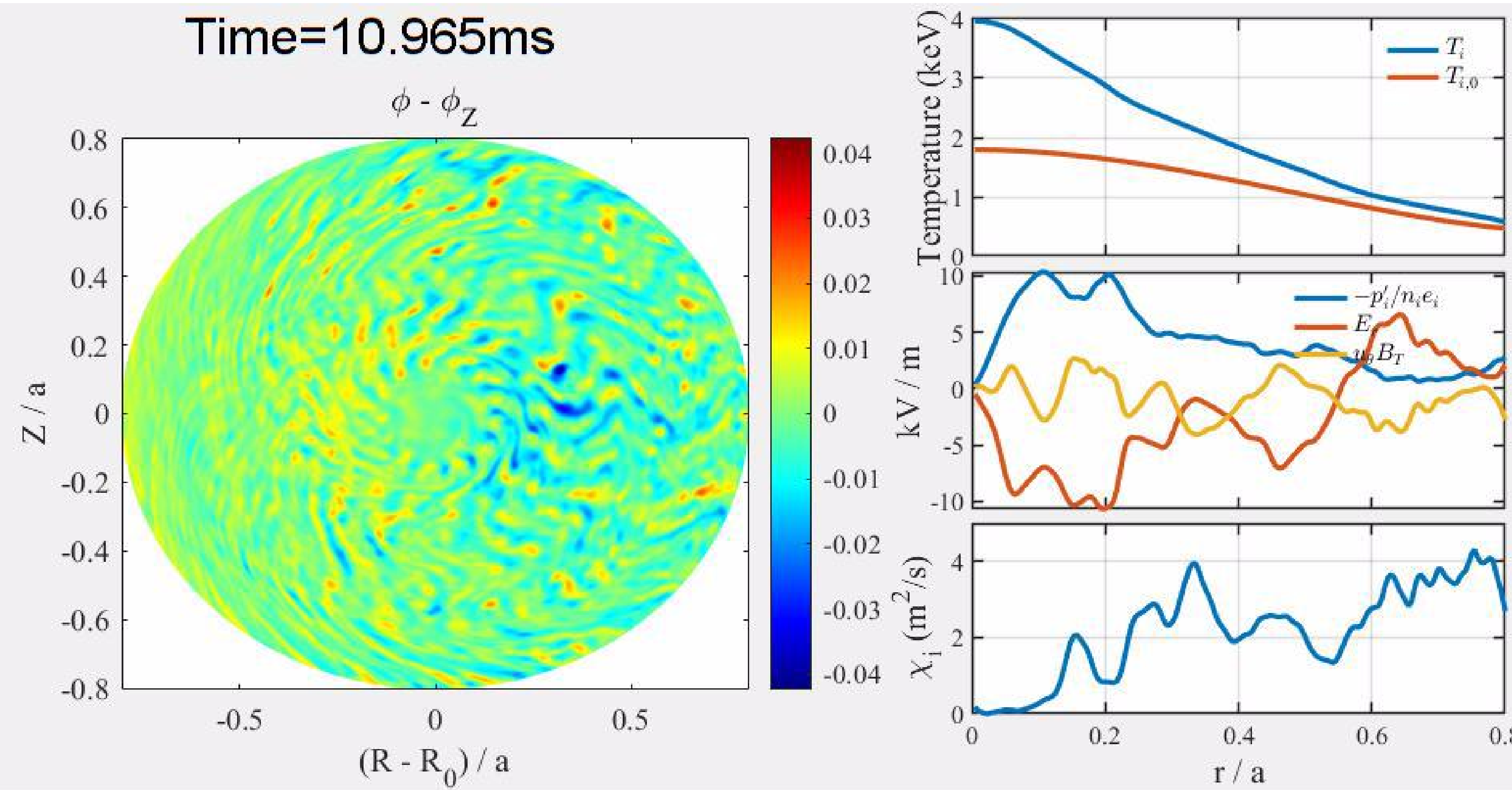
Why weak s is important for the near-axis ITB?

- The ExB shear of the near-axis Er-well， is cancelled by the geometric curvature effect in the inner half of the well.
- Er-well is not sufficient to suppress the ITG in the near-axis region, and the stabilizing effect of weak s (<0.1) is necessary there.

Self-organized ITB



Self-organized ITB



Thanks for your attention.