Approaches to Enhanced Fusion Energy Gain

Zheng-Mao Sheng (盛正卯)

Institute for Fusion Theory and simulation (IFTS), School of Physics, Zhejiang University

Collaborators: Shujun LIU, Yangchun LIU, Yijie ZHAO and D.Wu （SJTU）

IWLS 2026, April 09-12, 2026 Hangzhou, China

outline

- Introduction (Background and Motivation)
- **Non-Thermal Equilibrium ($T_e < T_i$) Nuclear Fusion**
- **Non-Maxwellian Ion distributions**
- **Summary and Discussion**

Introduction (motivation)

- Nuclear fusion energy is ideal future energy source because of its environment friendly, resource rich, … …
- Great achievements have been made in fusion plasmas research:

  MCF: JET(Europe) Q~0.3, EAST (China) long pulse discharged,

  KSTAR(Korea） high ion temperature, ITER (fusion in 2039), …

  ICF: NIF has realized ignition, fusion energy/laser energy ~ 5
- Many private companies are entering this field!

Introduction (motivation)

- **There are still many challenges before nuclear fusion become energy source!**
- **How to enhance energy gain Q? How to recycle Tritium in deuterium- tritium fusion reaction? How to keep steady state operation?**
- **It is still important to investigate alternative routes for fusion energy!**
- **Nuclear fusion Possibility** $\propto n_1 n_2 < \sigma v >$

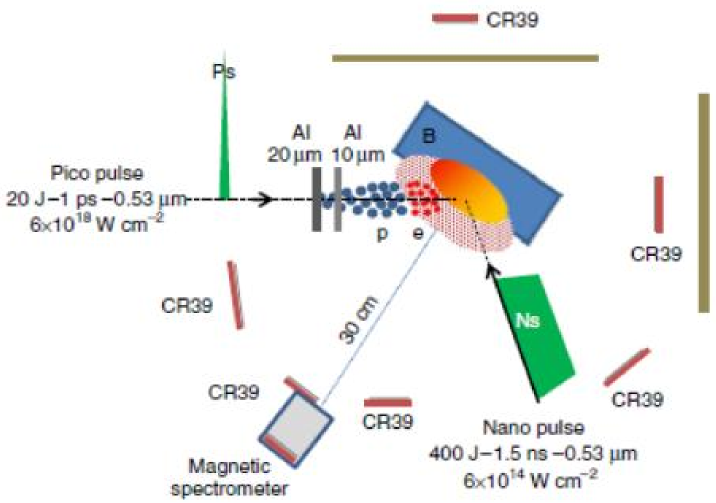
  High density + larger Cross section (suitable mass center energy)
- **Ion Beam with Thermal Plasma Fusion?**

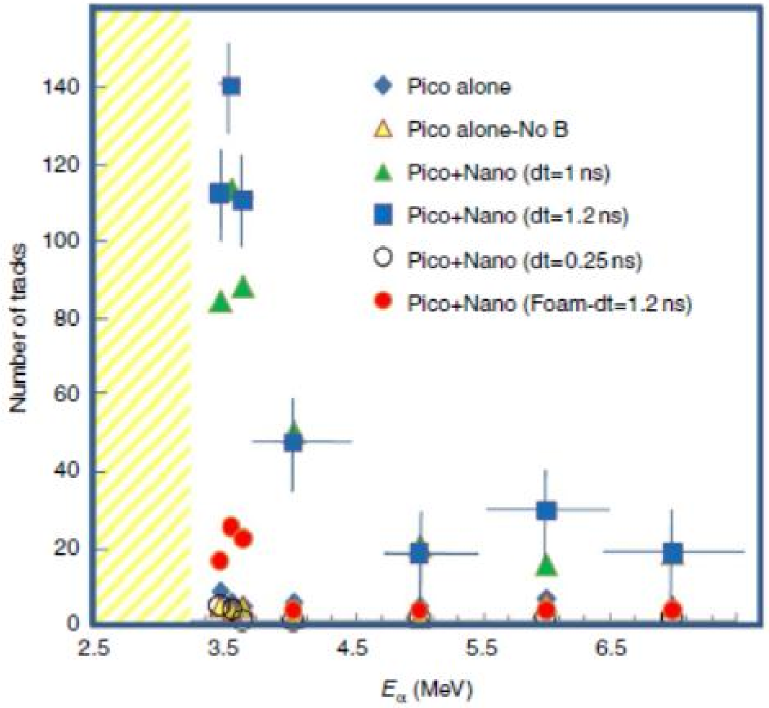
  How to reduce the stopping power of ion beam in the densed plasma ?

  Have to consider quantum effects !

➢ Beam with Thermal Plasma Fusion

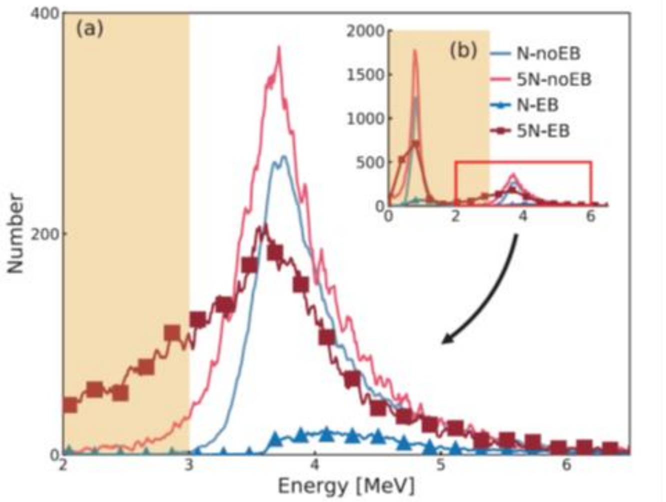
Proton-Boron Fusion Reaction



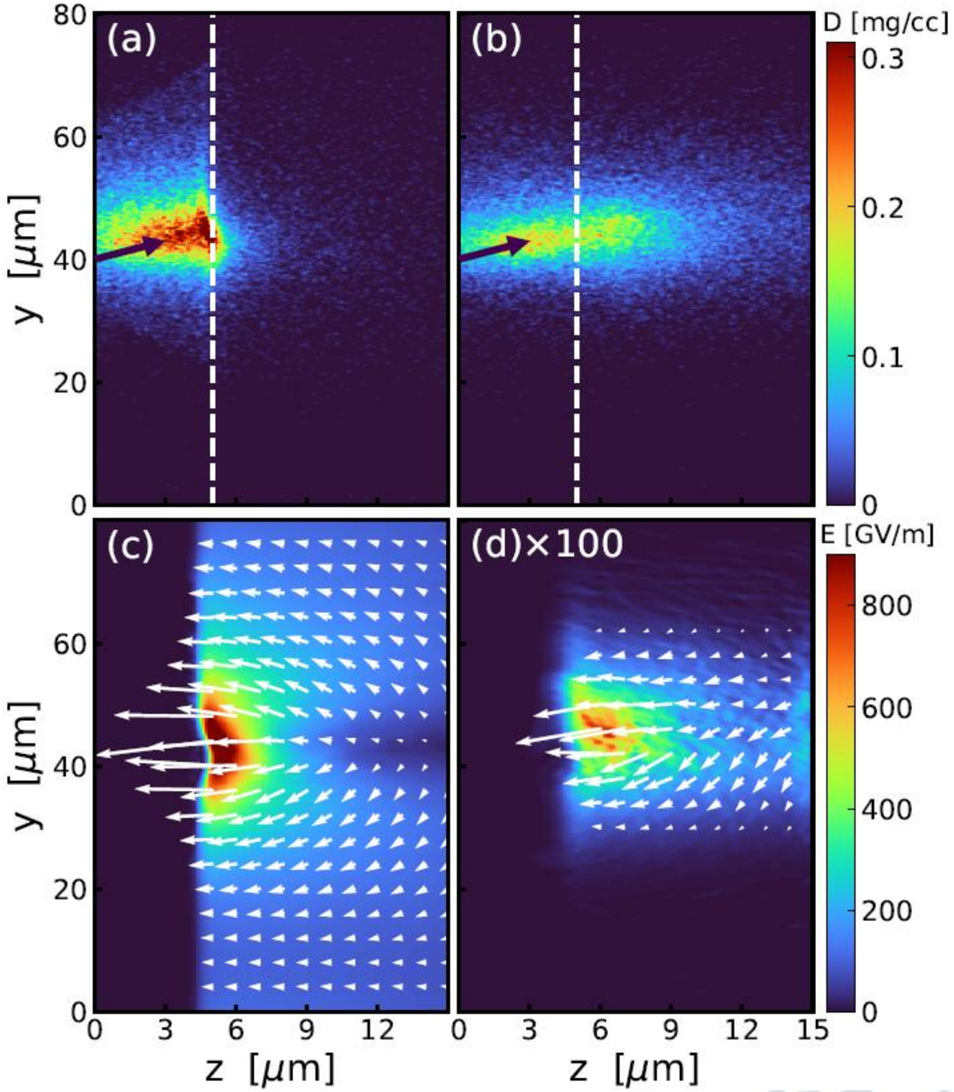


Fusion reactions initiated by laser-accelerated particle beams in a laser-produced plasma

Labaune et.al., Nature communication 2013



Quantum degeneracy effects and collective electromagnetic effects can increase the number of fusion reactions through reduction of the stopping power of the ion beam in plasma.



X. Ning, T. Liang, D. Wu*, S. Liu, Y. Liu, T.X. Hu, Z.-M. Sheng*, J. Ren, B. Jiang, Y.T. Zhao, D. H. H. Hoffmann, and X. T. He, Laser driven proton-boron fusions: influences of the boron state; Laser and Particle Beams, 9868807 (2022),

Mass density distributions of the proton beam and the electric field distributions for the normal boron solid in (a) and (c), and for the laser-ablated boron solid (boron plasma) in (b) and (d), respectively.

The scheme taking into account the effects of degeneracy

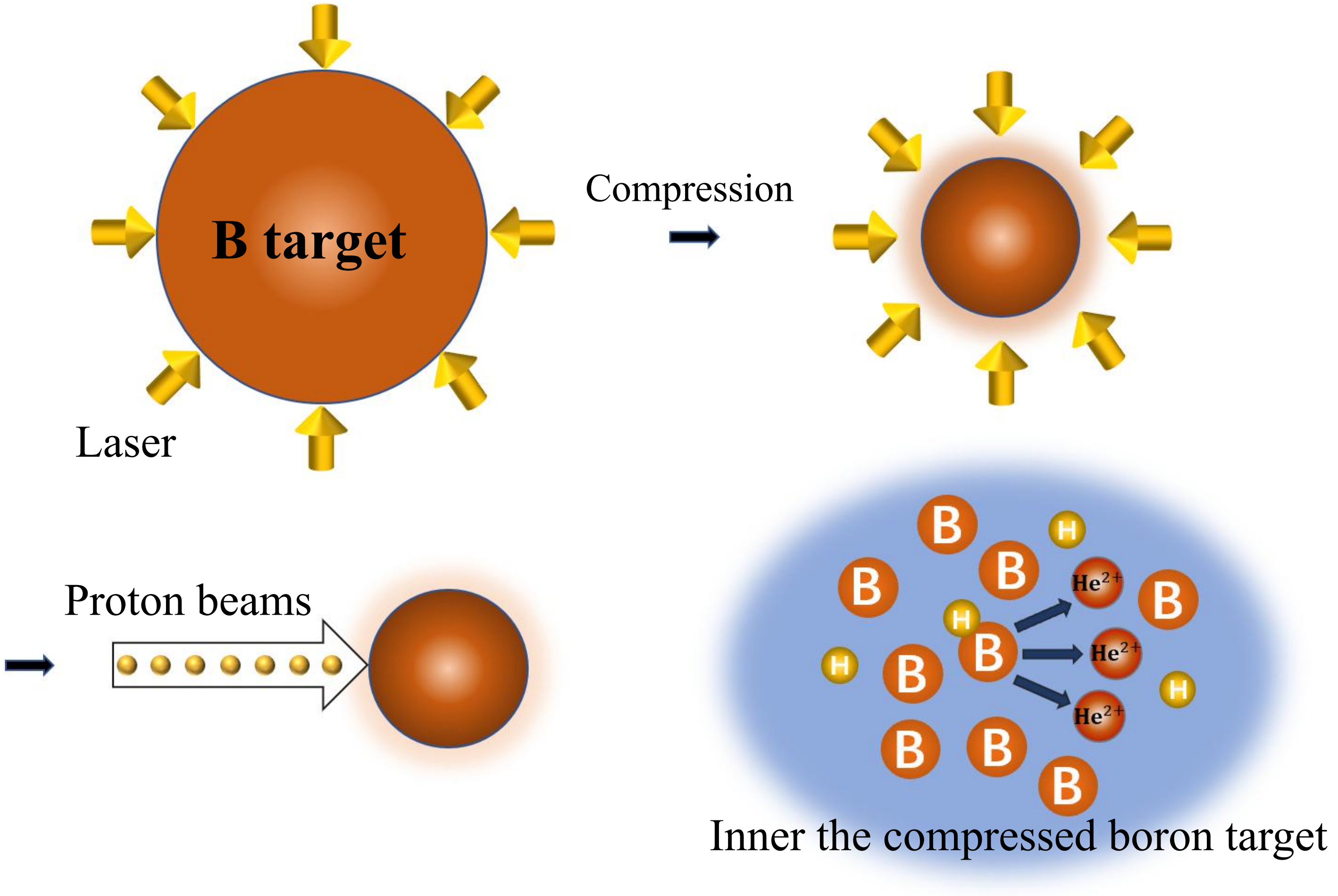
Considering the influence of strong degeneracy effect

The degree of degeneracy that characters the conditions of boron targets is defined as

$$\Theta = T_F/T_e$$

with T_F and T_e representing the Fermi energy and thermal temperature, respectively.

In order to achieve high degeneracy, the boron target need be compressed. Such a boron target can be achieved via quasi-isentropic compression of a solid boron by using precisely shaped laser pulses.



The scheme taking into account the effects of degeneracy

浙江大學
ZHEJIANG UNIVERSITY

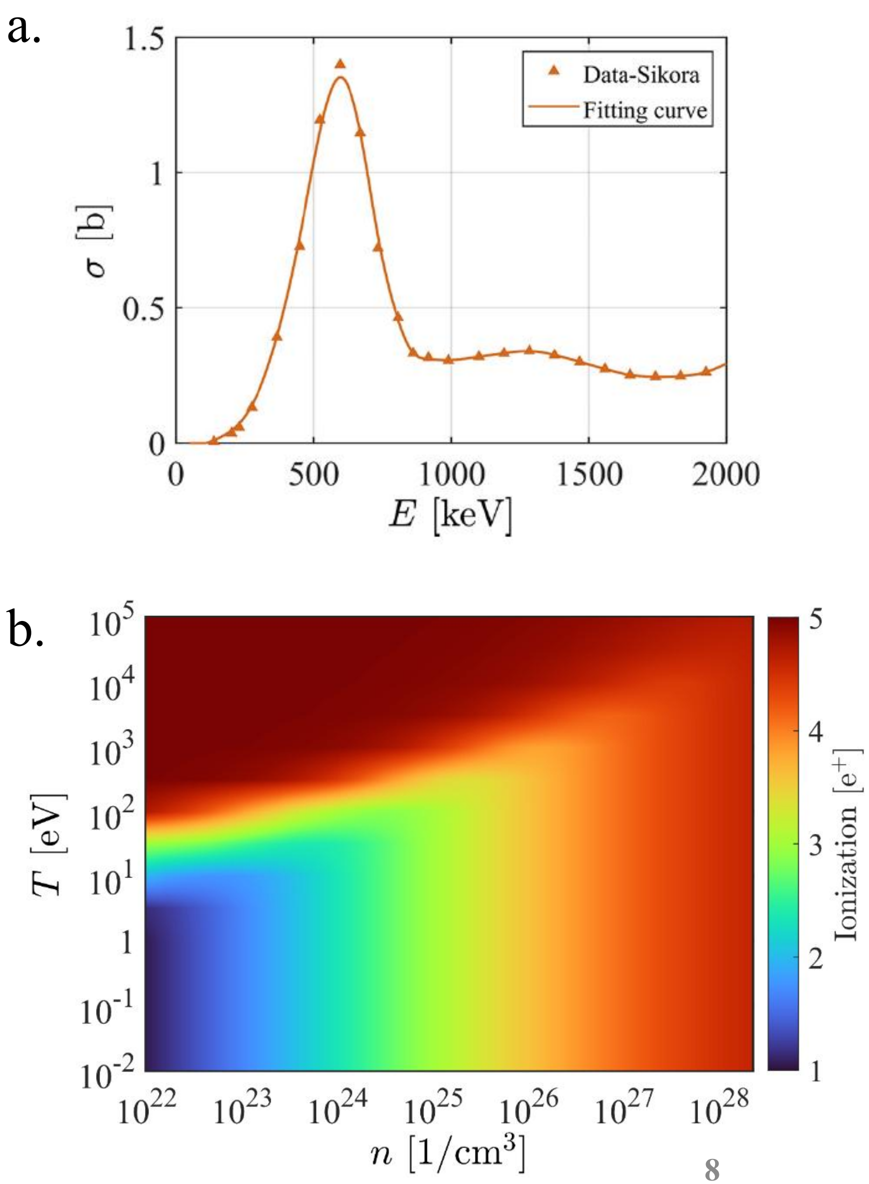
As for the proton-boron beam-target fusion, α grain yield satisfies

$$N_\alpha = 3N_p \int_0^{E_p} \frac{\sigma(E)}{S/n} dE = 3N_p P,$$

where $\sigma(E)$ is cross section of the proton-boron fusion, n is the number density of boron target, S is the stopping power of the incident proton beams,

$$S = S_f + S_b + S_n,$$

where S_f, S_b and S_n are the stopping power contributions of free electrons, bound electrons and nuclei respectively. The ionization state is helpful to calculate the stopping power of boron target.

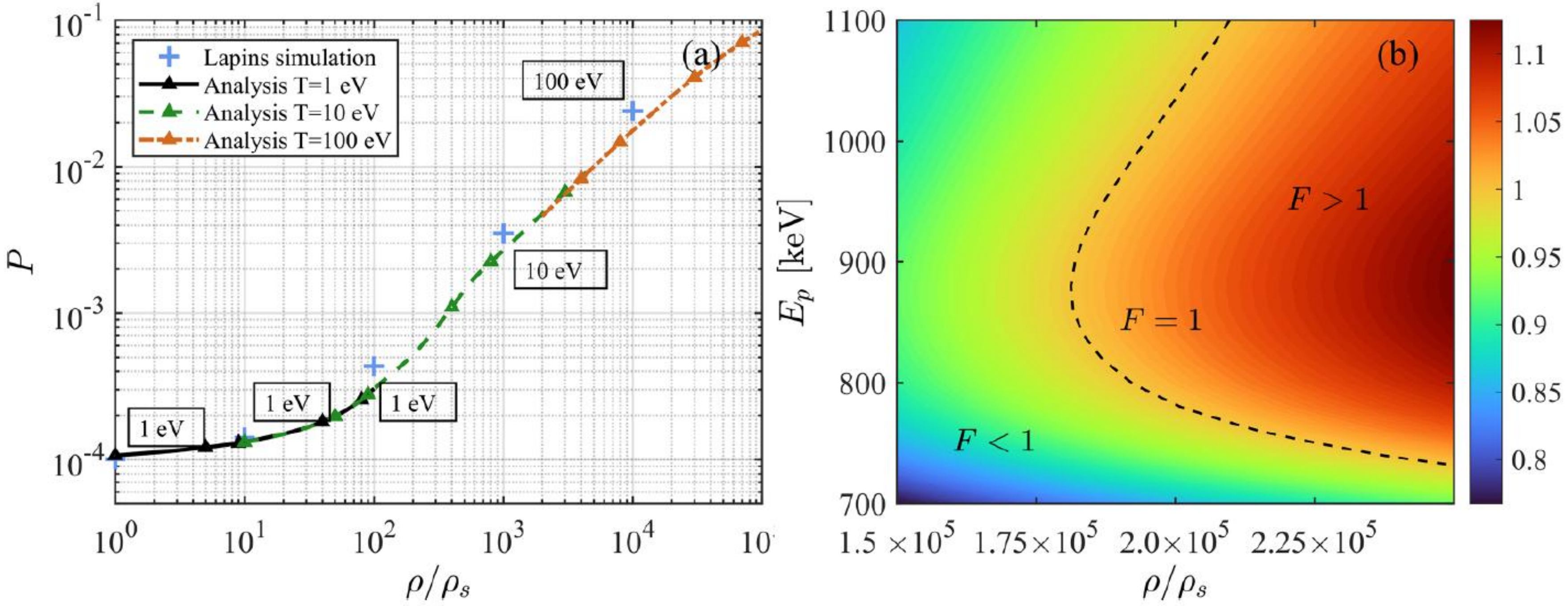


M. H. Sikora and H. R. Weller, J. Fusion Energy 35, 538 (2016).
T. Heltemes and G. Moses, Computer Physics Communications 183, 2629 (2012).

The scheme taking into account the effects of degeneracy

(a) According to the integral $P = \int_0^{E_p} \frac{\sigma(E)}{S/n} dE$, give the rate of the proton-boron beam-target fusion with initial protons energy 1 MeV as a function of the boron target density, for the interesting temperature.

(b) Figure the F factor versus the density of boron target and incident velocity of proton beams at $T = 100$ eV.



S. J. Liu, D. Wu*， T. X. Hu, T. Y. Liang, X. C. Ning, J. H. Liang, Y. C. Liu, P. Liu, X. Liu, Z. M. Sheng*, Y. T. Zhao, D. H. H. Hoffmann, X. T. He, and J. Zhang, Phys. Rev. Research 6, 013323 (2024)

Kinetic theory including finite-temperature exchange-correlation effects

Many body Schrodinger Equation → Kinetic Equation

Wigner distribution function

$$\left[\sum_{i}^{N}\left(-\frac{\hbar^2\nabla^2}{2m}+v_{\mathrm{ext}}(\boldsymbol{r}_i,t)\right)+\sum_{i<j}U\left(\boldsymbol{r}_i,\boldsymbol{r}_j\right)\right]\Psi(\boldsymbol{R},t)=i\hbar\partial_t\Psi(\boldsymbol{R},t).$$

$$\boldsymbol{R}=(\boldsymbol{r}_1,\boldsymbol{r}_2,\dots\boldsymbol{r}_N).$$

Density operator:

$$\hat{\rho}_{1..N}(t)=\sum_a P_a|\Psi^a_{1...N}\rangle\langle\Psi^a_{1...N}|,\qquad Tr\hat{\rho}_{1..N}(t)=1.$$

Quantum Liuville Eq.

$$i\hbar\frac{\partial}{\partial t}\hat{\rho}_{1..N}-\left[\hat{H}_{1..N},\hat{\rho}_{1..N}\right]=0.$$

By Wigner transformation: Transfer the operator to distribution function

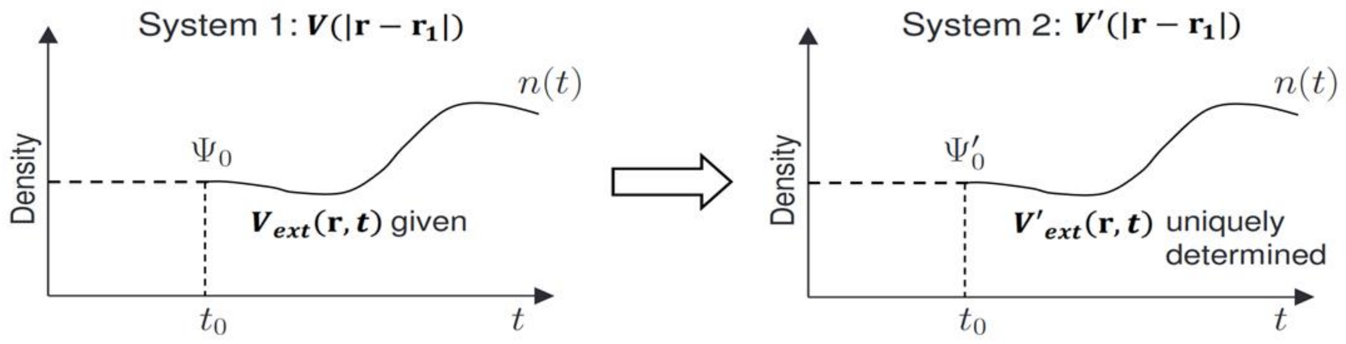
$$f(\mathbf{p},\mathbf{R},t)=\int\frac{d\mathbf{r}}{(2\pi\hbar)^3}\exp\left(\frac{i\mathbf{p}\cdot\mathbf{r}}{\hbar}\right)\langle\mathbf{R}+\mathbf{r}/2|\,\hat{F}_1(t)\,|\mathbf{R}-\mathbf{r}/2\rangle$$

We obtain the quantum kinetic equation:

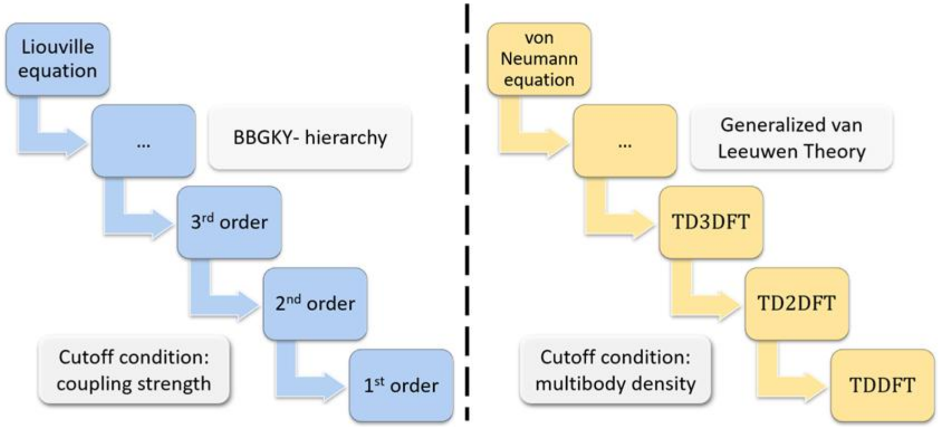
$$\left\{\frac{\partial}{\partial t}+\frac{\mathbf{p}_1}{m}\nabla_{\mathbf{R}_1}\right\}f(\mathbf{R}_1,\mathbf{p}_1,t)-\frac{1}{i\hbar}\int d\mathbf{r}_1\frac{d\mathbf{p}_1'}{(2\pi\hbar)^3}e^{-i(\mathbf{p}_1-\mathbf{p}_1')\cdot\frac{\mathbf{r}_1}{\hbar}}\{U_{\text{eff}}(\mathbf{R}_1+\mathbf{r}_1/2)-U_{\text{eff}}(\mathbf{R}_1-\mathbf{r}_1/2)\}f(\mathbf{R}_1,\mathbf{p}_1',t)=I_{corr}.$$

$$U_{\text{eff}}=V(\mathbf{r})+U^H(\mathbf{r},t),\qquad U^H(\mathbf{r},t)=g_s\int d\bar{r}V(\mathbf{r}-\bar{\mathbf{r}})f(\bar{r},\bar{r},t)$$

From van Leeuwen theorem: (Robert van Leeuwen, PRL 1999)



Extended Time-Dependent Density Functional Theory for Multibody Densities:



J.-H. Liang, T.-X. Hu, D. Wu, Z.-M. Sheng, and J. Zhang，Phys. Rev, Lett. 133, 263001 (2024)

- For proton beam + high density thermal boron plasma, we may obtain net energy gain when the density of boron target is higher than 10^4 times of the density of solid boron;
- For MCF case (density is much lower than solid state), is it possible to obtain net energy gain for proton- boron fusion?
- To reduce the radiation lost, the electron temperature should be lower than ion temperature, **Non-Thermal Equilibrium!**

Non-Thermal Equilibrium:

Use beam and temperature anisotropy to improve reaction rate by ~20% (Xie2023PPCF).

Apply external heating (NBI, ICRF) or high-energy α-particles to create high-energy ion tails (Putvinski2019NF, Kolmes2022PoP).

Exploit macroscopic ion velocity differences to increase reaction rates.

- **Power balance:** $P_{aux} + P_{fus} = P_C + P_R$, **Scientific Q factor** $Q = P_{fus}/P_{aux}$ **(Atzeni04book).**

We need as higher fusion power as possible with limited auxiliary power (all power required to sustain the fusion system) to increase the Q value.

- We have to pay cost for maintaining the Non-Thermal Equilibrium state!

Analysis from power

- **Achieving net energy gain for p-B11 fusion under equilibrium conditions is virtually impossible. (Nevins98JFE)!**

  In all cases where $T_i = T_e$, $P_{fus} < P_{brem}$.

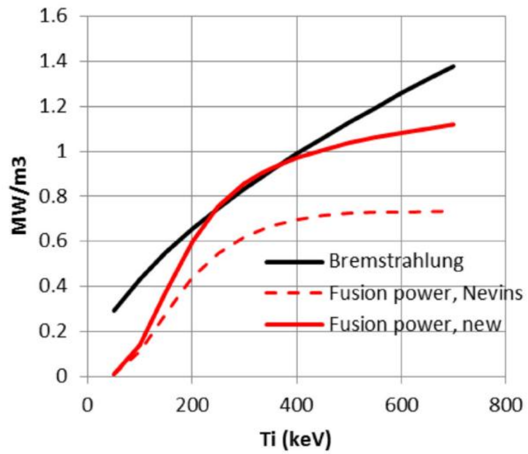
- **Power balance of electrons follows $P_{ie} + P_{\alpha e} = P_{brem}$, according to the cross-section data from Nevins00NF, the bremsstrahlung radiation loss power still exceeds the fusion output power. (Putvinski19NF)**

  At this point, in the optimal case, $P_{aux}(0.2) + P_{fus}(0.6) = P_R(0.8)$,

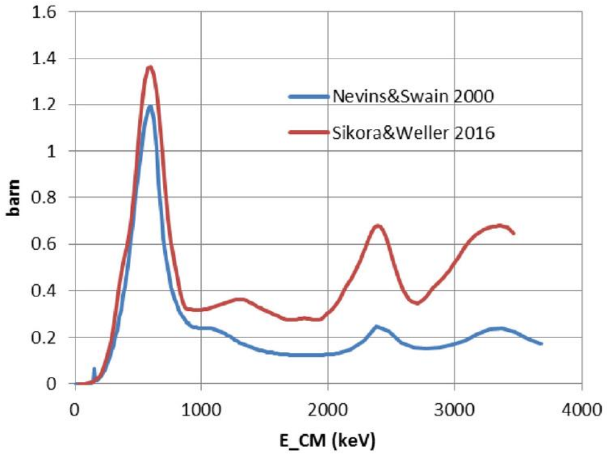
  The maximum Q value is approximately 3.

- **The updated reaction cross-section provides a theoretical ignition range (Putvinski19NF).**

The corresponding $T_i \approx 300$ keV.



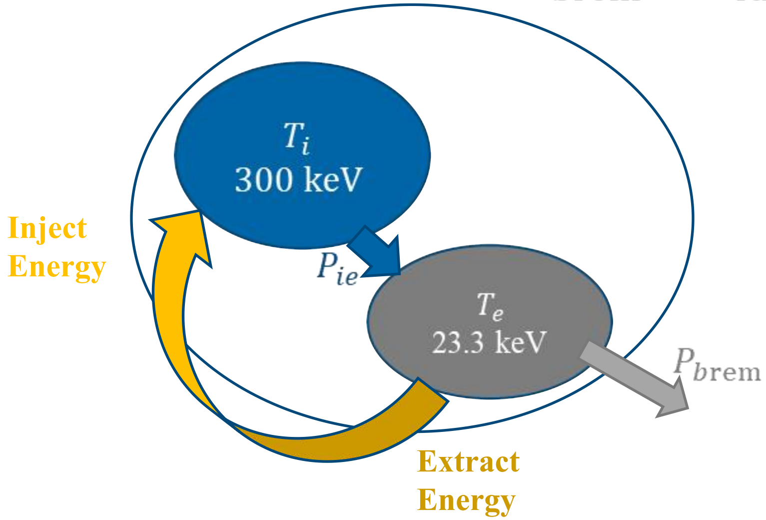
Power density of Putvinski19



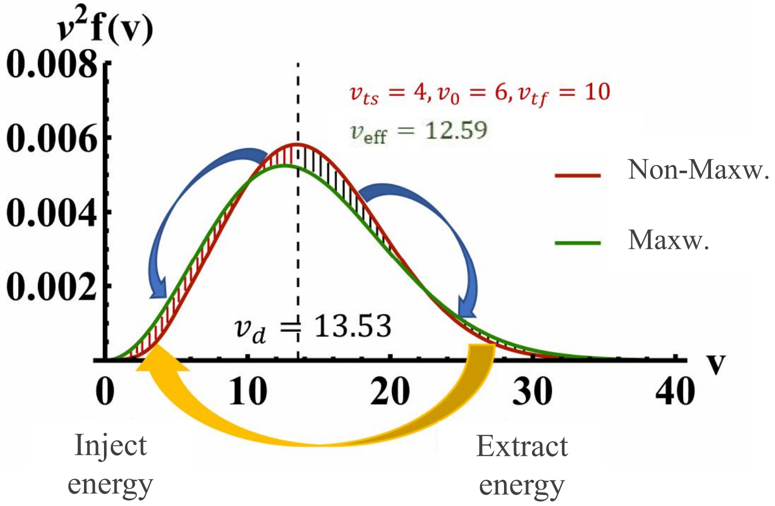
Cross-section of p-B11 (Putvinski19)

- **Todd H. Rider (PhD 1995, PoP 1997) studied the feasibility of non-equilibrium systems for nuclear fusion reactions. He proposed that a far-from-equilibrium system could achieve a steady state through continuous energy cycles.**

To maintain ion temperature and limit bremsstrahlung loss, electrons were kept at a effective temperature about 1/10 of the ions, therefore $P_{\mathrm{brem}} = P_{\mathrm{fus}}/2$.



T_i = 300 keV , Maxwellian
T_e = 23.33 keV, Maxwellian

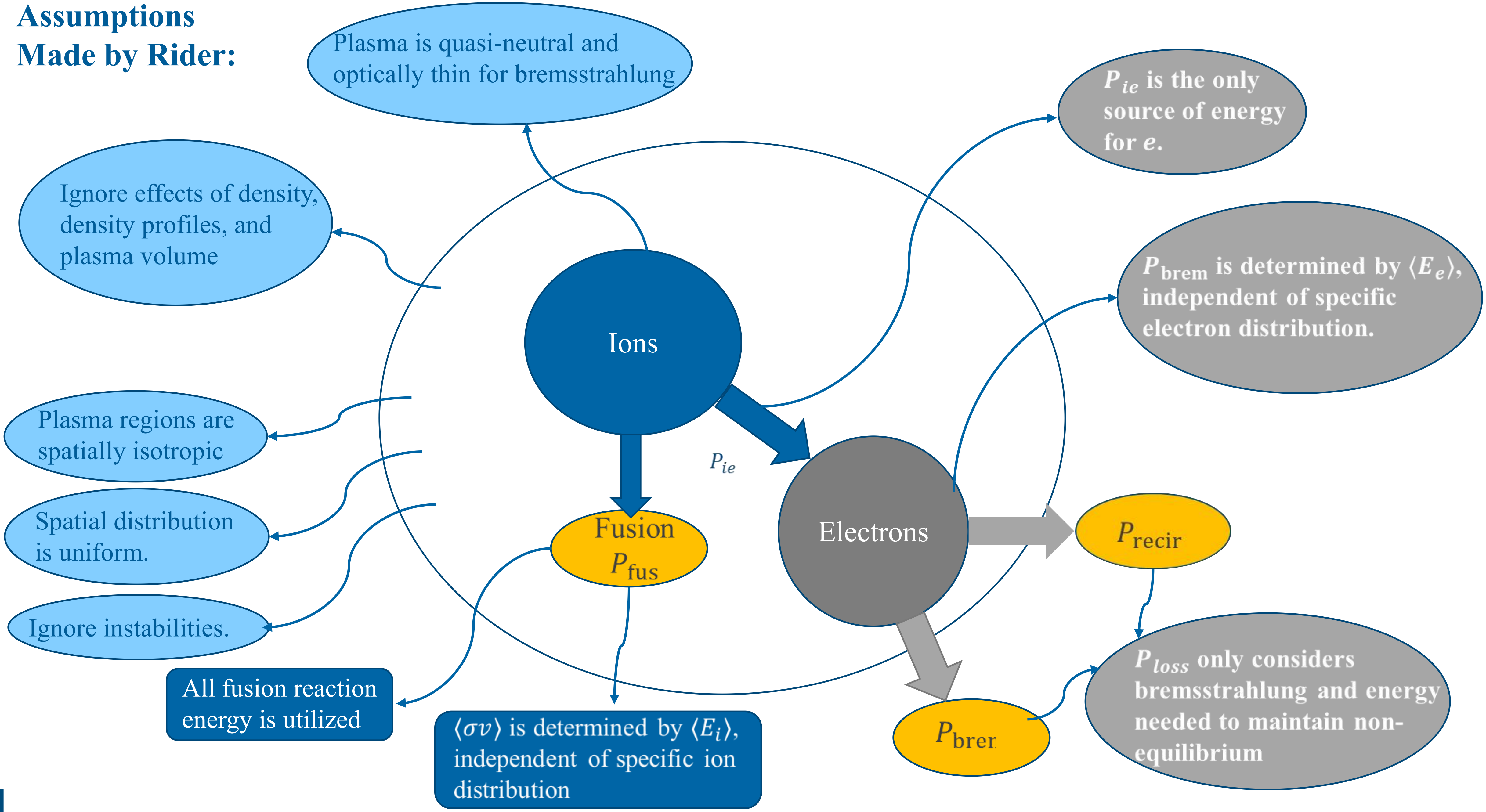


T_i =300 keV , Maxwellian
T_e^{eff} =23.33 keV, non-Maxwellian

Non-Equilibrium Systems in Theoretical Models:

Ions	Electrons
Maxwellian	Maxwellian
Non-Maxwellian	Non-Maxwellian

Assumptions Made by Rider:



- **Considering conduction losses, the total balance equation in steady-state is**

$$P_H + P_{\text{fus}} = P_{\text{brem}} + P_{\text{recir}} + P_C,$$

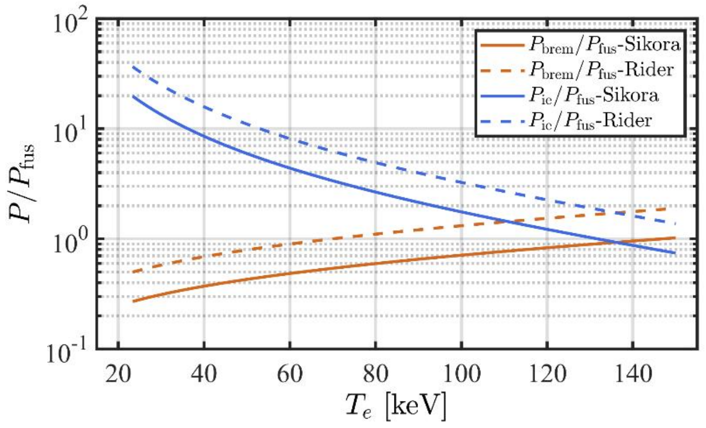
where $P_C = U_K/\tau_E$, and then, it is clear that

$$Q = \frac{P_{\mathbf{fus}}}{P_H} = \frac{P_{\mathbf{fus}}}{P_{\mathbf{brem}} + P_{\mathbf{recir}} + U_K/\tau_E - P_{\mathbf{fus}}},$$

To achieve Self heating, the minimum energy confinement time required is

$$\tau_E^\star \equiv \tau_E\Big|_{Q\to\infty} = \frac{U_K}{P_{\mathbf{fus}} - (P_{\mathbf{brem}} + P_{\mathbf{recir}})}$$

Given density parameters and their components, under a given ion temperature, the lower the value of $n_i\tau_E^\star$, the more favorable it is.

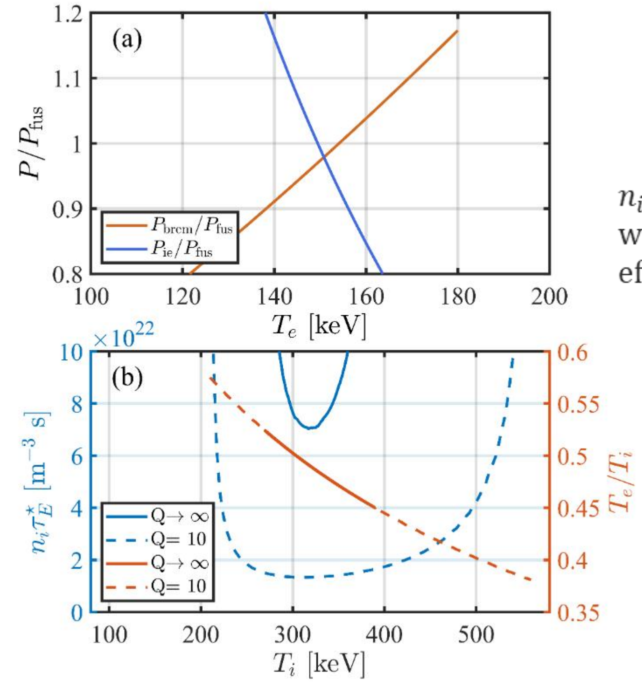


$T_i = 300$ keV,
$\ln\Lambda = 15$

- **Energy confinement time without considering P_{recir} is**

$$\tau_E^\star \equiv \tau_E\Big|_{Q_{\text{fuel}}\to\infty} = \frac{U_K}{P_{\mathbf{fus}} - P_{\text{brem}}}$$

Given n and T_i, determine T_e and P_{brem} from $P_{ie} = P_{\mathbf{brem}}$, and a simplified parameter map for p-B11 fusion under near-ignition conditions can be constructed.



$n_i = 10^{20}$ m^{-3}, $n_i = 85\%\, n$, without considering the effects of kinetic effects.

Shujun Liu, Dong Wu, Bing Liu, Yueng-Kay Martin Peng, Jiaqi Dong, Tianyi Liang, Hairong Huang, and Zheng-Mao Sheng, Phys. Plasmas 32, 012101 (2025)

Non-Maxwellian Ion distributions

High-energy alpha particles or neutral beam can lead to the formation of a high-energy ion tail, increasing fusion reaction rates (Putvinski 2019). Without external heating, within a certain temperature range, it is theoretically possible to achieve $P_{fus} > P_{brem}$, allowing for self heating.

External heating (NBI/ICRF) can theoretically limit heating power while enhancing fusion power using non-Maxwellian proton distributions, providing a sufficiently high Q value (Kirov 2020, Mantical 2024).

- What kind of non-Maxwellian proton distributions can enhance fusion reaction rates?
- Considering only the evolution of initially non-Maxwellian protons (in an isolated system), what is the minimum recirculating power required to maintain the non-Maxwellian distribution compared to the fusion gain it produces?
- In a more practical multi-component case, where inter-particle energy transfer power plays the role of recirculating power, what is the ratio of inter-particle energy transfer power to recirculating power?

The collision term between two particles can be written in the Fokker-Planck form

$$C_{\alpha\beta} = \Gamma_{\alpha\beta}\nabla_v \cdot \left[\frac{1}{2}(\nabla_v f_\alpha)\cdot\nabla_v\nabla_v g_{\alpha\beta} - \frac{m_\alpha}{m_\alpha + m_\beta} f_\alpha \nabla_v h_{\alpha\beta}\right]$$

Under isotropic conditions

$$C_{\alpha\beta} = \Gamma_{\alpha\beta}\frac{1}{v^2}\frac{\partial}{\partial v}v^2\left[\frac{1}{2}\frac{\partial f_\alpha}{\partial v}\frac{\partial^2 g_{\alpha\beta}}{\partial v^2} - \frac{m_\alpha}{m_\alpha + m_\beta} f_\alpha \frac{\partial h_{\alpha\beta}}{\partial v}\right]$$

$$= \frac{16\pi^2 Z_\alpha^2 Z_\beta^2 e^4 \ln\Lambda}{3m_\alpha^2}\left\{\frac{\partial^2 f_\alpha}{\partial v^2}\left[\frac{1}{v^3}\int_0^v du\ f_\beta(u)u^4 + \int_v^\infty du\ f_\beta(u)u\right]\right.$$

$$\left. + \frac{\partial f_\alpha}{\partial v}\left[\int_0^v du\ f_\beta(u)\left(3\frac{m_\alpha u^2}{m_\beta v^2} - \frac{u^4}{v^4}\right) + \frac{2}{v}\int_v^\infty du\ f_\beta(u)u\right] + 3\frac{m_\alpha}{m_\beta} f_\alpha(v) f_\beta(v)\right\}.$$

The kinetic equation simplifies to

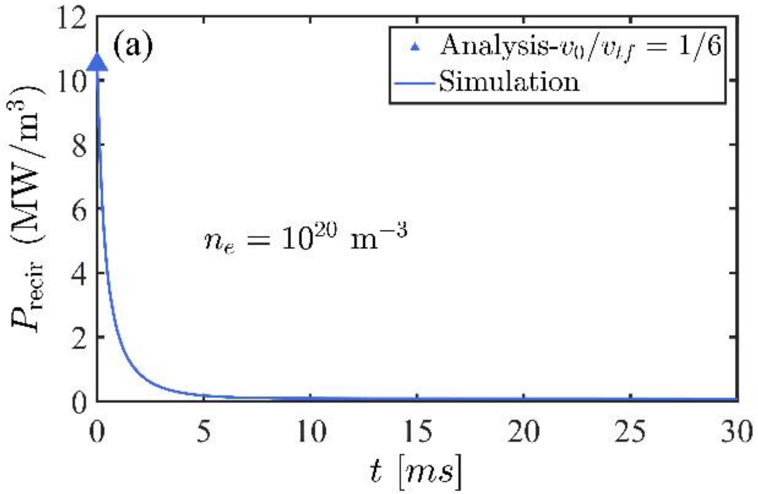
$$\frac{\partial f_\alpha}{\partial t} = \left(\frac{\partial f_\alpha}{\partial t}\right)_{col},$$

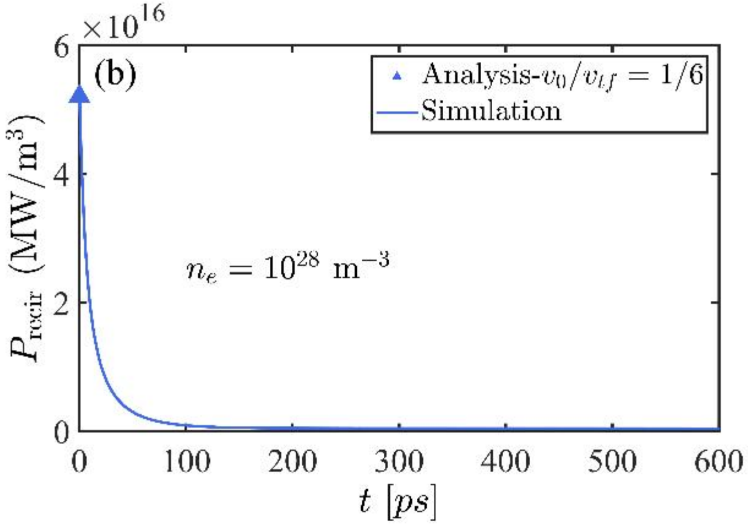
Write MATLAB code under isotropic conditions, considering the non-Maxwellian distribution of ions (zero-dimensional case, ignoring spatial inhomogeneity).

Normalization $n = \frac{n}{1e20\ m^{-3}}, m = \frac{m}{m_e}, v = \frac{v}{v_{tho}}, t = \frac{t}{\tau_{col}}, \quad \tau_{col} = \frac{3\sqrt{m_i}T_i^{3/2}}{4\sqrt{2\pi}n_i Z^2 e^4 \ln\Lambda}.$

Proton distribution

Isotropic $f(v) = K1\left(\exp\left(-\frac{v^2}{v_{th}^2}\right) + \xi \exp\left(-\frac{(v-v_f)^2}{v_{thf}^2}\right)\right).$





Calculate the recirculating power

- **During the evolution of an initially non-Maxwellian distribution, some particles gain energy while others lose energy. There exists at least one point where the particle velocity remains unchanged. Ideally, it can be assumed that the number of particles on either side of the dividing velocity v_d is conserved, with only energy being exchanged.**

$$\int_0^{v_d} 4\pi v^2 \left(\frac{\partial f}{\partial t}\right)_{col.} \mathrm{d}v = \int_{v_d}^{\infty} 4\pi v^2 \left(\frac{\partial f}{\partial t}\right)_{col.} \mathrm{d}v = 0.$$

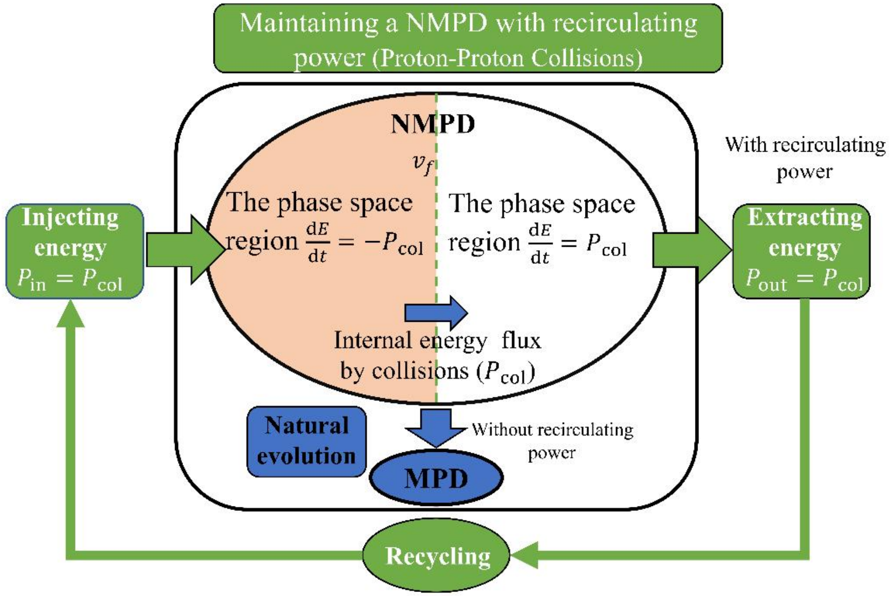
- **The power exchange between the two sides of the dividing velocity due to particle collisions at time t, that is, the recirculating power is given by**

$$P_{recirc} \equiv \left| \int_0^{v_d} (dv4\pi v^2)\left(\frac{1}{2}mv^2\right)\left(\frac{\partial f}{\partial t}\right)_{col} \right|$$
$$= \left| \int_{v_d}^{\infty} (dv4\pi v^2)\left(\frac{1}{2}mv^2\right)\left(\frac{\partial f}{\partial t}\right)_{col} \right|.$$

For multiple dividing velocities, the total recirculating power is defined by

$$P_{recirc} = \sum \int_{v_1}^{v_2} (dv4\pi v^2)\left(\frac{1}{2}mv^2\right)\left(\frac{\partial f}{\partial t}\right)_{col},$$

where v_1 and v_2 are the boundaries of each segment, with $v_1 = 0$ and $v_2 = \infty$ for the first and last segments, respectively.



Specific Mechanism for Recirculating Power Compensation

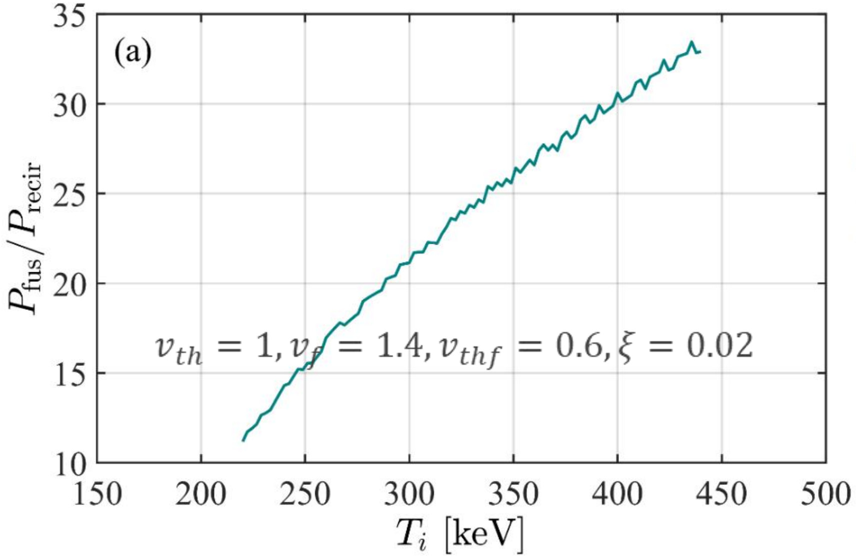
Assumptions:

- The input side of the recirculating power can be compensated by external heating power.
- The output side of the recirculating power can be supplemented by proton loss power.

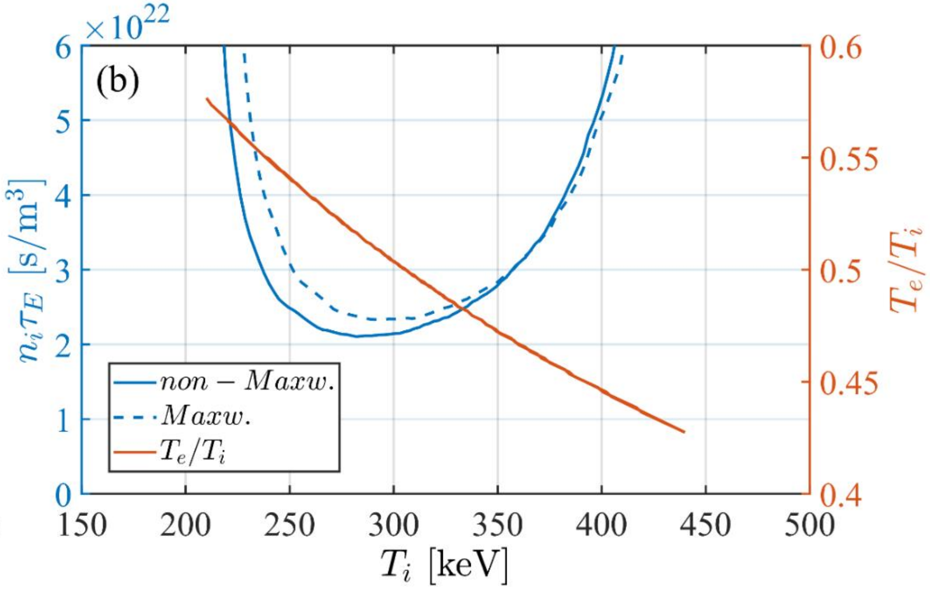
Subsequently, calculate the P_{ie} and P_{brem} through the electron power balance equation $P_{ie} = P_{\mathrm{brem}}$, then calculate the value of $n_i\tau_E$ from the overall power balance equation

$$P_H + P_{fus} = P_{brem} + P_C,$$

where $P_H = P_{recir}$, $P_C = U_K/\tau_E$. The value of Q is $P_{\mathrm{fus}}/P_{\mathrm{recir}}$. $\tau_E = \frac{U_K}{P_{\mathrm{fus}}+P_{\mathrm{recir}}-P_{\mathrm{brem}}}$.



P_{fus}/P_{recir} under different T_i



The figure of $n_i\tau_E$ versus T_i to achieve $Q = P_{fus}/P_{recir}$.

$n_p = 8.5e19\ m^{-3}$,
$n_B = 1.5e19\ m^{-3}$.

Assumptions:

- ◆ The input side of the recirculating power can be compensated by a portion of the fusion power ζP_{fus}.
- ◆ The output side of the recirculating power can be compensated by proton loss power.

The overall power balance equation $P_{fus} = P_{brem} + P_C$ is used to calculate the value of $n_i\tau_E$. The energy confinement time satisfies $\tau_E = \frac{U_K}{P_{fus} - P_{brem}}$.

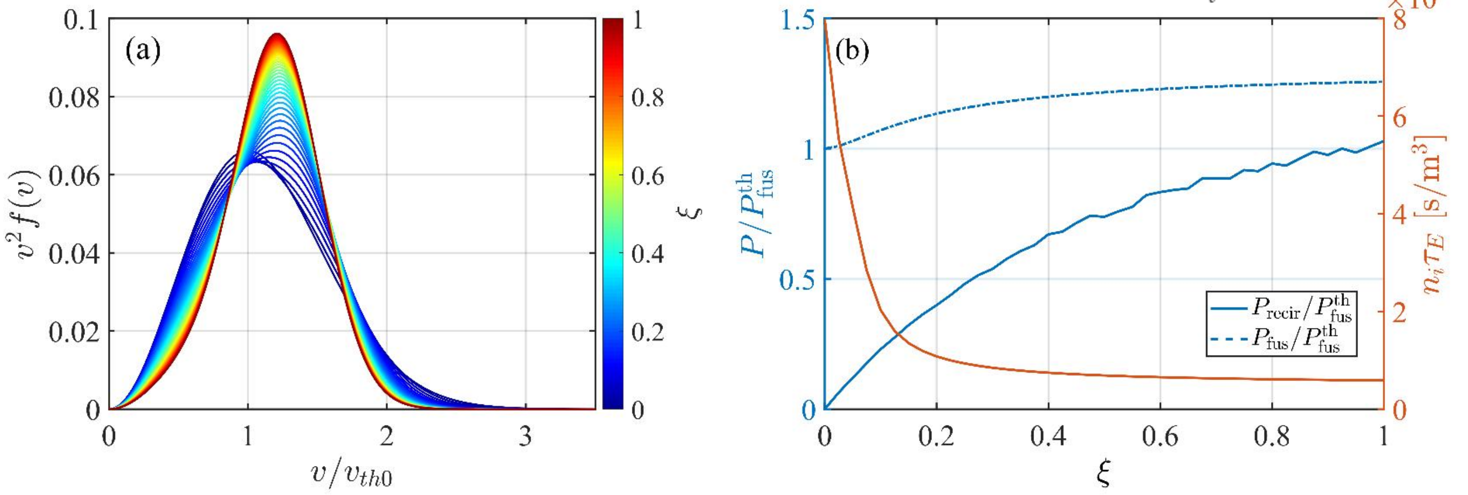
The value of P_{fus}/P_{recir} can be measured by the degree of deviation of protons from the Maxwellian distribution.

$$f(v) = K1\left(\exp\left(-\frac{v^2}{v_{th}^2}\right) + \xi \exp\left(-\frac{\left(v - v_f\right)^2}{v_{thf}^2}\right)\right)$$

The shape parameters are $v_{th} = 1, v_f = 1.4, v_{thf} = 0.6$, The value of ξ is adjusted to control the ratio P_{recir}/P_{fus}.

The non-Maxwellian distribution function enhances fusion power and leads to a reduction in energy confinement time requirement.

$v_{th} = 1, v_f = 1.4, v_{thf} = 0.6, \xi = 0 - 1$



Ignition parameters can be reduced by an order of magnitude.

Summary & Discussion

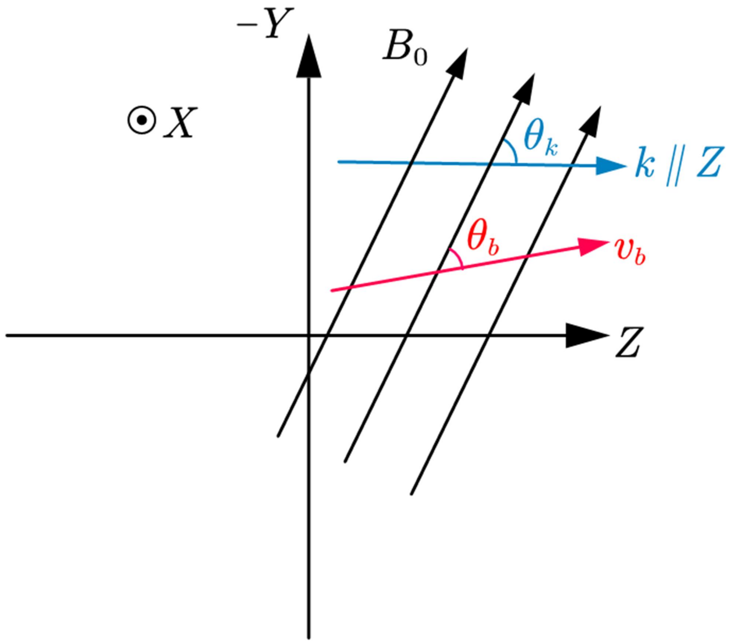
- Higher electron temperatures from Rider's setting increase Bremsstrahlung radiation losses but reduce electron-ion energy transfer, therefore, near ignition conditions ($Q \geq 10$) are found when ion temperatures (T_i) range from 200 to 600 keV in a thermonuclear steady-state accounting for Bremsstrahlung losses. At these conditions, the optimal electron-to-ion temperature ratio (T_e/T_i) is between 0.4 and 0.6.
- Using the Fokker-Planck equation, we constructed an 0D kinetic evolution model for non-Maxwellian proton distributions. When proton distribution evolution is dominated by proton-proton collisions, maintaining a tailored NMPD can reduce the energy confinement time required for Q>10 and ignition.
- Considering effect of proton others collisions (protons, electrons, boron ions, and α particles), the recirculating power density required to sustain the distribution-when inter-species collisions are included-is 2-3 times higher than that from proton-proton collisions alone.

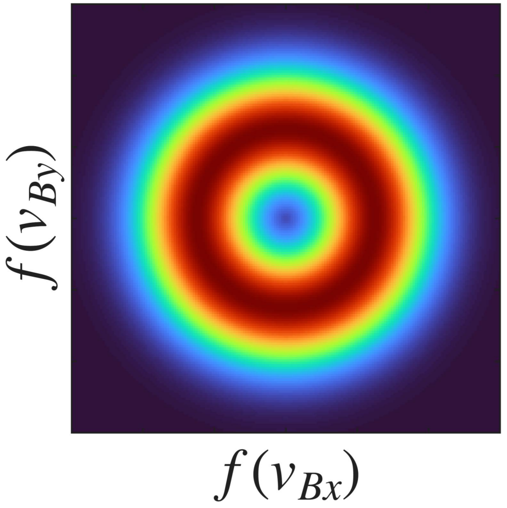
Shujun Liu, Di Luo, Yueng-Kay Martin Peng, Jiaqi Dong, Huasheng Xie, Khoo Nee Don, Hairong Huang, Zhi Li, Bing Liuand Zheng-Mao Sheng, Feasibility of p-11B Fusion Gain via the non-Maxwellian Proton Distribution, Submitted to PPCF

Energization of Proton via Beam-Driven Ion Bernstein Waves in p11B Plasmas

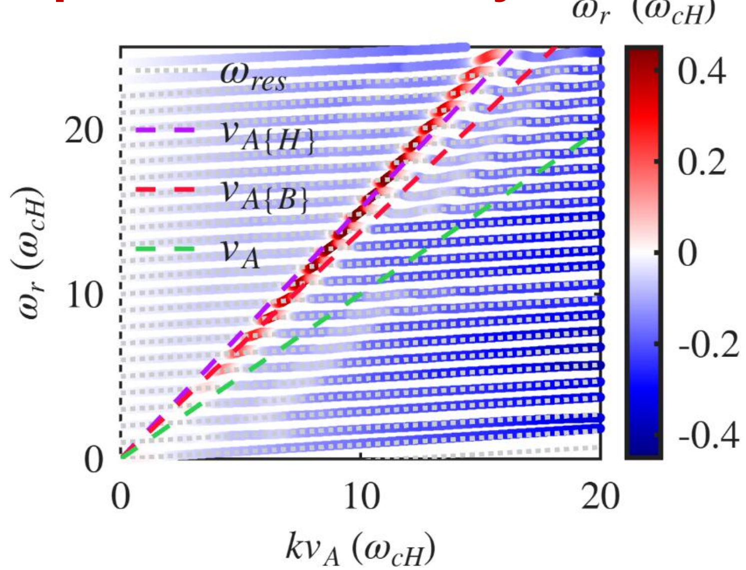
Parameter	Value
Magnetic field	2 T
Temperature	20 keV
Boron density	9.2e18 m^{-3}
Hydrogen boron ratio	9:1
Neutral beam density	8e18 m^{-3}
Neutral beam energy	200 keV

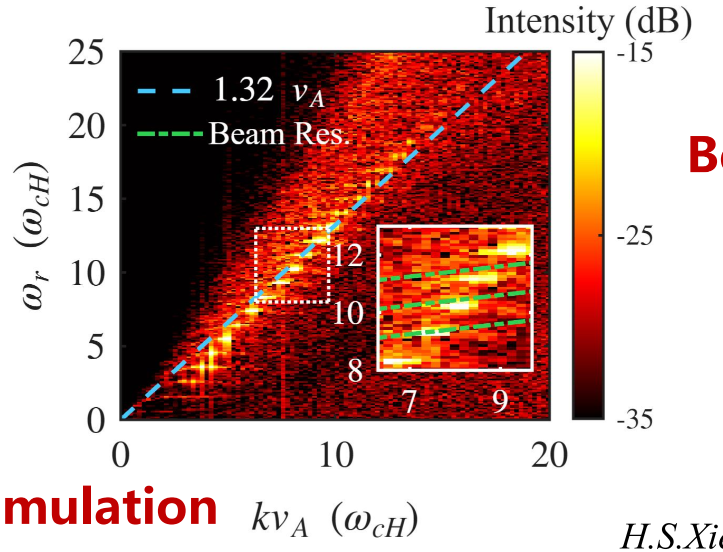
$$f \sim \exp\left[-\frac{(v_{\parallel}-v_{b\parallel})^2}{v_{tb}^2}\right]\exp\left[-\frac{(v_{\perp}-v_{b\perp})^2}{2v_{tb}^2}\right]$$





Dispersion Relation by BO/PDRK





Simulation

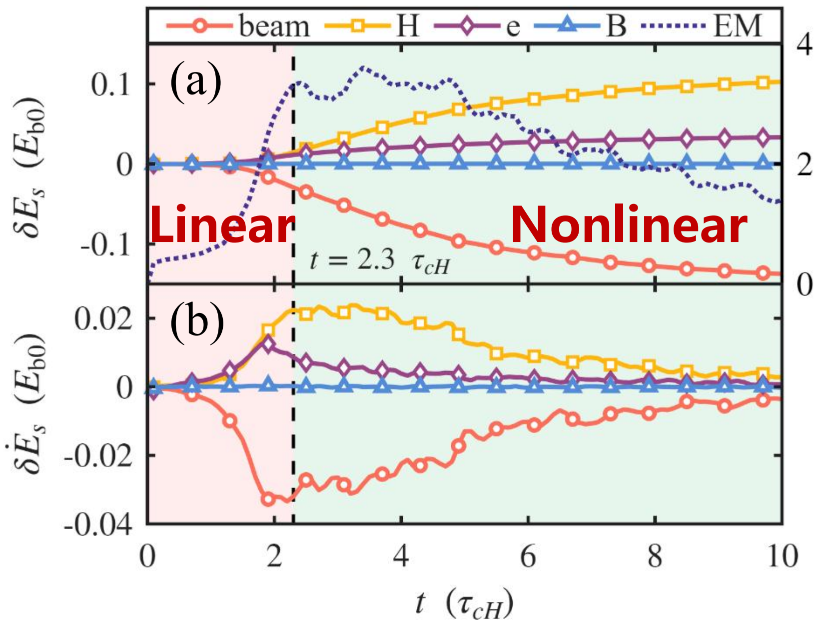
Beam Resonance Condition

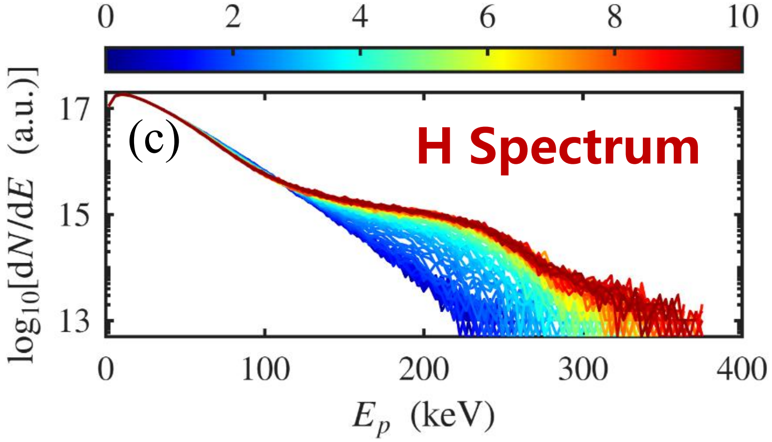
$$\omega = k_{\parallel} v_{b\parallel} \pm n\omega_{cb}$$

H.S.Xie. COMPUT PHYS COMMUN 244, 343–371 (2019)

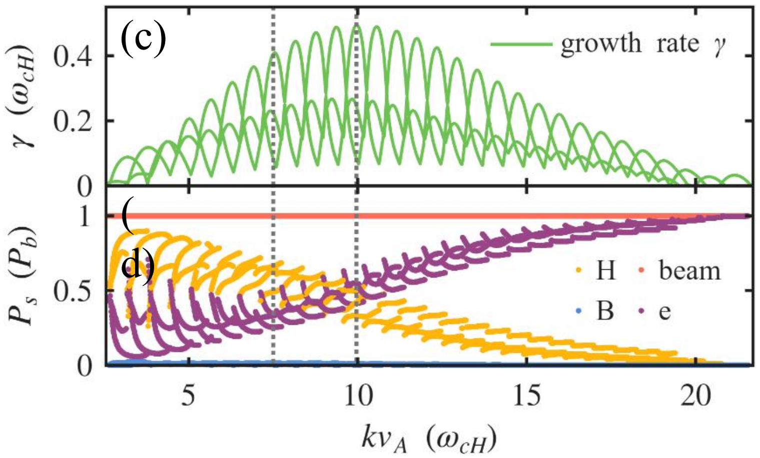
Energization of Proton via Beam-Driven Ion Bernstein Waves in p11B Plasmas

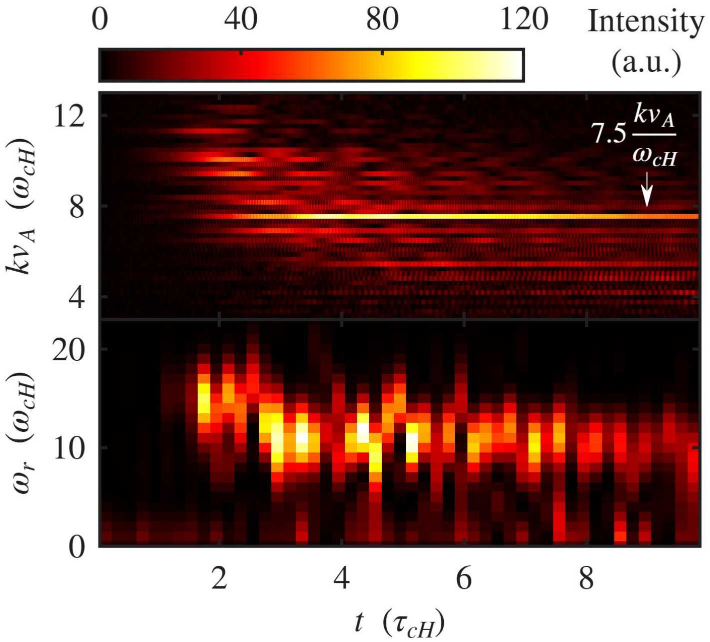
Energy Transfer Ratio from Simulation



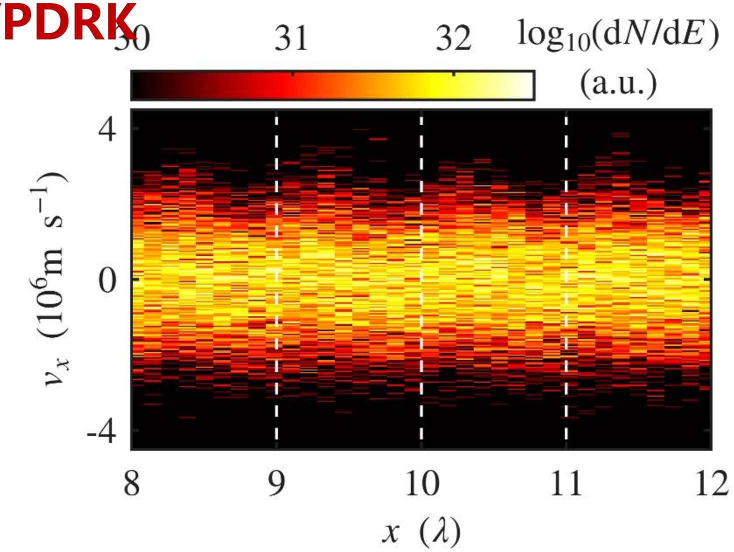


Energy Transfer Ratio from BO/PDRK





Inverse Spectrum Cascade

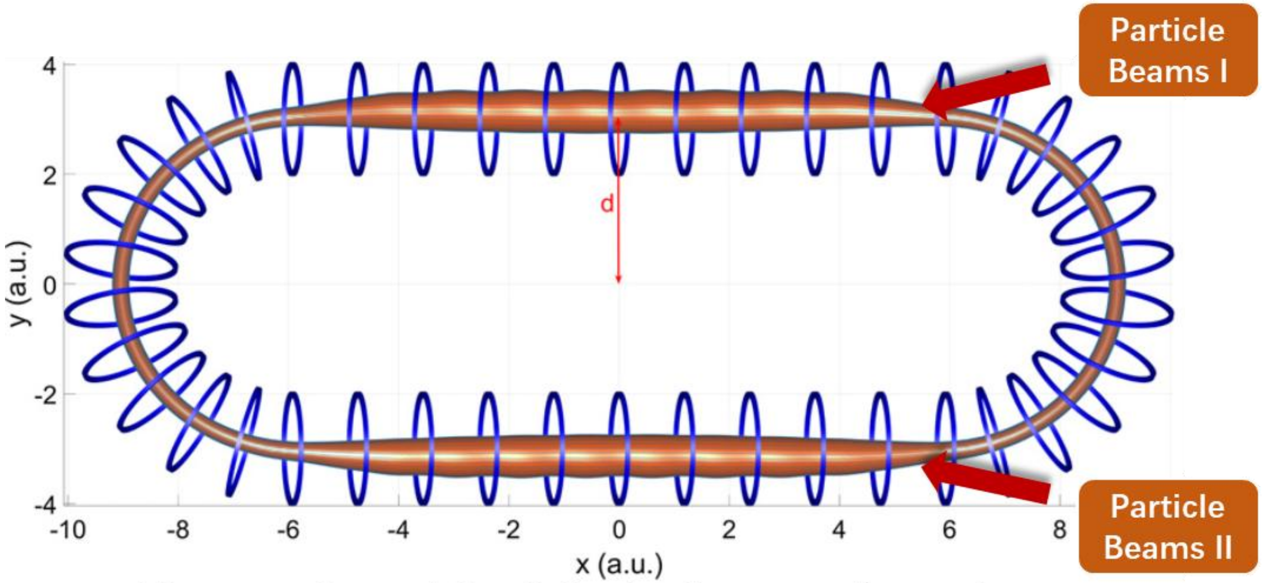


Wakefield-like Acceleration

Inverse spectral cascade in nonlinear stage enhances the wave and hydrogen ions coupling.

Thank you for your attention!

Fusion Scheme using Ion Beam Injected to Magnetic Confinement Plasma



The top view of the linked mirror configuration

(1) Proton Beam Energy≈ 800 keV

(2) 20 keV < T_i < 100 keV

(3) Low energy proton beam to heat plasmas

Ref. Zhichen FENG et.al. Nuclear Fusion (2021)

Consider the following non-Maxwellian distribution of protons

$$f(v) \equiv nK \left\{ \exp\left[-v^2/v_{ths}^2\right] + \xi \exp\left[-\frac{(v-v_f)^2}{v_{thf}^2}\right] \right\}$$

Fokker-Planck collision operator

$$\left(\frac{\partial f_\alpha}{\partial t}\right)_{col} = -\sum_\beta \Gamma_\alpha \nabla_{\boldsymbol{v}} \cdot \left[f_\alpha \nabla_{\boldsymbol{v}} h_{\alpha\beta} - \frac{1}{2}\nabla_{\boldsymbol{v}} \cdot (f_\alpha \nabla_{\boldsymbol{v}} \nabla_{\boldsymbol{v}} g_{\alpha\beta}) \right]$$

$$\equiv \sum_\beta C_{\alpha\beta} \equiv -\nabla_{\boldsymbol{v}} \cdot \sum_\beta \boldsymbol{J}_{\alpha\beta}$$

The power of energy transfer per unit volume due to collisions between two types of particles

$$P_{\alpha\beta} = -\int d^3\boldsymbol{v} \left(\frac{1}{2} m_\alpha v^2\right) C_{\alpha\beta}$$

$$= -m_\alpha \Gamma_{\alpha\beta} \int d^3\boldsymbol{v} f_\alpha \left[\boldsymbol{v} \cdot \nabla_v h_{\alpha\beta} + \left(\frac{m_\beta}{m_\alpha + m_\beta}\right) h_{\alpha\beta} \right]$$

Under isotropic conditions, the energy transfer power

$$P_{\alpha\beta} = 64\pi^3 Z_\alpha^2 Z_\beta^2 e^e \ln\Lambda \int_0^\infty dv v^2 f_\alpha \left[\frac{1}{m_\beta v} \int_0^v du f_\beta(u) u^2 - \frac{1}{m_\alpha} \int_0^\infty du f_\beta(u) u \right]$$

n: Proton number density, *K*: Normalization coefficient, v_{ths}: width of the Maxwellian background,

v_f: Position of the high-energy proton tail, v_{thf}: Width of the high-energy proton tail,

ξ:represents the proportion of the high-energy proton tail relative to the Maxwellian distribution background

$$\Gamma_{\alpha\beta} \equiv \frac{4\pi Z_\alpha^2 Z_\beta^2 e^4 \ln\Lambda}{m_\alpha^2} \quad h_{\alpha\beta}(\boldsymbol{v}) \equiv \frac{m_\alpha + m_\beta}{m_\beta} \int d^3\boldsymbol{u} \frac{f_\beta(\boldsymbol{u})}{|\boldsymbol{v} - \boldsymbol{u}|} \quad g_{\alpha\beta}(\boldsymbol{v}) \equiv \int d^3\boldsymbol{u} f_\beta(\boldsymbol{u}) |\boldsymbol{v} - \boldsymbol{u}|$$

Phys. Plasmas 32, 012101 (2025); Phys. Plasmas 4, 1039 (1997)

Considering that the non-Maxwellian distribution is maintained through energy exchange between protons, the collision operator

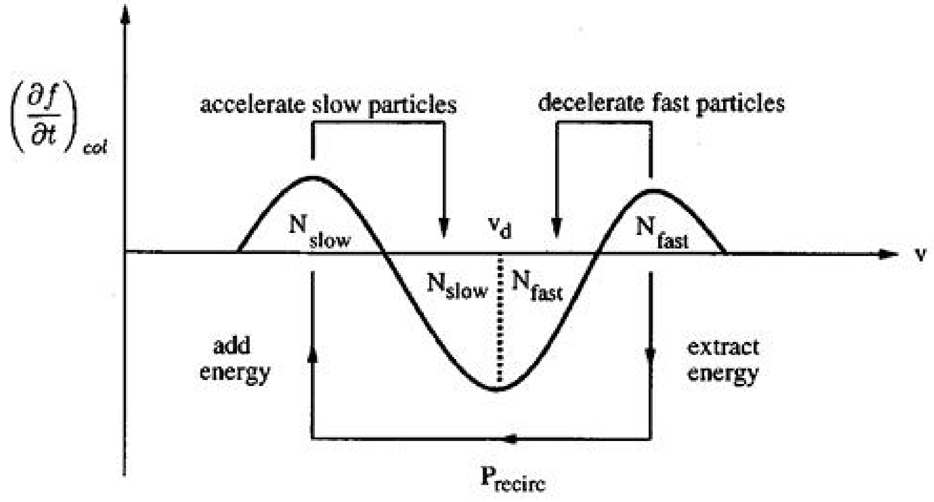
$$\left(\frac{\partial f}{\partial t}\right)_{col} = \frac{8\pi^2 (Ze)^4 \ln\Lambda}{m^2} \left\{ \frac{2}{3} \frac{\partial^2 f}{\partial v^2} \left[\frac{1}{v^3} \int_0^v du\, f(u) u^4 + \int_v^\infty du\, f(u) u \right] + 2[f(v)]^2 + \frac{4}{3v} \frac{\partial f}{\partial v} \left[\int_0^\infty du\, f(u) u - \int_0^v du\, f(u) u \left(1 - \frac{u}{v}\right)^2 \left(1 + \frac{u}{2v}\right) \right] \right\}$$

Assuming that protons on both sides of the boundary velocity v_d only exchange energy, then due to conservation of particle number, we have

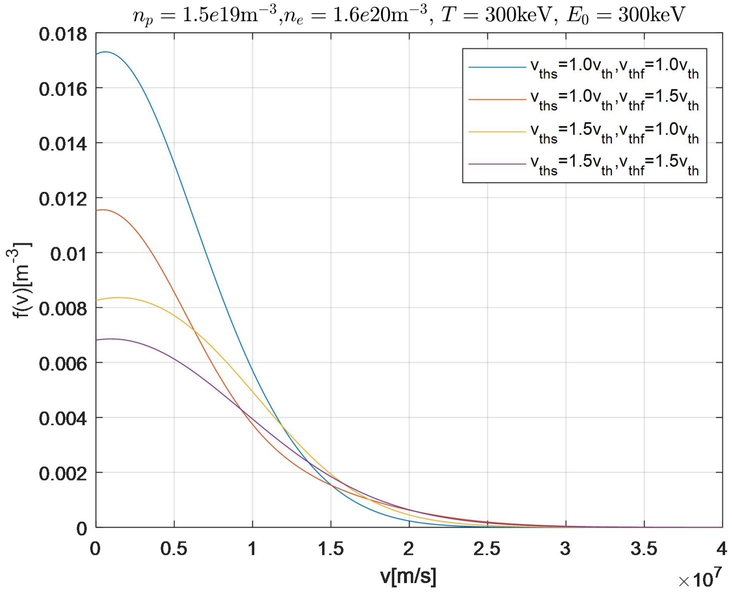
$$\int_0^{v_d} dv 4\pi v^2 \left(\frac{\partial f}{\partial t}\right)_{col} = 0$$

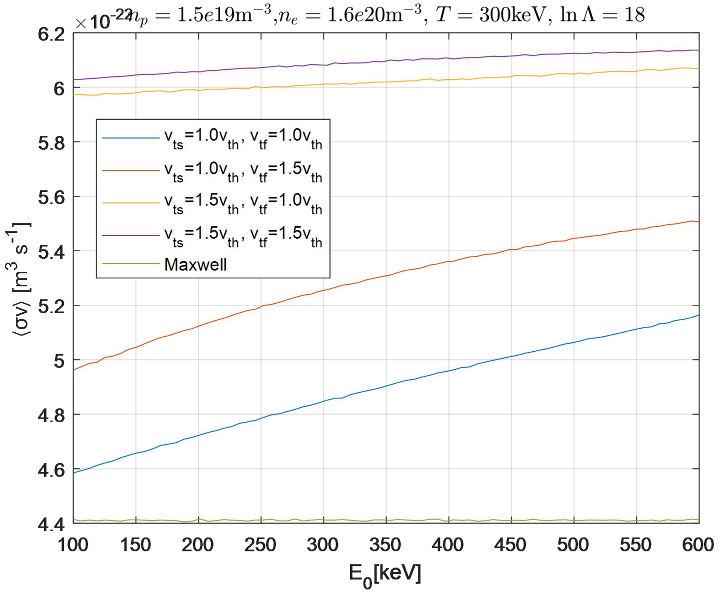
The energy transfer power

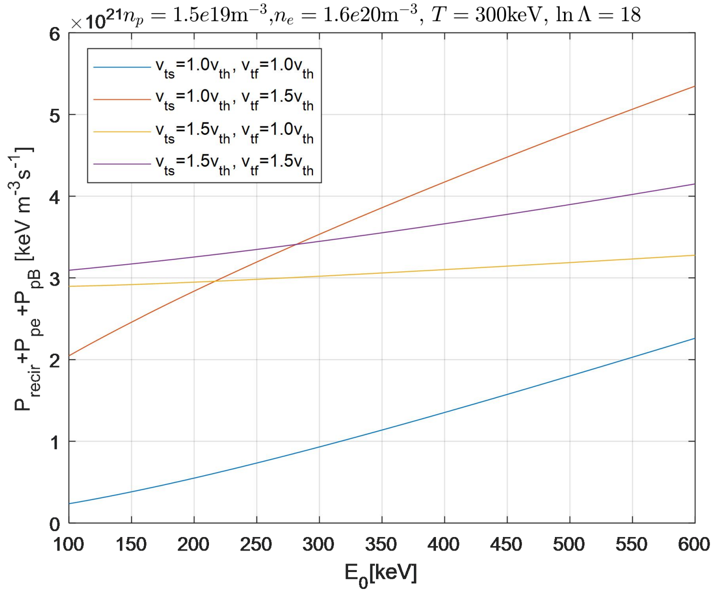
$$P_{recir} \equiv -\int_0^{v_d} (dv 4\pi v^2) \left(\frac{1}{2} m v^2\right) \left(\frac{\partial f}{\partial t}\right)_{col}$$

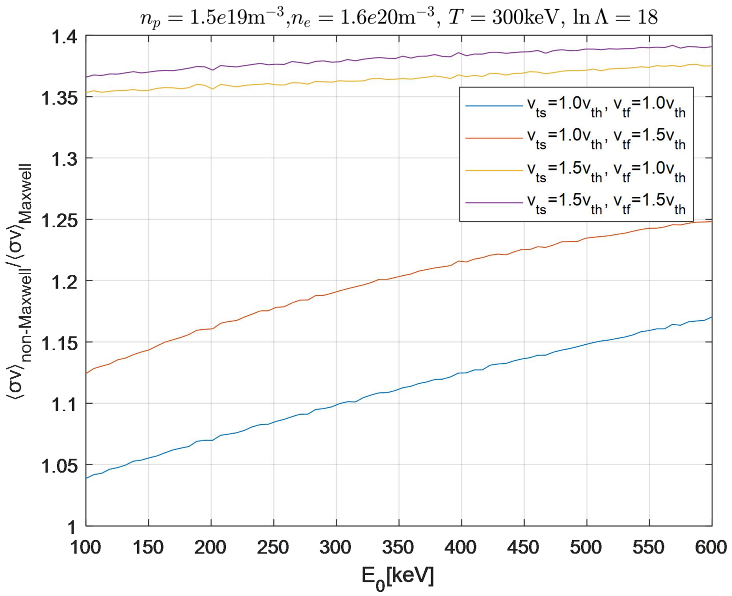


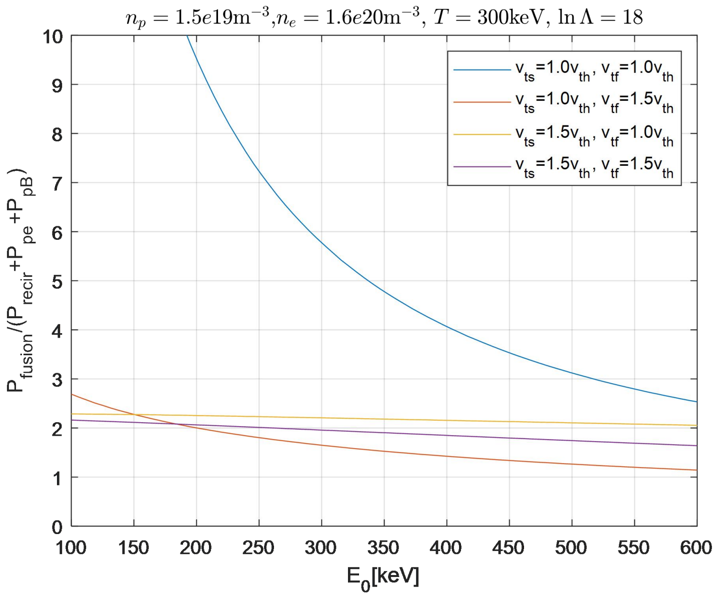
$P_{fusion} = n_p n_B \langle \sigma v \rangle Q_F$ $\langle \sigma v \rangle$ is calculated using the Monte Carlo method

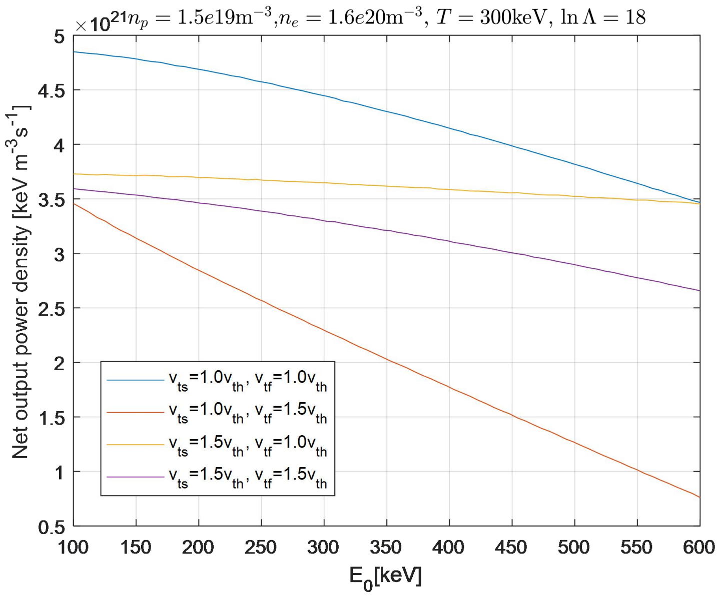






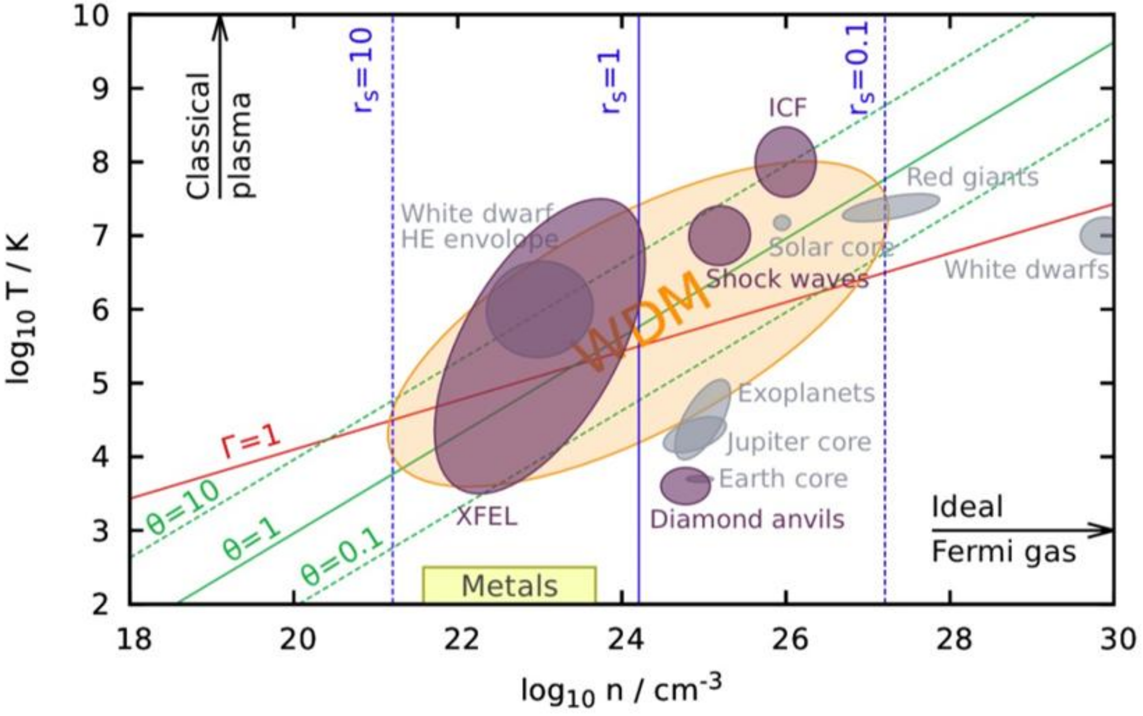






Quantum effects for Warm Dense Plasma

- **Quantum effects for instabilities, stopping power**



Degeneracy Prameter $\theta = \frac{k_B T}{E_F}$

De Broglie wavelength $\lambda_D > \frac{n_e^{-1}}{3}$

Coupling parameter $\Gamma = \frac{(Ze)^2}{r_s a_B k_B T}$

Fusion cross section parametrization

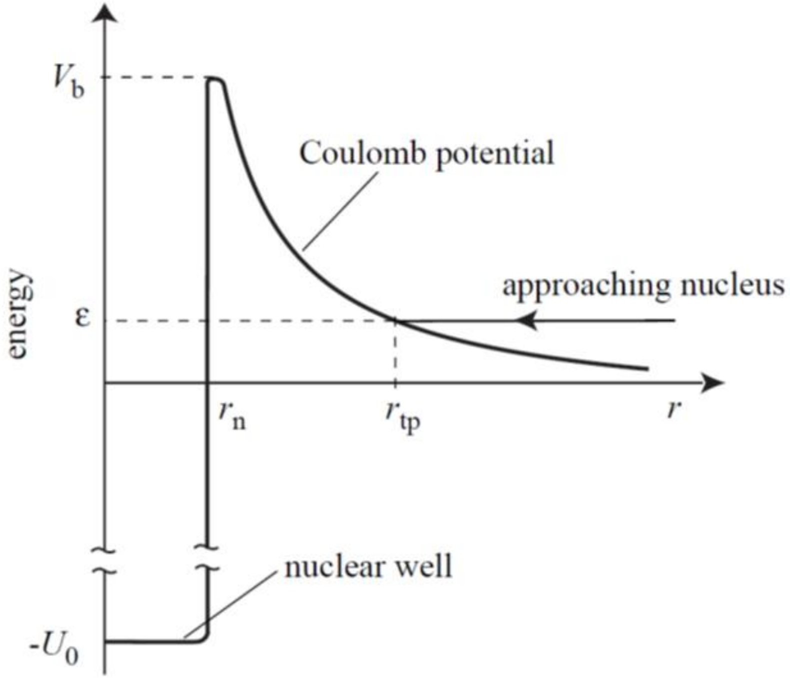
The Coulomb potential

$$V_c = \frac{Z_1 Z_2 e^2}{r}$$

at distances greater than nucleus size (Fermi)

$$r_n \cong 1.44\times 10^{-13}(A_1^{1/3} + A_2^{1/3})\ \text{cm}$$

The Coulomb barrier is of the order of 1 MeV.



A widely used parametrization of fusion reaction cross section is

$$\sigma \approx \sigma_{\text{geom}} \times T \times R$$

$$T \approx T_G = \exp(-\sqrt{\varepsilon_G/\varepsilon}),\quad \varepsilon_G = (\pi\alpha_f Z_1 Z_2)^2 2m_r c^2$$

quantum tunneling effect

$$\sigma_{\text{geom}} \approx \lambda^2 = \left(\frac{\hbar}{m_T v}\right)^2 \propto \frac{1}{\varepsilon}$$

$$\alpha_{\text{f}} = e^2/\hbar c = 1/137.04 \qquad m_{\text{r}} = \frac{m_1 m_2}{m_1 + m_2}$$

$$\sigma(\varepsilon) = \frac{S(\varepsilon)}{\varepsilon}\exp(-\sqrt{\varepsilon_G/\varepsilon})$$

Fusion reaction cross section

reaction	σ (10 keV) (barn)	σ (100 keV) (barn)	σ_{max} (barn)	ϵ_{max} (keV)
$D + T \rightarrow \alpha + n$	2.72×10^{-2}	3.43	5.0	64
$D + D \rightarrow T + p$	2.81×10^{-4}	3.3×10^{-2}	0.096	1250
$D + D \rightarrow {}^3He + n$	2.78×10^{-4}	3.7×10^{-2}	0.11	1750
$T + T \rightarrow \alpha + 2n$	7.90×10^{-4}	3.4×10^{-2}	0.16	1000
$D + {}^3He \rightarrow \alpha + p$	2.2×10^{-7}	0.1	0.9	250
$p + {}^6Li \rightarrow \alpha + {}^3He$	6×10^{-10}	7×10^{-3}	0.22	1500
$p + {}^{11}B \rightarrow 3\,\alpha$	(4.6×10^{-17})	3×10^{-4}	1.2	550
$p + p \rightarrow D + e^+ + \nu$	(3.6×10^{-26})	(4.4×10^{-25})		
$p + {}^{12}C \rightarrow {}^{13}N + \gamma$	(1.9×10^{-26})	2.0×10^{-10}	1.0×10^{-4}	400
${}^{12}C + {}^{12}C$ (all branches)		(5.0×10^{-103})		

From， Stefano Atzeni & Jurgen Meyer-ter-Vehn, Inertial Fusion