Toward Predictive Understanding of Drift-Wave Microinstabilities and Transport in Tokamaks

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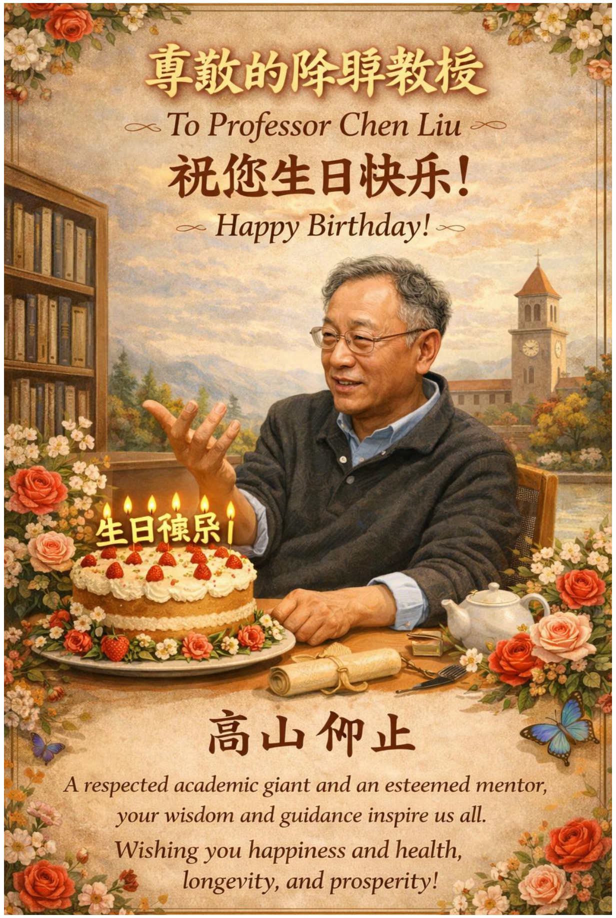
Happy Birthday to you, Professor Chen!

- This talk is devoted to Professor Liu Chen's 80th birthday.



尊敬的除聍教棱

To Professor Chen Liu

祝您生日快乐！

Happy Birthday!

高山仰止

A respected academic giant and an esteemed mentor,
your wisdom and guidance inspire us all.
Wishing you happiness and health,
longevity, and prosperity!

Picture taken around 2009

Trained PhD Students @ IFTS

Talk also devoted to IFTS 20-Year Anniversary 2011-2026

1. Huasheng Xie, chief scientist, ENN Fusion
2. Xishuo Wei, associate professor, SJTU
3. Chen Zhao， research scientist at General Atomics
4. Yueyan Li， Institute of Coal Industry, Tianjin
5. Hongwei Yang， postdoc, CAS-IPP
6. Gengxian Li， postdoc, Max-Planck Institute, Garching
7. Shenming Li， Honghu Fusion Company
8. Baobao Jia， Postdoc, CAS-IPP

Outline

- Introduction
- Gyrokinetic Tools
  - GTC
  - DAEPS
  - Reduced Model
- Gyrokinetic Simulation Findings
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Introduction

- Drift-wave microinstabilities, including ion temperature gradient (ITG), trapped electron mode (TEM), and kinetic ballooning mode (KBM), are key drivers of turbulent transport in tokamak plasmas.

- Developing predictive capability requires not only high-fidelity simulations, but also a unified framework that connects numerical results with underlying physics and reduced models.

- In this work, we demonstrate that an integrated gyrokinetic approach—combining first-principles simulation, eigenvalue analysis, and reduced model—provides a powerful pathway toward this goal.

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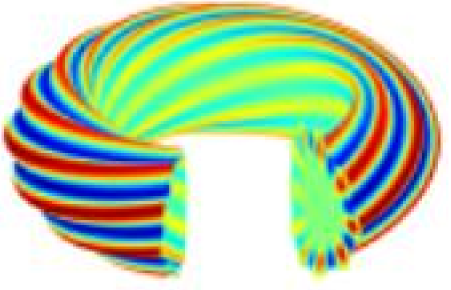
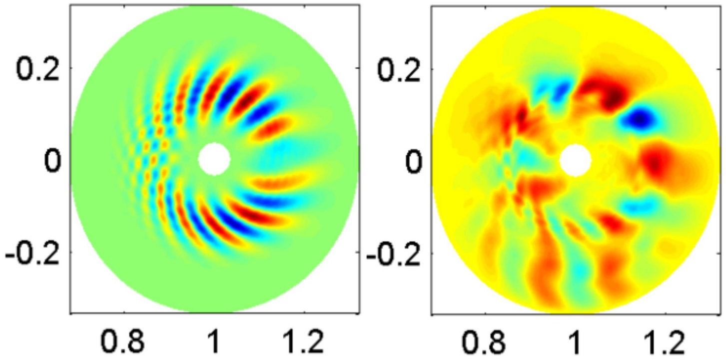
GTC :Global Gyrokinetic Simulation Code

- GTC (Gyrokinetic Toroidal Code, Z. Lin)
- Perturbative (delta-f) & Non-perturbative (full-f) simulation
- General geometry using EFIT & TRANSP data
- Kinetic electrons & electromagnetic simulation
- Neoclassical effects using Fokker-Planck collision operators conserving energy & momentum
- Toroidal & poloidal rotations; Multiple ion species
- Applications: microturbulence, EP physics & MHD modes
    - Parallelization >100,000 cores
    - Global field-aligned mesh
    - Parallel field solver PETSc

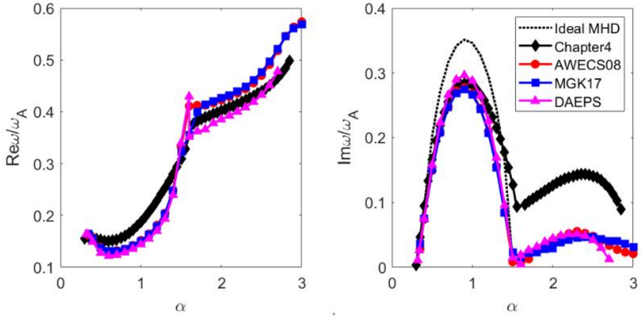




DAEPS Eigenvalue Solver

Drift Alfvén Energetic Particle Stability (DAEPS)

- The Drift Alfvén Energetic Particle Stability (DAEPS) code is an independently developed non-perturbative finite-element linear eigenvalue code[Li & Hu 2020].
- DAEPS is built on the general fishbone-mode dispersion relation framework and gyrokinetic theory, proposed by Prof. Chen Liu. [Chen and Zonca, 2016 RMP]
- DAEPS is developed to investigate and elucidate key low-frequency instabilities in fusion plasmas excited by energetic particles, such as shear Alfvén waves and drift Alfvén waves.



Dispersion relation benchmark of DAEPS

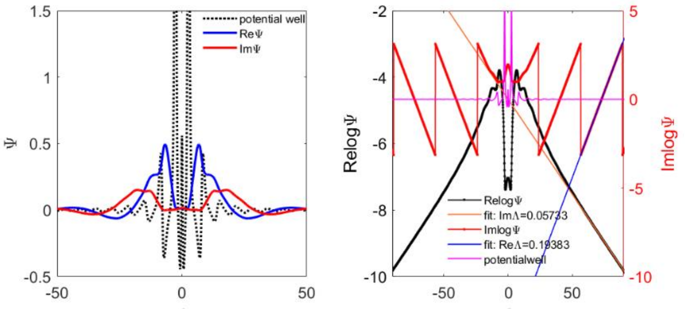
- δK_j is acquired from gyrokinetic equation:

$$\left(\frac{|v_\parallel|\partial_\theta}{qR} - i\omega + i\omega_d\right)\delta K_j = i\frac{q_j}{m_j}QF_{0j}\frac{\omega_{dj}}{\omega}J_0(k_\perp\rho_j)\delta\psi$$

- Vorticity equation in ballooning representation:

$$\left[\partial_\theta^2 + \frac{\omega(\omega - \omega_{\star pi})}{\omega_A^2} + V(\theta)\right]\Psi = \sum_j \left\langle\frac{4\pi q_j q^2 R^2}{k_\theta^2 c^2 \kappa_\perp}J_0(k_\perp\rho_j)\omega\omega_{dj}\delta K_j\right\rangle_v$$

where $V(\theta) = \frac{\alpha g}{\kappa_\perp^2} - \frac{\kappa_\perp''}{\kappa_\perp}$, $\Psi = \kappa_\perp\delta\psi$.



Mode structure and asymptotic behavior fitting of αTAE

Reduced ITG Gyrokinetic Model

Average ω_d ITG Gyrokinetic Model

- The average magnetic drift frequency in the bad-curvature region permits a satisfactory simplification of the ITG equation.

$$\left\{1+\frac{1}{\tau}-\frac{2}{\sqrt{\pi}}\int\int dx_\parallel dx_\perp x_\perp J_0^2\left(\sqrt{2b}x_\perp\right)\exp\left(-x^2\right)\right.$$
$$\left.\frac{\left\{\omega-\omega_{*in}\left[1+\eta_i\left(x^2-\frac{3}{2}\right)\right]\right\}}{\omega-\frac{\sqrt{2}zv_{ti}x_\parallel}{qR}-\bar{\omega}_{di}f\left(\hat{s}\right)\left(x_\perp^2/2+x_\parallel^2\right)}\right\}\hat{\delta\phi}\left(z\right)=0.$$

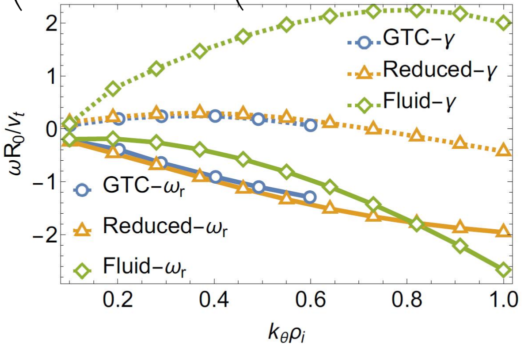
Partial FLR Expansion (L. Chen)

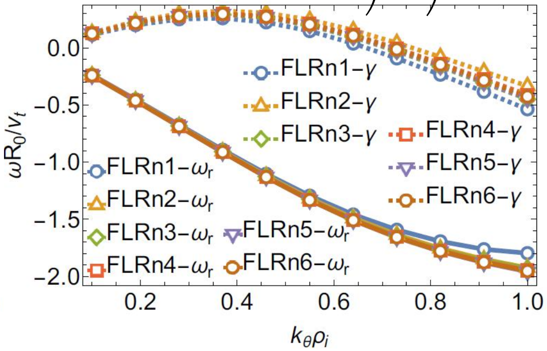
- Owing to the convergence of the first-order FLR expansion scheme, the equation can be further reduced to a Schrödinger-type eigenmode equation [BB Jia 2025].

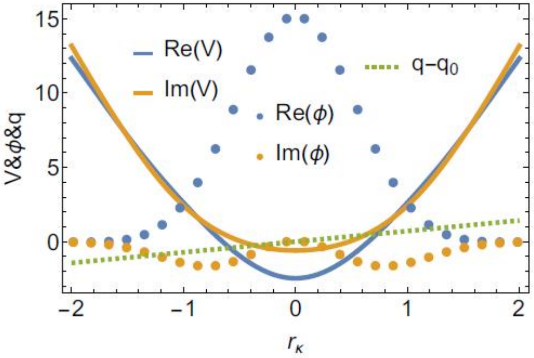
$$\left(\frac{\partial^2}{\partial z^2}+\frac{\overline{\omega_{di}}f(s)\left(1+\frac{1}{\tau}\right)+\left(\omega-\omega_{*i}\left(1-\frac{3}{2}\eta_i\right)\right)\mathcal{M}_{(1,0,0)}-\eta_i\omega_{*i}\left(\mathcal{M}_{(3,0,0)}+\mathcal{M}_{(1,2,0)}\right)}{\sqrt{2b_\theta\widehat{s^2}}\left(\left(\omega-\omega_{*i}\left(1-\frac{3}{2}\eta_i\right)\right)\mathcal{M}_{(2,0,1)}-\eta_i\omega_{*i}\left(\mathcal{M}_{(4,0,1)}+\mathcal{M}_{(2,2,1)}\right)\right)}\right)\delta\phi=0$$

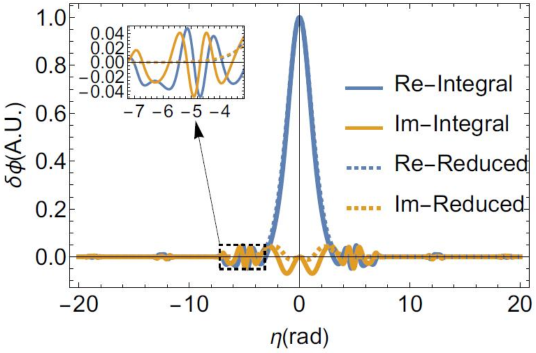
Ballooning representation

- Ballooning representation is extended to include reversed-shear case [Zonca & Chen 2002] [BB Jia 2026]









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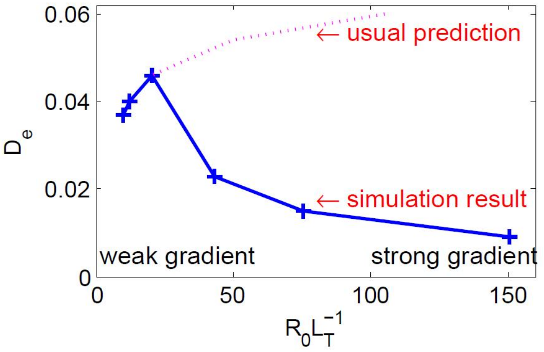
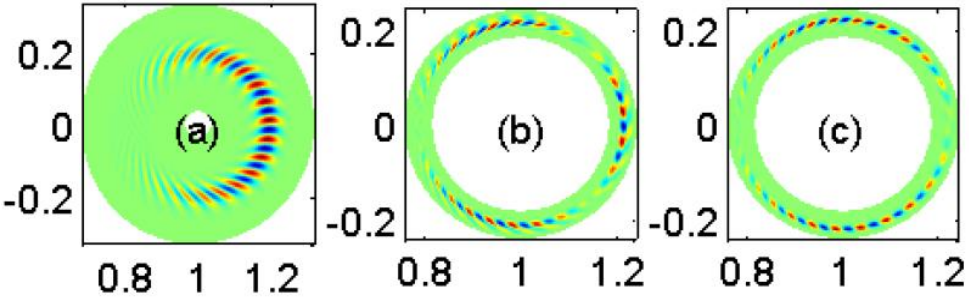
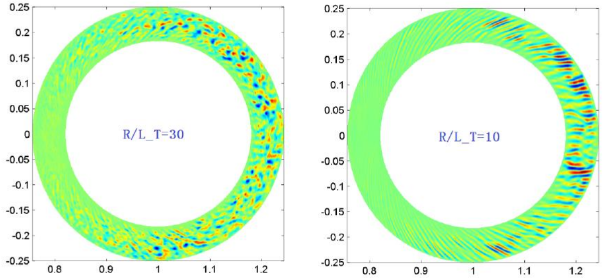
Mode Structures: Strong Gradient

GTC simulations find unconventional Mode Structures

- Weak gradient L-mode parameter gives conventional ballooning structures of TEM in GTC simulations(a).
- Strong gradient H-mode parameters give unconventional structures(b-c).





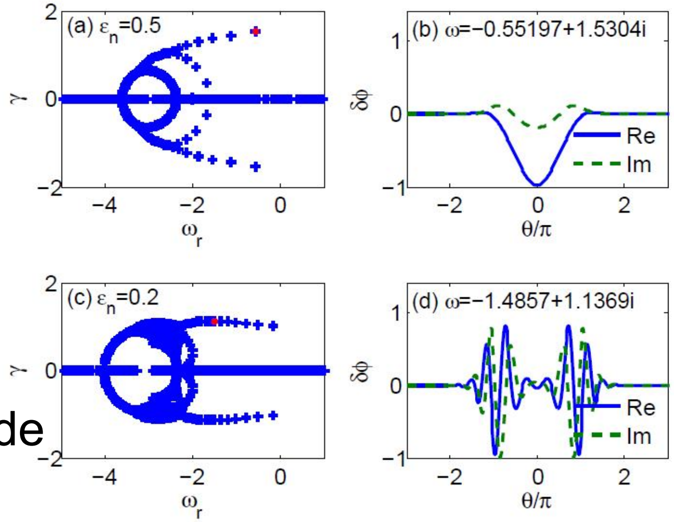


Modified Transport Behavior in Steep-Gradient

- A turning point (critical gradient) exists for the revers trend of the transport coefficients.[HS Xie 17 PRL]
- Strong gradients (left) small eddy size. Weak gradier (right) large eddy size

Reduced model:

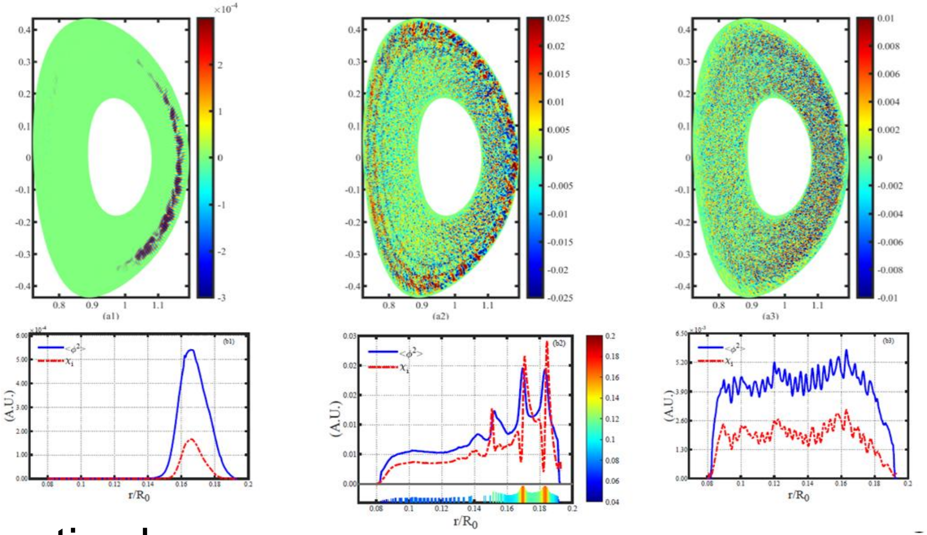
- Model eigenmode equation of drift wave predicts mode structures in different parameters. [HS Xie 15]



Mode Structure: Reversed Shear

CFETR Simulation with Reversed Shear

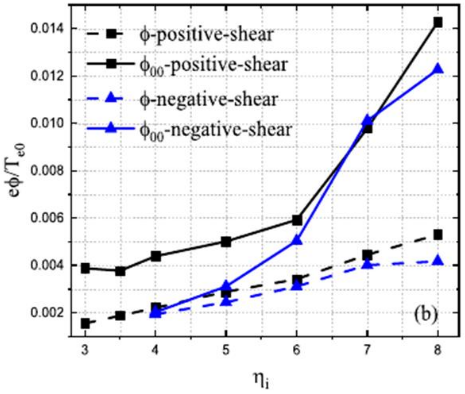
- Negative magnetic shear suppresses linear instabilities. Instability grows in the positive-shear region.
- In nonlinear stage (w/o zonal flow), turbulence accumulates at regions with the high mode rational-surface density.

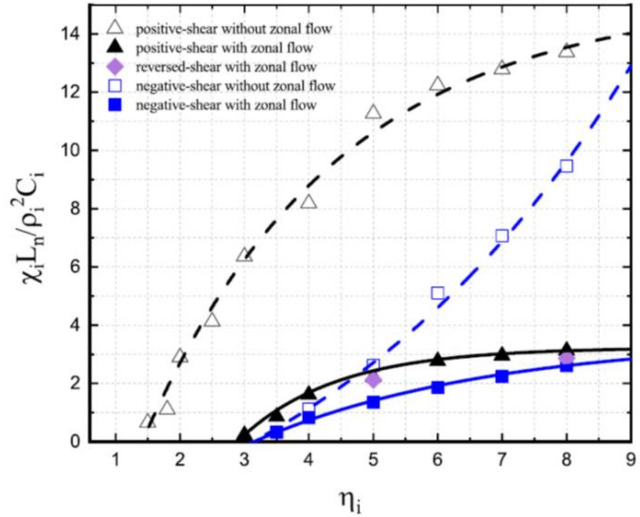


Dimits Shift vs Magnetic Shear

- Dimits shift is significant for positive magnetic shear, gets smaller for zero shear (weak shear), and totally vanishes for negative shear. [DK Yang 24]

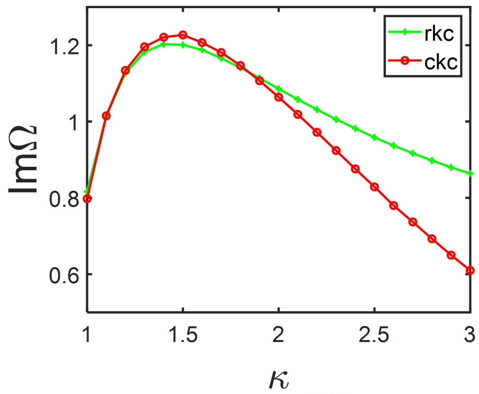
- It is the weak zonal flow generation close to marginal stability for the case of negative weak shear, leading to disappearance of Dimits shift.





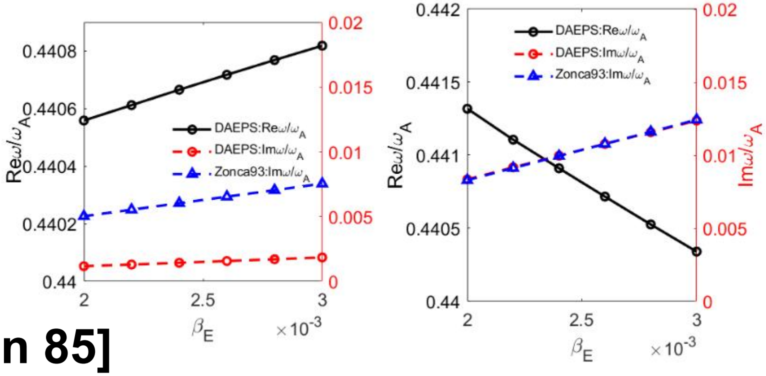
Eigenvalue Analysis

- GK simulation captures only most unstable modes and consumes a lot of computational resource. Eigenvalue code provide fast and accurate linear dispersion and mode structure, good for parameter scan.



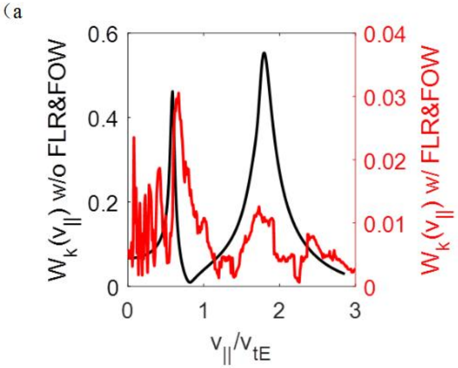
Effect of Elongation on BAE Instability [Zonca 96]

- Growth rate firstly increases and then decreases with elongation κ . [GX Li 23]
- Decrease of growth rate in the latter range is dominated by reduction of wave-particle resonance by elongation κ and FLR/FOW effect
- Increase of growth rate in the beginning range is dominated by the ideal fluid response.



TAE Physics Excited by Energetic Particles [Chen 85]

- Incorporating FLR/FOW effects into the DAEPS code leads to predicted growth rates lower than theoretical predictions.
- FLR/FOW effects modifies the structure of velocity-space resonances and overall exhibits a stabilizing effect.[Li 20]



ITG Reduced Models for Reversed Shear

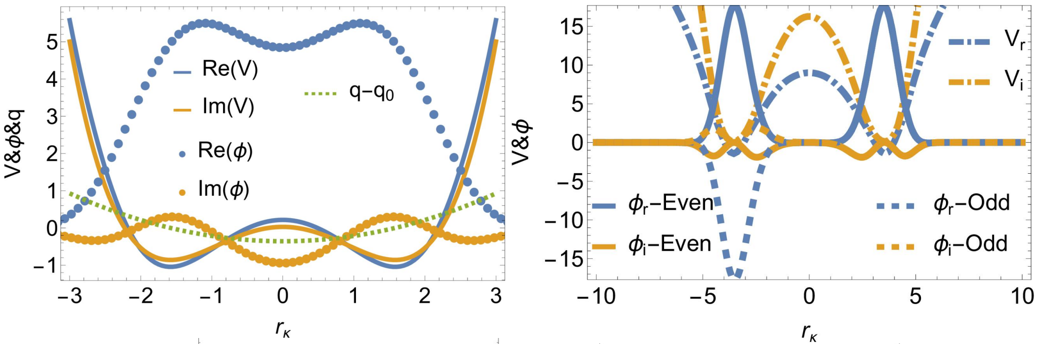
Reduced Model for Reversed Shear

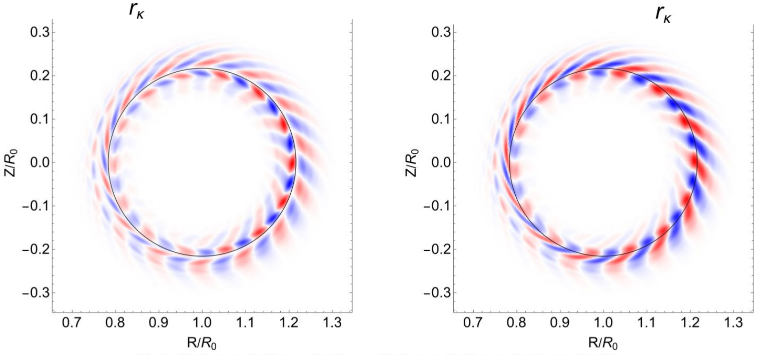
- Generalized translational invariance [Zonca 2002] extend the validity of ballooning representation, which is checked by GTC simulation

$$\left(\frac{\partial^2}{\partial r_\kappa^2}+\frac{\bar{\omega}_{di} f\left(\hat{s}\right)\left(1+\frac{1}{\tau}\right)+\mathcal{K}_0\left(\omega,\omega_*,\bar{\omega}_d,\zeta_\alpha,\zeta_\beta,\int d\vec{v}\ldots\right)}{\sqrt{2b_\theta}\mathcal{K}_1\left(\omega,\omega_*,\bar{\omega}_d,\zeta_\alpha,\zeta_\beta,\int d\vec{v}\ldots\right)}\right)\delta\phi\left(r_\kappa\right)=0.$$

Double-Well and Degenerate states

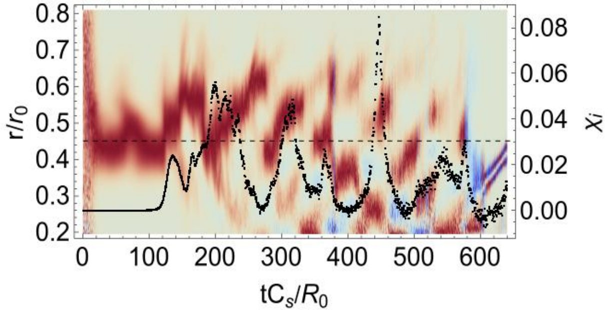
- Reverse shear, double rational surface leads to double-well potential structure.
- Double-well potential may introduce two degenerate eigenstates with nearly identical eigenvalues.





GTC Validation of Reduced Model Findings

- GTC simulations show both even and odd parity eigenstates in reversed shear case.
- The radial mode oscillation persists into the nonlinear saturation phase if w/o zonal flow.[Jia 26]



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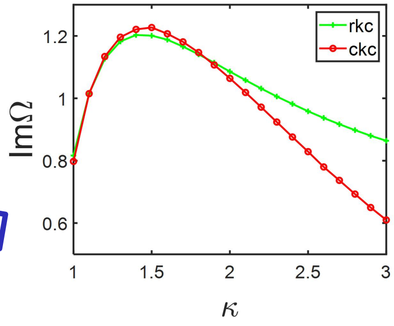
Integration and Closed-Loop Methodology

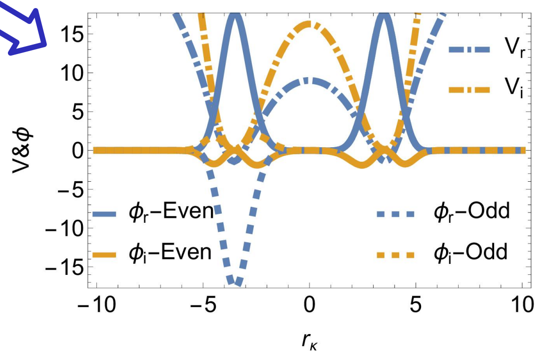
The integrated gyrokinetic approach establishes a closed-loop methodology linking computation, interpretation, and prediction.

- First-principles simulations identify robust phenomena, eigenvalue code provide fast and accurate linear dispersion and mode structure, and reduced models provide a lot of physics insights.
- This hierarchy provides both physical insight and practical capability, offering a general paradigm for turbulence and transport modeling and supporting predictive studies for next-generation fusion devices.

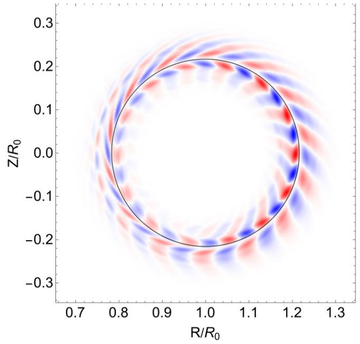
- **Use GK simulation more economically**
- **Understand physics better**



Eigenvalue



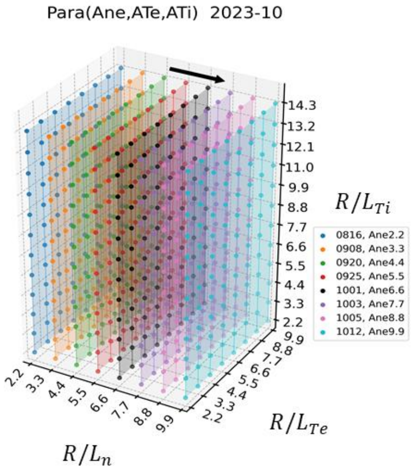
Reduced Model

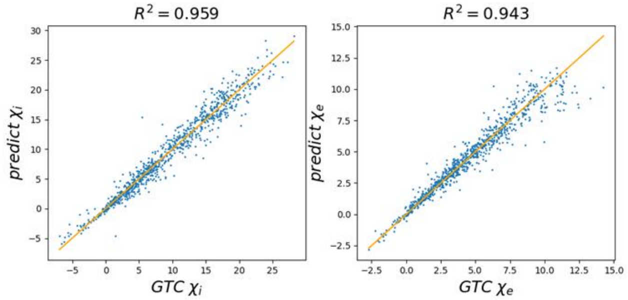


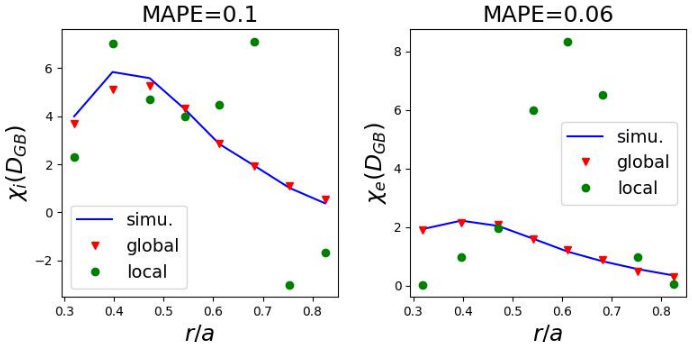
Simulation

Toward Predictive Turbulent Transport

AI Fusion—Turbulent transport Surrogate Model

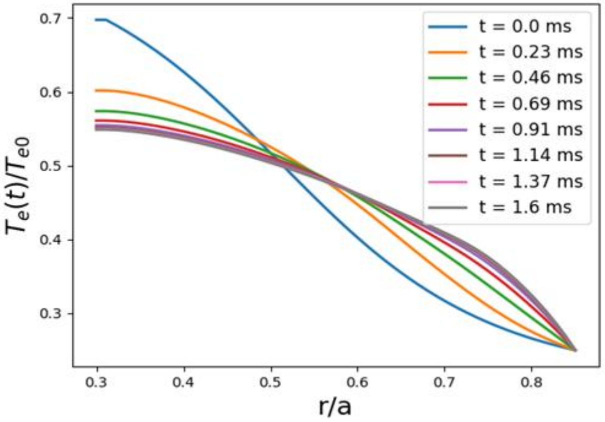






HJ Zhao, submitted 26

- Built a turbulence surrogate model based on 762 nonlinear GTC local-mode simulations.
- Invented a local-global mapping based on Neural Network.
- Turbulence surrogate model after local-global mapping can effectively predict turbulent transport at sampled points, which agrees well with global GTC simulations.



Thank You!