On Nonlinear Scattering of Drift Wave by Toroidal Alfvén Eigenmode in Tokamak Plasmas

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Using electron drift wave (eDW) as a paradigm model, we have investigated analytically direct wave-wave interactions between a test DW and ambient toroidal Alfvén eigenmodes (TAE) in toroidal plasmas. The nonlinear effects enter via scatterings to short-wavelength electron Landau damped kinetic Alfvén waves (KAWs). Specifically, it is found that scatterings to upper-sideband KAW lead to stimulated absorption of eDW. Scatterings to the lower-sideband KAW, on the contrary, lead to its spontaneous emission. As a consequence, for typical parameters and fluctuation intensity, nonlinear scatterings by TAE have negligible net effects on the stability of eDW.

I. INTRODUCTION

Drift wave (DW) [1] and shear Alfvén wave (SAW) [2–7] are two fundamental electromagnetic oscillations in magnetized plasmas such as tokamaks [8]. DWs are, typically, electrostatic fluctuations excited by thermal plasma density and/or temperature nonuniformities. Consequently, DWs have frequencies, perpendicular wavelengths and parallel wavelengths comparable, respectively, to the thermal plasma diamagnetic drift frequencies, thermal ion Larmor radii and the system size. SAWs, meanwhile, are electromagnetic fluctuations and, typically, manifest as Alfvén eigenmodes (AEs) locating within the frequency gaps of SAW continuum spectra [2]. For typical tokamak parameters, AE frequencies could be an order of magnitude higher than those of DWs, and, thus, spontaneous excitations of AEs often involve resonances with superthermal energetic particles (EPs); e.g., alphas in a D-T fusion plasma. AEs, thus, have perpendicular wavelengths in the order of EP Larmor radii and parallel wavelengths in the order of system size. In short, we may describe DWs as low-frequency micro-scale fluctuations; while AEs as high-frequency meso-scale fluctuations. Since both DWs and AEs are intrinsic fluctuations in magnetic confined fusion plasmas and have routinely been observed in tokamak plasmas, it is, thus, natural to inquire if and how these two fluctuations may interact and its implications. Recently, we have investigated such interactions via the channel of nonlinear wave scatterings between toroidal Alfvén eigenmode (TAE) [2] and, as a paradigm model, electron drift wave (eDW).

There are two types of direct nonlinear interactions between TAE and eDW. The first type involves the scattering of a test TAE by ambient eDWs [9]. Here, we demonstrate that the TAE will suffer significant damping via nonlinearly generated upper and lower sidebands of short-wavelength electron Landau damped kinetic Alfvén waves (KAWs) [10]. This scattering process, thus, may be regarded as stimulated absorption. For typical parameters, it is found, furthermore, that the nonlinear damping rate could be comparable to the growth rate of TAE instability excited by EPs. The second type of nonlinear wave-wave interactions involve the scattering of a test eDW by ambient TAEs, and is the focus of the present work. As will be shown in the following analysis, while the second type of scattering may be considered as the "reverse" of the first type, the induced nonlinear damping/growth rate in this case is, in fact, negligible for typical parameters. Qualitatively speaking, while the nonlinearly generated upper sideband KAW (UKAW) still gives rise to stimulated absorption, the nonlinearly generated lower sideband KAW (LKAW), however, gives rise to stimulated emission (i.e., as in a parametric decay instability) [11]. Quantatively, these two effects tend to nearly cancel each other; leading to negligible net effect on the stability of eDW.

The theoretical model and governing equations are given in Sec. II. Section III presents the nonlinear generation of upper and lower KAW sidebands. Nonlinear dispersion relation of eDW in the presence of the finite-amplitude TAE is then derived and analyzed in Sec. IV. Section V gives the final conclusions and discussions.

II. THEORETICAL MODEL AND GOVERNING EQUATIONS

We consider a large-aspect-ratio and low-β tokamak plasma with circular magnetic surfaces. Thus, $\epsilon \equiv r/R \ll 1$ with r and R being, respectively, the minor and major radii of the torus, and $\beta \sim O(\epsilon^2) \ll 1$ being the ratio between plasma and magnetic pressure. We, furthermore, take the thermal background plasma to be Maxwellian, and adopt the eDW paradigm model with finite density gradient but negligible temperature gradient as well as trapped particle effects.

The perturbed distribution function, δf_j with $j = e, i$, obeys the nonlinear gyrokinetic equation [12]

$$\delta f_j = -(e/T)_j \delta\phi F_{Mj} + \exp(-\boldsymbol{\rho}\cdot\nabla)\delta g_j, \tag{1}$$

and

$$\begin{aligned}&\left(\partial_t + v_\parallel \mathbf{b}\cdot\nabla + \mathbf{v}_d\cdot\nabla + \langle\delta\mathbf{u}_g\rangle_\alpha\cdot\right)\delta g_j \\ =\ & (e/T)_j F_{Mj}\left(\partial_t + i\omega_{*j}\right)\langle\exp(\boldsymbol{\rho}_j\cdot\nabla)\delta L\rangle_\alpha.\end{aligned} \tag{2}$$

Here, F_{Mj} is the Maxwellian distribution, $\boldsymbol{\rho}_j = \mathbf{v}\times\mathbf{b}/\Omega_j$, $\mathbf{b} \equiv \mathbf{B}_0/B_0$, $\Omega_j = (eB_0/mc)_j$, δg_j is the non-adiabatic particle response, $\mathbf{v}_d = \mathbf{b}\times[(v_\perp^2/2)\nabla\ln B_0 + v_\parallel^2\mathbf{b}\cdot\nabla\mathbf{b}]$ is the magnetic drift velocity, $\langle\mathbf{A}\rangle_\alpha$ denotes the gyrophase averaging of $\mathbf{A}$, $\omega_{*j} = -i(cT/eB_0)\mathbf{b}\times\nabla\ln N_j\cdot\nabla$ is the diamagnetic drift frequency due to the finite density gradient,

$$\langle\delta\mathbf{u}_j\rangle_\alpha = (c/B_0)\mathbf{b}\times\nabla\langle\exp(-\boldsymbol{\rho}_j\cdot\nabla)\delta L\rangle_\alpha, \tag{3}$$

and

$$\delta L = \delta\phi - v_\parallel\delta A_\parallel/c \tag{4}$$

with $\delta\phi$ and $\delta A_\parallel$ being, respectively, the scalar and parallel component of the vector potential. Note that, with $\beta \ll 1$, magnetic compression may be neglected; i.e., $\delta B_\parallel \simeq 0$.

Meanwhile, the governing field equations are the quasi-neutrality condition

$$\sum_{j=e,i}\left[(N_0e^2/T)_j\delta\phi - e_j\left\langle(J_k\delta g)_j\right\rangle_v\right] = 0, \tag{5}$$

and the parallel Ampere's law $\nabla_\perp^2\delta A_\parallel = -(4\pi/c)\delta J_\parallel$. Here, we note $J_k = J_0(k_\perp\rho) = \langle\exp(i\boldsymbol{\rho}\cdot\mathbf{k}_\perp)\rangle_\alpha$ and $k_\perp^2 = -\nabla_\perp^2$ should be understood as an operator. Furthermore, we note that, for SAW and KAW, instead of the Ampere's law, it is more convenient to use the following nonlinear gyrokinetic vorticity equation [13, 14]

$$\begin{aligned}
&ik_\parallel\delta J_{\parallel k} + (N_0e^2/T)_i\left(1-\Gamma_k\right)\left(\partial_t + i\omega_{*i}\right)_k\delta\phi_k \\
&-\sum_j\left\langle e_jJ_k\omega_d\delta g_j\right\rangle_v = \sum_{\mathbf{k}'+\mathbf{k}''=\mathbf{k}}\Lambda^{k'}_{k''}\left\{\delta A_{\parallel k'}\delta J_{k''}/c\right. \\
&\left.-e_j\left\langle\left(J_kJ_{k'} - J_{k''}\right)\delta L_{k'}\delta g_{k''j}\right\rangle_v\right\}.
\end{aligned} \tag{6}$$

Here, $\Gamma_k \equiv I_0(b_k)\exp(-b_k)$, $b_k = k_\perp^2\rho_i^2$, $\rho_i^2 = T_i/(m_i\Omega_i^2)$ and I_0 is the modified Bessel function. The first and second terms on the left hand side correspond, respectively, to the field line bending and inertia terms. Meanwhile, the third term corresponds to the curvature-pressure coupling term including the ballooning-interchange term and finite plasma compression. Note that, for TAE/KAW physics considered here, it can generally be ignored. The right hand side contains, the nonlinear terms; where $\Lambda^{k'}_{k''} = (c/B_0)\mathbf{b}\cdot(\mathbf{k}''\times\mathbf{k}')$, and the first and second terms correspond, respectively, to the Maxwell and generalized gyrokinetic ion Reynolds stresses. Note that, while the Maxwell stress makes negligible contribution in the TAE-eDW interaction since eDW is predominantly electrostatic, the Maxwell stress, as will be shown later, does play an important role in the TAE-KAW interaction.

We now consider the effects on eDW linear stability due to nonlinear scattering by TAE. Letting $\Omega_0(\omega_0,\mathbf{k}_0)$ and $\Omega_s = (\omega_s,\mathbf{k}_s)$ denote, respectively, a small but finite-amplitude TAE with toroidal mode number, n_0, and a test eDW with toroidal mode number, n_s. Thus, $|\omega_0| \simeq V_A/(2qR)$ with V_A being the Alfvén speed and q the safety factor, $\omega_s \sim \omega_{*e}$ the electron diamagnetic drift frequency, and $|k_{s\theta}\rho_i| = |n_sq\rho_i/r| \sim O(1)$. Furthermore, we have, typically, $|\omega_s/\omega_0| < 1$ and $|n_0/n_s| < 1$. That is, TAE and eDW are disparate both in spatial and temporal scales. Consequently, the sidebands nonlinearly generated by TAE and eDW; i.e., $\Omega_\pm = (\omega_\pm,\mathbf{k}_\pm) = \Omega_s\pm\Omega_0$, tend to have $|\omega_\pm| \simeq |\omega_0|$ and $|\mathbf{k}_\pm| \simeq |\mathbf{k}_s|$, and may be regarded as short-wavelength (high-n) KAWs. $\Omega_\pm$, in turn, can interact with Ω_0; resulting in the nonlinear modification of eDW dispersion relation and, thereby, the stability properties. The two-step scattering processes are illustrated schematically in Fig. 1. The first-step scattering process, i.e., the nonlinear generation of KAW sidebands is analyzed in the following Sec. III. Section IV analyzes the second step scattering process and the resultant nonlinear eDW dispersion relation.

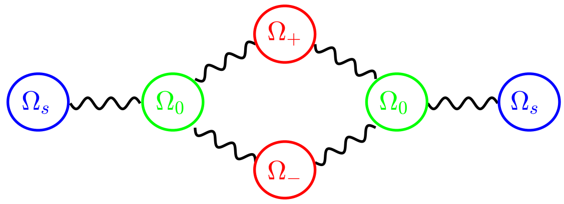

FIG. 1: Schematic diagram of the two-step scattering processes analyzed in the present work. The test eDW, ambient TAE and nonlinearly generated KAW sidebands are in blue, green and red, respectively.

III. NONLINEAR GENERATION OF UPPER AND LOWER SIDEBANDS OF KINETIC ALFVÉN WAVES

Let us first analyze the nonlinear generation of Ω_+; i.e, UKAW. The analysis for LKAW is similar. For electrons, we have, from Eq. (2) and letting $\delta g_{ke} = \delta g^{(1)}_{ke} + \delta g^{(2)}_{ke}$, with superscripts "(1)" and "(2)" denoting, respectively, the linear and nonlinear responses; thus,

$$\delta g^{(1)}_{ke} \simeq -\frac{e}{T_e}F_{Me}\left(1-\frac{\omega_{*e}}{\omega}\right)_k\delta\psi_k, \tag{7}$$

where $\delta\psi_k = (\omega\delta A_\parallel/ck_\parallel)_k$ is the effective potential due to the induced parallel electric field, $-\partial_t\delta A_\parallel/c$ and we have taken the massless-electron $|\omega_k/k_\parallel v_{te}| \ll 1$ limit. In Eq. (7), k stands for the TAE/KAW modes; Ω_0 and $\Omega_\pm$, and $\delta g^{(1)}_{se} \simeq 0$ as Ω_s is an electrostatic eDW mode. It then follows

$$\delta g^{(2)}_{+e} \simeq 0. \tag{8}$$

Meanwhile, for ions, with $|\omega/k_\parallel v_{ti}| \gg 1$ for all the modes considered here, TAE, KAW and eDW, we have

$$\delta g^{(1)}_{ki} \simeq \frac{e}{T_i}F_{Mi}J_k\delta\phi_k\left(1-\frac{\omega_{*i}}{\omega}\right)_k, \tag{9}$$

and

$$\delta g^{(2)}_{+i} \simeq -i\frac{\Lambda^s_0}{2\omega_+}J_0J_s\frac{e}{T_i}F_{Mi}\left(\frac{\omega_{*i}}{\omega}\right)_s\delta\phi_s\delta\phi_0. \quad (10)$$

Substituting Eqs. (7) to (10) into the quasi-neutrality condition, Eq. (5), we readily derive

$$\delta\psi_+ = \sigma_{*+}\delta\phi_+ + i\frac{\Lambda^s_0}{2\omega_+}D_+\delta\phi_0\delta\phi_s, \quad (11)$$

where

$$\sigma_{*+} = \left[1+\tau-\tau\Gamma_+(1-\omega_{*i}/\omega)_+\right]/(1-\omega_{*e}/\omega)_+, \quad (12)$$

and

$$D_+ = \tau(\omega_{*i}/\omega)_sF_+/(1-\omega_{*e}/\omega)_+, \quad (13)$$

$\tau = T_e/T_i$, and $F_+ = \langle J_0J_+J_sF_{Mi}\rangle_v/N_0$. Meanwhile, the nonlinear gyrokinetic vorticity equation, Eq. (6), readily yields

$$\begin{aligned}&\tau b_+\left[\left(1-\frac{\omega_{*i}}{\omega}\right)_+\frac{1-\Gamma_+}{b_+}\delta\phi_+ - \left(\frac{V_A^2k_\parallel bk_\parallel}{b\omega^2}\right)_+\delta\psi_+\right]\\ &= -i\frac{\Lambda^s_0}{2\omega_+}\gamma_+\delta\phi_s\delta\phi_0, \quad (14)\end{aligned}$$

where

$$\gamma_+ = \tau\left[\Gamma_s-\Gamma_0+(\omega_{*i}/\omega)_s(F_+-\Gamma_s)\right]. \quad (15)$$

Combining Eqs. (11) and (14) then yields the equation describing the nonlinear generation of Ω_+ by Ω_0 and Ω_s; i.e.,

$$\tau b_+\epsilon_{A+}\delta\phi_+ = -i(\Lambda^s_0/2\omega_+)\beta_+\delta\phi_s\delta\phi_0, \quad (16)$$

where

$$\epsilon_{Ak} = \left(1-\frac{\omega_{*i}}{\omega}\right)_k\frac{1-\Gamma_k}{b_k} - \left(\frac{V_A^2}{b}\frac{k_\parallel bk_\parallel}{\omega^2}\right)_k\sigma_{*k} \quad (17)$$

is the linear SAW/KAW operator, and

$$\begin{aligned}\beta_+ &= \tau(\Gamma_s-\Gamma_0)+\tau\left(\frac{\omega_{*i}}{\omega}\right)_s\\ &\times\left[F_+-\Gamma_s-\left(\frac{k_\parallel bk_\parallel}{\omega^2}\right)_+\frac{\tau V_A^2F_+}{(1-\omega_{*e}/\omega)_+}\right]. \quad (18)\end{aligned}$$

Nonlinear generation of Ω_- follows that of Ω_+, and we, therefore, present only the main results. For electrons, we have, again, $\delta g^{(2)}_{-e}\simeq 0$, and, for ions,

$$\delta g^{(2)}_{-i} \simeq i\frac{\Lambda^s_0}{2\omega_-}J_0J_s\frac{e}{T_i}F_{Mi}\left(\frac{\omega_{*i}}{\omega}\right)_s\delta\phi_s\delta\phi^*_0. \quad (19)$$

The quasi-neutrality condition, Eq. (5), yields,

$$\delta\psi_- = \sigma_{*-}\delta\phi_- - i(\Lambda^s_0/2\omega_-)D_-\delta\phi_s\delta\phi^*_0, \quad (20)$$

with

$$D_- = \tau(\omega_{*i}/\omega)_sF_-/(1-\omega_{*e}/\omega)_-, \quad (21)$$

and $F_- = \langle J_0J_-J_sF_{Mi}\rangle_v/N_0$. Meanwhile, the nonlinear gyrokinetic vorticity equation, Eq. (6), yields

$$\begin{aligned}&\tau b_-\left[\left(1-\frac{\omega_{*i}}{\omega}\right)_-\frac{(1-\Gamma_-)}{b_-}\delta\phi_- - \left(\frac{V_A^2k_\parallel bk_\parallel}{b\omega^2}\right)_-\delta\psi_-\right]\\ &= i\frac{\Lambda^s_0}{2\omega_-}\gamma_-\delta\phi_s\delta\phi^*_0, \quad (22)\end{aligned}$$

and

$$\gamma_- = \tau\left[\Gamma_s-\Gamma_0+(\omega_{*i}/\omega)_s(F_--\Gamma_s)\right]. \quad (23)$$

Finally, from Eqs. (20) and (22), we have

$$\tau b_-\epsilon_{A-}\delta\phi_- = i(\Lambda^s_0/2\omega_-)\beta_-\delta\phi_s\delta\phi^*_0, \quad (24)$$

and

$$\begin{aligned}\beta_- &= \tau(\Gamma_s-\Gamma_0)+\tau\left(\frac{\omega_{*i}}{\omega}\right)_s\\ &\times\left[F_--\Gamma_s-\left(\frac{k_\parallel bk_\parallel}{\omega^2}\right)_-\frac{\tau V_A^2F_-}{(1-\omega_{*e}/\omega)_-}\right]. \quad (25)\end{aligned}$$

We remark, again, that $\epsilon_{A\pm}$ in Eqs. (16) and (24) are KAW operators. That is, in terms of physics, Eqs. (16) and (24) describe mode-converted KAWs ($\Omega_\pm$) driven by the nonlinear coupling between a TAE (Ω_0) and eDW (Ω_s).

IV. NONLINEAR DISPERSION RELATION OF ELECTRON DRIFT WAVE

We now analyze the second scattering process between $\Omega_\pm$ and Ω_0 back into Ω_s. Again, let us first consider the Ω_+ channel; i.e., $\Omega_+ + (-\Omega_0)\to\Omega_s$. From the nonlinear gyrokinetic equation, Eq. (2), we have, for electrons in the massless $|\omega/k_\parallel v_{te}|\ll 1$ limit and noting Eqs. (7) and (8),

$$\delta g^{(2)}_{se,+} \simeq -i\frac{\Lambda^s_0}{2\omega_+}\frac{e}{T_e}F_{Me}\delta\psi_+\delta\psi^*_0\left[1+\frac{k_{\parallel 0}}{k_{\parallel s}}\frac{(\omega_{*e}-\omega)_s}{\omega_0}\right] (26)$$

Here, $\delta g^{(2)}_{se,+}$ denotes nonlinear electron response of Ω_s due to Ω_+ and Ω^*_0 coupling. For ions, meanwhile, we have

$$\delta g^{(2)}_{si,+} \simeq i(\Lambda^s_0/2\omega_s)\left(J_+\delta\phi_+\delta g^{(1)*}_{0i} - J_0\delta\phi^*_0\delta g_{+i}\right). \quad (27)$$

Here, we note that $\delta g_{+i} = \delta g^{(1)}_{+i}+\delta g^{(2)}_{+i}$ given, respectively, by Eqs. (9) and (10). $\delta g^{(2)}_{si,+}$ is then given by

$$\begin{aligned}\delta g^{(2)}_{si,+} &\simeq \left[i\frac{\Lambda^s_0}{2\omega_+}J_0J_+\delta\phi_+\delta\phi^*_0 - \frac{(\Lambda^s_0)^2}{4\omega_s\omega_+}J_0^2J_s|\delta\phi_0|^2\delta\phi_s\right]\\ &\times\left(\frac{\omega_{*i}}{\omega}\right)_s\frac{e}{T_i}F_{Mi}. \quad (28)\end{aligned}$$

The analysis is similar for the $\Omega_- + \Omega_0 \to \Omega_s$ scattering channel. Then, we have

$$\delta g^{(2)}_{se,-} \simeq i\frac{\Lambda_0^s}{2\omega_-}\frac{e}{T_e}F_{Me}\delta\psi_-\delta\psi_0\left[1+\frac{k_{\parallel 0}}{k_{\parallel s}}\frac{(\omega_{*e}-\omega)_s}{\omega_0}\right] \tag{29}$$

and

$$\begin{aligned}\delta g^{(2)}_{si,-} \simeq & -\left[i\frac{\Lambda_0^s}{2\omega_-}J_0J_-\delta\phi_-\delta\phi_0 + \frac{(\Lambda_0^s)^2}{4\omega_s\omega_-}J_0^2J_s|\delta\phi_0|^2\delta\phi_s\right] \\ & \times \left(\frac{\omega_{*i}}{\omega}\right)_s\frac{e}{T_i}F_{Mi}.\end{aligned} \tag{30}$$

Substituting the $\delta g_{sj} = \delta g^{(1)}_{sj} + \delta g^{(2)}_{sj,+} + \delta g^{(2)}_{sj,-}$ for $j = e, i$ into the quasi-neutrality condition, Eq. (5), of the Ω_s mode, we then readily derive the following governing equation for $\delta\phi_s$;

$$\begin{aligned}\epsilon_s\delta\phi_s = & \ i(\Lambda_0^s/2\omega_+)\beta_{s+}\delta\phi_0^*\delta\phi_+ - i(\Lambda_0^s/2\omega_-)\beta_{s-}\delta\phi_0\delta\phi_- \\ & -\epsilon_s^{(2)}|\delta\phi_0|^2\delta\phi_s.\end{aligned} \tag{31}$$

Here, ϵ_s is the eDW linear dielectric operator and, in the limit of adiabatic circulating electrons and neglecting trapped electrons, is given by

$$\epsilon_s = 1+\tau-\tau\left\langle\left(\frac{\omega-\omega_{*i}}{\omega-k_\parallel v_\parallel-\omega_d}\right)_s\frac{F_{Mi}}{N_0}J_s^2\right\rangle_v; \tag{32}$$

and, in the lowest order,

$$\epsilon_s \simeq 1+\tau(1-\Gamma_s)+\tau\Gamma_s(\omega_{*i}/\omega)_s. \tag{33}$$

Meanwhile,

$$\beta_{s\pm} = \tau\left(\frac{\omega_{*i}}{\omega}\right)_s F_\pm + \sigma_{*0}\sigma_{*\pm}\left[1+\frac{k_{\parallel 0}}{k_{\parallel\pm}}\frac{(\omega_{*e}-\omega)_s}{\omega_0}\right] \tag{34}$$

and

$$\begin{aligned}\epsilon_s^{(2)} = & \sum_{l=+,-}\left\{\frac{F_2}{\omega_s\omega_l}+\sigma_{*0}\left[1+\frac{k_{\parallel 0}}{k_{\parallel l}}\frac{(\omega_{*e}-\omega)_s}{\omega_0}\right]\right. \\ & \left.\times\left[\frac{F_l}{\omega_l^2(1-\omega_{*e}/\omega)_l}\right]\right\}\frac{(\Lambda_0^s)^2}{4}\tau\left(\frac{\omega_{*i}}{\omega}\right)_s.\end{aligned} \tag{35}$$

Noting Eqs. (16) and (24) for, respectively, $\delta\phi_+$ and $\delta\phi_-$, Eq. (31) can be formally expressed as

$$\begin{aligned}\left(\epsilon_s+\epsilon_s^{(2)}|\delta\phi_0|^2\right)\delta\phi_s = & \left[\left(\frac{\Lambda_0^s}{2\omega_+}\right)^2\frac{\beta_s^+\delta\phi_0^*\beta_+}{\tau b_+\epsilon_{A+}}\delta\phi_0\right. \\ & \left.+\left(\frac{\Lambda_0^s}{2\omega_-}\right)^2\frac{\beta_s^-\delta\phi_0\beta_-}{\tau b_-\epsilon_{A-}}\delta\phi_0^*\right]\delta\phi_s;\end{aligned} \tag{36}$$

which may be regarded as the nonlinear eigenmode equation of Ω_s (eDW) in the presence of finite-amplitude Ω_0 (TAE) fluctuations.

Equation (36), in general, need to be solved numerically. We can, however, make analytical progress by employing the scale separation and obtain an analytical dispersion relation variationally. First, we adopt the ballooning-mode representation for $\delta\phi_s$;

$$\delta\phi_s = \exp(in_s\xi)\sum_{m_s}\exp(-im_s\theta)\Phi_s(n_sq-m_s\equiv z_s), \tag{37}$$

where ξ and θ are, respectively the toroidal and poloidal angles, and denote the spatial scales of TAE and eDW as, respectively, $\mathbf{x}_0$ and $\mathbf{x}_s$; such that $|\mathbf{x}_s|/|\mathbf{x}_0| \sim O(n_0/n_s) \ll 1$. Multiplying Eq. (36) by $\delta\phi_s^*$ and integrating over $\mathbf{x}_s$, we readily derive

$$D_s + \chi_s^{(2)}|\delta\phi_0(\mathbf{x}_0)|^2 = R_+ + R_-, \tag{38}$$

where

$$D_s = \langle\Phi_s^*(z_s)\epsilon_s\Phi_s\rangle_s \tag{39}$$

is the linear dielectric constant of Ω_s,

$$\begin{aligned}\langle\Phi_s^*[A]\Phi_s\rangle_s \equiv & \int_{-1/2}^{1/2}dz_s\sum_{m_s}\Phi_s^*[A]\Phi_s \\ = & \int_{-\infty}^{\infty}dz_s\Phi_s^*[A]\Phi_s\end{aligned} \tag{40}$$

with the normalization $\langle|\Phi_s|^2\rangle_s = 1$,

$$\chi_s^{(2)} = \langle\Phi_s^*\epsilon_s^{(2)}\Phi_s\rangle_s, \tag{41}$$

and

$$R_\pm = \left\langle\Phi_s^*\left(\frac{\Lambda_s^s}{2\omega_\pm}\right)^2\beta_s^\pm\begin{Bmatrix}\delta\phi_0^*\\ \delta\phi_0\end{Bmatrix}\frac{\beta_\pm}{(\tau b\epsilon_A)_\pm}\begin{Bmatrix}\delta\phi_0\\ \delta\phi_0^*\end{Bmatrix}\Phi_s\right\rangle_s. \tag{42}$$

Equation (38) is formally the variational nonlinear eDW dispersion relation in the presence of a finite-amplitude TAE given by $\delta\phi_0$. We will later analyze it further using a trial function for $\Phi_s(z_s)$. We now make same qualitative observations. We note that $\chi_s^{(2)}$ is real and, in general, $R_\pm = Re(R_\pm) + iIm(R_\pm)$. Thus, $\chi_s^{(2)}$ and $Re(R_\pm)$ lead to nonlinear frequency shift; while $Im(R_\pm)$ gives rise to nonlinear damping or growth. Focusing on $Im(R_\pm)$ first, we observe, from Eq. (42), that $Im(R_\pm) \propto Im(1/\epsilon_\pm)$; i.e., the imaginary component of the SAW/KAW operator, $\epsilon_{A\pm}$, given, by Eq. (17) along with Eqs. (16) and (24), respectively. Looking at Eq. (16) and letting

$$\begin{aligned}\delta\phi_+ = & \ A_+(\mathbf{x}_0)\exp(in_s\xi_s) \\ & \times\sum_{m_s}\exp(-im_s\theta)\Phi_+(z_s\equiv n_sq-m_s),\end{aligned} \tag{43}$$

we then have, recalling the scale separation between $\mathbf{x}_0$ and $\mathbf{x}_s$,

$$A_+(\mathbf{x}_0)\tau b_s\epsilon^s_{A+}\Phi_+(z_s) = -i\frac{\Lambda_0^s}{2\omega_+}\beta_+\Phi_s(z_s)\delta\phi_0(\mathbf{x}_0), \tag{44}$$

where

$$\epsilon^s_{A\pm} = \left(1-\frac{\omega_*}{\omega}\right)_\pm \frac{1-\Gamma_s}{b_s} - \left(\frac{V_A^2}{b_s}\frac{k_{\parallel s}b_s k_{\parallel s}}{\omega_\pm^2}\right)\sigma^s_{*\pm},$$
$$\sigma^s_{*\pm} \simeq \left[1+\tau-\tau\Gamma_s\left(1-\omega_{*i}/\omega\right)_\pm\right]/(1-\omega_{*e}/\omega)_\pm, \quad (45)$$

$b_s = b_{s\theta}(1+\hat{s}^2\partial^2_{z_s})$, $b_{s\theta} = k^2_{s\theta}\rho_i^2$, $\hat{s} = rq'/q$ denotes magnetic shear, and $k_{\parallel s} = (n_s q - m_s)/(qR) = z_s/(qR)$. Since $|\hat{s}^2\partial^2_{z_s}| < 1$ for moderately/strongly ballooning modes, $\epsilon^s_{A\pm}$ further reduces to

$$\epsilon^s_{A\pm} \simeq b_{s\theta}\hat{s}^2\partial^2_{z_s}\frac{\partial\epsilon^s_{A\pm}}{\partial b_{s\theta}} - \left(\frac{\omega_A}{\omega_\pm}\right)^2\sigma_{\pm s}(z_s^2 - z_\pm^2). \quad (46)$$

Here, $\omega_A = V_A/(qR)$,

$$\sigma_{\pm s} = \left[1+\tau-\tau\Gamma_s(b_{s\theta})(1-\omega_{*i}/\omega)_\pm\right]/(1-\omega_{*e}/\omega)_\pm, \quad (47)$$

and

$$z^2_\pm = \left(\frac{\omega}{\omega_A}\right)^2_\pm\left(1-\frac{\omega_{*i}}{\omega}\right)_\pm\frac{1-\Gamma_s(b_{s\theta})}{b_{s\theta}}\frac{1}{\sigma_{\pm s}} < \frac{1}{4}; \quad (48)$$

as $|\omega/\omega_A|^2_\pm \simeq 1/4$. Equation (44) along with ϵ^s_{A+} given by Eq. (46) indicates that the upper sideband is a KAW mode converted at the high-n Alfvén resonance layer $z_s = \pm z_+$. As noted in previous study of mode-converted KAW [10], for $\tau = T_e/T_i \sim 1$, the finite electron Landau damping as well as the Airy swelling of the amplitude dictate that the damping occur predominantly around $z = \pm z_+$ and the energy absorption rate approximates that of the local Alfvén resonance via the causality constraint $Im(\omega_s, \omega_+, \omega_-) > 0$; i.e.,

$$Im\left(\frac{1}{\epsilon_{A+}}\right) \simeq -\pi\delta(\epsilon_{A+}) \simeq -\pi\left(\frac{\omega_+}{\omega_A}\right)^2\frac{\delta(z_s^2 - z_+^2)}{\sigma_{+s}}. \quad (49)$$

Similar processes occur for the Ω_- KAW; i.e.,

$$Im\left(\frac{1}{\epsilon_{A-}}\right) \simeq \pi\delta(\epsilon_{A-}) \simeq \pi\left(\frac{\omega_-}{\omega_A}\right)^2\frac{\delta(z_s^2 - z_-^2)}{\sigma_{-s}}. \quad (50)$$

Consequently,

$$Im(R_+) = -\left\langle \Phi^*_s\left(\frac{\Lambda^s_0}{2\omega_+}\right)^2\frac{\beta^+_s\delta\phi^*_0\beta_+}{\tau b_s} \times\left(\frac{\omega_+}{\omega_A}\right)^2\frac{\delta(z_s^2-z_+^2)}{\sigma_{+s}}\delta\phi_0\Phi_s\right\rangle, \quad (51)$$

and, omitted here, a similar corresponding expression can be obtained for $Im(R_-)$.

To proceed further, we take the $|k_{0\perp}\rho_i|^2 \ll 1$ limit but keep the finite $|\omega_s/\omega_0| < 1$ correction, it is then straightforward to derive

$$\beta^\pm_s\beta_\pm\delta(\epsilon_{A\pm}) \simeq \tau(1-\Gamma_{s\theta})\sigma_{s\theta}\frac{(\omega_{*e}-\omega)_s(\omega-\omega_{*i})_s}{\omega_0^2} \times\frac{k_{\parallel\pm}}{k_{\parallel s}}\delta(\epsilon_{A\pm}). \quad (52)$$

Here, $\Gamma_{s\theta} = \Gamma_s(b_{s\theta})$ and $\sigma_{s\theta} = 1+\tau(1-\Gamma_{s\theta})$.

The variational nonlinear eDW dispersion relation, Eq. (38), then yields, letting $\omega_s = \omega_{sr} + i\gamma_s$ and $D_{sr}(\omega_{sr}) = 0$,

$$\left(\gamma_s + \gamma^l_s\right)\frac{\partial}{\partial\omega_{sr}}D_{sr} = Im(R_+ + R_-)$$
$$\simeq -\frac{\pi}{4\beta_i}\left(\frac{\Omega_i}{\omega_0}\right)^2\left|\frac{\delta B_{0\theta}}{B_0}\right|^2(1-\Gamma_{s\theta})\sigma_{s\theta}\frac{(\omega_{*e}-\omega)_s(\omega-\omega_{*i})_s}{\omega_0^2}$$
$$\times\left\langle\left[\left(\frac{\omega_+}{\omega_A}\right)^2\frac{\delta(z_s^2-z_+^2)}{\sigma_{+s}} - \left(\frac{\omega_-}{\omega_A}\right)^2\frac{\delta(z_s^2-z_-^2)}{\sigma_{-s}}\right]|\Phi_s|^2\right\rangle_s \quad (53)$$

Here, γ^l_s is the linear damping/growth of eDW. Noting that $\partial D_{sr}/\partial\omega_{sr} > 0$, $Im(R_+) < 0$ and $Im(R_-) > 0$, scatterings to UKAW and LKAW, thus, lead to, respectively, damping and growth of eDW. As illustrated in Fig. 2, one may qualitatively regard UKAW scattering as stimulated absorption, and LKAW scattering as spontaneous emission, similar to the familiar parametric decay instability via a quasi-mode. In Eq. (53), we have also noted $ck_{0r}\delta\phi_0/B_0 \simeq V_A\delta B_{0\theta}/B_0$.

To estimate the nonlinear damping/growth rate quantitatively, we adopt a trial function for Φ_s as $|\Phi_s|^2 = (1/\sqrt{\pi}\Delta_s)\exp(-z_s^2/\Delta_s^2)$ with $\Delta_s > 1$ for a typical moderately ballooning eDW. Equation (53) then yields

$$Im(R_+ + R_-) \simeq -\frac{\sqrt{\pi}}{4\beta_i}\left(\frac{\Omega_i}{\omega_0}\right)^2\left|\frac{\delta B_{0\theta}}{B_0}\right|^2(1-\Gamma_{s\theta})\sigma_{s\theta}$$
$$\times\frac{(\omega_{*e}-\omega)_s(\omega-\omega_{*i})_s}{\omega_0^2}$$
$$\times\left[\left(\frac{\omega_+}{\omega_A}\right)^2\frac{1}{\sigma_{+s}z_+\Delta_s} - \left(\frac{\omega_-}{\omega_A}\right)^2\frac{1}{\sigma_{-s}z_-\Delta_s}\right]. \quad (54)$$

Taking typical tokamak parameters, $\Omega_i/\omega_0 \sim O(10^2)$, $\beta_i \sim O(10^{-2})$, $b_{s\theta} \sim O(1)$, $|\omega_s/\omega_0|^2 \sim O(10^{-1})$, and $|\Delta_s z_\pm| \sim O(1)$, we then find

$$|Im(R_+ + R_-)| < O(10^5)\left|\frac{\delta B_{0\theta}}{B_0}\right|^2. \quad (55)$$

Noting that $\partial D_{sr}/\partial\omega_{sr} \sim 1/\omega_{sr}$ and $\gamma^l_s/\omega_{sr} \sim O(10^{-1})$ as, e.g., in the trapped electron mode [15], we then find that, for TAE fluctuations with $|\delta B_{0\theta}/B_0|^2 \lesssim O(10^{-7})$ [16], the nonlinearly induced damping/growth rate, $\gamma_s/\omega_{sr} \sim |Im(R_+ + R_-)| \lesssim O(10^{-2})$, and should have negligible effects on the eDW stability. We also remark that one can, furthermore, straightforwardly show that the nonlinear frequency shift due to $\chi^{(2)}_s|\delta\phi_0|^2$ and $Re(R_+ + R_-)$ is also typically negligible.

V. CONCLUSIONS AND DISCUSSIONS

In this work, we have employed the nonlinear gyrokinetic equations and investigated analytically direct

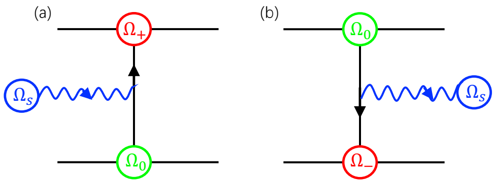

FIG. 2: Sketch illustrating the nonlinear scattering processes of (a) stimulated absorption and (b) spontaneous emission. Here, Ω_0 is the finite-amplitude TAE, Ω_s is the test eDW, and $\Omega_\pm$ are, respectively, the upper and lower sideband KAWs. .

wave-wave interactions between a test electron drift wave (eDW) and ambient finite-amplitude toroidal Alfvén eigenmodes (TAEs) in low-β circular tokamak plasmas. Here, nonlinear scatterings generate upper and lower sidebands of mode-converted kinetic Alfvén waves (KAWs) at high todoial mode numbers which are typically damped by electrons around the mode conversion positions. Furthermore, we find that scattering to upper-sideband KAW gives rise to stimulated absorption and, hence, damping of the eDW. Scattering to lower-sideband KAW, on the other hand, gives rise to spontaneous emission and, thereby, growth of the eDW; i.e., TAE parametrically decays to eDW via the lower-sideband KAW quasi-mode. For typical tokamak parameters and TAE fluctuation intensity, our analysis indicates that the net effects on eDW stability properties should be negligible. We remark again that, as noted in Sec. I, the present results are in contrast to those obtained previously for the case of direct wave-wave interactions between a test TAE and ambient eDW [9]. In that case, both channels of scatterings to KAWs lead to stimulated absorption and, thereby, significant damping of the TAE.

As noted above, our analysis adopts the electron drift waves without temperature gradients as a paradigm model in order to simplify the analysis and delineate more clearly the underlying nonlinear physics mechanisms. It is clearly desirable to extend the investigations to include ion-temperature-gradient (ITG) modes as well as other types of AEs; such as reversed shear Alfvén eigenmodes (RSAEs) [17, 18] and beta-induced Alfvén eigenmodes (BAEs) [4, 19]. While detailed analyses for such cases remain to be carried out, one may conjecture that the physical pictures outlined in the current paradigm model should hold at least qualitatively.

Finally, that the present results indicating negligible effects on eDW via direct wave-wave interactions with TAE suggests the possible significance of indirect route of interaction via, e.g., the zonal structures consisting of flow, field and phase space nonlinearly generated by AEs [20–22]. This interesting subject remains to be further investigated in the future.

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