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Theory and modeling of nonlinear spectrum evolution and density limit by parametric instabilities during the injection of lower hybrid waves

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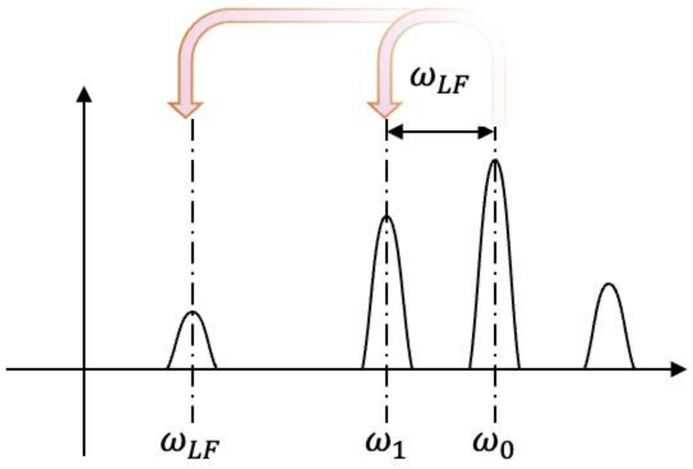
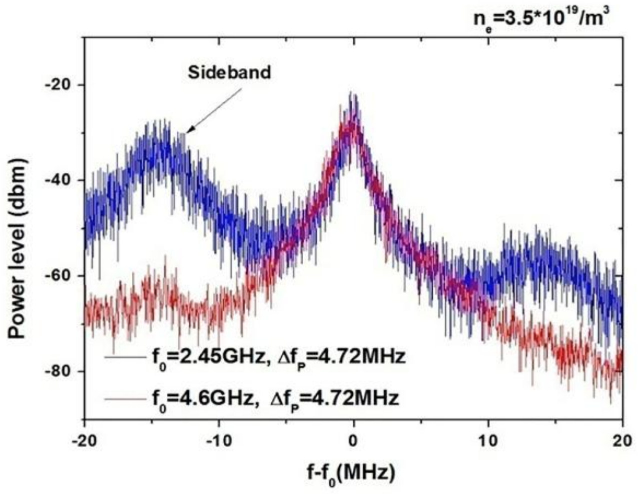
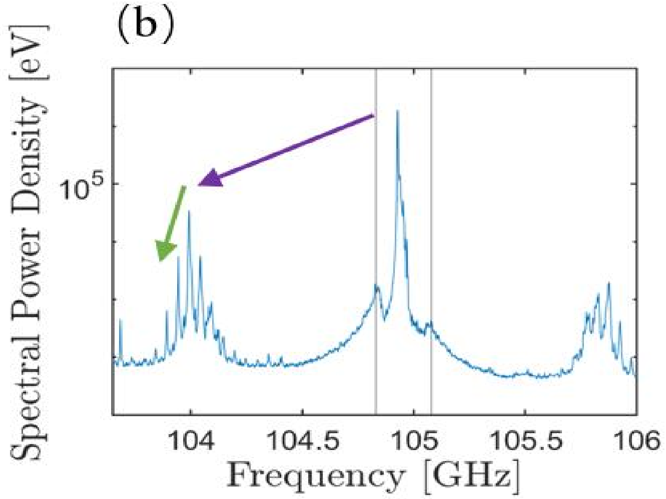
Background

- Parametric Instability (PI or PDI) is a common and important nonlinear wave-wave interaction.
- PDIs result in unexpected power deposition and/or degradation of efficiency of H&CD. For example, PDIs at the edge may responsible for the failure of LHCD in high density plasma.





LHW@EAST MH Li 2022



ECRH@AUG Hansen 2019

Background

- Early theories of parametric instability established in early 1970s, mainly focusing on stimulated scattering in laser plasma [Rosenbluth 1972, 1973, White 1974, Cohen 1976]

✓ Usually resonant decay with only quasilinear treatment

✓ Convective instability saturated by wavenumber mismatch due to plasma inhomogeneity.

✓ Finite pump width even drives absolute instability & other nonlinearities needed

Background

- Early theories of parametric instability established in early 1970s, mainly focusing on stimulated scattering in laser plasma [Rosenbluth 1972, 1973, White 1974, Cohen 1976]

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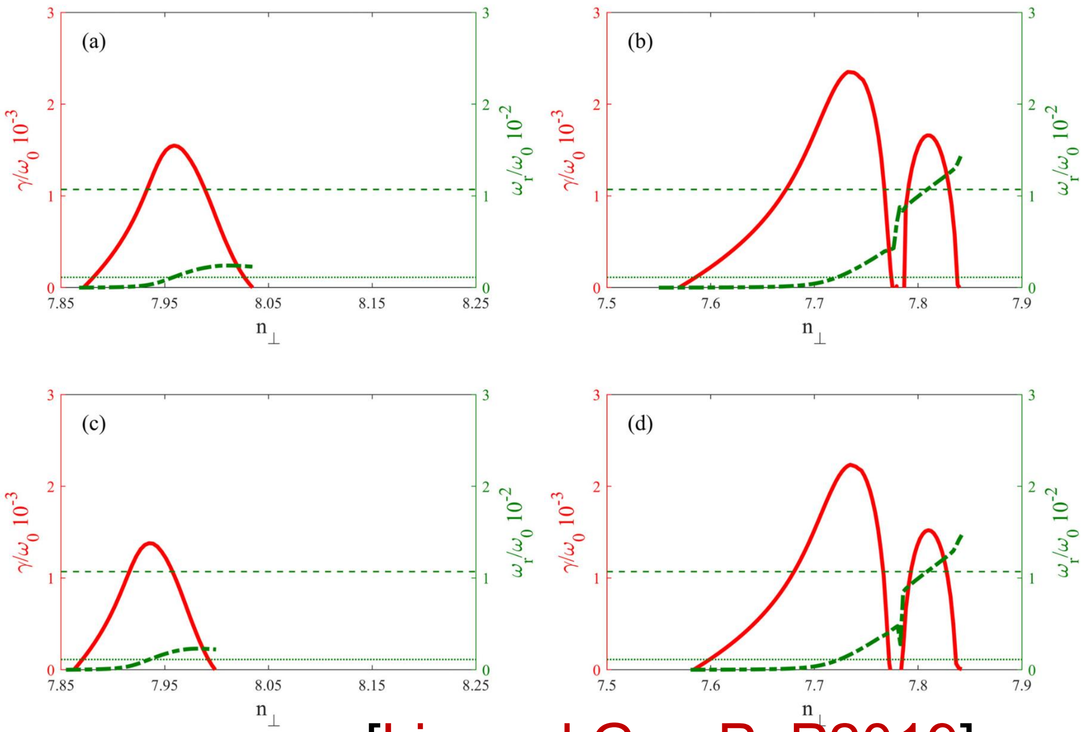
✓ Finite pump width even drives absolute instability & other nonlinearities needed

- In MCF plasma, PDI is typically displaying as quasi-mode decay, such as nonlinear ion landau/cyclotron damping（ISQM/ICQM）.

✓ coupling: quasilinear is not enough（A fluid-kinetic hybrid approach [Liu 1986] and a approach by integrating along an unperturbed orbit with pump field [Porkolab 1974]）

✓ saturation: mismatch of the wavenumber hardly calculated

Coupling for quasi-mode decay



[Liu and Gao PoP2019]

$$f_{sj} = f_{sj}^{\mathrm{L}} + f_{sj}^{\mathrm{NL}}$$

$$f_{sj}^{\mathrm{L}} = \mathbb{L}\left(f_{\mathrm{M}s}, \boldsymbol{E}_j\right)$$

$$f_{s,\mathrm{LF}}^{\mathrm{NL}} = \mathbb{C}\left(f_{s,0}, \boldsymbol{E}_1\right) + \mathbb{D}\left(f_{s,1}, \boldsymbol{E}_0\right)$$

$$f_{s,1}^{\mathrm{NL}} = \mathbb{E}\left(f_{s,0}, \boldsymbol{E}_{\mathrm{LF}}\right) + \mathbb{F}\left(f_{s,\mathrm{LF}}, \boldsymbol{E}_0\right)$$

QL-QL：

$$f_{s,\mathrm{LF}}^{\mathrm{QL}} = \mathbb{C}\left(f_{s,0}^{\mathrm{L}}, \boldsymbol{E}_1\right) + \mathbb{D}\left(f_{s,1}^{\mathrm{L}}, \boldsymbol{E}_0\right)$$

$$f_{s,1}^{\mathrm{QL}} = \mathbb{E}\left(f_{s,0}^{\mathrm{L}}, \boldsymbol{E}_{\mathrm{LF}}\right) + \mathbb{F}\left(f_{s,\mathrm{LF}}^{\mathrm{L}}, \boldsymbol{E}_0\right)$$

resonant decay： $f^{\mathrm{L}} \gg f^{\mathrm{NL}}$

QL-NL：

$$f_{s,\mathrm{LF}}^{\mathrm{NL}} \approx f_{s,\mathrm{LF}}^{\mathrm{QL}} = \mathbb{C}\left(f_{s,0}^{\mathrm{L}}, \boldsymbol{E}_1\right) + \mathbb{D}\left(f_{s,1}^{\mathrm{L}}, \boldsymbol{E}_0\right)$$

$$f_{s,1}^{\mathrm{NL}} \approx \mathbb{E}\left(f_{s,0}^{\mathrm{L}}, \boldsymbol{E}_{\mathrm{LF}}\right) + \boxed{\mathbb{F}\left(f_{s,\mathrm{LF}}^{\mathrm{L}} + f_{s,\mathrm{LF}}^{\mathrm{QL}}, \boldsymbol{E}_0\right)}$$

quasi-mode decay： $f_{\mathrm{LF}}^{\mathrm{L}} \sim f_{\mathrm{LF}}^{\mathrm{NL}}$, $f_1^{\mathrm{L}} \gg f_1^{\mathrm{NL}}$

Fortunately, the coupling is mainly ES and fluid

$$\begin{bmatrix} \boldsymbol{D}_{\mathrm{LF}} & \tilde{\boldsymbol{\sigma}}^{(1)}_{\mathrm{LF}\leftarrow 1} \\ \tilde{\boldsymbol{\sigma}}^{(1)}_{1\leftarrow \mathrm{LF}} & \boldsymbol{D}_1 + \tilde{\boldsymbol{\sigma}}^{(2)}_{1\leftarrow \mathrm{LF}\leftarrow 1} \end{bmatrix} \begin{bmatrix} \boldsymbol{E}_{\mathrm{LF}} \\ \boldsymbol{E}_1 \end{bmatrix} = \boldsymbol{0}$$

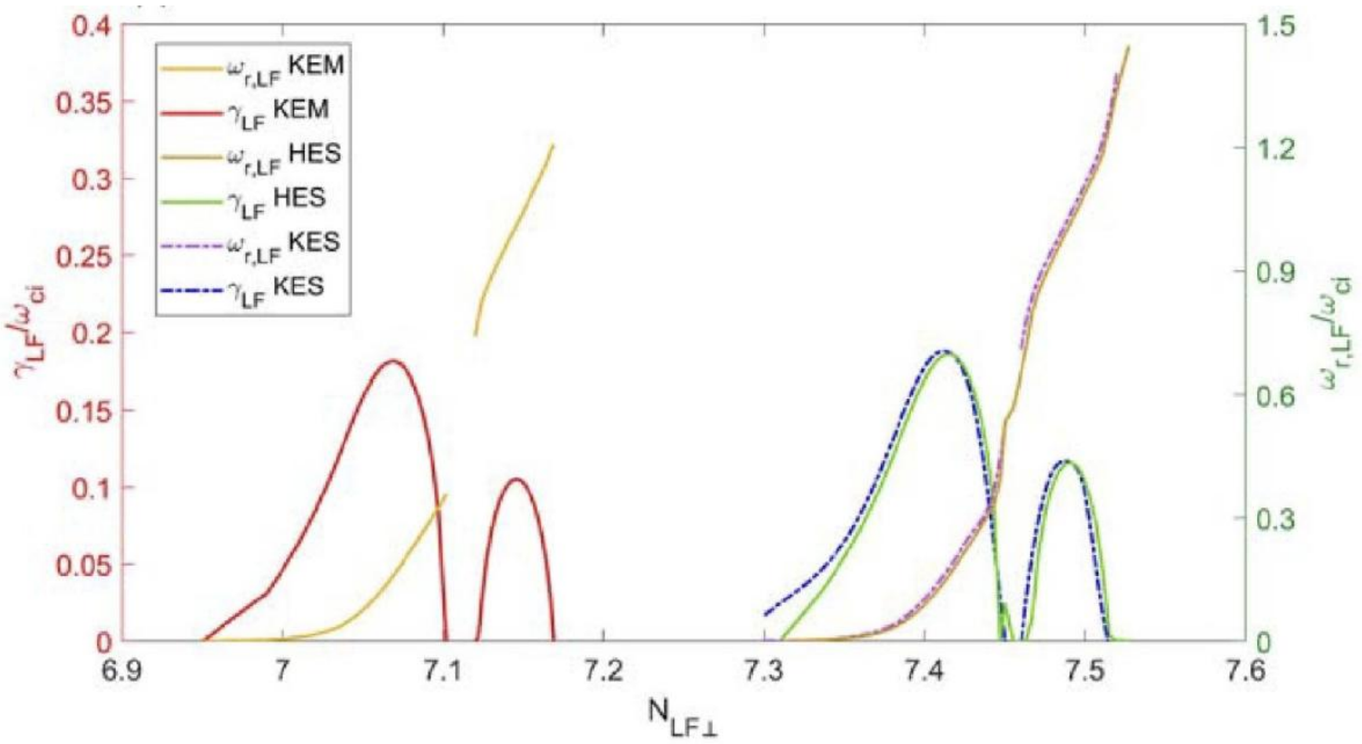
ES Limit

$$\begin{vmatrix} \varepsilon_{\mathrm{LF}} & \chi^{(1)}_{\mathrm{LF}\leftarrow 1} \\ \chi^{(1)}_{1\leftarrow \mathrm{LF}} & \varepsilon_1 + \chi^{(2)}_{1\leftarrow \mathrm{LF}\leftarrow 1} \end{vmatrix} = 0$$

$$\mu_1^* = \chi^{(1)}_{1\leftarrow \mathrm{LF}}\chi^{(1)}_{\mathrm{LF}\leftarrow 1} - \varepsilon_{\mathrm{LF}}\chi^{(2)}_{1\leftarrow \mathrm{LF}\leftarrow 1}$$

$$\varepsilon_{\mathrm{LF}} = \frac{\mu_1^*}{\varepsilon_1}$$

same explicit form as from the fluid-kinetic hybrid approach [Liu 1986]



LHW@JET $N_{1z} = 3$ $\delta_1 = 90°$

- Noted that kinetic effect in the linear response and EM effect in the linear dispersion relation of the pump should be kept.

[Gao et al NF 2025]

How to saturate the instability of quasimode decay

- Mismatch of the wavenumber due to plasma inhomogeneity cannot be given by the linear dispersion relation

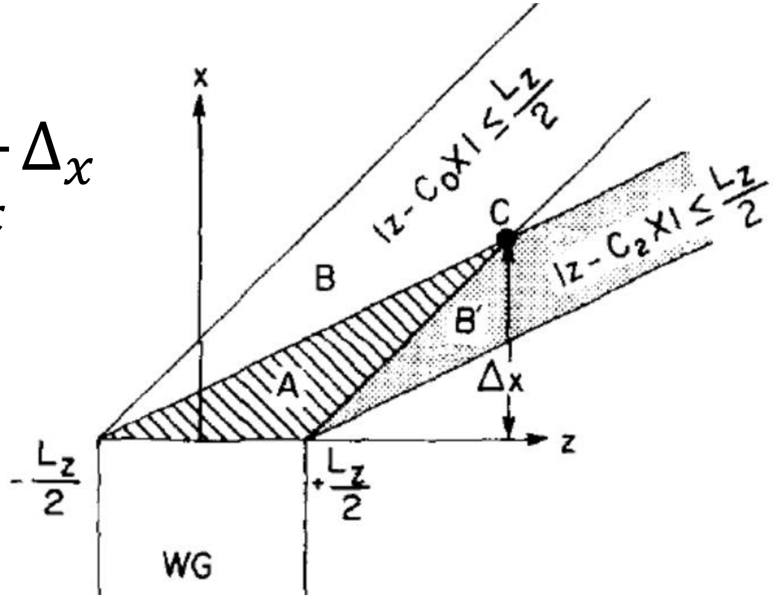
$$\kappa = \Delta k_L - \Delta k_1 - \Delta k_0 \approx \kappa' x$$

?

$$A_{INH} \sim \frac{\pi\gamma^2}{v_{1gx} v_{gx} \kappa'}$$

- Saturation by finite pump width proposed [Liu Chen et al 1977]

$$A_{FIN} \sim \frac{\gamma}{v_{1gx}} \Delta_x$$



✓ Only consider finite pump width saturation by set κ=0 in Takase 1983

✓ Try to include two mechanisms by A= $min\,(A_{INH}, A_{FIN})$ but $\Delta k_L = 0$ used without any physics explanation in Cesario 2006

To characterize the inhomogeneity

PDI Equation（ES）:

$$\begin{cases} \varepsilon_{\mathrm{LF}}\phi_{\mathrm{LF}} = \alpha_{\mathrm{LF}\leftarrow 1}\phi_0^*\phi_{\mathrm{LF}} \\ \varepsilon_1\phi_1 = \alpha_{1\leftarrow \mathrm{LF}}\phi_0\phi_1 \end{cases}$$

Rosenbluth approach: for resonant decay ($\varepsilon_j \sim 0$)

$$\Phi_j = \phi_j(x,t)\exp\left[\mathrm{i}\boldsymbol{k}_j\cdot\boldsymbol{x} - \mathrm{i}\omega_j t + \mathrm{i}\int \Delta k_j\, x\right]$$

$$\varepsilon_{\mathrm{j}}(\omega_{\mathrm{j}}, k_j) \rightarrow \varepsilon_{\mathrm{j}}(\omega_{\mathrm{j}} + i\partial_t, k_j - i\partial_x)$$

$$\left[\frac{v_j}{v_{jg}} + \partial_x + \frac{1}{v_{jg}}\cdot\partial_t\right]\phi_j \exp\left[i\int \Delta k_j dx\right] = \frac{i\alpha_{i\rightarrow j}}{\partial\epsilon_j/\partial k_j}\phi_0\phi_i \exp\left[i\int(\Delta k_0 + \Delta k_i)dx\right]$$

$$\kappa = \Delta k_{\mathrm{LF}} - \Delta k_1 - \Delta k_0$$

To characterize the inhomogeneity

PDI Equation（ES）:

$$\begin{cases} \varepsilon_{\mathrm{LF}}\phi_{\mathrm{LF}} = \alpha_{\mathrm{LF}\leftarrow 1}\phi_0^*\phi_{\mathrm{LF}} \\ \varepsilon_1\phi_1 = \alpha_{1\leftarrow \mathrm{LF}}\phi_0\phi_1 \end{cases}$$

Rosenbluth approach: for resonant decay ($\varepsilon_j \sim 0$)

$$\Phi_j = \phi_j(x,t)\exp\left[\mathrm{i}\boldsymbol{k}_j\cdot\boldsymbol{x} - \mathrm{i}\omega_j t + \mathrm{i}\int \Delta k_j\, x\right]$$

$$\varepsilon_{\mathrm{j}}(\omega_{\mathrm{j}}, k_j) \to \varepsilon_{\mathrm{j}}(\omega_{\mathrm{j}} + i\partial_t, k_j - i\partial_x)$$

Our approach: for quasi-mode decay ($\varepsilon_{\mathrm{LF}} \gg 1$)

$$\Phi_j = \phi_j(x,t)\exp\left[\mathrm{i}\boldsymbol{k}_j\cdot\boldsymbol{x} - \mathrm{i}\omega_j t\right]$$

$$\varepsilon_{\mathrm{j}}(\omega_{\mathrm{j}}, k_j, x = 0) \to \varepsilon_{\mathrm{j}}(\omega_{\mathrm{j}} + i\partial_t, k_j - i\partial_x, x)$$

$$\left[\varepsilon_j(\omega_j, k_j) + \frac{\partial\varepsilon_j}{\partial\omega_j}\cdot\mathrm{i}\partial_t - \frac{\partial\varepsilon_j}{\partial k_j}\cdot\mathrm{i}\partial_x + \frac{\partial\varepsilon_j}{\partial x}\cdot x\right]\phi_j(x,t) = \alpha_{j\leftarrow i}\phi_0(x,t)\phi_i(x,t)$$

Local term | temporal and spatial evolution | Plasma inhomogeneity to replace Δk_j

Eigenvalue problem of the nonlocal equations

- Eliminating time, the nonlocal coupling equations turn to Schrödinger form:

$$\left[\frac{\partial^2}{\partial X^2}-\left(\frac{1}{4}\left(\left(\mathrm{i}K_1+K_2\right)X+\sigma\right)^2-\lambda-\frac{\mathrm{i}K_1}{2}\right)\right]A_1\left(X,p\right)=A_{\mathrm{LF}}\left(X\right)|_{t=0}$$

The Hamiltonian, $V(X)$, is non Hermitian! Initial value

- Dimensionless parameters: coordinate X, amplitude A, plasma inhomogeneity K_1, finite pump profile K_2, growth and/or damping rate σ (complex), coupling coefficient λ
- convective instability: with initial value (Laplace transform $t\rightarrow p$)
- absolute instability: eigenmode problem (natural boundary condition)

On the absolute instability

- Absolute instability: PDI can't be saturated by plasma inhomogeneity, the solutions are the superposition of time eigenmodes

$$\phi_j(x,t) = \sum_{p_k>0} \phi_{j,k}(x) e^{p_k t}$$

- Estimating the threshold of a quasi-mode decay with a WKB analysis on the complex plane for finite pump profile:

$$\lambda > \exp\left[\frac{1}{4}\mathrm{Re}\left(\sigma^2\right)\right]$$

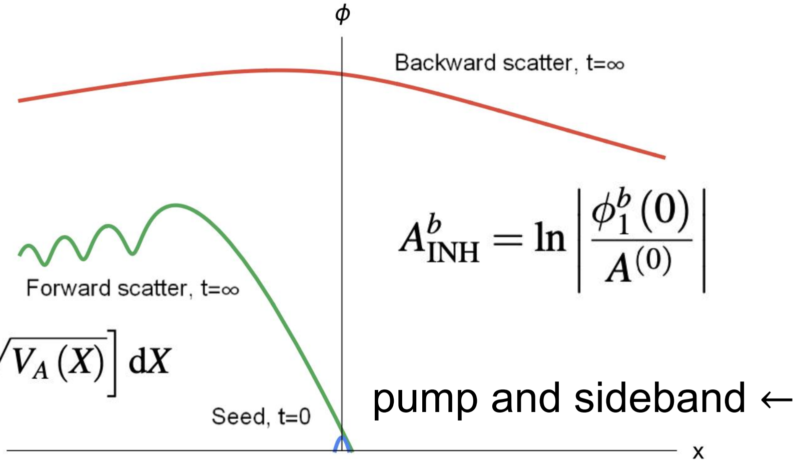
  - Due to exponential relation between coupling coefficient and damping rate, there is no absolute instability for quasi-mode decay in MCF plasma
  - Back to resonant decay $\lambda > 1 + \frac{1}{4}\sigma^2$ with weak damping [White 1974]

[Details in Chen and Gao, Communications in Theoretical Physics, 2025]

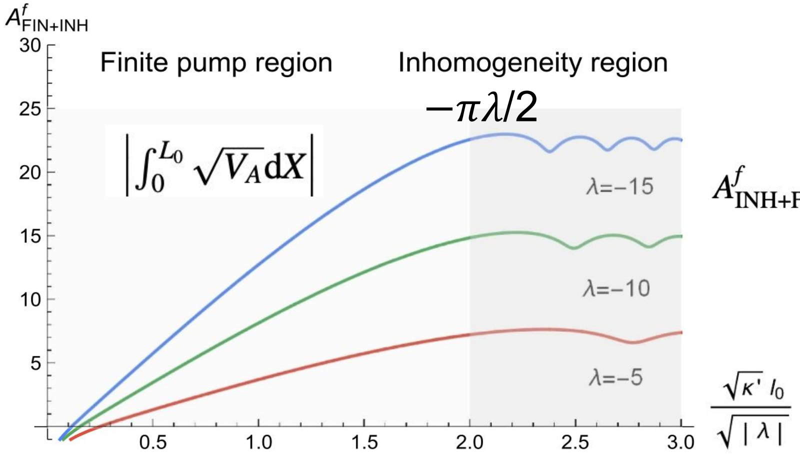
On the convective instability: resonant decay

- Consider the steady-state solution of convective instability
- different cases for co-direction scatter ($\left|v_{1gx}v_{LFgx}\right| > 0$) and counter-direction ($\left|v_{1gx}v_{LFgx}\right| < 0$)

$$A_{\rm INH}^{f} = \ln\left|\frac{\phi_1^f(-\infty)}{A^{(0)}}\right|$$
$$= \int_{X_0}^{-\infty} -{\rm Re}\left[\sqrt{V_A(X)}\right]{\rm d}X$$



$$A_{\rm INH}^{b} = \ln\left|\frac{\phi_1^b(0)}{A^{(0)}}\right|$$



$$A_{\rm INH+FIN}^{f} = \ln\left|\frac{\phi_1(-l_0)}{A^{(0)}}\right|$$

✓ only plasma inhomogeneity:

$$A_{\rm f} = -\frac{\pi\lambda}{2} = \frac{\pi\gamma_0^2}{2v_{1gx}v_{Lgx}\kappa'}$$

$$A_{\rm b} = \pi\lambda = \frac{\pi\gamma_0^2}{v_{1gx}v_{Lgx}\kappa'}$$

similar results obtained as in Rosenbluth with a tiny correction

✓ both plasma inhomogeneity and finite pump width

- for counter-direction scatter, two mechanisms are competitive and finite pump width may excite the absolute instability [White 1974]
- for co-direction scatter, two saturation mechanisms mixed

[Details in Chen and Gao, PPCF 2025]

On the convective instability: quasi-mode decay

- No counter-direction scatter in QM decay due to strong damping.
- For co-direction scatter, the convective amplification factor is,

$$A = \int \left[\sqrt{V_B} + \frac{i}{2} \left(\Delta k_L^{(0)} + \Delta k_1^{(0)} \right) \right] dx$$

$$\approx \int \frac{dx}{v_{1gx}} \gamma_0 = \int dt \, \gamma_0$$

Finite pump width, actually finite trajectory length

plasma inhomogeneity (saturation when $\gamma_0 = 0$)

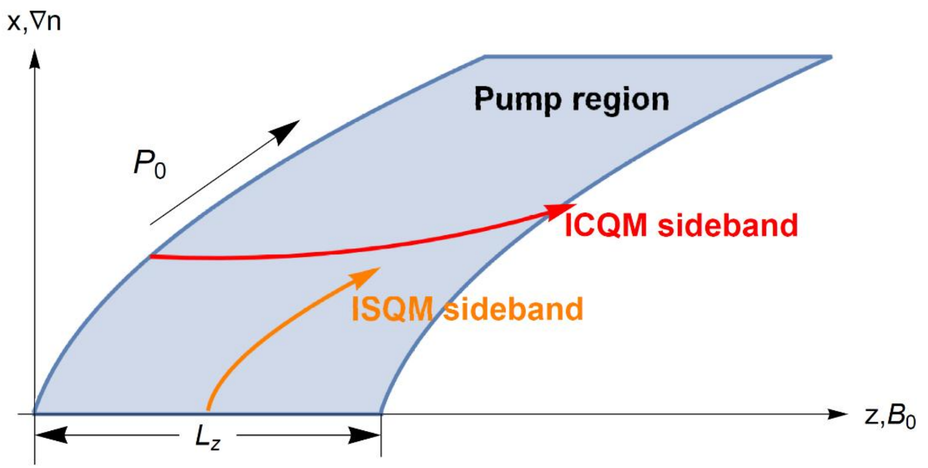
  – the PDI growth rate

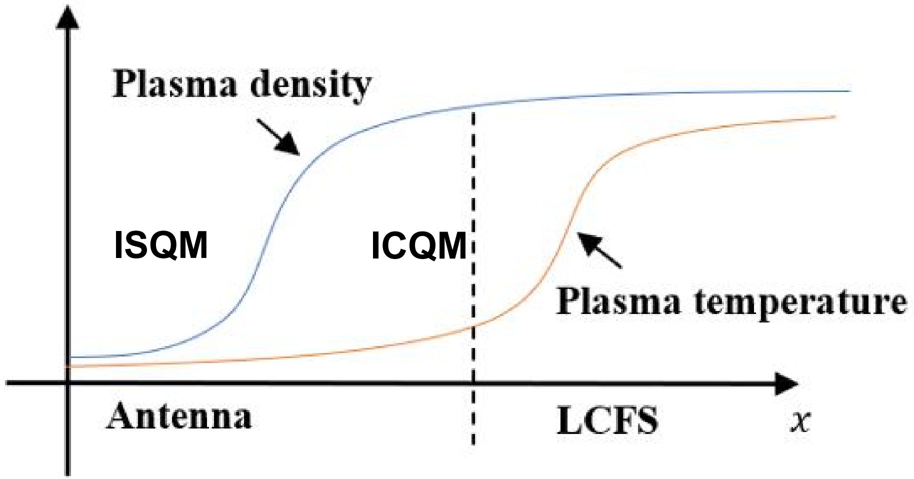
$$\gamma_0 = \mathrm{Im}\left[\frac{\alpha_{\mathrm{LF}\leftarrow 1} \alpha_{1\leftarrow \mathrm{LF}} \left| \phi_0 \right|^2}{\varepsilon_{\mathrm{LF}} \left(\partial \varepsilon_1 / \partial \omega_1 \right)} \right] - \nu_1$$

[Details in Chen and Gao, NF 2025a]

Nonlocal model around the SOL in tokamak with injection of LHWs

- Decay channels: ISQM+ICQM
- Plasma inhomogeneity exits in x-direction (radial)
- Finite pump profile exits mainly in z-direction (toroidal) $v_{1gz} >> v_{1gy}$





$$P_0 = \int W_0 L_y (v_{0gx} dz - v_{0gz} dx)$$
$$= \int W_0 L_y (v_{0gx} v_{1gz} - v_{0gz} v_{1gx}) dt$$

$$\mathrm{A} = \int dt\, \gamma_0 = \int \gamma_0 \cdot \frac{dP_0}{W_0 L_y (v_{0gx} v_{1gz} - v_{0gz} v_{1gx})}$$
$$= \left\langle n_{1z} \gamma_0 \cdot \frac{L_z}{c(1-\cos\delta)} \right\rangle$$

PIPERS code: ray tracing + PDI

- Energy conservation equations constrained by PDIs

$$\begin{cases} \nabla\cdot\boldsymbol{P}_0(\boldsymbol{r})+\sum_{\omega_1,\boldsymbol{k}_1}\left[\nabla\cdot\boldsymbol{P}_1(\boldsymbol{r},\omega_1,\boldsymbol{k}_1)-2\gamma_{1L}(\boldsymbol{r},\omega_1,\boldsymbol{k}_1)U_1(\boldsymbol{r},\omega_1,\boldsymbol{k}_1)\right]=0 \\ U_1(\boldsymbol{r},\omega_1,\boldsymbol{k}_1)=U_{\mathrm{th}}(\boldsymbol{k}_{\mathrm{LF}})\exp(2A) \end{cases}$$

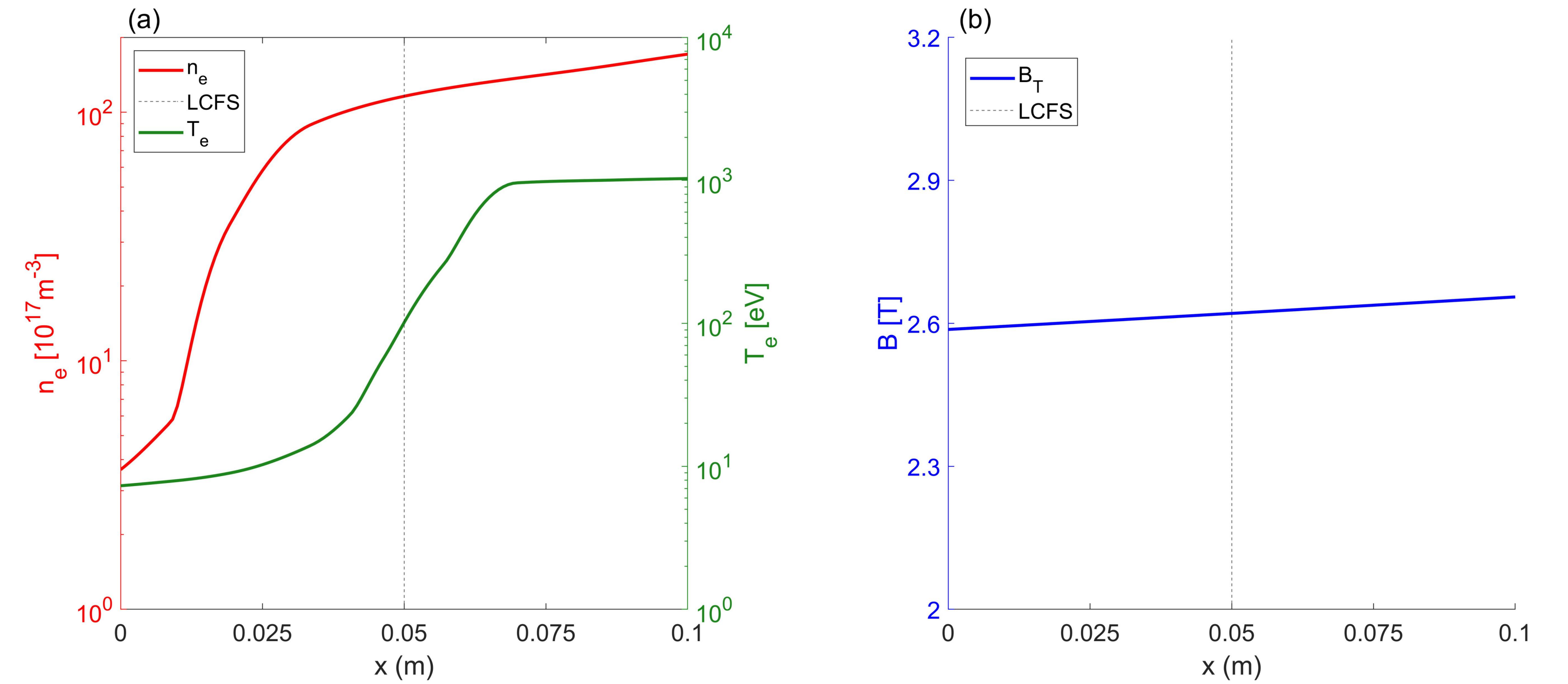
  - The pump damping and low frequency wave terms are omitted
  - The initial value of PDI is the electrostatic thermal noise

$$U_{\mathrm{th}}=\frac{1}{2}\left(1+\frac{\omega_{\mathrm{pe}}^2}{\omega_{\mathrm{ce}}^2}\right)\frac{T}{1+\lambda_{\mathrm{De}}^2k_{\mathrm{LF}}^2}\mathrm{d}\boldsymbol{k}_{\mathrm{LF}}$$

- The convective amplification factor A is integrated along the trajectories of the sideband waves in the SOL

Details in Huang…Gao, NF 2026

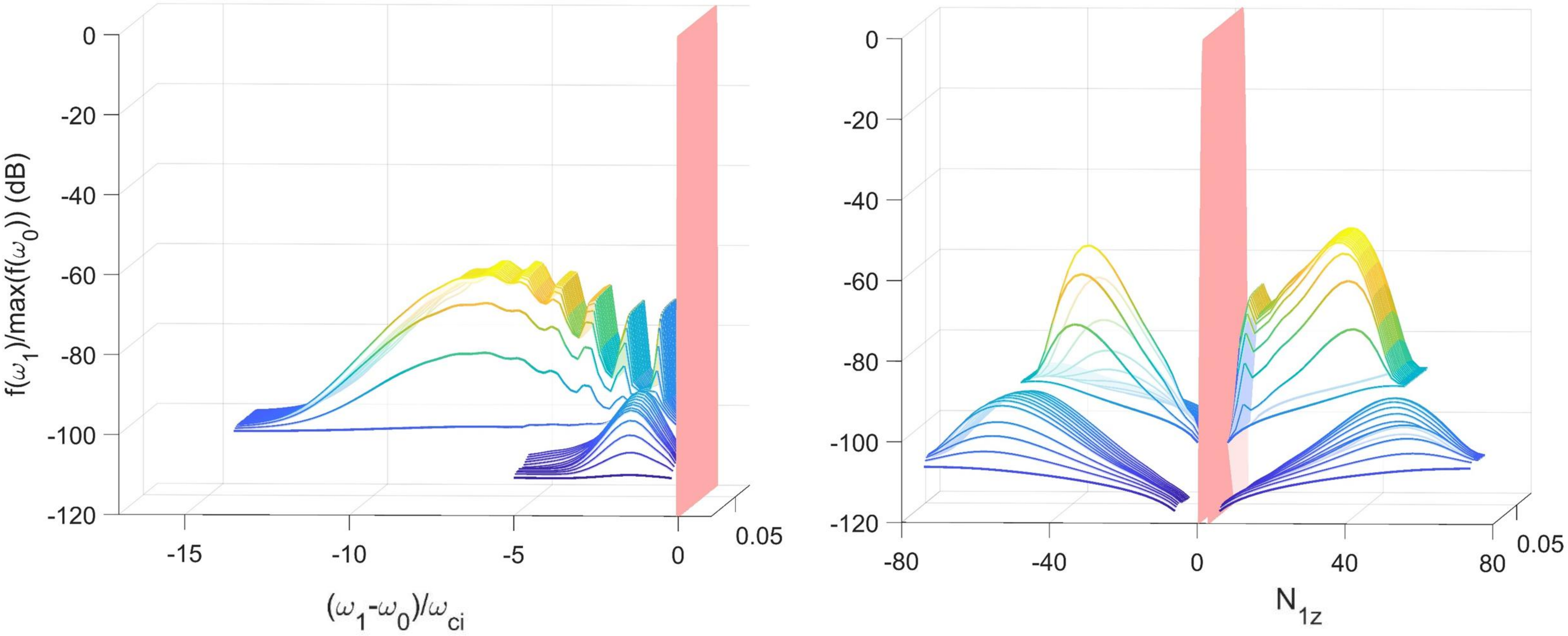
An Example for LHCD experiment



Item	Value
Frequency, f_0	3.7GHz
Parallel refractive index, n_{0z}	1.84
Large radius coordinates, $R_{antenna}$	3.89m
Power, P_0	2MW
The size of the antenna mouth, $L_y \times L_z$	0.884m×0.348m

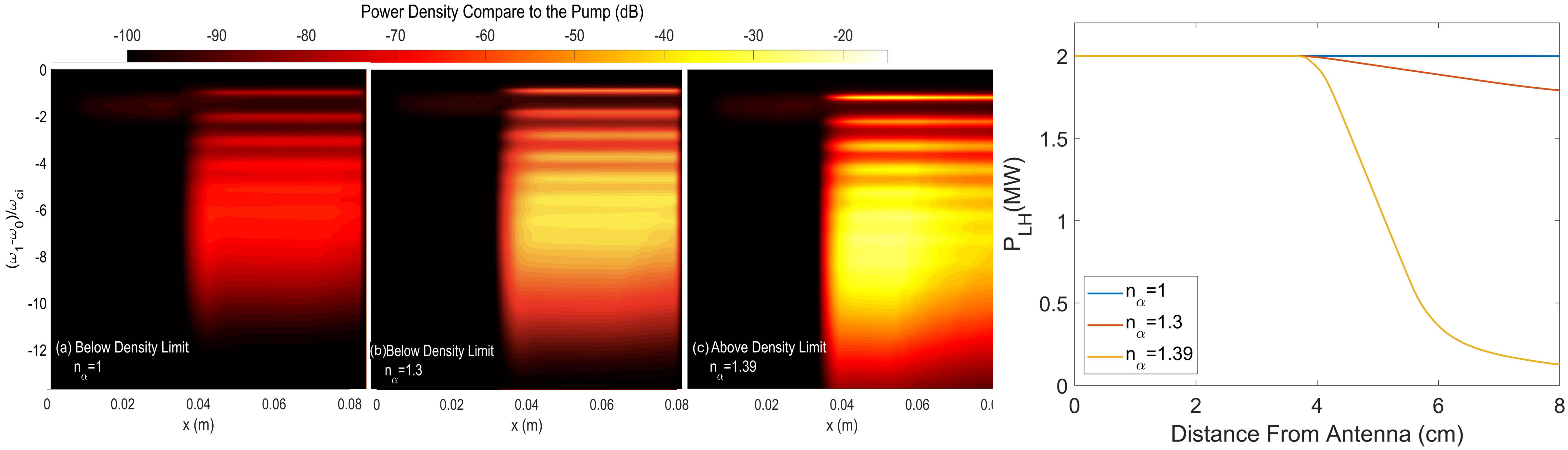
[parameter profiles @JET Cesario 2006]

Spectrum evolution and power transfer



Frequency sidebands and wavenumber broadening appears
But no significant power transfer occurs since PDI is weak

The density limit observed in simulation



When the density increase above a limit, PDI become stronger and the pump is exhausted due to significant power transferring to sidebands

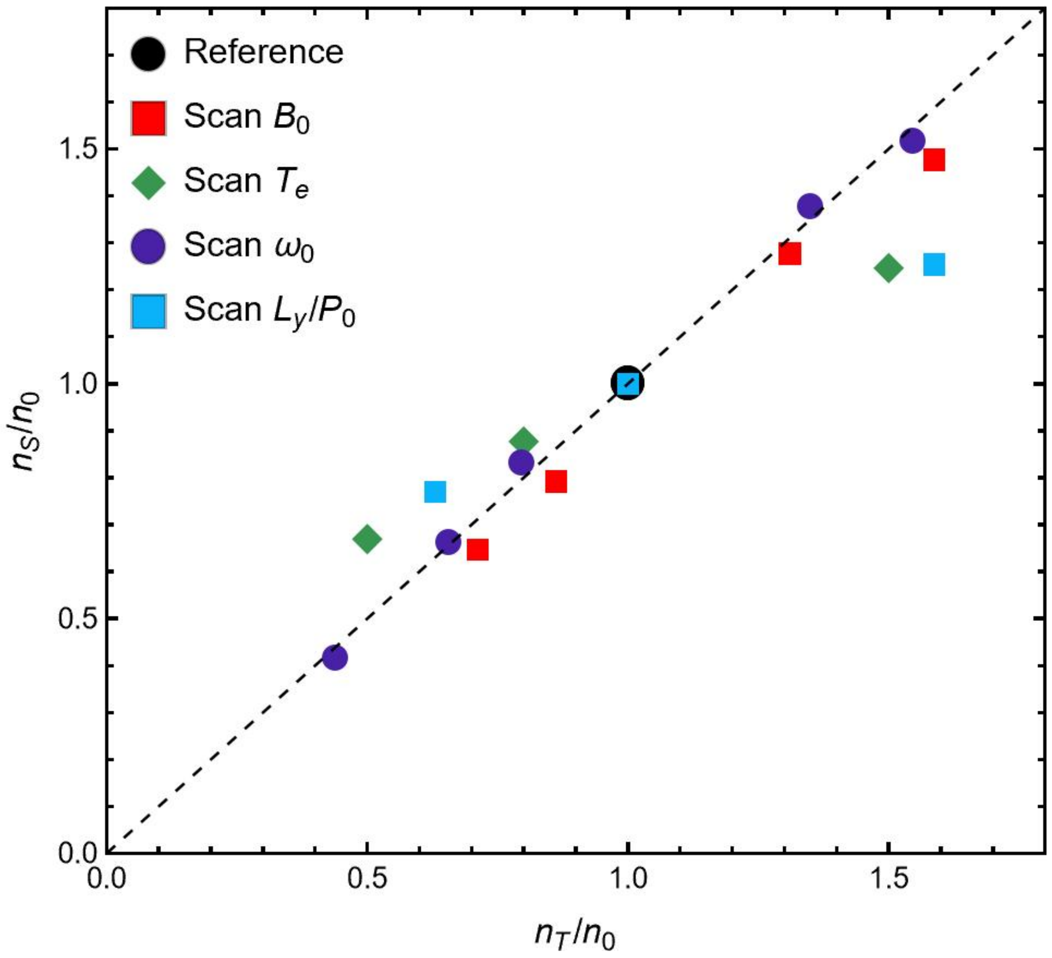
A theoretical scaling relation of the density limit

- When the convective amplification factor is above a certain value, the PDI become strong enough to exhaust the pump， a theoretical scaling relation of the density limit can be derived

$$A_{\mathrm{amp}} \propto P_0 L_y^{-1} \omega_{LH}^3 \omega_0^{-3} B_0^{-2} T_e^{-3/2}$$

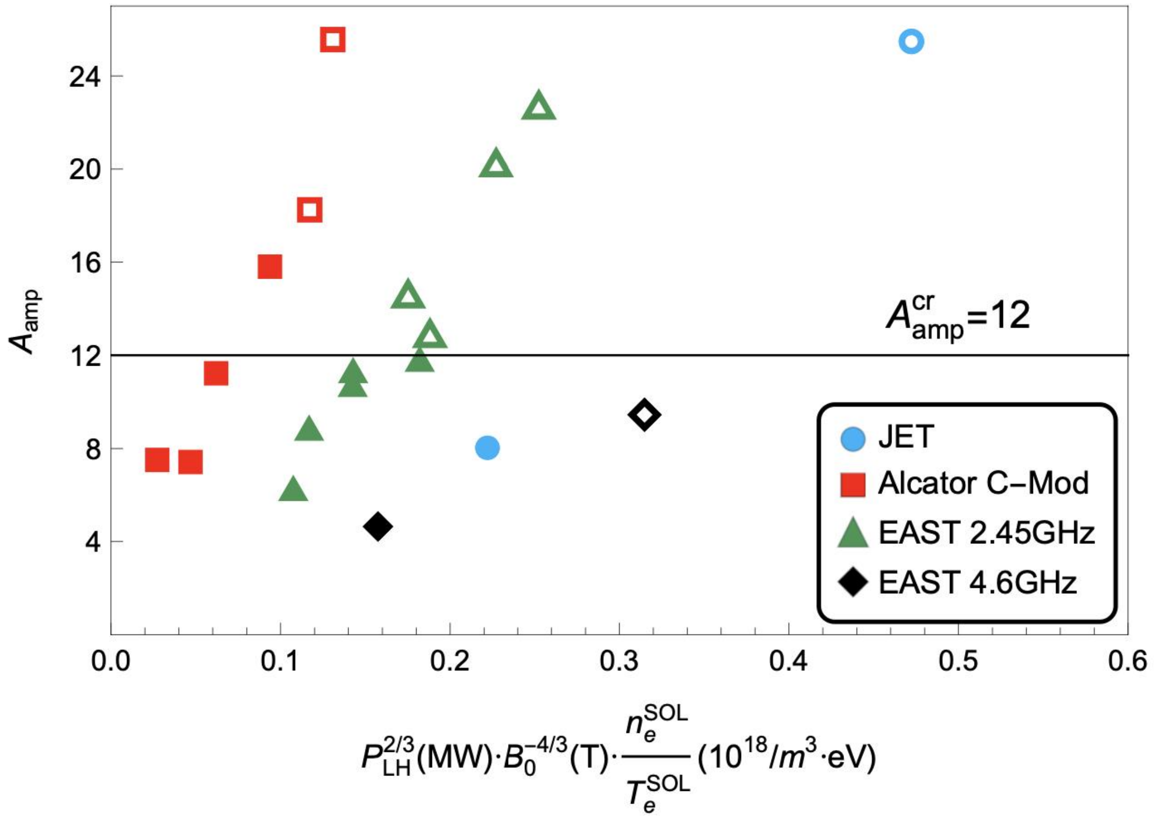
$$n_{\mathrm{PDI}} \propto P_0^{-2/3} L_y^{2/3} \omega_0^2 B_0^{4/3} T_e$$

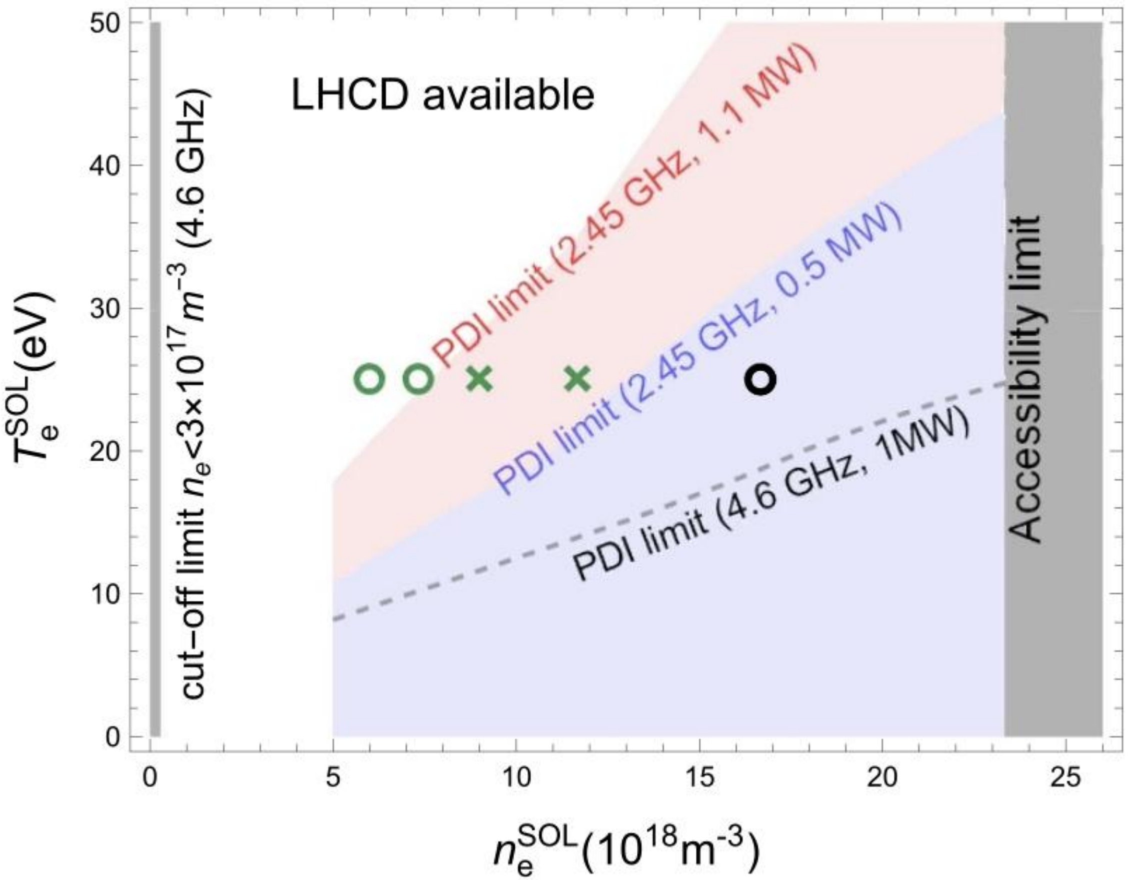
- we can also change the parameter to get the parameter dependence of the density limit from the simulation.
- The simulation results agree quite well with the theoretical scaling relation



Chen … Gao, NF 2025b

A critical convective amplification factor may be found



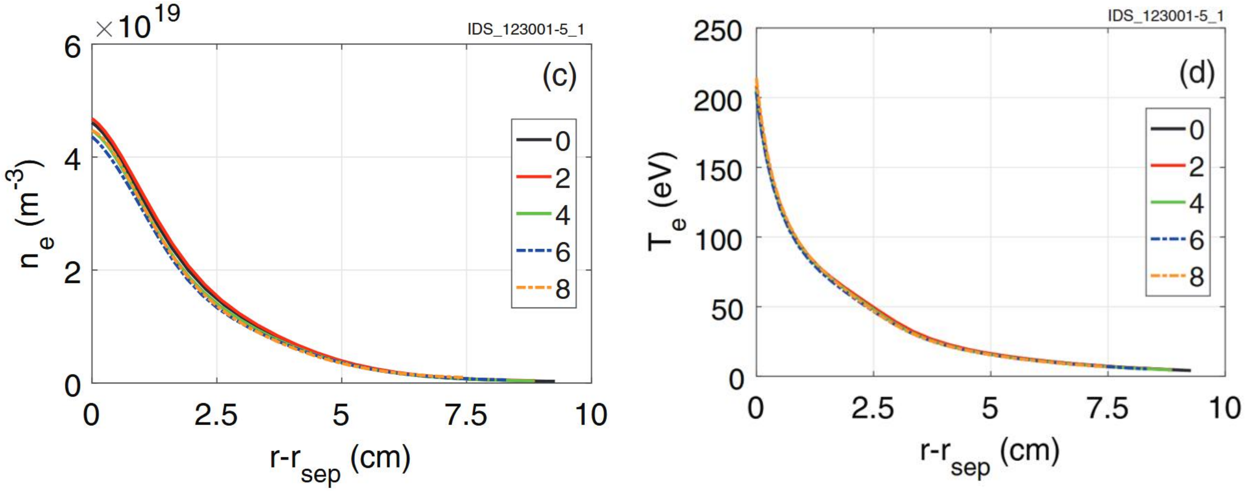


** hollow points indicate significant degradation of LHCD efficiency*

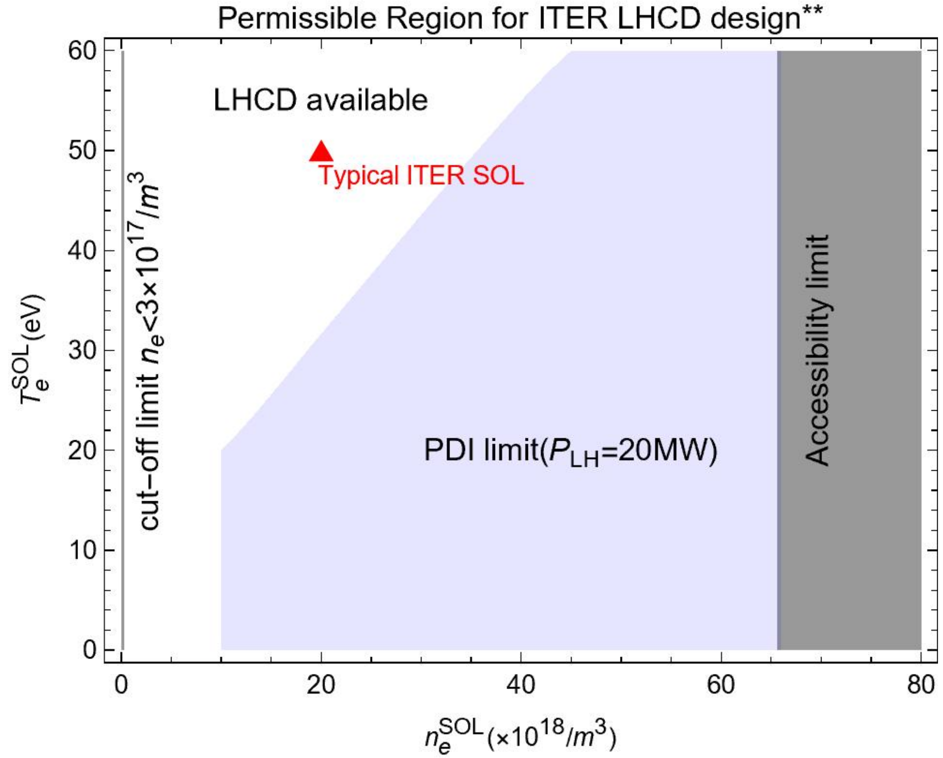
Permissible regions of LHCD for EAST

Discussion: LHCD on ITER

- Using the ITER SOL parameters to simulate, the density is far from the density limit on ITER due to the high SOL temperature, high B and high LHW frequency
- It may indicate that LHCD on ITER is still effective……

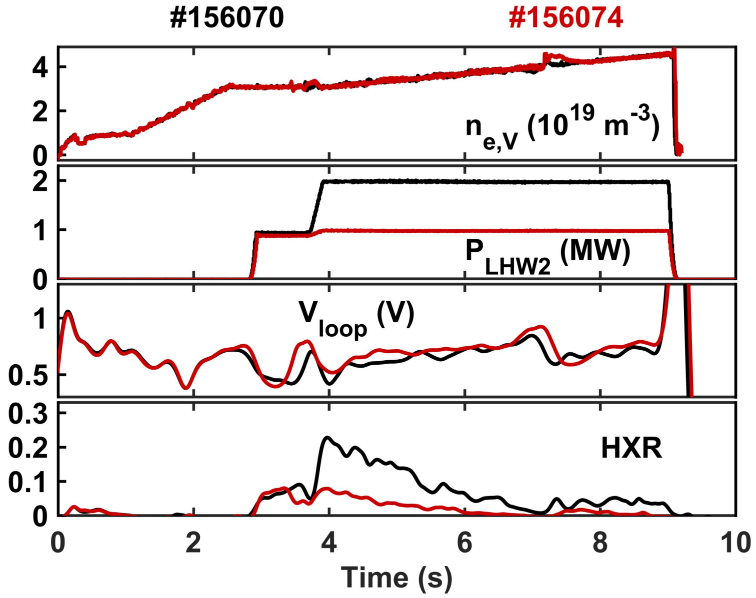


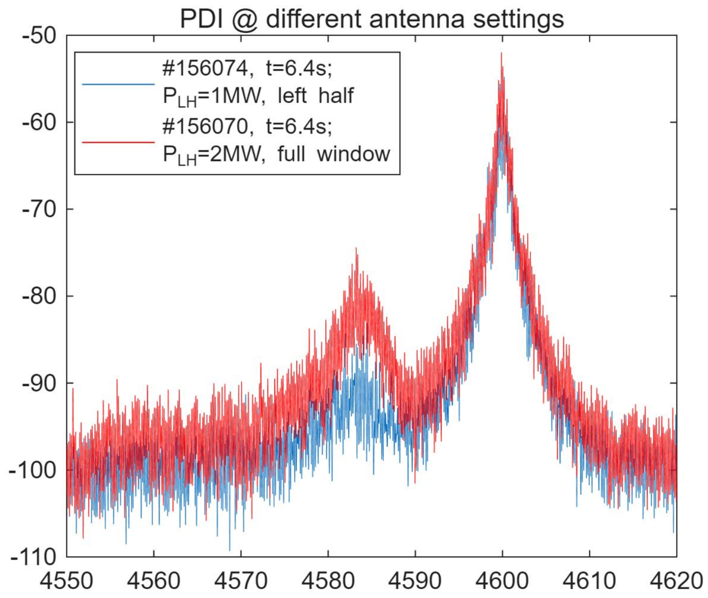
[Carli 2018]

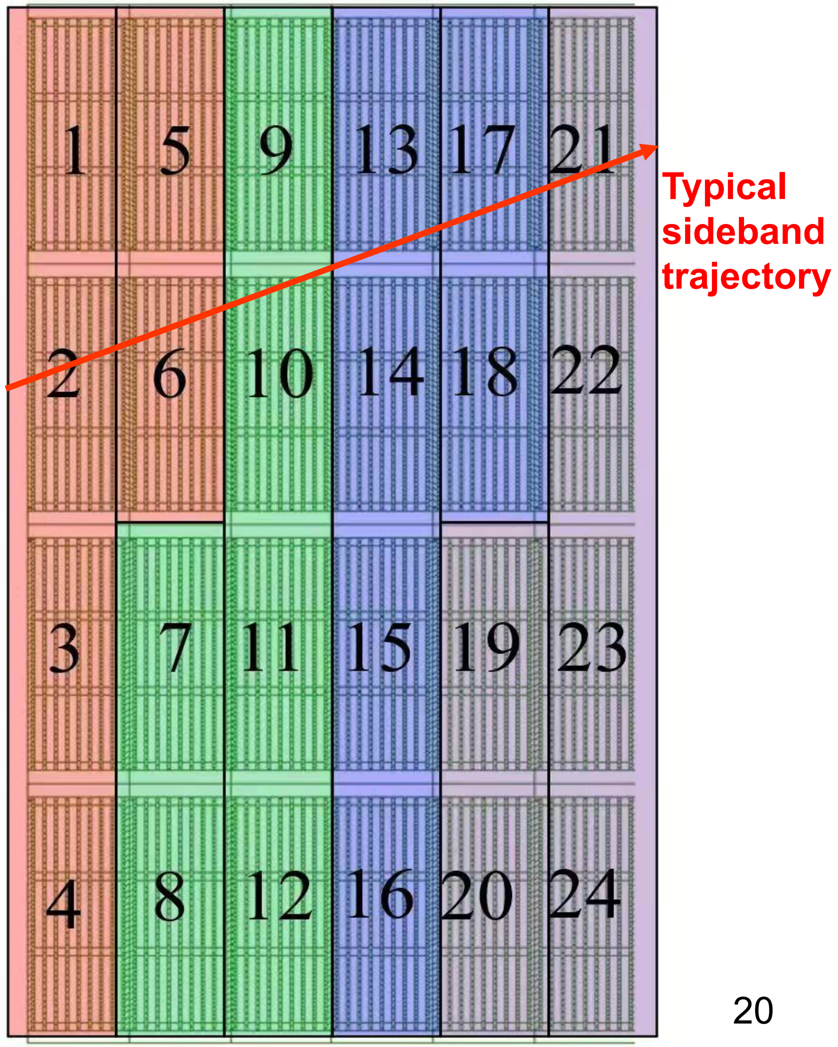


Recent experiments on EAST

- 1MW Toroidal half antenna array VS 2MW full array
- Same power density → same growth rate of PDI
- Shorter trajectory length → weaker sideband
- Convective instability saturated by finite pump width verified



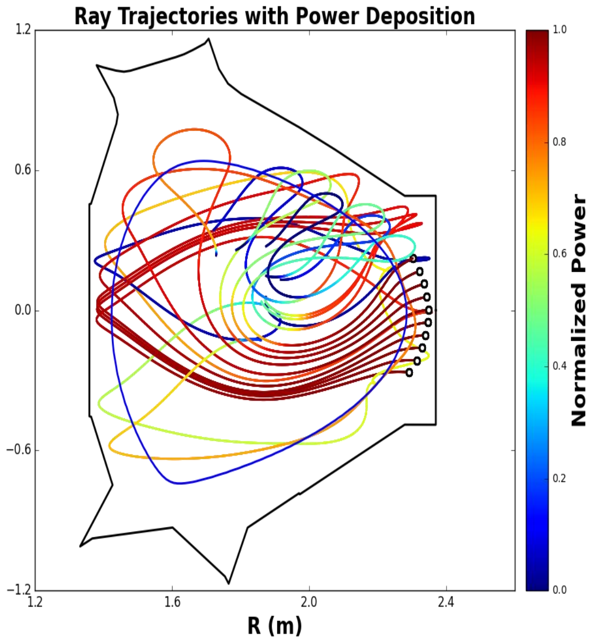


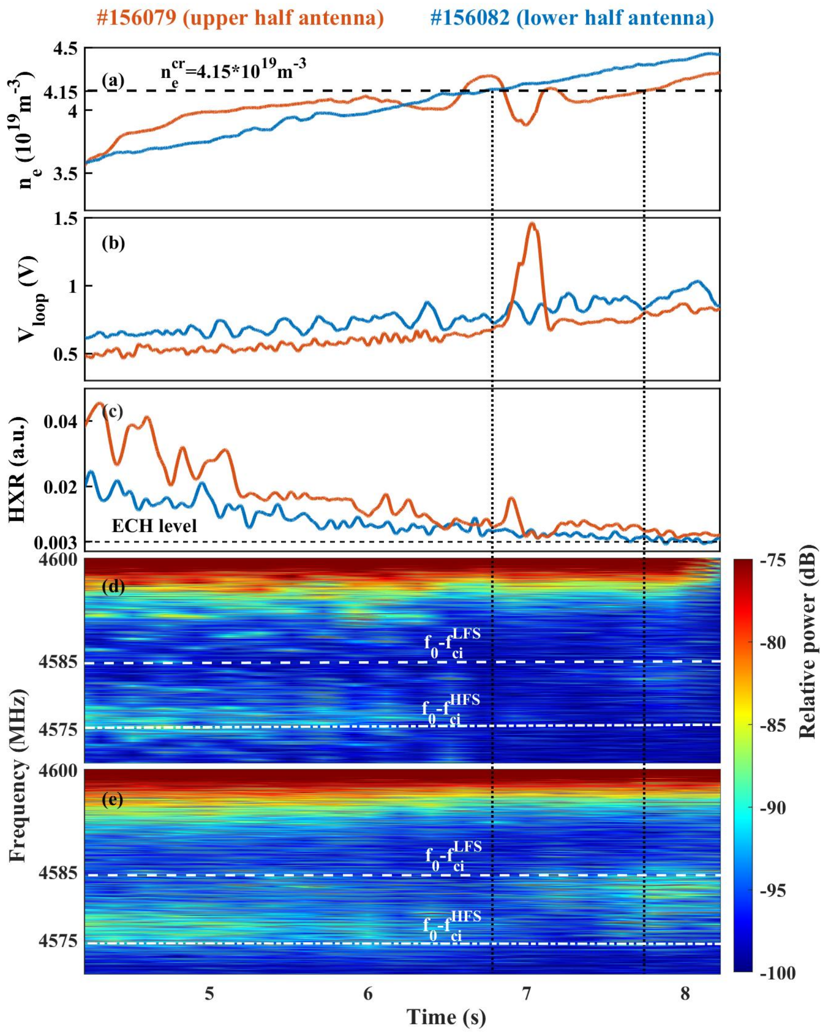


Recent experiments on EAST (ctd)

- $n_e \uparrow$, $f_{sideband} \uparrow$, Δf: $\omega_{ci}|_{HFS} \rightarrow \omega_{ci}|_{LFS}$
- During HFS PDIs, strong poloidal asymmetric
  - ✓ lower half antenna: stronger PDI and lower LHCD efficiency
  - ✓ physics: ray focusing due to poloidal asymmetry of plasma configuration
- During LFS PDIs, symmetric drive, which implies strong power deposition near the mouth.

[Chen….Gao submitted 2026]

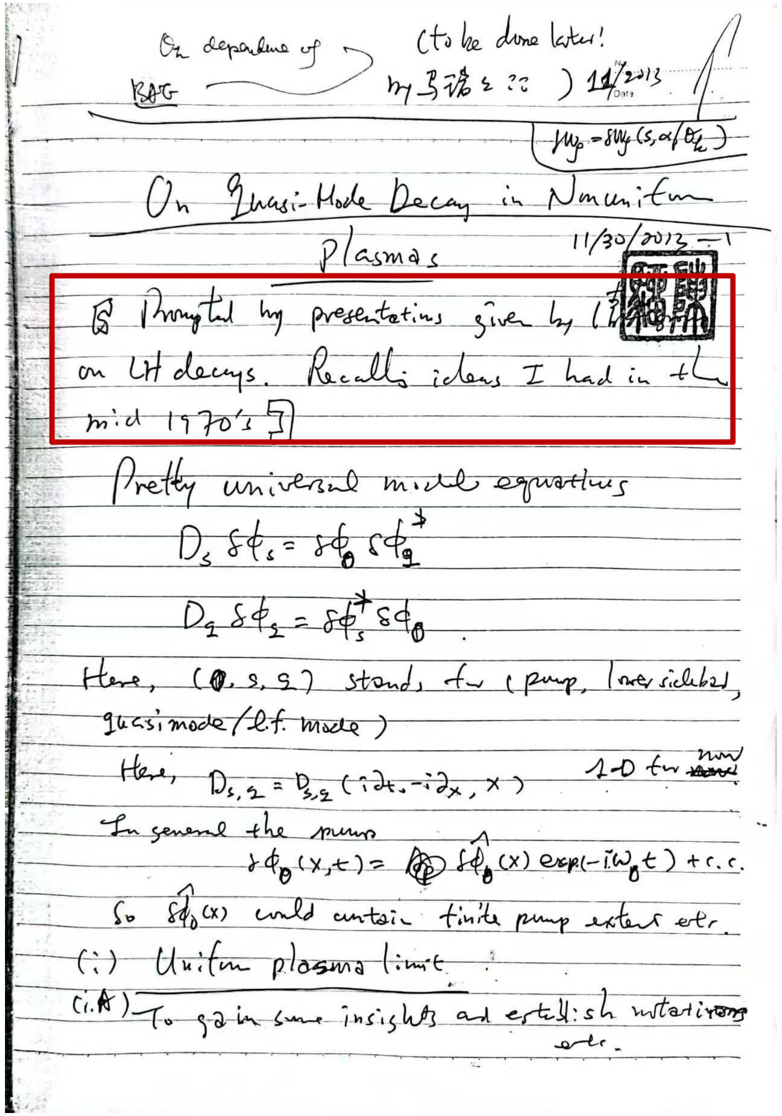




Summary

- The PDI theory is extended to adapt to the scenarios where non-resonant quasi-mode decays are dominant, and a self-consistent modeling and simulation of LHWs in the SOL plasma by coupling the propagation of waves to the power transfer among waves by PDIs is performed.
- Frequency sidebands and wavenumber broadening appear due to PDIs and a cool and dense SOL leads to considerable PDI growth rate and convective loss, which results in the density limit of LHCD
- A theoretical scaling relation of the density limit, which shows agreement with simulation and experimental results, which indicates that LHCD remains a promising method of driving plasma current for ITER and future fusion reactors.
- Dedicated experiments on EAST performed recently

Discussion with Prof. Liu Chen in 2013



On dependence of BAG → (to be done later! by 马瑞 ??) 11/2013

$\delta\omega_p = \delta\omega_p(s, \alpha / \theta_k)$

On Quasi-Mode Decay in Nonuniform Plasmas

11/30/2013 — 1

[Prompted by presentations given by (…) on LH decays. Recalling ideas I had in the mid 1970's]

Pretty universal model equations

$$D_s \delta\phi_s = \delta\phi_0 \delta\phi_q^*$$

$$D_q \delta\phi_q = \delta\phi_s^* \delta\phi_0 .$$

Here, (0, s, q) stands for (pump, lower sideband, quasimode/l.f. mode)

Here, $D_{s,q} = D_{s,q}(i\partial_t, -i\partial_x, x)$ 1-D for now

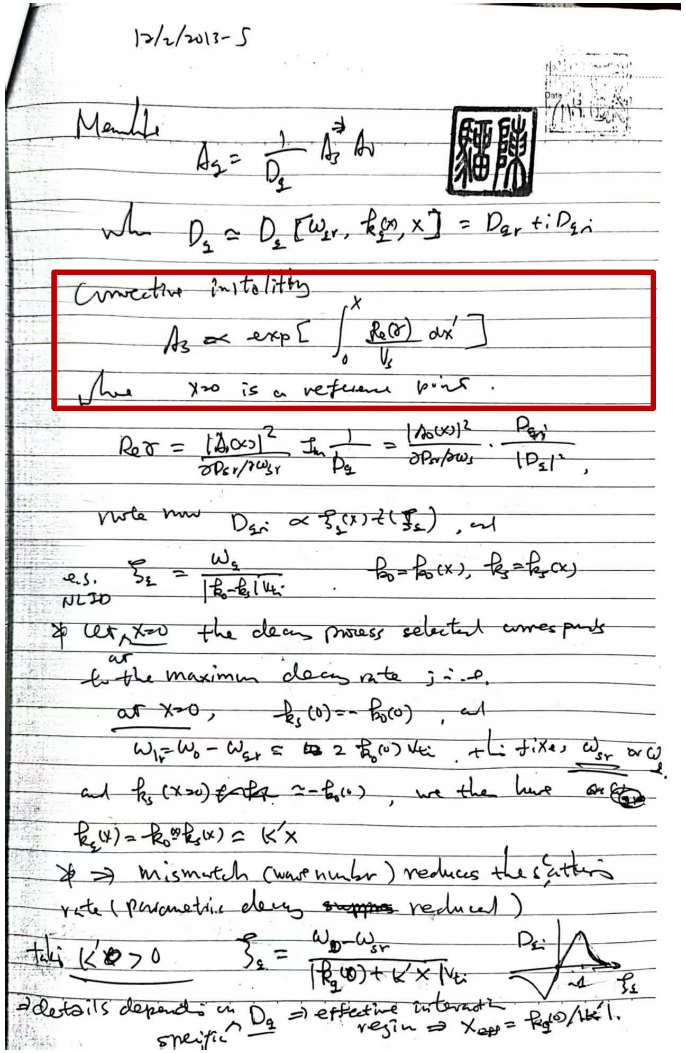
In general the pump

$$\delta\phi_0(x,t) = \hat{\delta\phi}_0(x)\exp(-i\omega_0 t) + c.c.$$

So $\hat{\delta\phi}_0(x)$ could contain finite pump extent etc.

(i) Uniform plasma limit

(i.A) To gain some insights and establish notations etc.



12/2/2013 - 5

Meanwhile

$$A_q = \frac{1}{D_q} A_s^* A_0$$

where $D_q \simeq D_q[\omega_{qr}, k_q(x), x] = D_{qr} + iD_{qi}$

Convective instability

$$A_s \propto \exp\left[\int_0^x \frac{Re(\gamma)}{V_s} dx'\right]$$

where $x=0$ is a reference point.

$$Re\,\gamma = \frac{|A_0(x)|^2}{\partial D_{sr}/\partial\omega_{sr}} Im\frac{1}{D_q} = \frac{|A_0(x)|^2}{\partial D_{sr}/\partial\omega_s} \cdot \frac{D_{qi}}{|D_q|^2},$$

note now $D_{qi} \propto \xi_q(x) Z(\xi_q)$, and

e.g. NLID $\xi_q = \frac{\omega_q}{|k_0 - k_s| v_{ti}}$. $k_0 = k_0(x)$, $k_s = k_s(x)$

let at $x=0$ the decay process selected corresponds to the maximum decay rate; i.e.

at $x=0$, $k_s(0) = -k_0(0)$, and

$\omega_{1r} = \omega_0 - \omega_{sr} \simeq 2k_0(0)v_{ti}$. this fixes ω_{sr} or ω_q.

and $k_s(x=0) \simeq -k_0(0)$, we then have

$k_q(x) = k_0(x) - k_s(x) \simeq K'x$

⇒ mismatch (wave number) reduces the scattering rate (parametric decay reduced)

taking $K' > 0$ $\xi_q = \frac{\omega_0 - \omega_{sr}}{|k_q(0) + K'x| v_{ti}}$

D_{qi} ξ_q

⇒ details depending on specific D_q ⇒ effective interaction region ⇒ $x_{eff} = k_q(0)/|K'|$.

Details can be found in following references

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