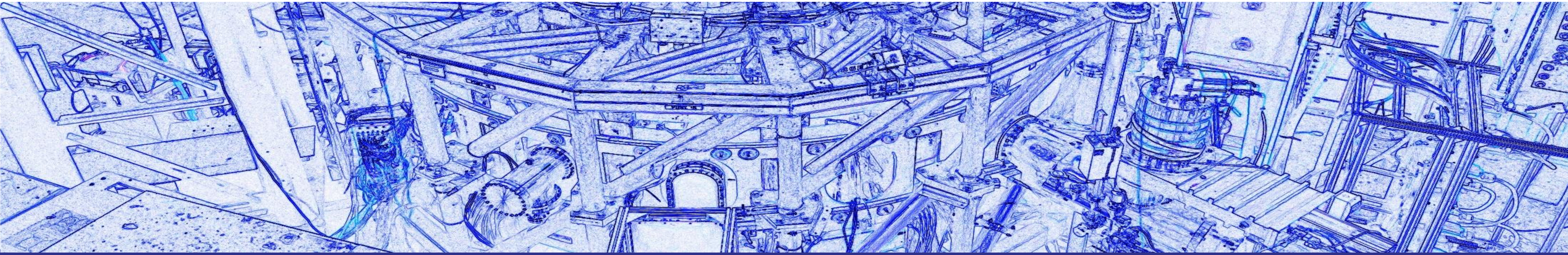
Drift-Alfvén wave and its induced particle and heat transport in I-mode pedestal plasmas

Lu Wang, Junang Zhang and Weixin Guo

Huazhong University of Science and Technology, Wuhan, China

E-mail: luwang@hust.edu.cn



Outline

- **Background and motivation**
- **Drift Alfven wave (DAW) in I-mode pedestal**
- **DAW induced particle and heat transport**
- **Summary and future plan**

Outline

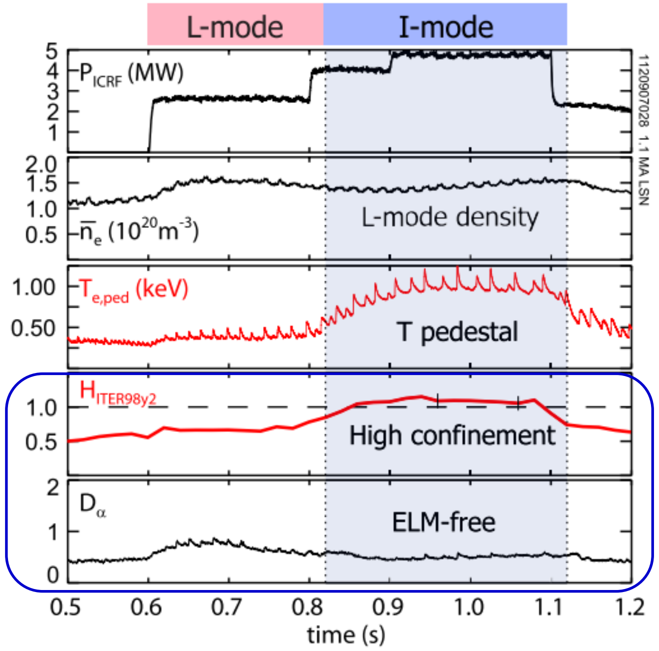
- **Background and motivation**
- **Drift Alfven wave (DAW) in I-mode pedestal**
- **DAW induced particle and heat transport**
- **Summary and future plan**

I-mode: appropriate operation scenario

- **Ideal operation scenarios: compatible with high confinement and low heat load**

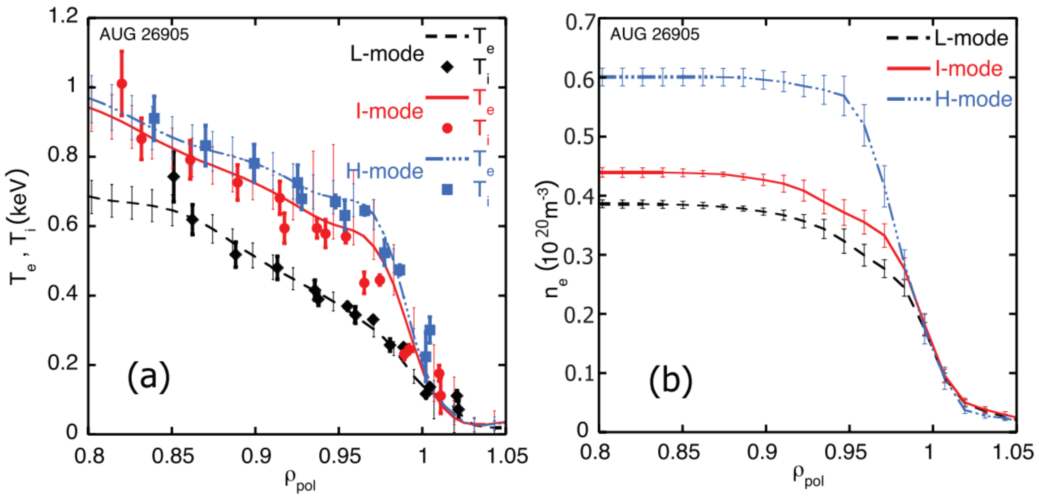
[Whyte NF 2010]

- **I-mode: a promising scenario**



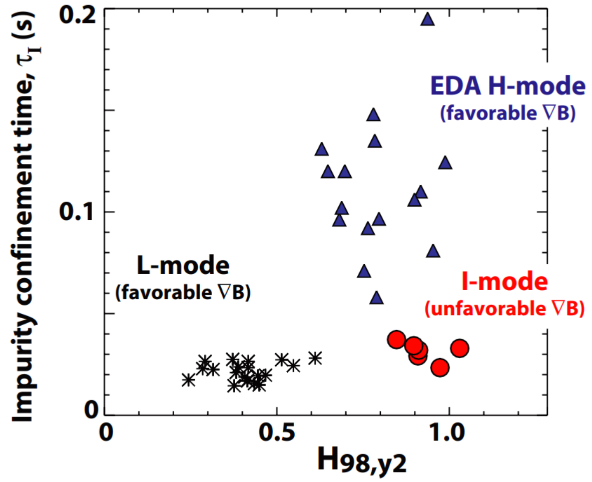
[Greenwald POP 2014]

H-mode like $T_{e,i}$ profile **L-mode like n_e profile**



[Hubbard NF 2016]

No impurity accumulation



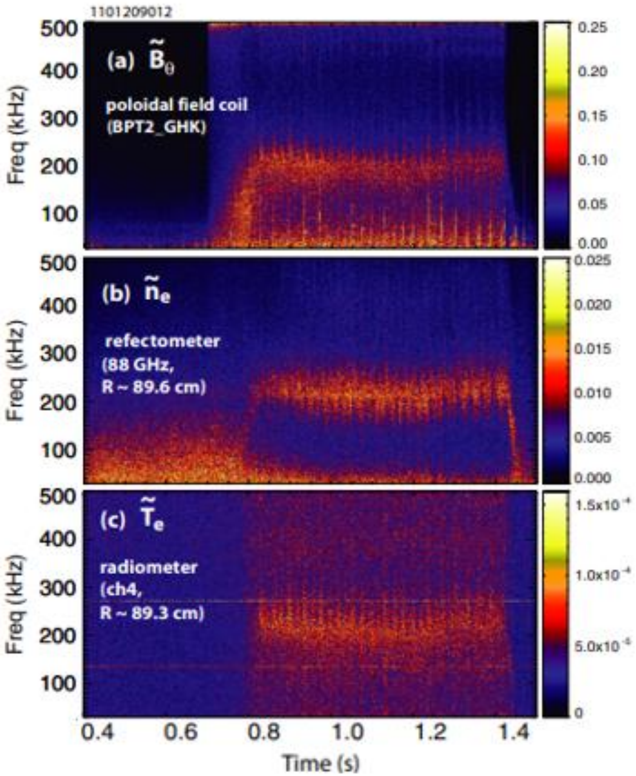
[Whyte NF 2010]

Transport mechanism behind the excellent confinement of I-mode pedestal plasmas remains unclear

Turbulence after L-I transition

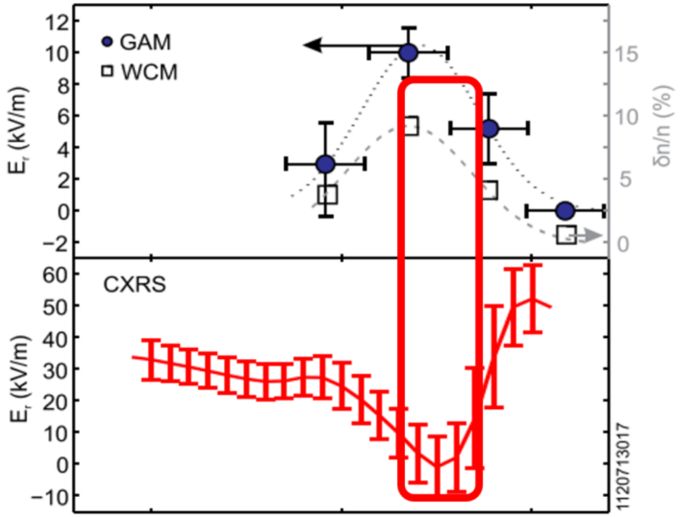
☐ Edge turbulence after L-I transition: weakly coherent mode (WCM)

- ➢ **High frequency (200-300 kHz) fluctuations increase → WCM**
- ➢ $\tilde{B}/B = 0.7\sim2\times10^{-4}$ → EM
  $\tilde{n}_e/n_0 \sim 0.15$
  $\tilde{T}_e/T_{e0} \sim 0.015$ → $\tilde{n}_e/n_0 \gg \tilde{T}_e/T_{e0}$
- ➢ **Phase velocity propagates in EDD:**
  $v_{pol} = 1\sim10$ km/s,
  $k_\theta = 0.1\sim1$ cm^{-1}



[White NF 2011]

- ➢ **The position of maximum amplitude of WCM close to the deepest E_r well → Doppler frequency shift**



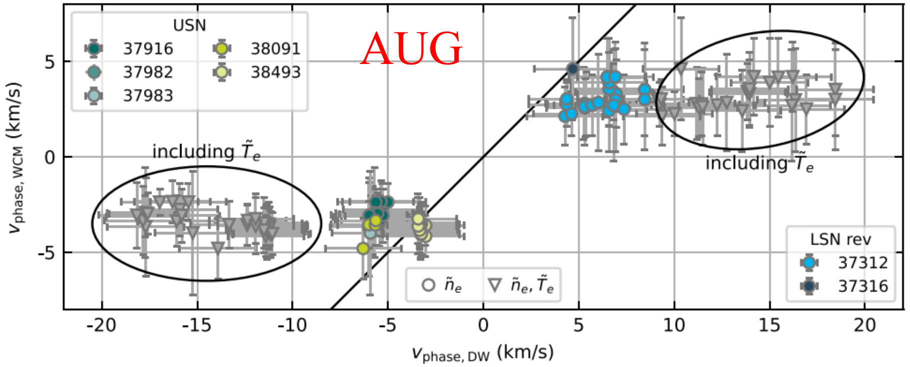
[Cziegler POP 2013]

Identify the nature of WCM and quantify the transport it drives

Mode identification for WCM from simulation and experiment

- **Based on the frequency, wavenumber, phase velocity and propagation direction**
  - **Simulation**: **Drift-Alfvén wave (DAW)**
    - **GEMR**: **parallel thermal conductivity enhancement of DAW explains the decoupling of transport** [Manz NF 2020]
    - **BOUT++**: **growth rate and density fluctuation of WCM related to DAW;** [Liu POP 2016; Liu NF 2022]
      frequency depends on the ∇n_e and ∇T_e, similar to DAW [Lang NF 2022]
  - **Experiment**: **drift wave:**

    $v_{ph} \propto v_{ph,DW}$ by ∇n_e



[Herschel NF 2024]

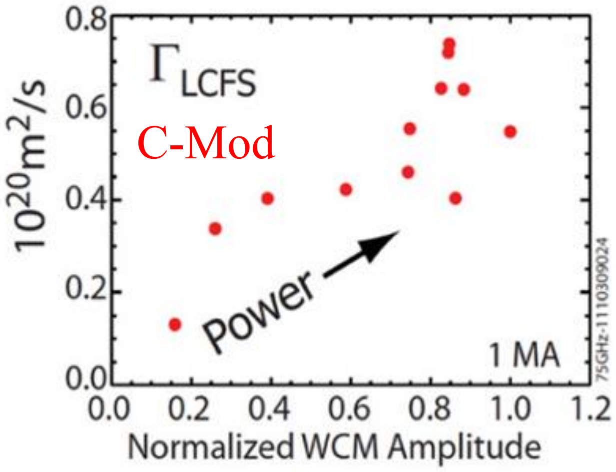
  - **Theory**: **heavy particle modes** [Coppi POP 2012], **sub-dominant ITG** [Terry NF 2023]

1st part of this work: theoretical research on DAW and compare it with the experimental characteristics of WCM

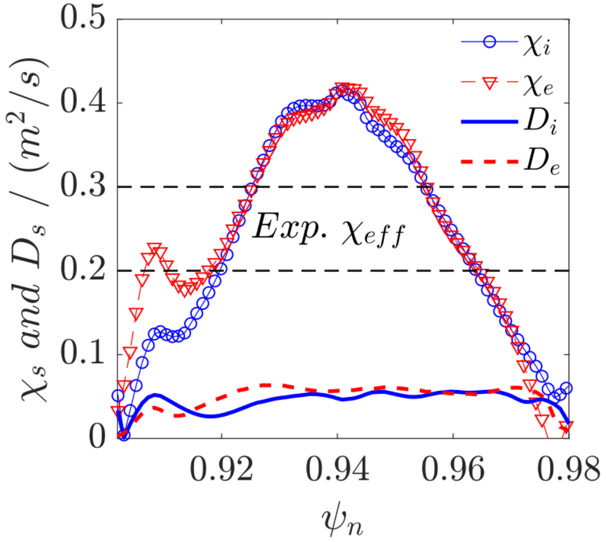
DAW induced transport

Particle and heat transport caused by DAW

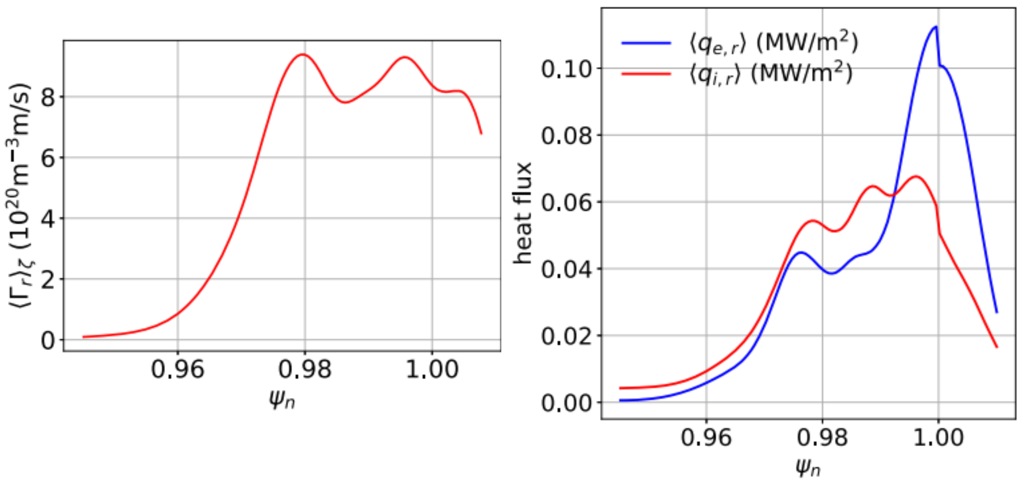
- **Exp: $\Gamma_r \propto$ WCM amp.**
- **GTC (ES):** $\chi_{eff} \sim \chi_{exp}$, $D \sim \frac{1}{2} D_{exp}$
- **BOUT++:** $\Gamma_r \sim \Gamma_{exp}$, $q_r \ll q_{exp}$



[Maingi NF 2014]



[Yang NF 2021]



[Lang NF 2021]

- **Theory: only qualitatively analyzed particle transport;**
  heat transport and quantitative comparisons were missing

[Coppi POP 2012]
[Terry NF 2023]

2^{nd} part of this work: DAW induced both particle and heat transport

Outline

- Background and motivation
- **Drift Alfven wave (DAW) in I-mode pedestal**
- DAW induced particle and heat transport
- Summary and future plan

Derivation of the DAW dispersion equation

- **Electron: the electromagnetic drift-kinetic equation including the equilibrium electric field E_r**

$$\left[\left(\frac{\partial}{\partial t}+\mathbf{v}_{\mathrm{E0}}\cdot\boldsymbol{\nabla}\right)+v_{\parallel}\nabla_{\parallel}\right]\tilde{f}+\tilde{\mathbf{v}}_E\cdot\boldsymbol{\nabla}f_0+v_{\parallel}\tilde{\mathbf{b}}\cdot\boldsymbol{\nabla}f_0+\frac{e}{m_e}\left[\boldsymbol{\nabla}_{\parallel}\tilde{\phi}+\frac{1}{c}\left(\frac{\partial}{\partial t}+\mathbf{v}_{\mathrm{E0}}\cdot\boldsymbol{\nabla}\right)\tilde{A}_{\parallel}\right]\frac{\partial f_0}{\partial v_{\parallel}}=0$$

- **Ion: fluid limit with FLR effects** $\dfrac{\tilde{n}_i}{n_0}=\dfrac{e\tilde{\phi}}{T_{i0}}\chi_i^L$

- **Quasi-neutrality condition:** $\tilde{n}_i(\mathbf{x},t)=\tilde{n}_e(\mathbf{x},t)$

- **Ampere's law:** $\nabla^2\tilde{A}_{\parallel}\simeq-\dfrac{4\pi}{c}\tilde{j}_{\parallel}=\dfrac{4\pi e}{c}\int\tilde{f}v_{\parallel}d\mathbf{v}$

→ Dispersion equation of DAW

$$1+\tau(1-\Gamma_0)-\frac{\widehat{\omega}_{*e}}{\widehat{\omega}'_k}\Gamma_0+\left(\frac{\widehat{\omega}'_k}{\hat{k}_{\parallel}}\right)^2\hat{\mu}\left(\frac{\widehat{\omega}_{*e}}{\widehat{\omega}'_k}-1-\eta_e\frac{\widehat{\omega}_{*e}}{\widehat{\omega}'_k}\right)-\left(\frac{\widehat{\omega}'_k}{\hat{k}_{\parallel}}\right)^2\frac{\hat{\beta}}{b_s}\left(1-\frac{\widehat{\omega}_{*e}}{\widehat{\omega}'_k}\right)\left(\tau+\frac{\widehat{\omega}_{*e}}{\widehat{\omega}'_k}\right)(1-\Gamma_0)$$
$$+\Gamma_0\eta_i\frac{\widehat{\omega}_{*e}}{\widehat{\omega}'_k}b_i\left[1-\left(\frac{\widehat{\omega}'_k}{\hat{k}_{\parallel}}\right)^2\frac{\hat{\beta}}{b_s}\left(1-\frac{\widehat{\omega}_{*e}}{\widehat{\omega}'_k}\right)\right]\left(1-\frac{I_1}{I_0}\right)=0$$

- $\hat{\mu}$: inertial effect; $\hat{\beta}$: electromagnetic effects

Solution of the DAW dispersion relation

- **Let** $\widetilde{\omega}'_k = \frac{\omega'_k}{\omega_{*e}}$ → **Dispersion equation:** $A_3(\widetilde{\omega}'_k)^3 + A_2(\widetilde{\omega}'_k)^2 + A_1\widetilde{\omega}'_k + A_0 = 0$
- **Assumptions:**
  - Ignore the electron inertia effect: $\hat{\mu} = 0$;
  - Long wavelength limit: $b_i \sim b_s \sim \varepsilon$; $C_A = \left(\frac{\omega_{*e}}{\omega_A}\right)^2 \approx \frac{1}{2}\frac{\hat{\beta}}{\hat{s}} b_s \sim \varepsilon$ → $A_0 \sim A_1 \sim \varepsilon^0$, $A_2 \sim A_3 \sim \varepsilon^1$
- **Two limiting cases:**
  - **AW branch:** $\omega'_k \sim \omega_A$, $\widetilde{\omega}'_k \sim \varepsilon^{-1/2}$ → $\underset{\varepsilon^{-1/2}}{A_3(\widetilde{\omega}'_k)^3} + \underset{\varepsilon^{0}}{A_2(\widetilde{\omega}'_k)^2} + \underset{\varepsilon^{-1/2}}{A_1\widetilde{\omega}'_k} + \underset{\varepsilon^{0}}{A_0} = 0$

$$\omega_{AW} = \pm\omega_A\left(1 \pm \frac{\omega_{*pi}}{2\omega_A} + \frac{\omega_{*pi}^2}{8\omega_A^2} + \frac{3b_i}{8} + \frac{b_s}{2}\right)$$

  - **DW branch:** $\omega'_k \sim \omega_{*e}$, $\widetilde{\omega}'_k \sim \varepsilon^0$ → $\underset{\varepsilon^{1}}{A_3(\widetilde{\omega}'_k)^3} + \underset{\varepsilon^{1}}{A_2(\widetilde{\omega}'_k)^2} + \underset{\varepsilon^{0}}{A_1\widetilde{\omega}'_k} + \underset{\varepsilon^{0}}{A_0} = 0$

$$\omega_{DW} = \omega_{*e}\left\{\{1 - (\tau + 1 + \eta_i)b_i + \left[\tau\left(\tau + \eta_i + \frac{7}{4}\right) + \frac{3}{4} + \frac{3\eta_i}{2}\right] b_i^2 - \frac{(\tau + 1 + \eta_i)^2}{2}\frac{b_i^2\hat{\beta}}{\hat{s}^2}\right\}$$

Identify WCM through the dispersion relationship

☐ Typical parameters of I-mode pedestal in C-Mod: [Lang NF 2022，Liu POP 2016]

Physical quantity	Value
Major/Minor radius (m)	0.68/0.22
Magnetic field B_0 (T)	5.789
Safety factor q	4.3
Magnetic shear $\hat{s}$	9.2
Poloidal wave number k_y (m^{-1})	180
Parallel wave number $k_\parallel \approx \frac{\hat{s}}{qR}$ (m^{-1})	3.15
Radial electric field E_r (kV/m)	−20
Density gradient scale length $\frac{1}{L_n}$ (m^{-1})	43.73
Electron and ion temperatures $T_{e0} = T_{i0}$ (eV)	300
Temperature gradient scale length $\frac{1}{L_{T_e}} = \frac{1}{L_{T_i}}$ (m^{-1})	173.3
Electron pressure ratio β_e	4.4×10^{-4}

➢ **Exp frequency**: $f_{WCM} = 350$ kHz **(laboratory frame)**

$\omega_{WCM} = 1.38 \times 10^6$ rad/s **(plasma frame)**

Theory: $\omega_{DW} = 5.38 \times 10^5$ rad/s , $\omega_{AW} = 2.45 \times 10^7$ rad/s

→ ω_{WCM} **closer to ω_{DW} and much lower than ω_{AW}**

➢ **Exp phase velocity**: $v_{WCM} = (8.5 \pm 3)$ km/s, EDD

Theory: $v_{DW} = f_{DW}/k_y \approx 7.0$ km/s, **EDD**

→ v_{WCM} **quantitatively consistent with v_{DW}**

The WCM in I-mode pedestal on C-Mod belongs to the DW branch

Comparison of normalized fluctuations

- **Compare the parallel induced electric field with the electrostatic field for DW branch**

$$\frac{1}{c}\frac{\partial \tilde{A}_{\parallel}}{\partial t}/\nabla_{\parallel}\tilde{\phi} = -2\beta_{\mu}\zeta_e^2\left(\frac{\omega_{*e}}{\omega_{DW}} - 1\right) \approx |6.4 \times 10^{-3}| \ll 1$$

  - **The parallel induced electric field much smaller than the electrostatic field → weak EM**

- **Normalized magnetic fluctuation $|\tilde{\mathbf{b}}|$ compared with normalized density fluctuation**

$$|\tilde{\mathbf{b}}|/\left|\frac{\tilde{n}_e}{n_0}\right| = \left|\frac{k_{\perp}\rho_s c_s}{c}\frac{\chi_{A\phi}}{\chi_e^L}\right| \approx 8.31 \times 10^{-4}$$

  - **Exp:** $\left|\frac{\tilde{n}_e}{n_0}\right| = 0.15 \rightarrow |\tilde{\mathbf{b}}| \approx 1.25 \times 10^{-4}$**, consistent with the experimental results $\mathbf{0.7 \sim 2 \times 10^{-4}}$**

[Hubbard POP 2011]

- **Normalized temperature/density fluctuations**

$$\left|\frac{\tilde{T}_e}{T_{e0}}\right|/\left|\frac{\tilde{n}_e}{n_0}\right| = \frac{2|\chi_T^L|}{3|\chi_e^L|} \approx 0.1$$

  - **Highly consistent with the experimental observations** [White NF 2011]

Brief summary of 1st part

- The dispersion relations of the two branches (DW and AW) of DAW were analyzed

	Analytical result	Experimental result	Simulation result
Frequency(kHz)	~200	300~400 [Liu POP 2016]	~250 [Lang NF 2022]
Phase velocity(km/s)	7.0	8.5 ± 3 [Hubbard POP 2011]	/
Propagating direction	EDD	EDD [Hubbard POP 2011]	EDD [Yang NF 2021]
$\frac{1}{c}\frac{\partial \tilde{A}_\parallel}{\partial t}/\nabla_\parallel \tilde{\phi}$	-6.4×10^{-3}	/	/
$\left\lvert\frac{\tilde{T}_e}{T_{e0}}\right\rvert / \left\lvert\frac{\tilde{n}_e}{n_0}\right\rvert$	0.1	0.1 [White NF 2011]	0.9 [Manz NF 2020]
$\lvert\tilde{\mathbf{b}}\rvert / \left\lvert\frac{\tilde{n}_e}{n_0}\right\rvert$	8.3×10^{-4}	10^{-3}~10^{-2} [Hubbard POP 2011]	/

Primary characteristics of the experimental WCM reproduced by DW branch

Outline

- Background and motivation
- Drift Alfven wave (DAW) in I-mode pedestal
- **DAW induced particle and heat transport**
- Summary and future plan

Particle and heat transport flux based on modulation methods

- The modulation method describe turbulence and transport respectively, $f = f_0 + \tilde{f} + \hat{f}$
  - Time scale separation: turbulence much faster ($|\omega_0| \ll |\omega|$)
  - Space scale separation: turbulence much smaller ($|k_0| \ll |k|$), $k' = k_0 - k \approx -k$
- Model equations: "numerical modulation experiments"

Continuity equation with modulation	$\frac{\partial \hat{f}}{\partial t} + \nabla \cdot \hat{\boldsymbol{\Gamma}} = \hat{\xi}$
Transport flux in phase space	$\hat{\boldsymbol{\Gamma}} = \left\langle (\tilde{\mathbf{v}}_E + v_\parallel \tilde{\mathbf{b}})\tilde{f} + \frac{e}{m_e}\tilde{\phi}\tilde{\mathbf{b}}\frac{\partial f_0}{\partial v_\parallel} \right\rangle$
$\tilde{f}$ decomposition	$\tilde{f} = \left(\frac{e\tilde{\phi}}{T_{e0}}\right) f_0 + \tilde{p} f_{0\perp} + \tilde{q} f_{0\perp} \frac{v_\perp^2 - 2v_e^2}{2v_e^2}$
NL coupling	$g_{k'}^{-1}\tilde{p}_{k'}^{(1)} + b_{k'}^{p} f_{0\parallel}\left(\frac{e\tilde{\Phi}_{k'}^{(1)}}{T_{e0}}\right) = -W_{k,k_0}\left(\frac{e}{T_{e0}}\right)\left(\tilde{\Phi}_k^* \hat{p}_{k_0} - \hat{\Phi}_{k_0}\tilde{p}_k^*\right)$ $g_{k'}^{-1}\tilde{q}_{k'}^{(1)} + b_{k'}^{q} f_{0\parallel}\left(\frac{e\tilde{\Phi}_{k'}^{(1)}}{T_{e0}}\right) = -W_{k,k_0}\left(\frac{e}{T_{e0}}\right)\left(\tilde{\Phi}_k^* \hat{q}_{k_0} - \hat{\Phi}_{k_0}\tilde{q}_k^*\right)$

Particle and heat transport flux based on modulation methods

- **Modulated phase space transport flux**

$$\hat{\boldsymbol{\Gamma}}_{\mathbf{mod}} = \mathrm{Re}\left\{-i\frac{cT_{e0}}{eB_0}\sum_k \mathbf{k}\times\mathbf{b}_0\left[\frac{e\widetilde{\Phi}_k^{(1)}}{T_{e0}}\left(\tilde{p}_{k'}+\frac{v_\perp^2-2v_e^2}{2v_e^2}\tilde{q}_{k'}\right)-\frac{e\widetilde{\Phi}_{k'}}{T_{e0}}\left(\tilde{p}_k^{(1)}+\frac{v_\perp^2-2v_e^2}{2v_e^2}\tilde{q}_k^{(1)}\right)\right]f_{0\perp}\right\}$$

- **Test particle approximation ($\widetilde{\Phi}_k^{(1)}=0$) → transport flux is written in terms of source terms**

$$\hat{\boldsymbol{\Gamma}}_{\mathbf{mod}}^{\mathbf{TP}} = \mathrm{Re}\left[i\frac{cT_{e0}}{eB_0}\sum_k \mathbf{k}\times\mathbf{b}_0\frac{W_{kk_0}}{-i\omega_0}g_k f_{0\perp}\left(\hat{\xi}_p+\frac{v_\perp^2-2v_e^2}{2v_e^2}\hat{\xi}_q\right)\left|\frac{e\widetilde{\Phi}_k}{T_{e0}}\right|^2\right]$$

- **Substitute the particle and heat source terms and integrate in velocity space → radial component of the particle and electron heat fluxes**

$$\hat{\Gamma}_{n,r}^{\mathrm{TP}} = \nabla_r\hat{n}_0\left(\frac{cT_{e0}}{eB_0}\right)^2\sum_k k_y^2\frac{1}{\omega_k'}\mathrm{Im}\left[G_0^0\left|\frac{e\tilde{\phi}_k}{T_{e0}}\right|^2-\left(G_0^1+G_0^{1*}\right)\left|\frac{e\tilde{\phi}_k^*}{T_{e0}}\right|\left|\frac{e\tilde{A}_{\parallel k}}{T_{e0}}\right|+G_0^2\left|\frac{e\tilde{A}_{\parallel k}}{T_{e0}}\right|^2\right]$$

$$\hat{\mathrm{q}}_r^{\mathrm{TP}} = n_0\nabla_\mathrm{r}\hat{T}_{e0}\left(\frac{cT_{e0}}{eB_0}\right)^2\sum_k k_y^2\frac{1}{\omega_k'}\mathrm{Im}\left[\left(G_0^0+G_2^0\right)\left|\frac{e\tilde{\phi}_k}{T_{e0}}\right|^2-\left(G_0^1+G_0^{1*}+G_2^1+G_2^{1*}\right)\left|\frac{e\tilde{\phi}_k^*}{T_{e0}}\right|\left|\frac{e\tilde{A}_{\parallel k}}{T_{e0}}\right|+\left(G_0^2+G_2^2\right)\left|\frac{e\tilde{A}_{\parallel k}}{T_{e0}}\right|^2\right]$$

Particle diffusivity and electron thermal conductivity by DW branch

- $\hat{\Gamma}_{n,r}^{\mathrm{TP}} = -\hat{D}_n \nabla_{\mathrm{r}} \hat{n}_0$, **particle diffusivity $\hat{D}_n$**

$$\hat{D}_n = -\left(\frac{cT_{e0}}{eB_0}\right)^2 \int \frac{d\mathbf{k}}{(2\pi)^3} k_y^2 \frac{1}{\omega_k'} \left[\mathrm{Im}G_0^0 - 2\mathrm{Re}G_0^1 \mathrm{Im}\chi_{A\phi} + \mathrm{Im}G_0^2 |\chi_{A\phi}|^2\right] \left|\frac{e\tilde{\phi}_k}{T_{e0}}\right|^2$$

$$= \int \frac{d\mathbf{k}}{(2\pi)^3} k_y^2 \rho_s^2 c_s^2 \frac{1}{\omega_k'} G_i \left[1 - 4\left(\frac{\omega_{*e}}{\omega_k'} - 1 - \frac{1}{2}\eta_e \frac{\omega_{*e}}{\omega_k'}\right)\beta_\mu \zeta_e^2\right] \left|\frac{e\tilde{\phi}_k}{T_{e0}}\right|^2$$

- $\hat{q}_r^{\mathrm{TP}} = -\hat{\chi}_e n_0 \nabla_{\mathrm{r}} \hat{T}_{e0}$, **electron thermal conductivity $\hat{\chi}_e$**

$$\hat{\chi}_e = -\left(\frac{cT_{e0}}{eB_0}\right)^2 \int \frac{d\mathbf{k}}{(2\pi)^3} k_y^2 \frac{1}{\omega_k'} \left[\left(\mathrm{Im}G_0^0 + \mathrm{Im}G_2^0\right) - 2\left(\mathrm{Re}G_0^1 + \mathrm{Re}G_2^1\right)\mathrm{Im}\chi_{A\phi} + \left(\mathrm{Im}G_0^2 + \mathrm{Im}G_2^2\right)|\chi_{A\phi}|^2\right] \left|\frac{e\tilde{\phi}_k}{T_{e0}}\right|^2$$

$$= \int \frac{d\mathbf{k}}{(2\pi)^3} k_y^2 \rho_s^2 c_s^2 \frac{1}{\omega_k'} G_i \left[\frac{5}{4} - 6\left(\frac{\omega_{*e}}{\omega_k'} - 1 - \frac{1}{2}\eta_e \frac{\omega_{*e}}{\omega_k'}\right)\beta_\mu \zeta_e^2\right] \left|\frac{e\tilde{\phi}_k}{T_{e0}}\right|^2$$

[Terry NF 2023]

- **Imaginary part G_i origins from the resonance of electron and DW, similar with $g_k = \pi\delta(\omega - k_\parallel v_\parallel)$**
- **The transport coefficient contributed by the EM part**

Particle diffusivity and electron thermal conductivity

- **The turbulence intensity is taken as the experimental value:** $\left|\frac{\tilde{T}_e}{T_{e0}}\right| = 0.015 \rightarrow \left|\frac{e\tilde{\phi}_k}{T_{e0}}\right|^2 = 0.022$
  - $\hat{D}_n \approx 0.21\ \mathrm{m^2/s}$, **about twice that experimental value** [Dominguez PHD thesis 2012]
  - $\hat{\chi}_e \approx 0.27\ \mathrm{m^2/s}$, **good agreement with the experimental and simulated values** [Liu POP 2016; Yang NF 2021]
  - **EM contribution is about 10% that ES part**

Comparison of the analytical transport coefficients with the experimental and simulation results

	Analytical result	Experimental result	Simulation result
$\hat{D}_n$ $(\mathrm{m^2/s})$	**0.21**	**0.1** [Dominguez PHD thesis 2012]	**0.045~0.05** [Lang NF 2022]
$\hat{\chi}_e$ $(\mathrm{m^2/s})$	**0.27**	**0.13~0.2** [Liu POP 2016]	**0.15~0.35** [Yang NF 2021]
EM contribution to $\hat{D}_n$	**12%**	/	/
EM contribution to $\hat{\chi}_e$	**15%**	/	/

Outline

- **Background and motivation**
- **Drift Alfven wave (DAW) in I-mode pedestal**
- **DAW induced particle and heat transport**
- **Summary and future plan**

Summary and future plan

Summary

- **The dispersion relations of the two branches (DW and AW) of DAW are calculated**
- **Confirm that WCM on C-Mod belongs to the DW branch, reproduce main characteristics of the experimental WCM: frequency 200 kHz, phase velocity 7.0 km/s, propagation along EDD,**

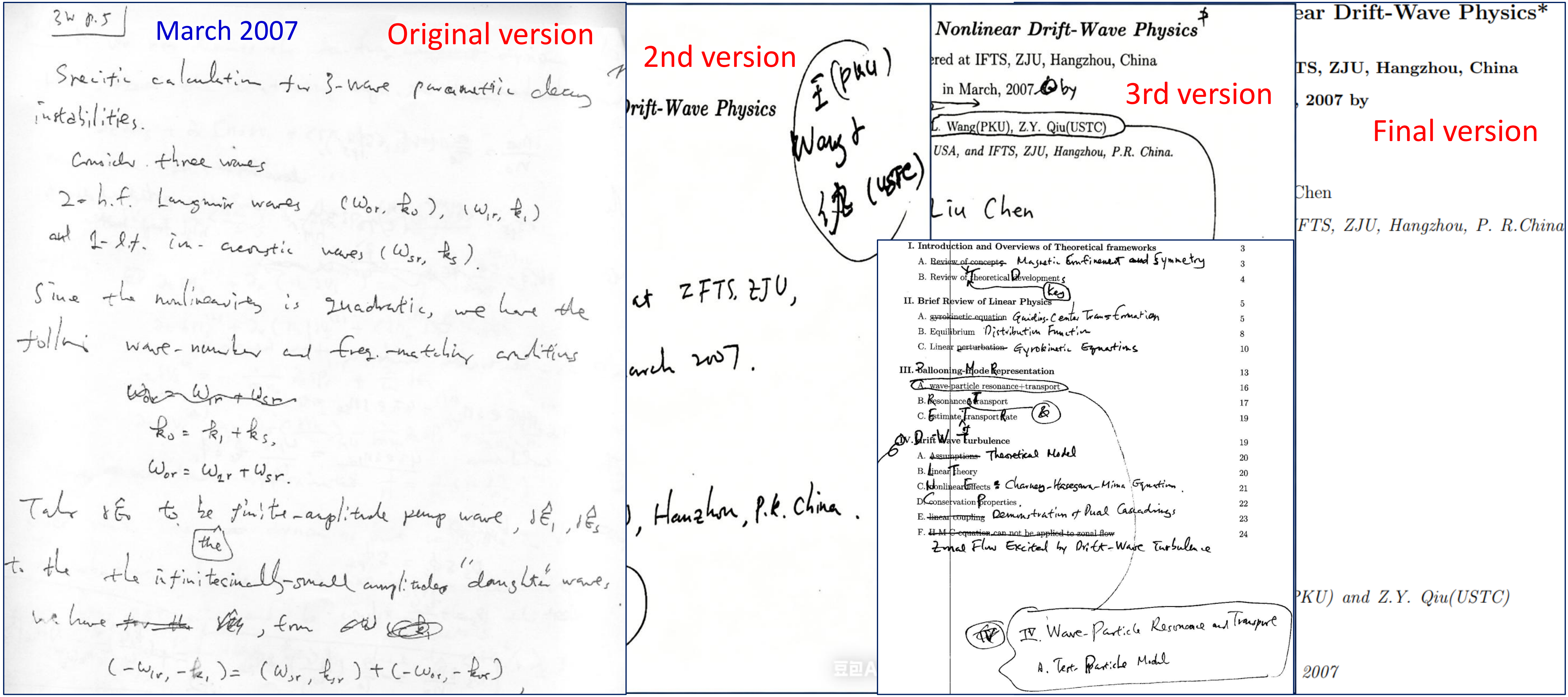
$$\left|\frac{\tilde{T}_e}{T_{e0}}\right| \Big/ \left|\frac{\tilde{n}_e}{n_0}\right| \approx 0.1 \text{ and } |\tilde{\mathrm{b}}| \Big/ \left|\frac{\tilde{n}_e}{n_0}\right| \approx 8.3 \times 10^{-4}$$

- **Transport coefficients: $\widehat{D}_n \approx 0.21\ \mathrm{m}^2/\mathrm{s}$ and $\hat{\chi}_e \approx 0.27\ \mathrm{m}^2/\mathrm{s}$ quantitatively good agreement with the experimental results** [Zhang, Wang and Guo, Nucl. Fusion 65 (2025) 026068]

Future plan

- **Nonlinear saturation level of turbulence intensity**
- **Extension to compare with EAST I-mode experimental results**

Thanks to Prof. Chen for your lectures ~20 years ago



八秩风华，桃李盈门；

学海领航，德业长青。

恭祝陈老师福寿康宁！

3.3 The relationship between the modulation and quasi-linear transport coefficients

- **Formal relationship between the modulated and the quasi-linear transport coefficients**

$$\Gamma_{\mathrm{QL}} = -D_n \nabla n_0 - D_T \nabla T_{e0} + U_n n_0 + U_T T_{e0}$$

variation → $\delta\Gamma_{\mathrm{QL}} = -D_n^{\mathrm{incr}} \delta\nabla n_0 + U_n^{\mathrm{incr}} \delta n_0$

$$D_n^{\mathrm{incr}} = D_n + \left(\frac{\partial D_n}{\partial \nabla n_0}\right)\nabla n_0 + \left(\frac{\partial D_T}{\partial \nabla n_0}\right)\nabla T_{e0} - \left(\frac{\partial U_n}{\partial \nabla n_0}\right) n_0 - \left(\frac{\partial U_T}{\partial \nabla n_0}\right) T_{e0}$$

- **Modulation particle diffusivity D_n^{incr} different from QL D_n**

- **Substituting the QL transport coefficients → D_n^{incr}**

$$D_n^{\mathrm{incr}} = \left(\frac{cT_{e0}}{eB_0}\right)^2 \sum_k k_y^2 \sqrt{\pi} |\zeta_e| e^{-\zeta_e^2} \frac{1}{\omega_k'} \left[1 - 2\beta_\mu \zeta_e^2 \left(\frac{4\omega_{*e}}{\omega_k'} - 4 - \eta_e \frac{\omega_{*e}}{\omega_k'}\right)\right] \left|\frac{e\tilde{\phi}_k}{T_{e0}}\right|^2$$

- **D_n^{incr} is the counterpart of $\widehat{D}_n$ obtained before, ES parts are the same**
- **The electron thermal transport coefficients are similar**