Reconnection Rate in Nonstationary Magnetic Reconnection

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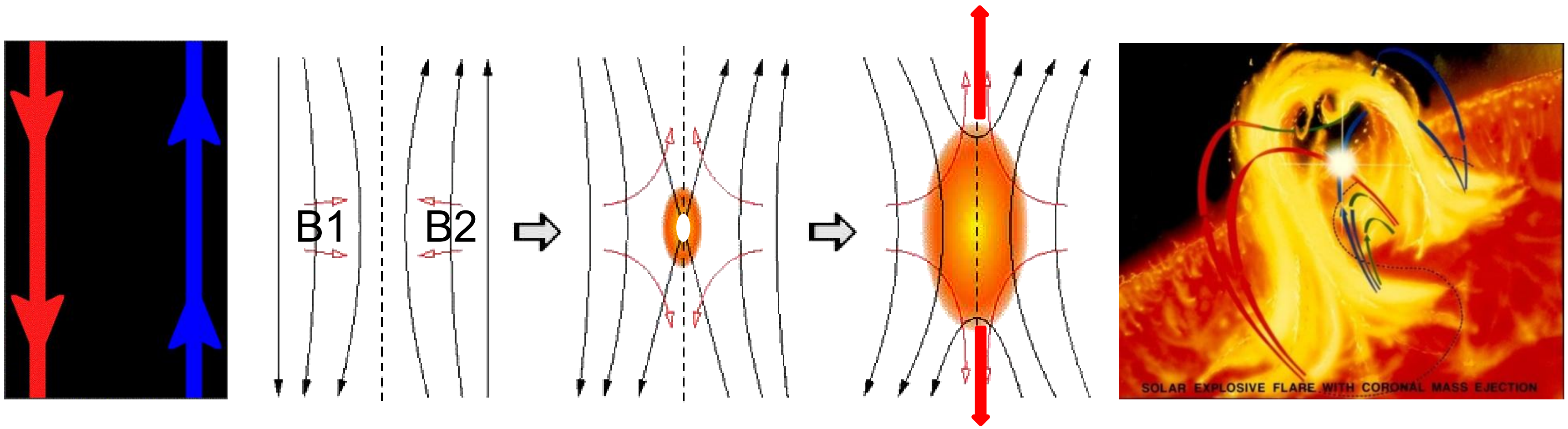
Magnetic Reconnection

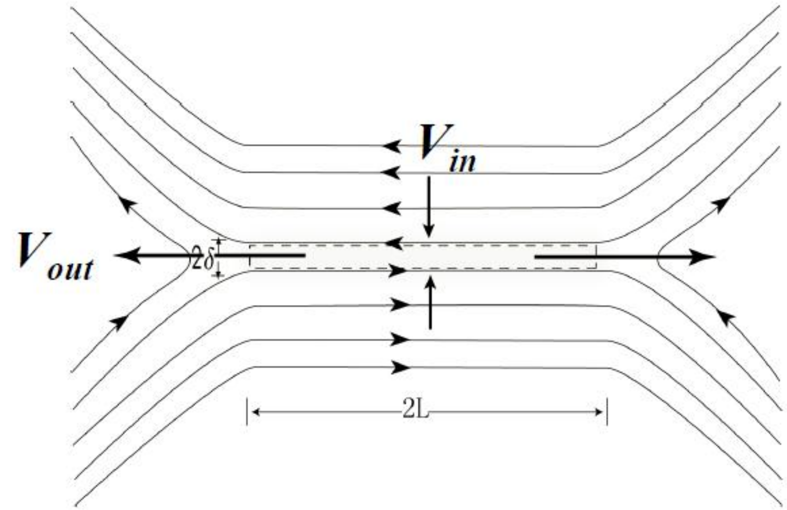
In magnetic reconnection, magnetic energy is transferred into plasma kinetic energy via topological rearrangement of magnetic field.



Reconnection rate: the amount of magnetic flux being reconnected per unit time.

Sweeter-Parker(SP) Model

- **2D MHD**
- **Steady-state**
- **Incompressibility**



Mass conservation: $V_{in} L \approx V_{out} \delta$

Pressure balance: $\frac{1}{2}\rho V_{out}^2 \approx \frac{B^2}{2\mu_0} \Rightarrow V_{out} \approx V_A$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \frac{\eta}{\mu_0}\nabla^2 \mathbf{B} \Rightarrow V_{in} B \approx \frac{\eta}{\mu_0}\frac{B}{\delta}$$

Reconnection rate R_0

$$R_0 = \frac{V_{in}}{V_A} = \frac{\delta}{L} = \frac{B_z}{B_0} = \frac{1}{\sqrt{S}}$$

$$S = \frac{\mu_0 L V_A}{\eta}$$

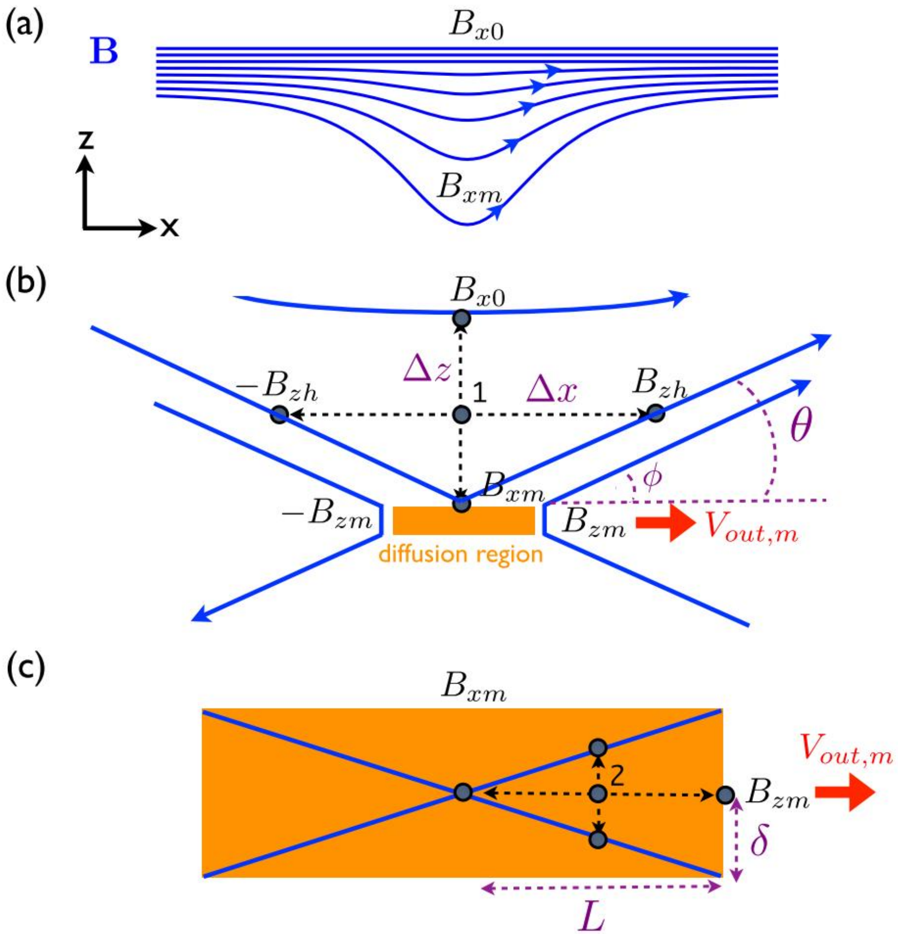
S: Lundquist number

In plasmas with large Lundquist, the model predicts extremely slow reconnection, conflicting with observations.

Reconnection Rate in Steady-state Reconnection

Derived from geometric relationships and the balance between magnetic pressure and magnetic tension in the upstream region.

$$\frac{B_{xm}}{B_0} = \frac{1 - (\frac{\Delta z}{\Delta x})^2}{1 + (\frac{\Delta z}{\Delta x})^2}$$



Reconnection Rate in Steady-state Reconnection

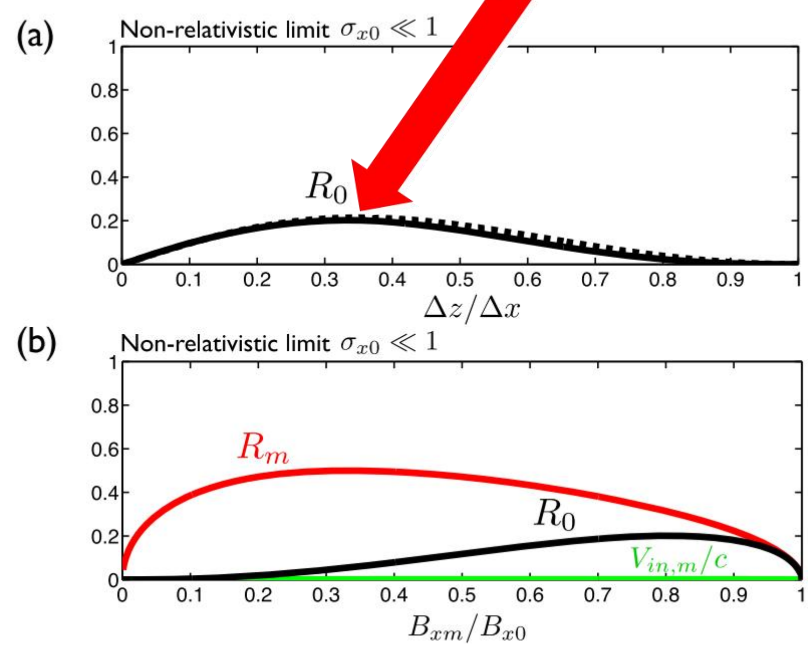
Reconnection Rate

$$R_0 = \frac{E_y}{B_{x0}V_{A0}} = \frac{B_{zm}}{B_{xm}}\frac{B_{xm}}{B_{x0}}\frac{V_{out}}{V_{A0}}$$

$$\approx \frac{\Delta z}{\Delta x}\left(\frac{1-(\frac{\Delta z}{\Delta x})^2}{1+(\frac{\Delta z}{\Delta x})^2}\right)^2\sqrt{1-(\frac{\Delta z}{\Delta x})^2}$$

$\frac{\Delta z}{\Delta x} \approx 0.31$时，

$R_{0,\max} \approx 0.2$

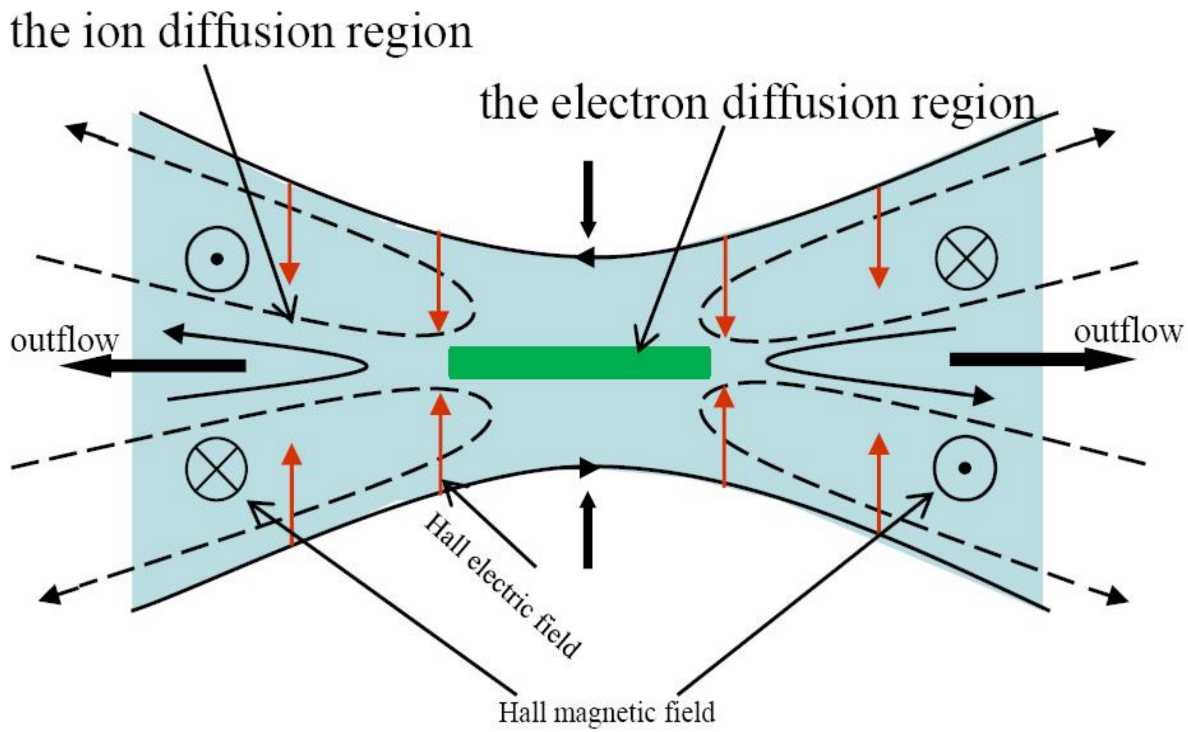


The reconnection rate is determined by the opening angle of the outflow region and is typically on the order of 0.1.

When the opening angle is sufficiently small, it is consistent with that of SP model.

[Liu et al., 2017]

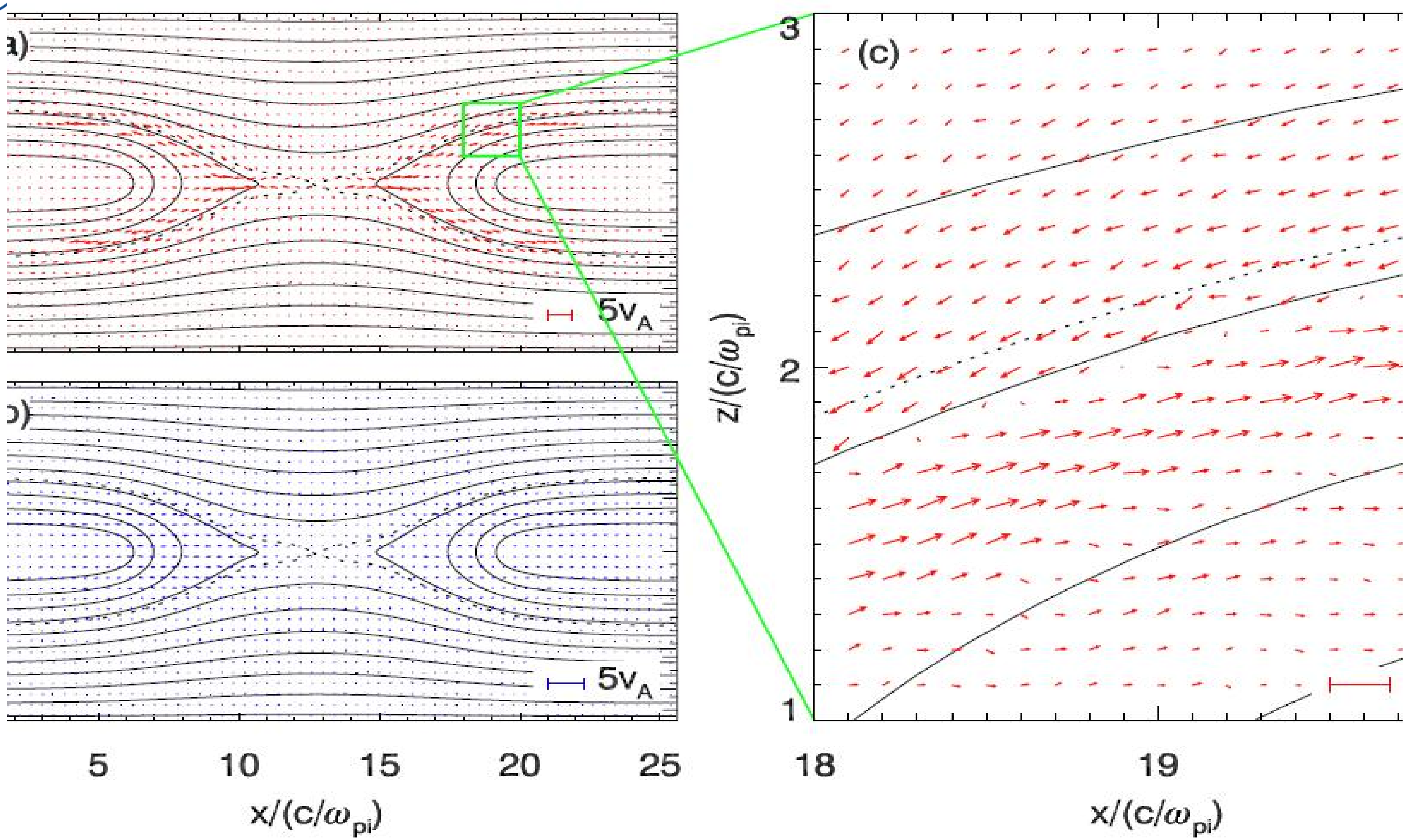
Collisionless Magnetic Reconnection



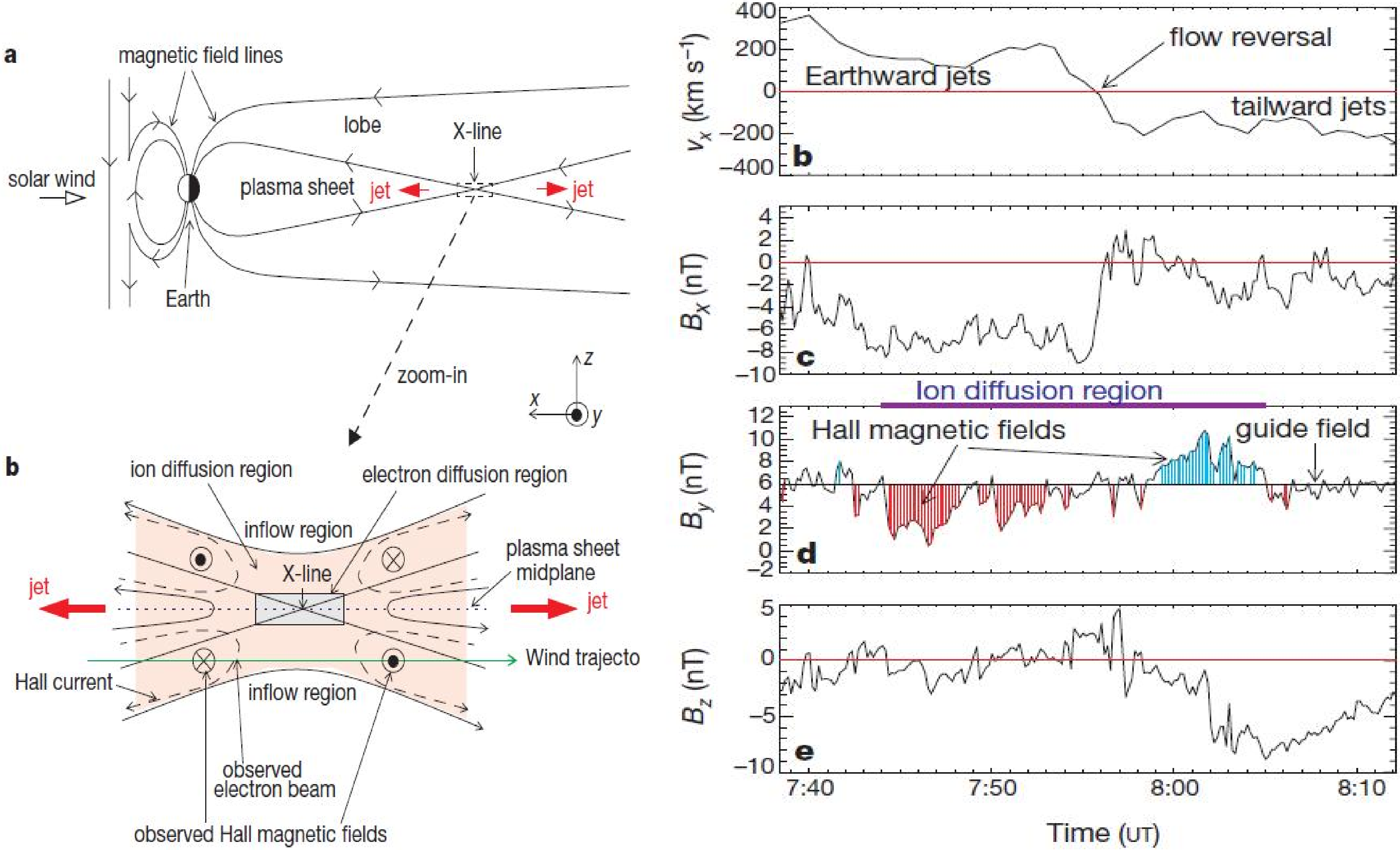
Ion diffusion Region(IDR): Electrons are magnetized, and ions are unmagnetized. The ion and electron motions are decoupled, resulting in quadrupole structure of out-of-plane magnetic field.

Electron Diffusion Region(EDR): Electrons are unmagnetized.

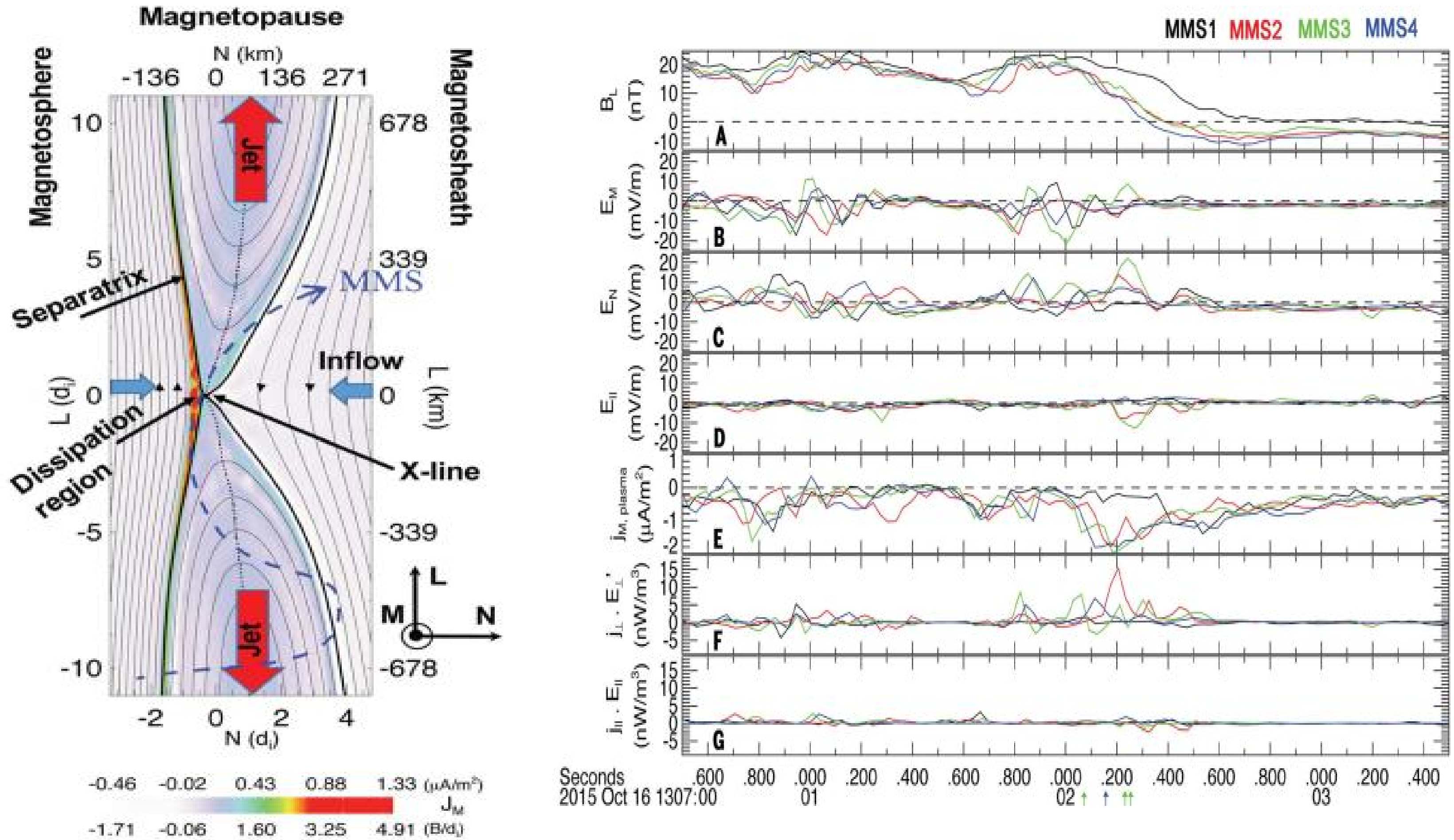
The reconnection rate is on the order 0.1.



The ion and electron motions from PIC simulations[Lu et al., 2010]

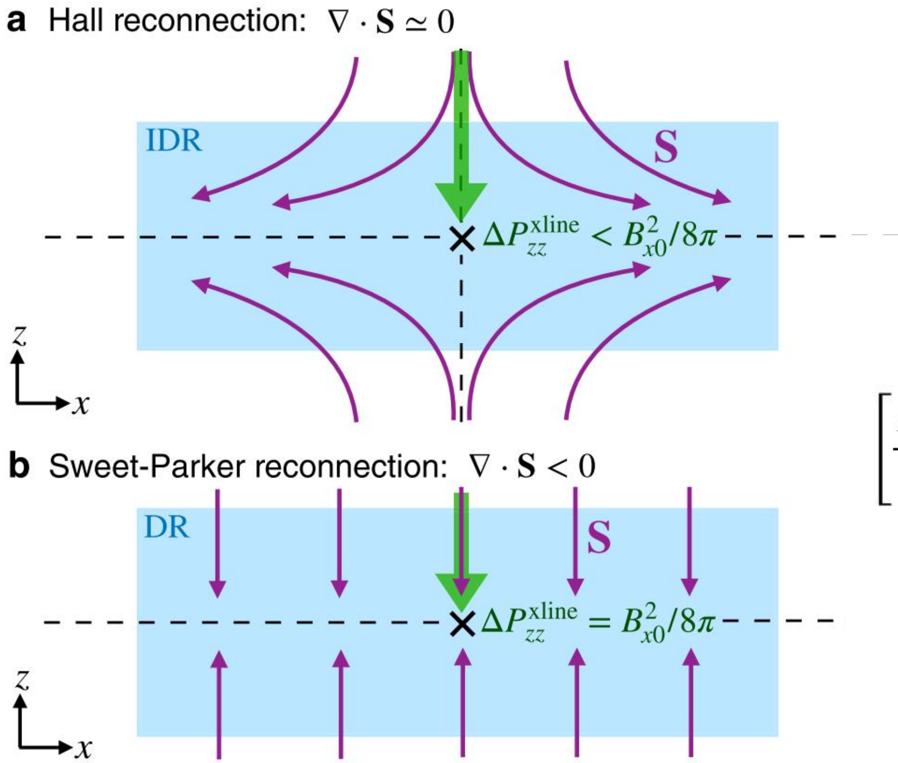


Satellite observations of ion diffusion region[Oieroset et al., 2001]

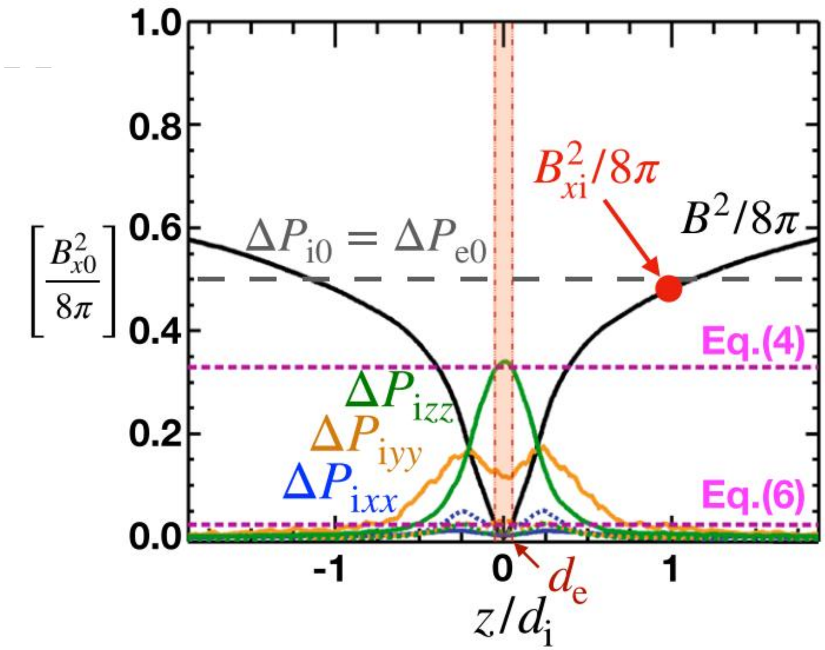


Satellite observations of electron diffusion region[Burch et al., 2016]

Reconnection Rate in Steady-state Reconnection



Thermal and magnetic pressure in the diffusion region

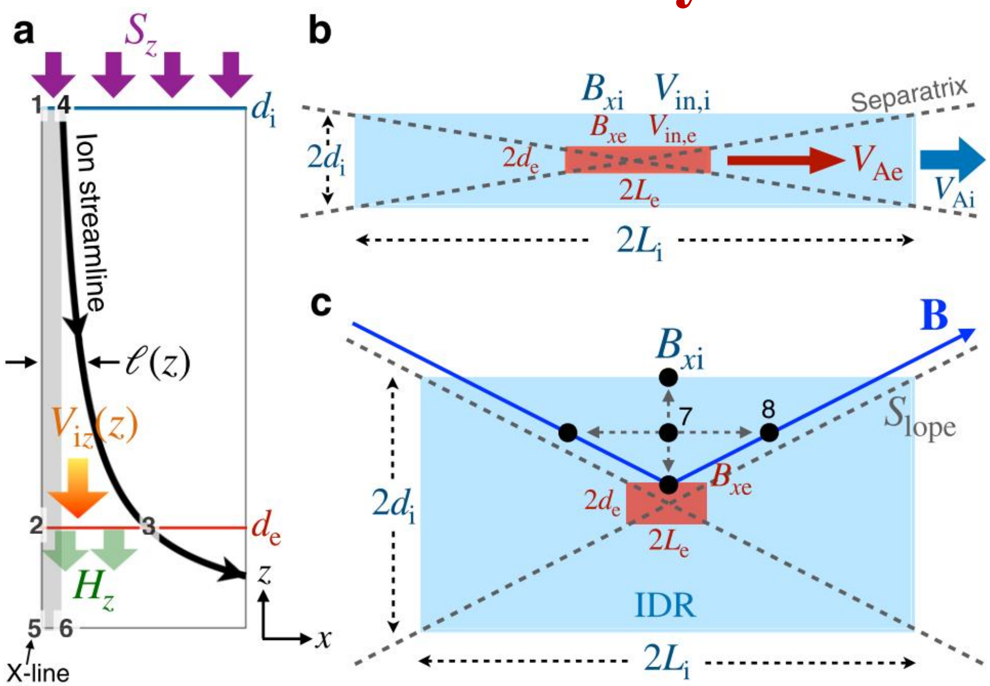


Steady-state + incompressible, then plasma density is a constant

Thermal pressure and magnetic pressure in the diffusion region satisfy the force balance condition.

$$\frac{B^2}{8\pi} + \Delta P_{zz} - \int^z \frac{\boldsymbol{B} \cdot \nabla B_z}{4\pi} dz' \approx \text{constant}$$

Reconnection Rate in Steady-state Reconnection



Assuming all J·E in the ion diffusion region (IDR) is converted into enthalpy flux.

$$S_{\text{lope}}^2 \approx \frac{1}{3}\left[\frac{1-\left(\frac{B_{x,e}}{B_{x,i}}\right)}{1+\left(\frac{B_{x,e}}{B_{x,i}}\right)}\right]$$

The aspect ratio in the EDR is the same as that in the IDR.

$$\frac{B_{x,e}}{B_{x,i}} \approx \left(\frac{m_e}{m_i}\right)^{1/4}$$

Reconnection Rate in Steady-state Reconnection

Based on the Equation in (Y. H. Liu, 2017), the reconnection rate is

$$R \approx S_{lope} \left(\frac{1 - S_{lope}^2}{1 + S_{lope}^2} \right)^2 \sqrt{1 - S_{lope}^2}$$

R=0.157, when m_i/m_e=1836
R=0.172, when m_i/m_e=400
R=0.188, when m_i/m_e=100

The reconnection rate depends on the mass ratio

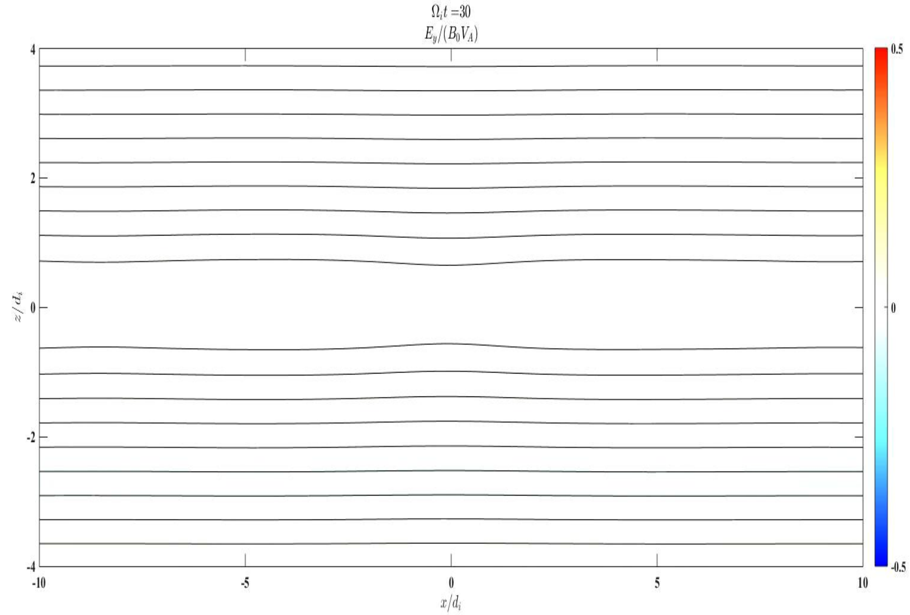
[Liu et al., 2022]

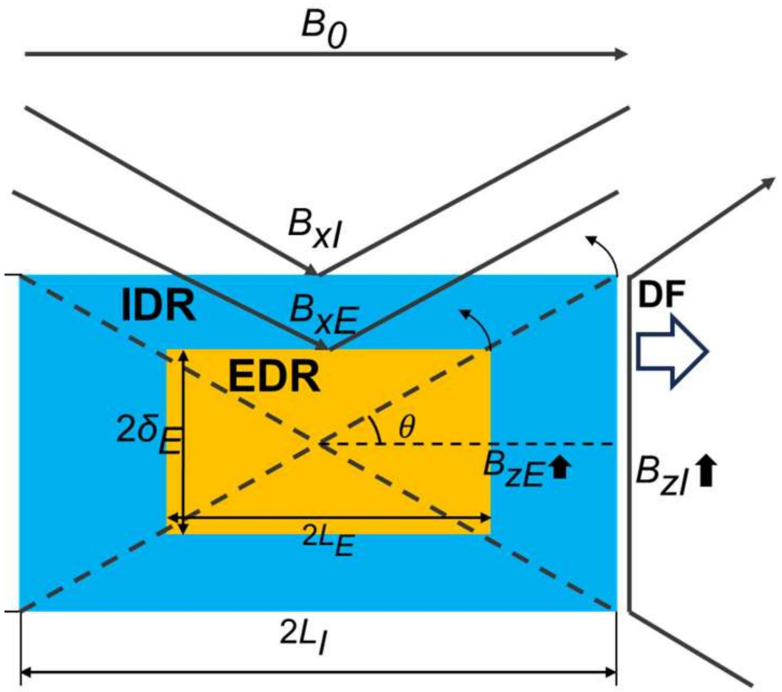
Reconnection Rate in Nonstationary Reconnection

In reality, magnetic reconnection proceeds in a nonstationary state.

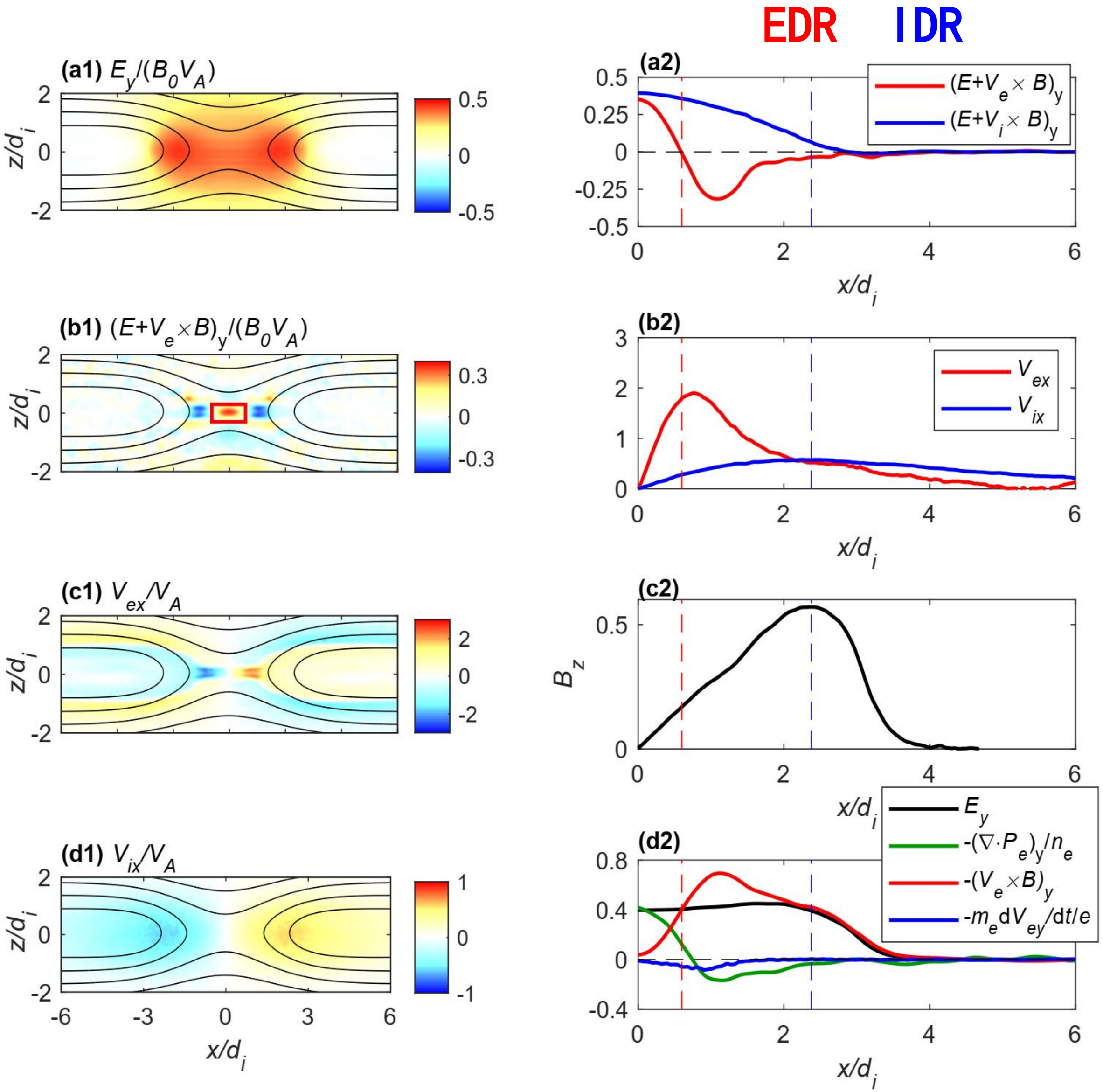
What does the reconnection rate in a steady state mean in a nonstationary reconnection?

Nonstationary Magnetic Reconnection





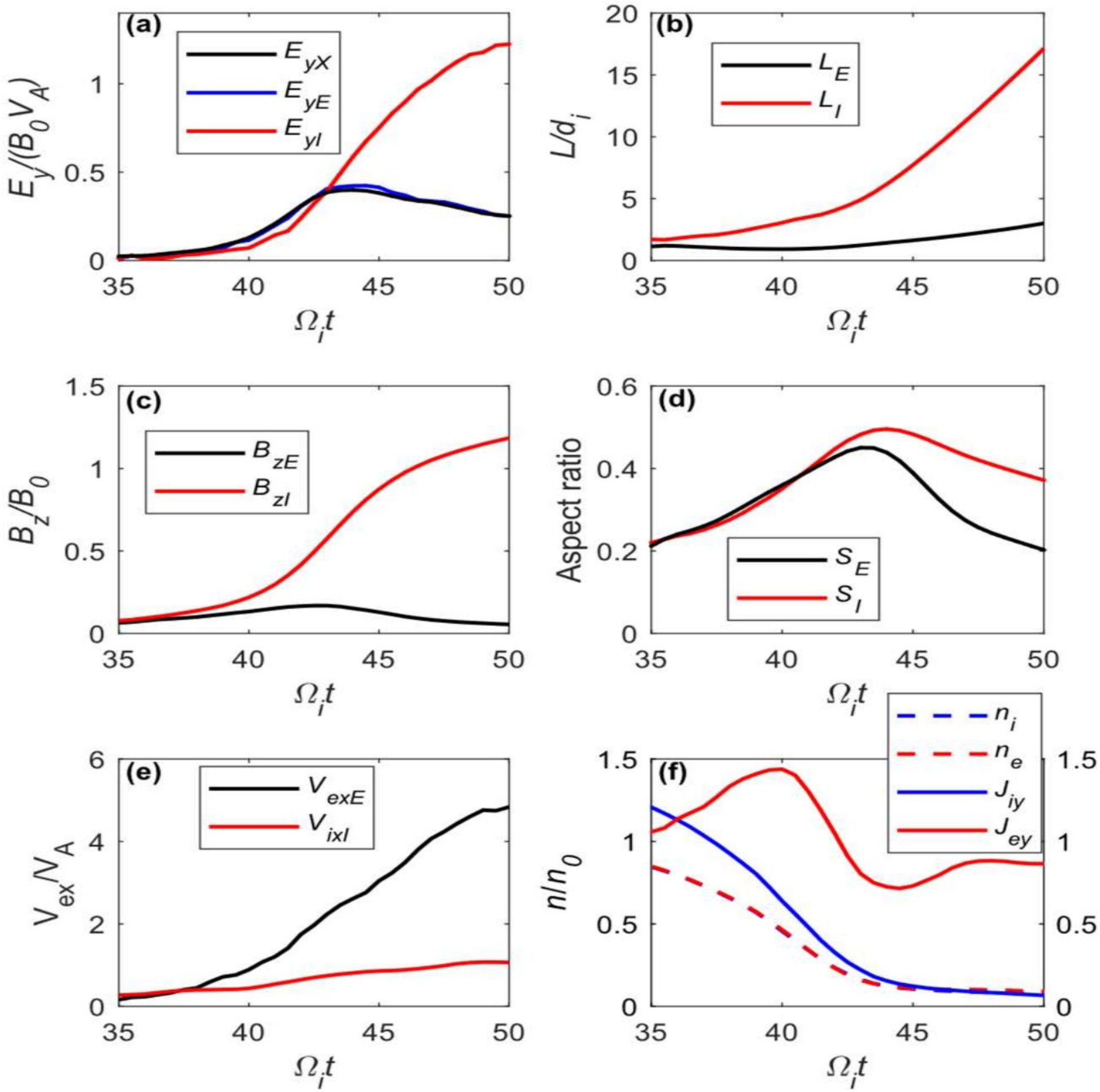
Diffusion Regions in nonstationary reconnection



The dipolarization front(DF) serves as the boundary of the IDR, where the motion of electrons and ions is coupled.

In the EDR, there are two subregions: Near the X-point, the electron pressure tensor term dominates the electric field. Far from the X-point, the Lorentz force term is the dominant factor.

The Evolution of Magnetic Reconnection



There are stages: Stage I, the boundaries of both EDR and IDR are almostly fixed, and their reconnection electric field is the same as that in the X point. The aspect ratio increases with the time.

The plasma density within the EDR continuously decreases, eventually reaching the background density level.

Electron current in the EDR increases while ion current decreases, leading to a state where the EDR current is ultimately dominated by the electrons.

Self-reinforcing Process of Magnetic Reconnection

The reconnection electric field at the center of the EDR is dominated by the electron pressure divergence term. The region where the electron pressure tensor term dominates is limited to the center of the EDR, and the reconnection electric field can be described as

$$E_{yX} \approx \frac{1}{e}\frac{\partial V_{ex}}{\partial x}\sqrt{2m_e T_e}$$

The reconnection electric field accelerates the electrons around the X line in the y direction, and the electron motions can be described as $d|V_{eyX}|/dt \propto |E_{yX}| \propto |V_{exE}|$.

When leaving away from the X line, the electrons perform meandering motions, their outflow speed $|V_{exE}|$ should be proportional to $|V_{eyX}|$. At last, we can get $d|V_{eyX}|/dt \propto |V_{eyX}|$.

Ion and Electron Outflow Speed

The x component of the electron and ion momentum equations along the line $z = 0$

$$n_s m_s \left(\frac{\partial V_{sx}}{\partial t} + V_{sx} \frac{\partial V_{sx}}{\partial x} \right) = j_{sy} B_z + \boldsymbol{q_s n_s} E_x - \frac{\partial P_{sxx}}{\partial x}$$

In general, the Lorentz term is dominant

electron momentum equation in the EDR

$$\boldsymbol{n_e m_e V_{ex} \frac{\partial V_{ex}}{\partial x} = j_{ey} B_z \approx j_y B_z}$$

$$V_{exE} \approx V_{Ae} \left(1 - S_L^2\right)^{\frac{1}{2}}$$

where $V_{Ae} = \dfrac{B_{xE}}{\sqrt{\mu_0 n_b m_e}}$

$$S_L \approx \frac{\delta_E}{L_E} \approx \frac{\delta_I}{L_I}$$

ion momentum equation in the IDR

$$\boldsymbol{n_i m_i V_{ix} \frac{\partial V_{ix}}{\partial x} = j_{iy} B_z \approx j_y B_z}$$

$$V_{ixI} \approx V_{Ai} \left(1 - S_L^2\right)^{\frac{1}{2}}$$

$$V_{Ai} = \frac{B_{xI}}{\sqrt{\mu_0 n_b m_i}}$$

The Reconnection Rate

Assume that the plasma is incompressible, and the reconnection electric field is uniform in the diffusion regions

$$\frac{B_{xI}}{B_0} \approx \frac{1-S_L^2}{1+S_L^2} \quad \textbf{and} \quad \frac{B_{xE}}{B_{xI}} \approx \left(\frac{m_e}{m_i}\right)^{1/4}$$

$$E_{yE} \approx S_L B_{xE} V_{Ae} \left(1-S_L^2\right)^{\frac{1}{2}} \approx \left(\frac{n_0}{n_b}\right)^{\frac{1}{2}} S_L \left(\frac{1-S_L^2}{1+S_L^2}\right)^2 \sqrt{1-S_L^2} B_0 V_{A0}$$

$$\Downarrow$$

$$R_{peak} \approx \left(\frac{n_0}{n_b}\right)^{\frac{1}{2}} S_{Lp} \left(\frac{1-S_{Lp}^2}{1+S_{Lp}^2}\right)^2 \sqrt{1-S_{Lp}^2} \quad \textbf{if} \quad V_{A0} = \frac{B_0}{\sqrt{\mu_0 n_0 m_i}}$$

$$R_{peak} \approx S_{Lp} \left(\frac{1-S_{Lp}^2}{1+S_{Lp}^2}\right)^2 \sqrt{1-S_{Lp}^2} \quad \textbf{if} \quad V_{A\mathbf{0}} = \frac{B_{x\mathbf{0}}}{\sqrt{\mu_0 n_b m_i}}$$

Electron Outflow Speed

Electron momentum equation in the EDR

$$n_e m_e V_{ex}\frac{\partial V_{ex}}{\partial x} \approx j_y B_z - e n_e E_x - \frac{\partial P_{exx}}{\partial x}$$

Based on Faraday's law, we can get $\oint \boldsymbol{E}\cdot d\boldsymbol{l} = \frac{\partial}{\partial t}\iint \boldsymbol{B}\cdot d\boldsymbol{s}$. When the reconnection rate approaches its peak value, $\frac{\partial}{\partial t}\iint \boldsymbol{B}\cdot d\boldsymbol{s} \approx 0$. At the same time, the parallel electric field is much smaller that the perpendicular electric field, the second term on the left hand is about $-\frac{1}{2}\mu_0 e^2 n_b^2 V_{exE}^2 \delta_E^2/L_E$.

The reconnection electric field is uniform in the EDR, and the work done to the electrons in the EDR is converted to the electron enthalpy flux away from the EDR. The third term on the left hand is about $-\frac{2B_{xE}^2}{5\mu_0 L_E}(1-\frac{\delta_E^2}{L_E^2})$

The electron outflow speed just downstream of the EDR can be easily obtained

$$V_{exE} \approx V_{Aeb}\left[\frac{1-S_{LE}^2}{5\left(1+\frac{\delta_E^2}{d_e^{*2}}\right)}\right]^{1/2}$$

where d_e^* is the electron inertial length based on the plasma density n_b.

and $V_{Aeb} = \frac{B_{xE}}{\sqrt{\mu_0 n_b m_e}}$

Ion Outflow Speed

Ion momentum equation in the IDR

$$n_i m_i V_{ix} \frac{\partial V_{ix}}{\partial x} \approx j_{iy} B_z - e n_i E_x - \frac{\partial P_{ixx}}{\partial x}$$

In the IDR (except the EDR), the electrons are magnetized, and then the electric field can be described as $\boldsymbol{E} = -\boldsymbol{V}_e \times \boldsymbol{B}$. Therefore, the equation can be written as

$$n_i m_i \left(\frac{\partial V_{ix}}{\partial t} + V_{ix} \frac{\partial V_{ix}}{\partial x} \right) = j_y B_z - \frac{\partial P_{ixx}}{\partial x}$$

In the similar way to that in the EDR, we can obtain the ion outflow speed just downstream of the IDR

$$V_{ixI} \approx \frac{V_{Aib}(1 - S_{LI}^2)}{\sqrt{5}}$$

and $V_{Aib} = \frac{B_{xI}}{\sqrt{\mu_0 n_b m_i}}$

The Reconnection Rate

$$\frac{B_{xI}}{B_0} \approx \frac{1-S_L^2}{1+S_L^2}$$ **and** $$\frac{B_{xE}}{B_{xI}} \approx \left(\frac{m_e}{m_i}\right)^{1/4}$$

$$E_{yE} \approx \left(\frac{n_0}{5n_b}\right)^{1/2} S_L \left(\frac{1-S_L^2}{1+S_L^2}\right)^2 \left(1-S_L^2\right)^{\frac{1}{2}} B_0 V_{A0}$$

↓

$$R_{peak} \approx \left(\frac{n_0}{5n_b}\right)^{\frac{1}{2}} S_{Lp} \left(\frac{1-S_{Lp}^2}{1+S_{Lp}^2}\right)^2 \sqrt{1-S_{Lp}^2}$$ **if** $$V_{A0} = \frac{B_{x0}}{\sqrt{\mu_0 n_0 m_i}}$$

$$R_{peak} \approx \frac{\mathbf{1}}{\sqrt{\mathbf{5}}} S_{Lp} \left(\frac{1-S_{Lp}^2}{1+S_{Lp}^2}\right)^2 \sqrt{1-S_{Lp}^2}$$ **if** $$V_{A0} = \frac{B_{x\mathbf{0}}}{\sqrt{\mu_0 n_b m_i}}$$

The maximum of R_{peak} is 0.09

Conclusions

1. The reconnection is a self-amplifying process, the the electron pressure tensor term near the X point leads to the increase of the electron outflow;
2. The plasma density in the EDR continuously decreases while the electron outflow velocity increases gradually. The Lorentz force term and other terms limit the outflow speed;
3. The peak rate is on the order of 0.1, which is independent of the ion-electron mass ratio but proportional to$(n_0/n_b)^{1/2}$ if the Alfven speed is defined by n_0.

Thanks！