A Unified View of Neoclassical Transport in 3D Magnetic Configurations

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Growing interest for neoclassical transport in 3D magnetic configurations

- External helical magnetic perturbations in tokamaks
  Evans 04, Park 09, Shaing 15, Park 21
- Magnetic ripple in tokamaks Kovrizhnykh 99, Kovrizhnykh 03, Saibene 07, Nave 10, Varennes 23
- Optimised stellarators Nührenberg 88, Mynick 06, Boozer 08, Helander 14
- Complex calculation → resort to drift kinetic codes
  Hirshman 86, Satake 08, Belli 12, Landreman 14.
- Reduced model : fast → parameter scan

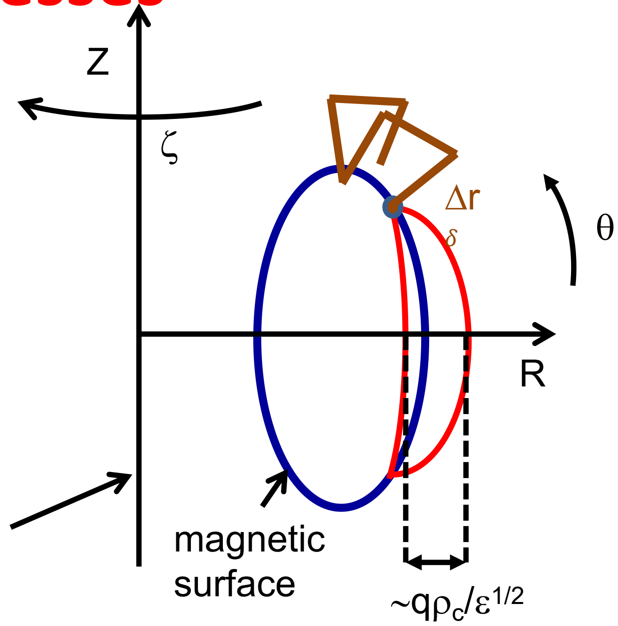
Outline

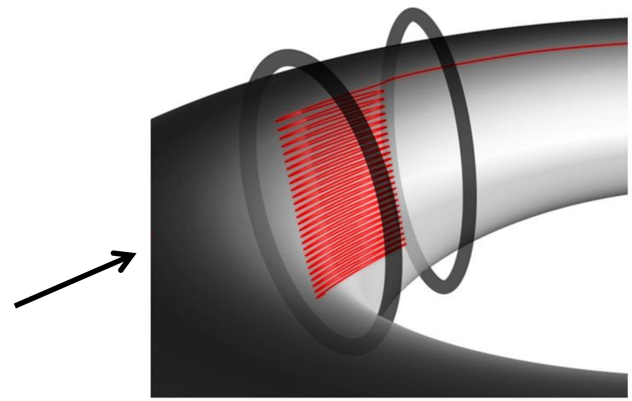
1. Basics in 3D neoclassical transport
2. Hamiltonian description
3. Principle of minimum of entropy production rate
4. Comparison to simulations

3D neoclassical transport comes from several additive processes

- Transport in a torus with 1 single helical perturbation → 4 basic contributions Mynick 06

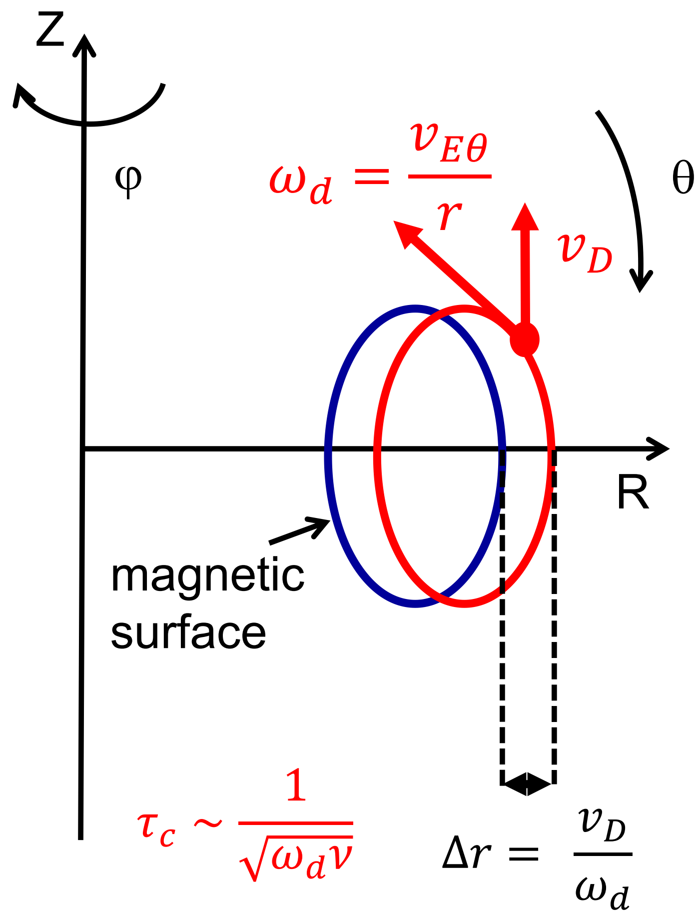
- toroidally symmetric tokamak
- uncompensated magnetic drift for bananas ("banana-drift")
- helically symmetric device ("straight" stellarator)
- uncompensated magnetic drift for helically trapped particles ("super-banana")



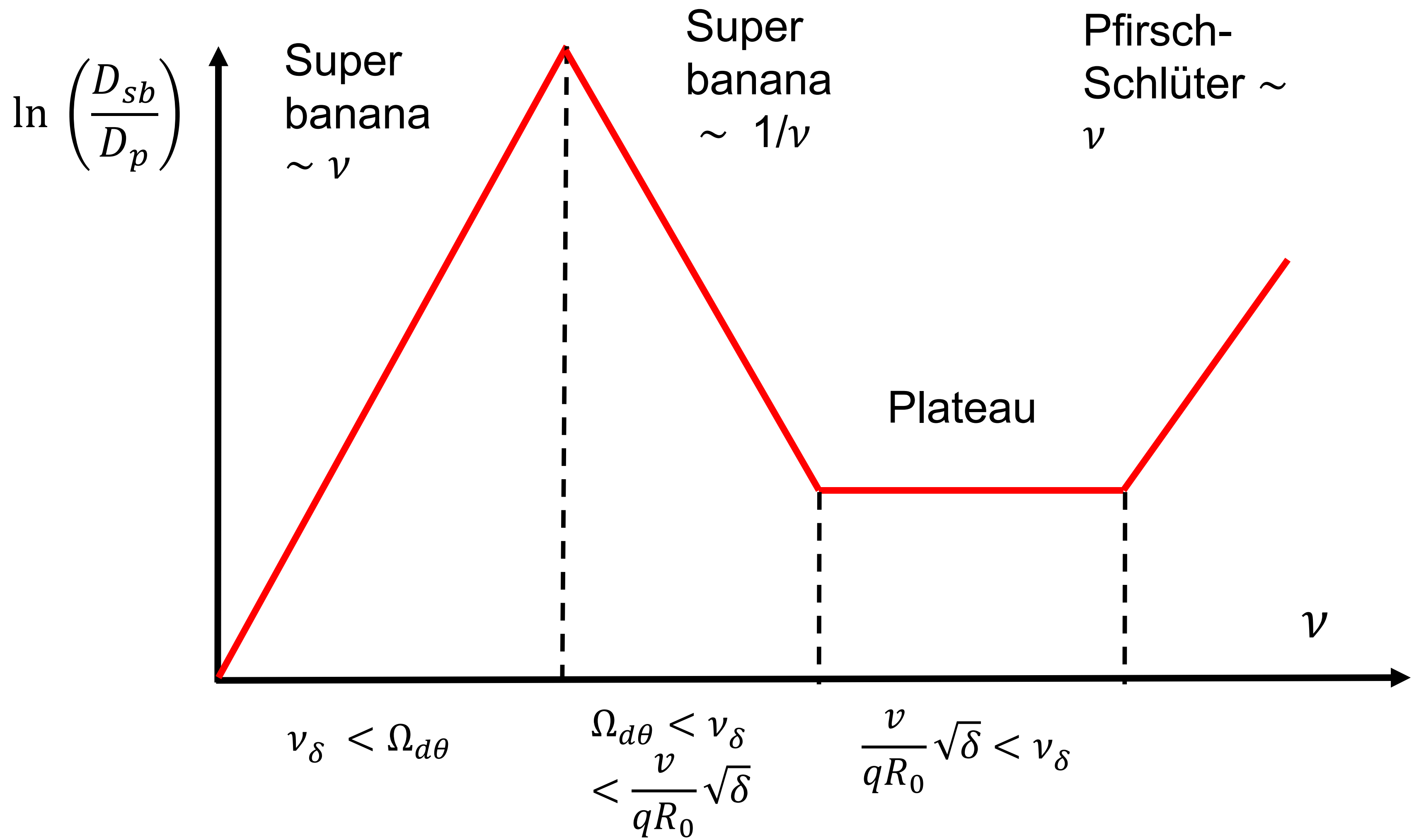


ExB drift plays an important role

- Uncompensated magnetic drift leads to unfavourable "1/ ν" scaling
- $E \times B$ + magnetic drifts save the day → precession frequency ω_d
- Precession → compensation recovered: ν scaling recovered at low collisionality



Wrapping-up for each energy class of particles



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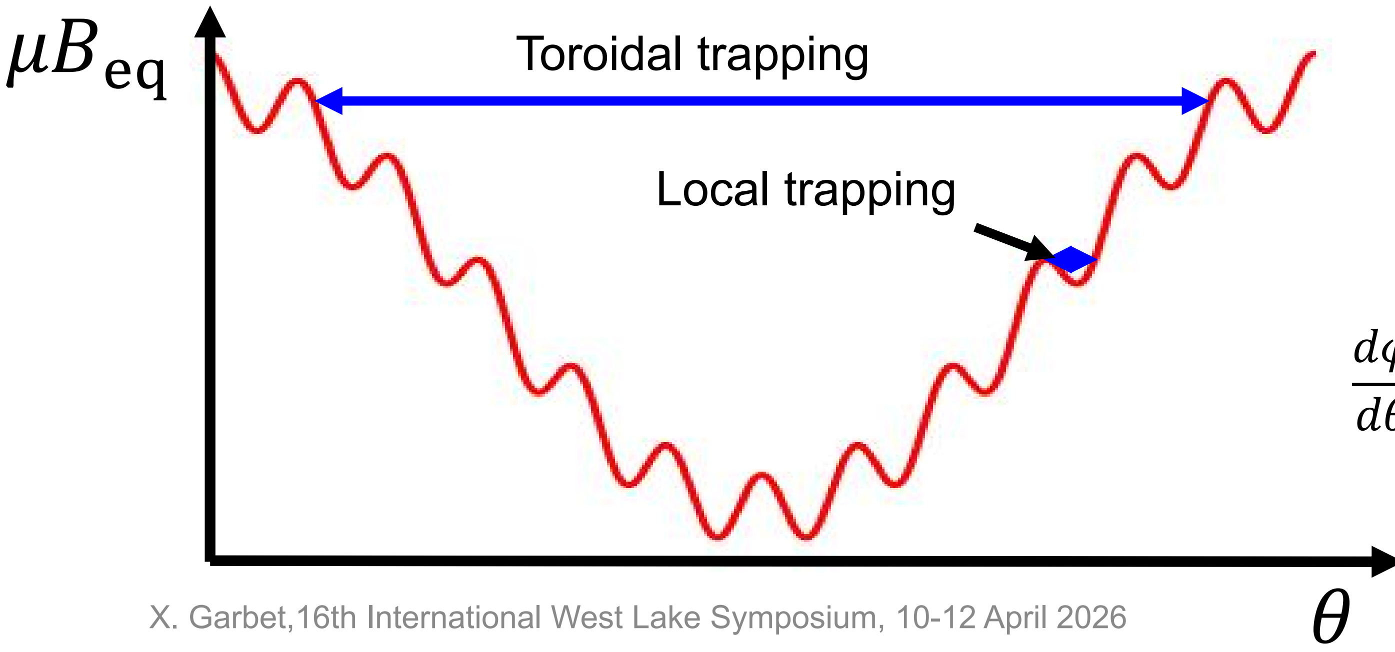
Basic model

- Hamiltonian guiding centres

$$H = H_{eq} - \underbrace{\mu B_0 \epsilon(r) \cos(\theta)}_{\nabla B \text{ perturbation}} - \underbrace{\mu B_0 \delta(r) \cos(M\theta + N\varphi)}_{\text{Helical perturbation}} + e_a \Phi(r)$$

- Unperturbed Hamiltonian in a screw pinch

$$H_{eq} = \frac{1}{2} m_a v_\parallel^2 + \mu B_{eq}(r)$$



$$\frac{d\varphi}{d\theta} = q(r)$$

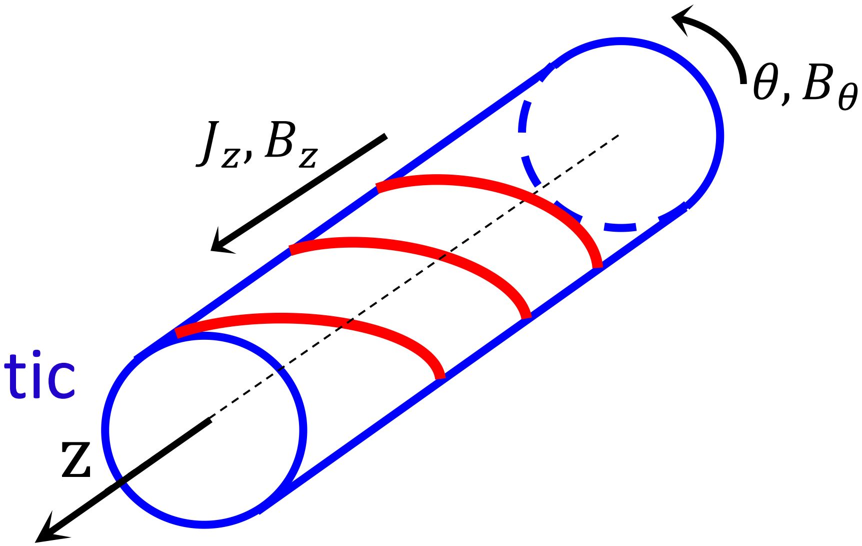
Generic Hamiltonian reformulation

- Introduce angle/action $(\boldsymbol{\alpha}, \boldsymbol{J})$

$\boldsymbol{\alpha}$: poloidal and toroidal angles

$J_2 = \mathrm{e_a}\chi \, , \, J_3 = -\mathrm{e_a}\psi + m_a v_\parallel R_0$

χ, ψ toroidal and poloidal magnetic fluxes



- Hamiltonian

$$H(\boldsymbol{\alpha}, \boldsymbol{J}) = H_{eq}(\boldsymbol{J}) - h(\boldsymbol{J})\cos(\boldsymbol{n}.\boldsymbol{\alpha}) - h'(\boldsymbol{J})\cos(\boldsymbol{n}'.\boldsymbol{\alpha})$$

- Resonant surface $\Omega(\boldsymbol{J}) = \boldsymbol{n}.\,\partial_J H_{eq} = 0$

- $\Omega \simeq n_h v_\parallel(\boldsymbol{J})/qR_0$ with helicity $n_h = m + nq$

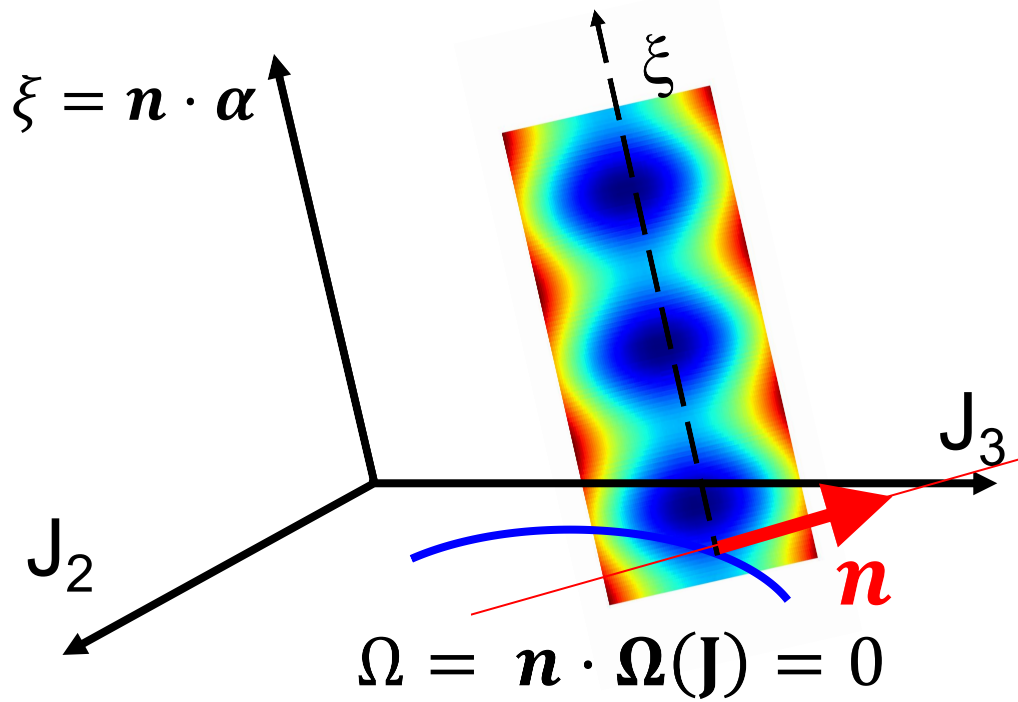
Single perturbation produces an island in the phase space

- Single perturbation produces an island – still integrable

$$H_\Omega = \frac{1}{2}\Omega^2 - \omega_b^2 \cos\xi$$

$\omega_b^2 = Ch$ where

$$C = n_i n_k \frac{\partial^2 \mathrm{H}_{eq}}{\partial J_i \partial J_k}\bigg|_{\Omega=0}$$

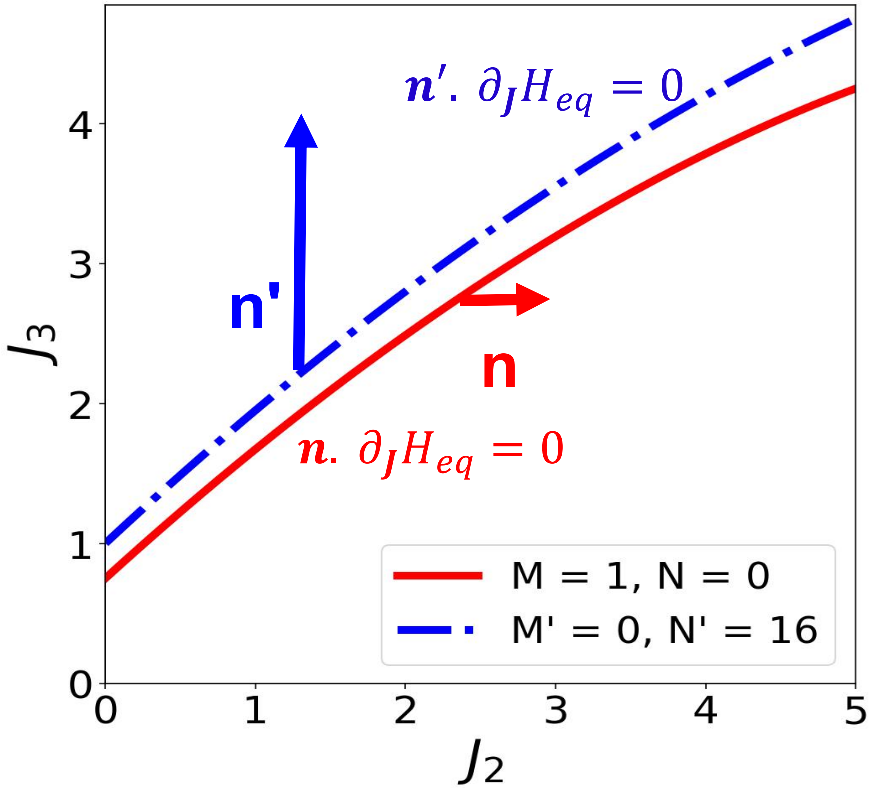


- Collisional boundary layer develops near separatrix
- Neoclassical transport comes from this boundary layer

Helander 14

Transport due to a secondary perturbation comes from collisional random walk

- Second Hamiltonian perturbations does not lead to chaos because of frequency hierarchy $\omega_b \gg \omega_d$
- Transport comes mainly from effect of collisions on trapped particles in the primary perturbation.



- Key parameters are $\sigma = \frac{n'_h}{n_h}$ and de-trapping collision frequency $\hat{\nu} = \frac{D_\Omega}{2\omega_b^2\omega_d}$

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Transport is best computed using a principle of minimum entropy production

- Distribution function

$$F(\boldsymbol{J},t) = F_0 e^{-\frac{H(\boldsymbol{J}) - U(\boldsymbol{J},\boldsymbol{t})}{T_*}}$$

- Entropy production rate

$$\dot{S} = \frac{2}{T_*^2} \int d\gamma\, \delta U_{eq}^{\dagger} F_{eq} \left(\frac{\partial U_{eq}}{\partial t} + \nabla_{\boldsymbol{J}} \cdot \boldsymbol{\Gamma}_{\boldsymbol{U}} - \mathcal{C}(U_{eq}) \right)$$

- Flux divergence

$$\nabla_{\boldsymbol{J}} \cdot \boldsymbol{\Gamma}_{\boldsymbol{U}} = - \left\langle \left\{ \widetilde{H}, \widetilde{U} \right\} \right\rangle_{\boldsymbol{\alpha}}$$

- Requires computing $\widetilde{U}$ knowing $F_{eq}, H_{eq}, \widetilde{H}$

Fluxes primarily due to effect of collisions on particles trapped in the primary perturbation

- Bounce average Fokker-Planck equation Galeev 69, Mynick 83, Beidler 87, Kovrizhnykh 90, Shaing 90, Satake 11, Sun 19

$$i\,\hat{\nu}\left(G(K)\frac{\partial^2\hat{F}}{\partial K^2}+\frac{\partial\hat{F}}{\partial K}\right)+\hat{F}=L_\sigma(K)$$

K = trapping parameter, $\hat{F}$ normalised perturbed distribution function – analytic solutions available

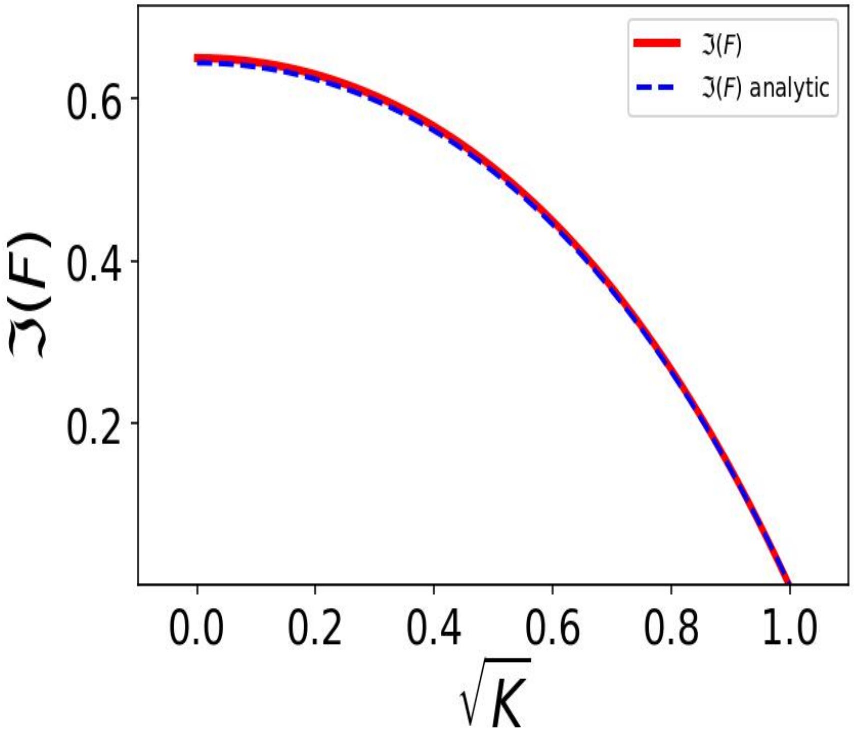
depends on 2 parameters $\sigma=\frac{n_h'}{n_h}$ and $\hat{\nu}=\frac{D_\Omega}{2\omega_b^2\omega_d}$

- Boundary condition at separatrix K=1 debated (stellarators). For cases of interest $\hat{F}(K=1)=0$

Analytic solution holds in the collisional case $\hat{\nu} \gg 1$ and $\sigma \ll 1$

- Yields 1/ν regime for rippled tokamaks or non resonant magnetic perturbations $\sigma \ll 1$
- Distribution function is smooth
- Analytic solution holds for all K

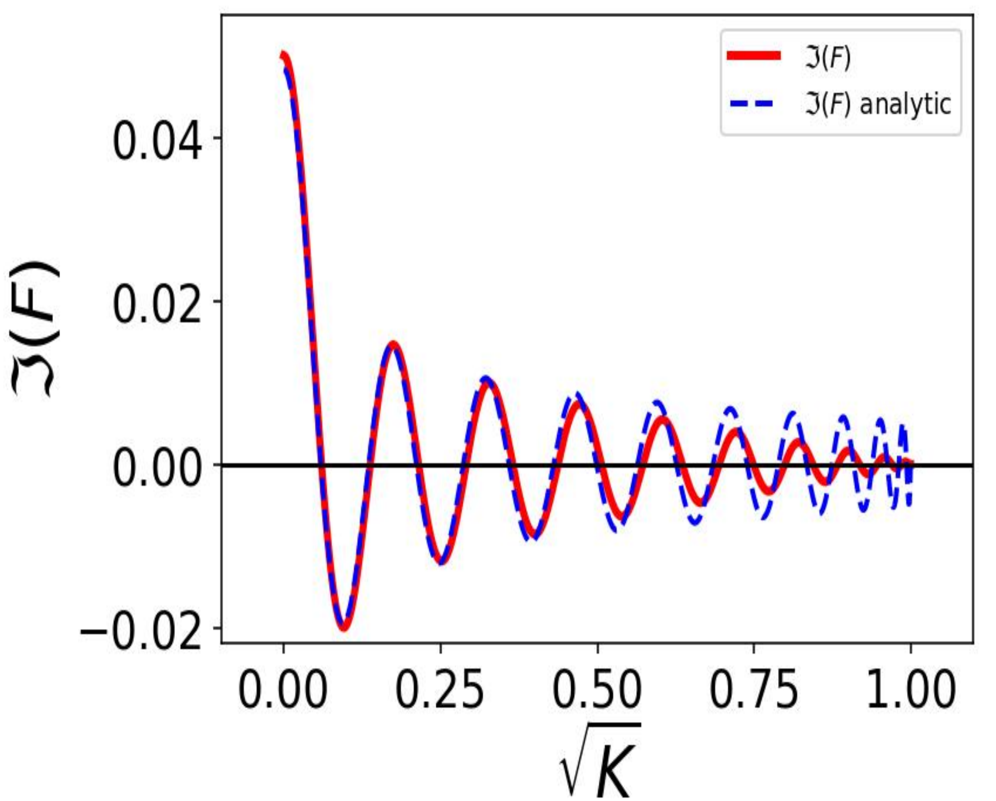
Imaginary part of F



Analytic solution holds in the collisional case $\hat{\nu} \gg 1$ and $\sigma \gg 1$ for deeply trapped particles only

- Yields the 1/ν regime for banana drift regime
- Distribution function is oscillatory
- Discrepancy for barely trapped particles K≈1. Still acceptable

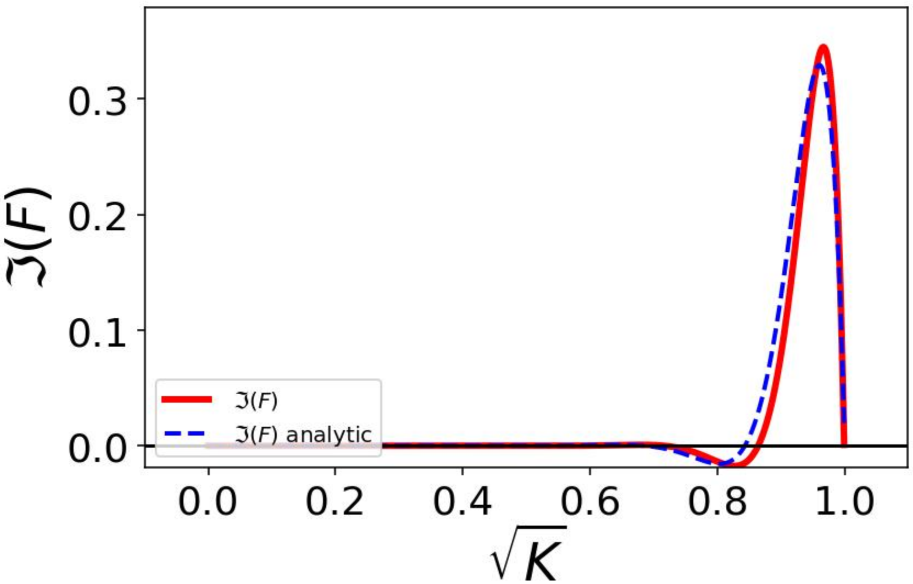
Imaginary part of F



Effect of ExB drift is more difficult to capture

- Analytic formula still fine – but:
- Narrow boundary layer near K =1 → condition $\widehat{F}(K=1)=0$ fails at low $\hat{\nu}$
- Still works for $\delta \ll \epsilon$ Kovrizhnykh 90

Imaginary part of F



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Theory predicts a transport matrix

- Relationship between fluxes and forces Connor 73, Kovrizhnykh 99, Shaing 15, Varennes 23, Ban 25

$$\begin{pmatrix} \frac{\Gamma}{N} \\ \frac{\mathcal{M}}{NeB_p} \\ \frac{\mathrm{Q}}{NT} \end{pmatrix} = -M \begin{pmatrix} \frac{\mathrm{dN}}{Ndr} + \frac{e}{T}\frac{d\Phi}{dr} \\ \frac{eB_pV_t}{T} \\ \frac{\mathrm{dT}}{Tdr} \end{pmatrix}$$

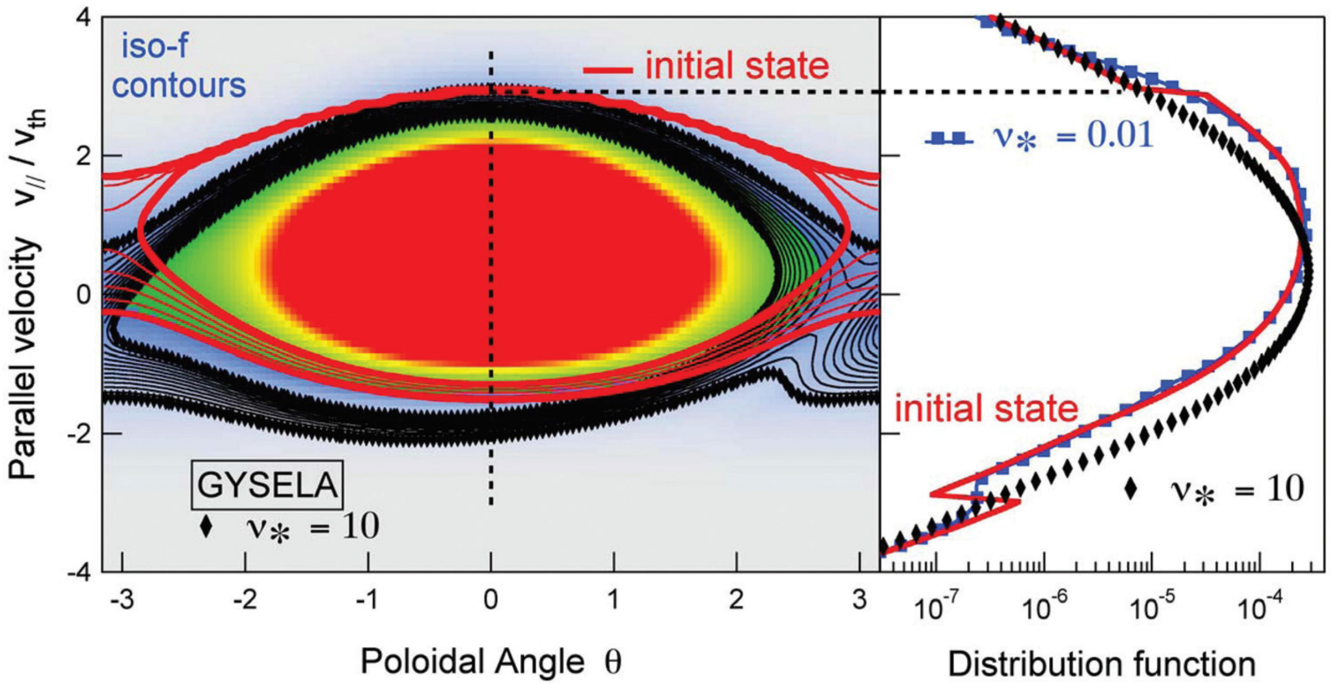
M is a 3x3 symmetric positive-definite transport matrix

- $\mathcal{M}$ is the Neoclassical Toroidal Viscosity torque – recovered by gyrokinetics ? Matsuoka 17

Primary entropy production agrees with conventional neoclassical results

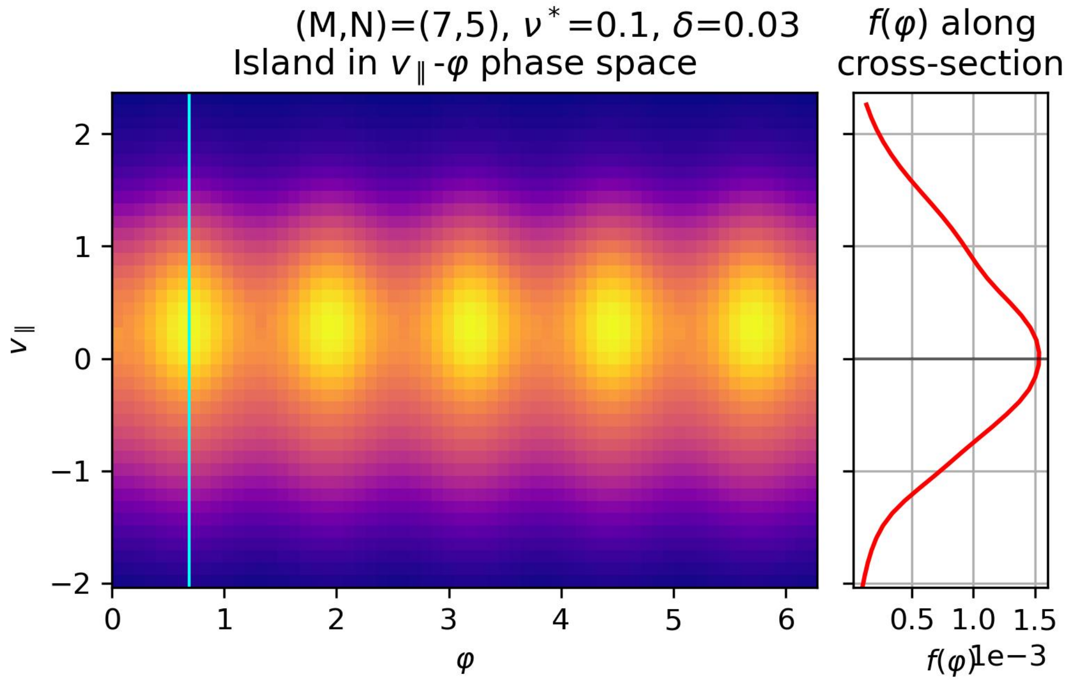
- Tested in GYSELA gyrokinetic code Dif-Pradalier 11

- Viscous damping and ion heat diffusivity Samain 77, Garbet 09 in accordance with theory Hinton-Hazeltine 76, Galeev-Sagdeev 79, Hirshman-Sigmar 81



Secondary island appears in phase space as expected for a rippled tokamak

- N lobes clearly seen in the $(v_\parallel, \varphi)$ space
- 2nd magnetic island as expected
- smooth deformation of the distribution function



Ban 25

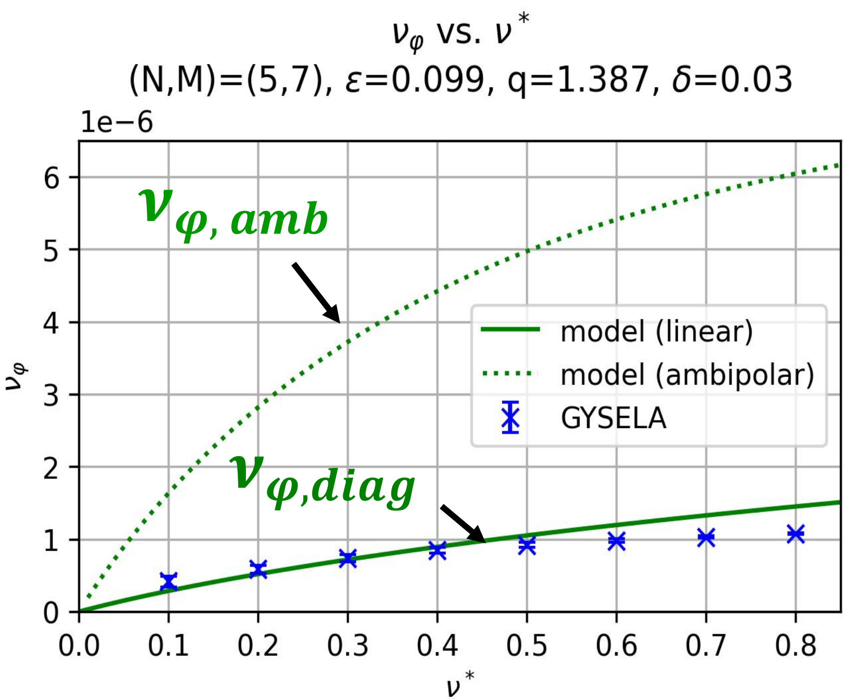
NTV agrees with simulations for diagonal coefficient only in a rippled tokamak

- Magnetic drag can be defined as $\nu_{\varphi,diag}$ = M_{22} , or by enforcing ambipolarity

$$\Gamma_{V_\varphi} = -\nu_{\varphi,amb}\left(V_\varphi - k_{V_T}\frac{\nabla T}{eB_\theta}\right)$$

- GYSELA toroidal friction rate ν_φ (NTV) fits better $\nu_{\varphi,diag}$

- Same was found for rippled tokamak Varennes 23

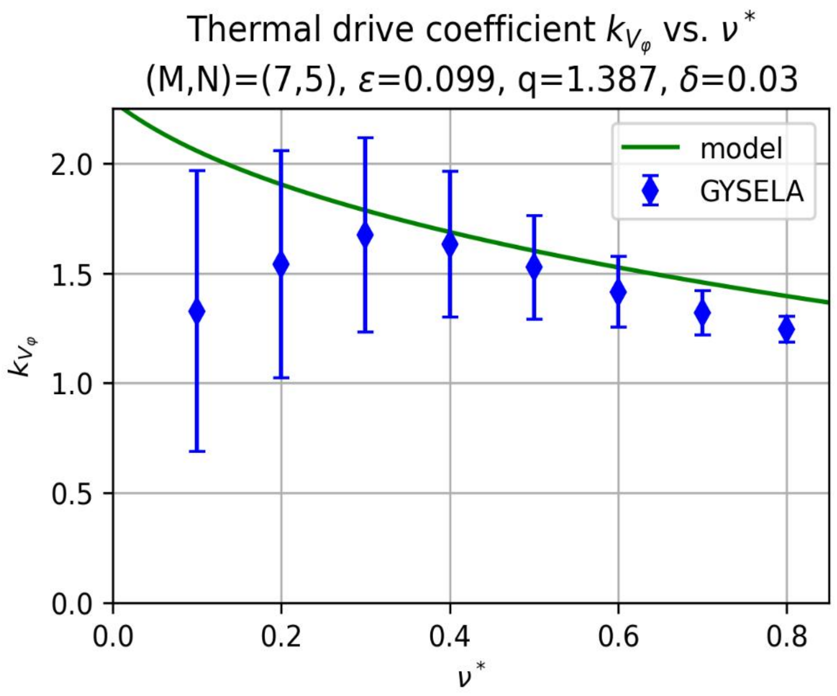


Ban 25

Relaxed velocity agrees reasonably well with analytic theory

- Coefficient k_{V_T} agrees with GYSELA simulations at intermediate collisionality – same result with ripple
  Varennes 23

- Discrepancy at low collisionality (drift effect?)



Ban 25

Conclusion

- Hamiltonian description combined with a principle of minimum entropy production rate reproduces main results derived in the field
- Numerical simulations agree qualitatively with model – still far from quantitative agreement
- Next step is to add turbulence and investigate synergies with helical perturbations (see Varennes 22 for rippled tokamak)

Back-up slides

Compact formulation

- Entropy production rate primary perturbation

$$\dot{S}_{res,I} = \frac{1}{T_*^2} \int d\gamma F_{eq}(\mathbf{J}) \delta(\mathbf{n}\cdot\boldsymbol{\Omega}) h^2 \Lambda(\mathbf{J}) \left(\mathbf{n} \cdot \frac{\partial U_{eq}^{\dagger}}{\partial \mathbf{J}} \right)^2$$

Form factor

Primary resonance

Primary perturbation

Gradient and flows

- Entropy production rate secondary perturbation

$$\dot{S}_{res,II} = \frac{1}{T_*^2} \int d\gamma\, \omega_b\, \delta(\mathbf{n}\cdot\boldsymbol{\Omega})\, \mathcal{F}_{eq} h'^2 \frac{1}{\omega_d} \Im[\Phi] \left(\sigma, \frac{\nu_d}{\omega_d} \right) \left((\mathbf{n}' - \mathbf{n}\sigma) \cdot \frac{\partial U_{eq}^{\dagger}}{\partial \mathbf{J}} \right)^2$$

Primary resonance

Secondary perturbation

Form factor – ExB flow

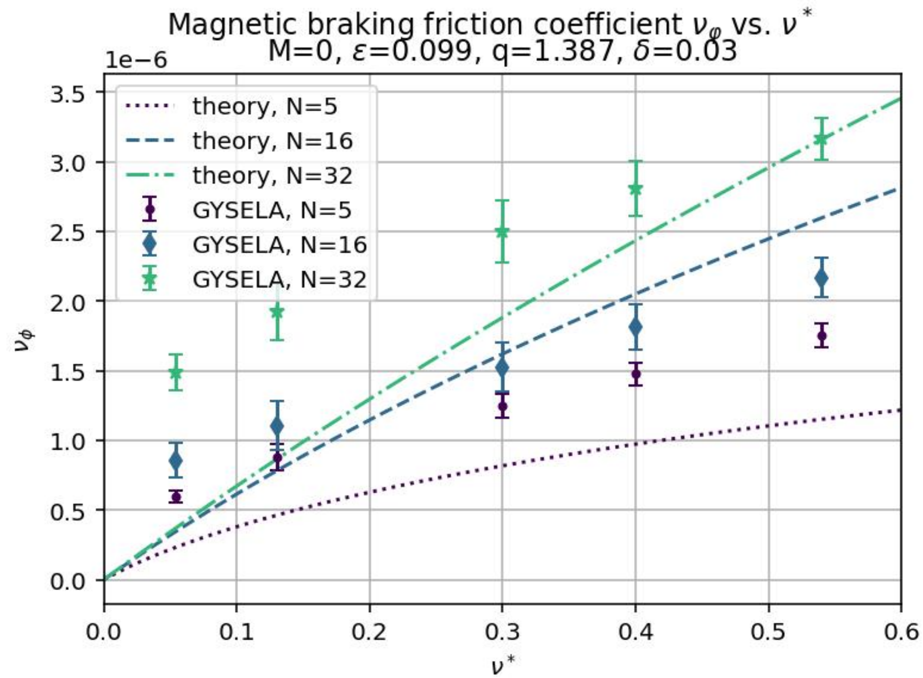
Gradient and flow in new equilibrium

Rippled tokamak (cont.)

- Prediction Connor 73, Kovrizhnykh 99, Shaing 15, Varennes 23, Ban 25

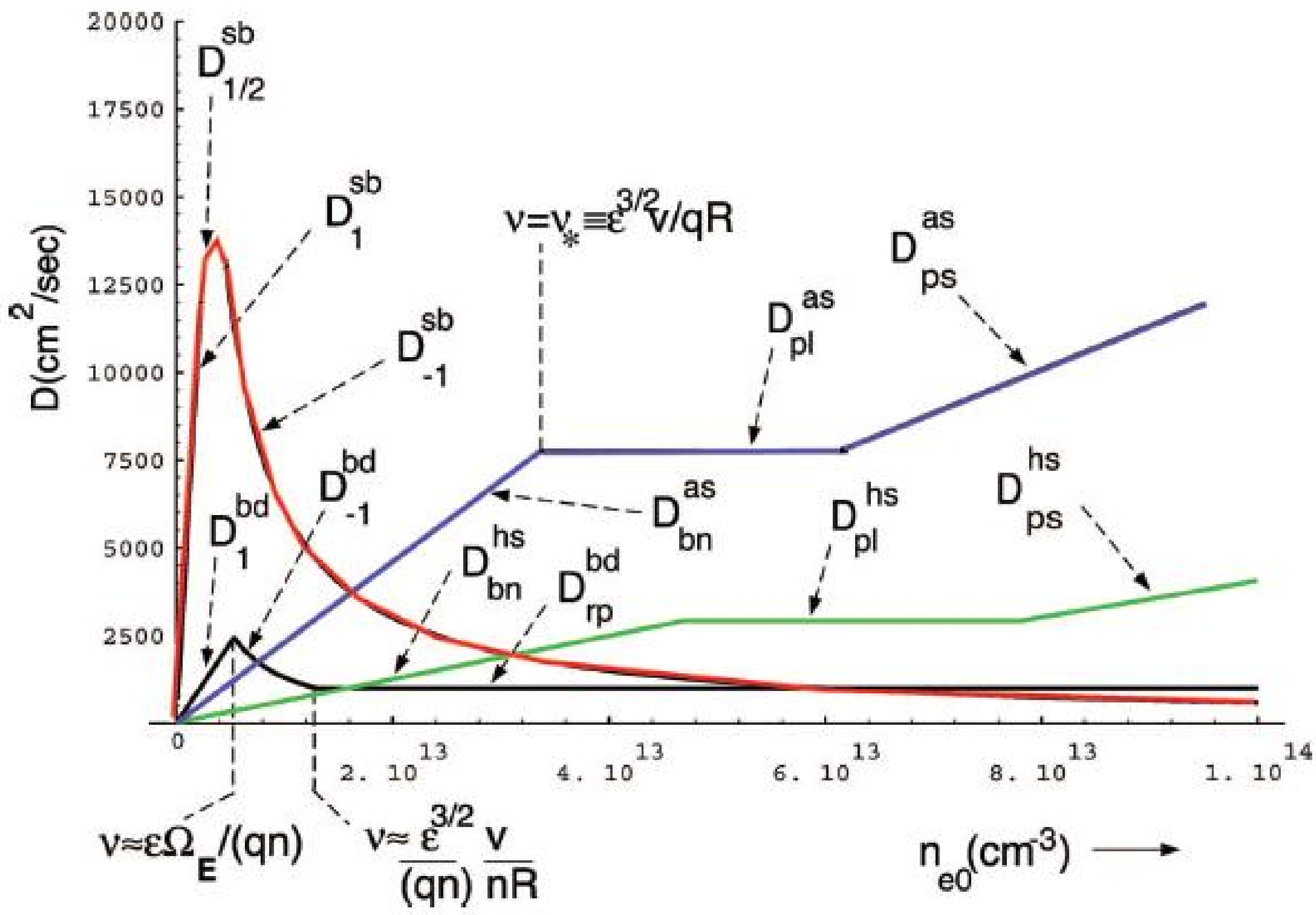
$$\Gamma_{V_\varphi} = -\nu_\varphi \left(V_\varphi - k_{V_T} \frac{\nabla T}{eB_\theta} \right)$$

GYSELA toroidal friction rate ν_φ (Neoclassical Toroidal Viscosity) is within factor 2 with model



Ban 25

Summary of all contributions



Mynick 06

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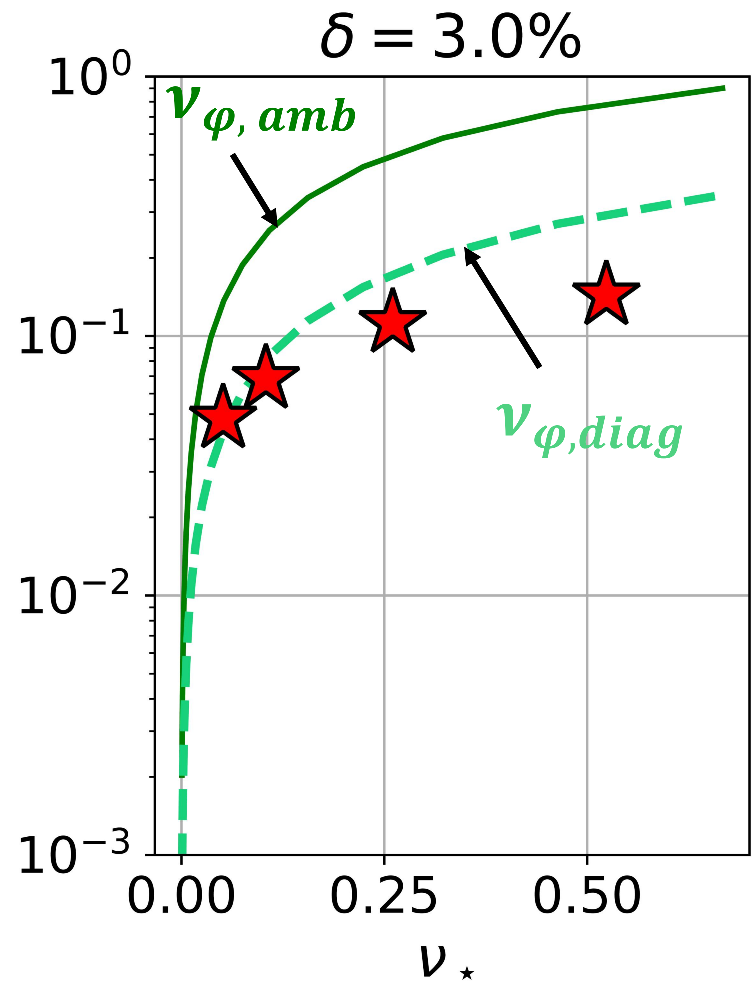
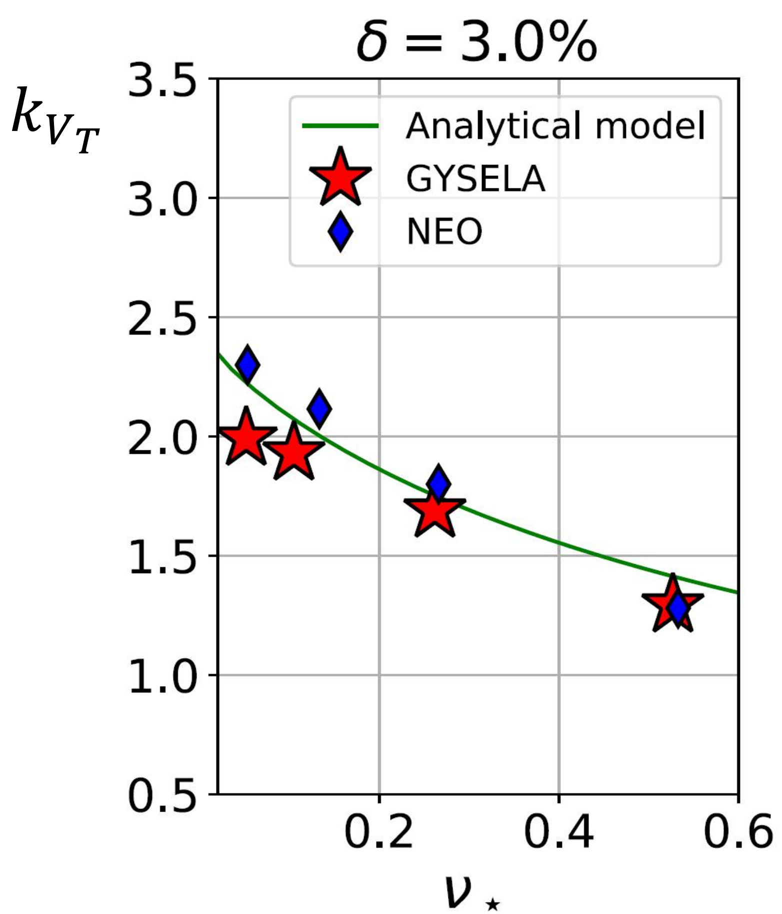


Varennes 23

Relaxed velocity agrees reasonably well with analytic theory

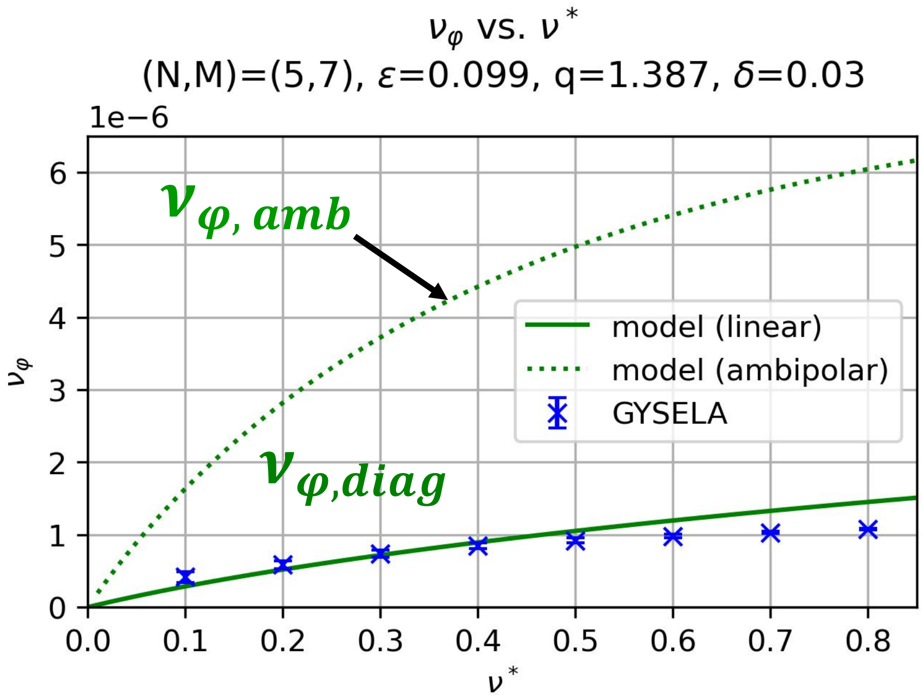
Varennes 23

- Coefficient k_{V_T} agrees with NEO3D and GYSELA
  Varennes 23
- This is with ambipolar constraint
- Discrepancy at low collisionality (drift effect?)



Similar behaviour seen for GYSELA vs theory in the helical case

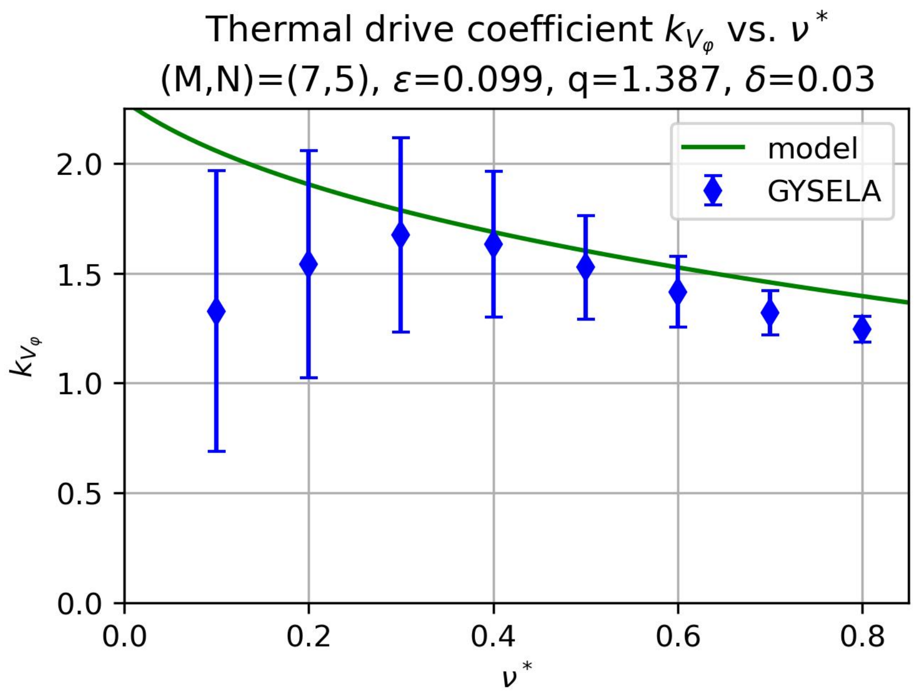
- GYSELA toroidal friction rate ν_φ (Neoclassical Toroidal Viscosity) agrees with diagonal matrix element
- But disagrees with expected value when constrained by ambipolarity



Ban 25

Relaxed flow satisfactory

- Agreement at intermediate collisionality is satisfactory
- Disagreement at low frequency not understood yet



Ban 25