中国科学技术大学
University of Science and Technology of China

# Simulations of nonlinear interaction between beta-induced Alfven eigenmode and tearing mode


Huishan Cai[1]

**Sizhe Duan[1]**, Ding Li[2]

[1]University of Science and Technology of China

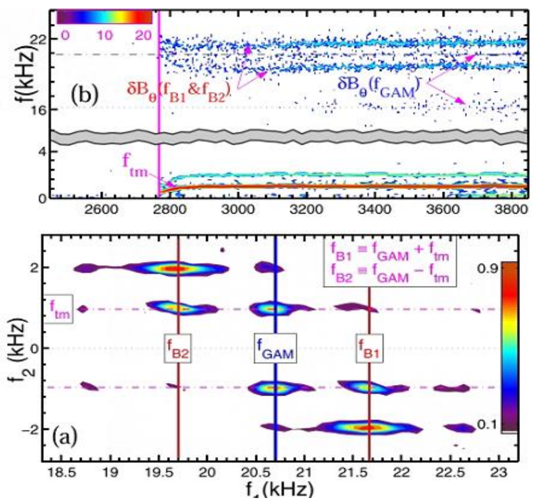
[2]Chinese Academy of Sciences

# Outline

- **Background and Motivation**
- **M3D-K model and parameters setup**
- **Nonlinear interaction between BAE and TM**
  - Influence of TM activity on BAE
  - Influence of BAE on magnetic island evolution
- **Discussion and summary**

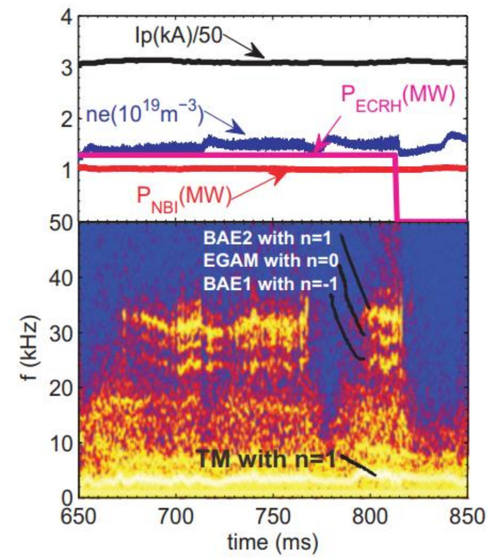
# Background and Motivation

- **Synergistic effect of AE and TM enhance EP redistribution and losses** (B. Madsen *et al* 2020 *Nucl. Fusion* 60 066024 ).
- **Interactions between BAEs and TM were found on FTU, MAST, EAST, JET, HL-2A…**




$$\begin{cases} f_{BAE,Co} = f_{GAM} + f_{TM} \\ f_{BAE,Cn} = f_{GAM} - f_{TM} \end{cases}$$

EAST (Ming X, *et al*, NF, 2021)

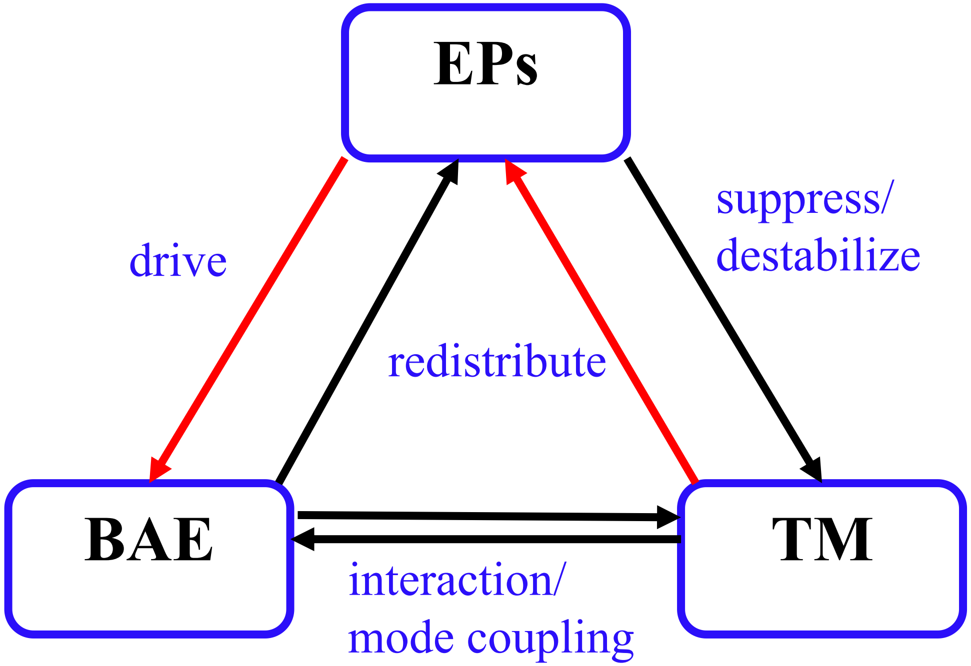



$$f_{n=0\ mode} = f_{AE2} - f_{TM}$$
$$f_{n=0\ mode} = f_{AE1} + f_{TM}.$$

HL-2A (Chen W, *et al*, EPL, 2014)

# Physics Interpretation

➢ **Simple Physics Picture**




**Notes: BAEs and TM are both localized near the rational surface.**
**They can interact strongly.**

# M3D-K model

## Resistive MHD equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0$$

$$\rho \frac{dv}{dt} = J \times B - \nabla P - \nabla \cdot P_h$$

$$\frac{\partial B}{\partial t} = -\nabla \times E$$

$$\frac{dp}{dt} = -\gamma p \nabla \cdot v + \rho \nabla \cdot \kappa \cdot \nabla \frac{p}{\rho}$$

$$J = \nabla \times B$$

$$E + v \times B = \eta J$$

$$\nabla \cdot B = 0$$

## Momentum equation with particle stress tensor*:

$$P_h = P_\perp I + (P_\parallel - P_\perp) bb$$

$$P_\parallel(x) = \int M v_\parallel^2 \delta(x - X - \rho_h) F(X, v_\parallel, \mu) d^3X dv_\parallel d\mu d\theta$$

$$P_\perp(x) = \int \frac{1}{2} M v_\perp^2 \delta(x - X - \rho_h) F(X, v_\parallel, \mu) d^3X dv_\parallel d\mu d\theta$$

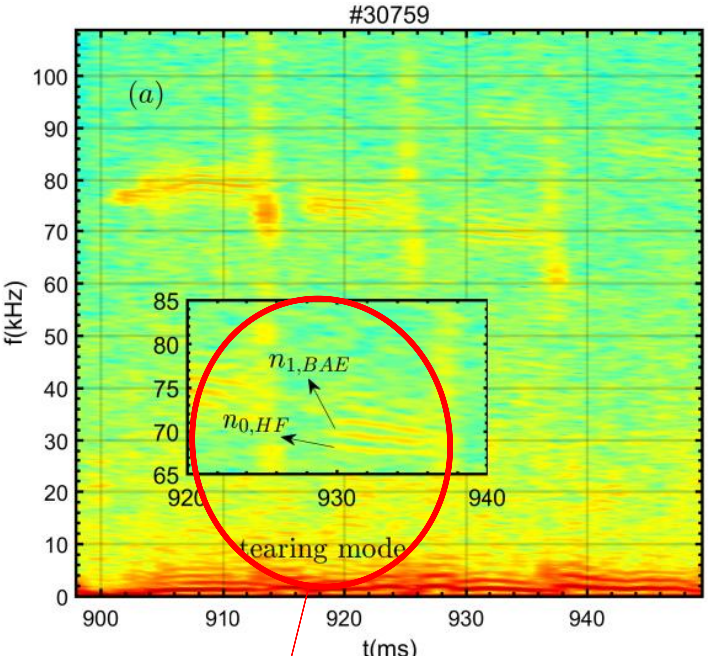
$$\frac{dX}{dt} = \frac{1}{B^{**}} \left[ v_\parallel \left( B^* - b_0 \times \left( \langle E \rangle - \frac{1}{e} \mu \nabla (B_0 + \langle \delta B \rangle) \right) \right) \right]$$

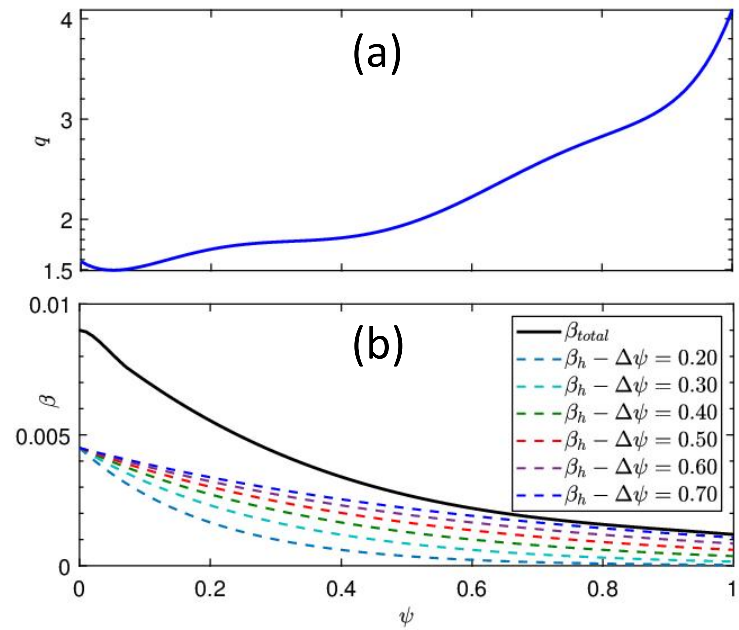
$$m \frac{dv_\parallel}{dt} = \frac{e}{B^{**}} B^* \cdot \left( \langle E \rangle - \frac{1}{e} \mu \nabla (B_0 + \langle \delta B \rangle) \right)$$

*G. Fu, et al, POP, 2006

# Simulation setup

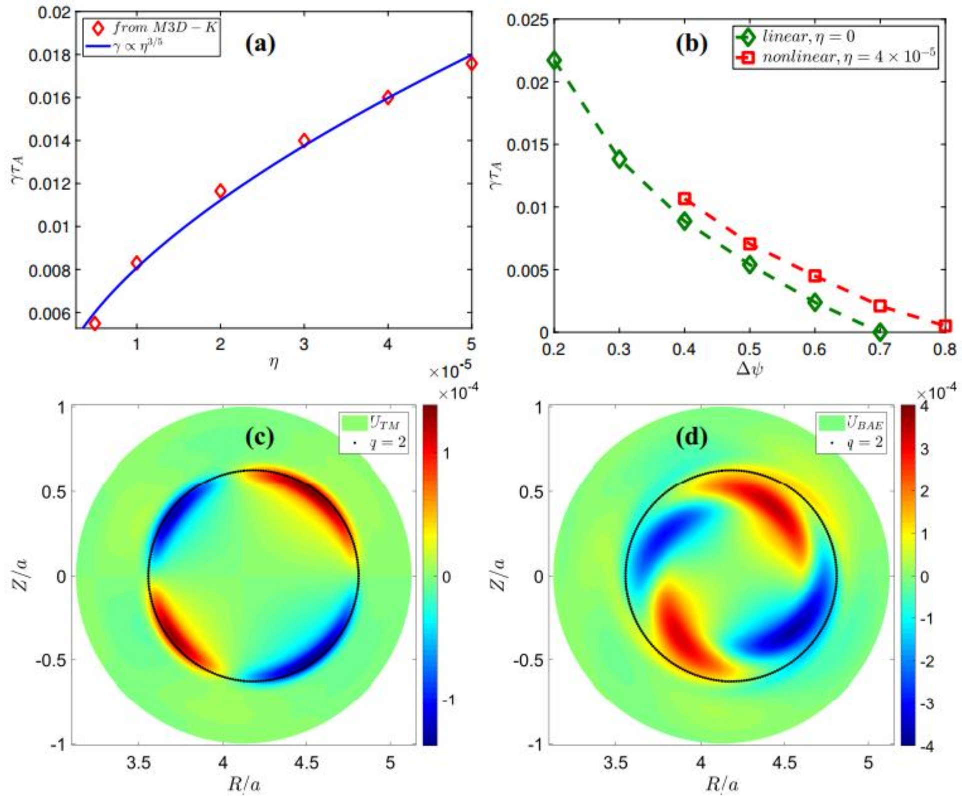
$$f=\frac{cH(v_0-v)}{(v^3+v_c^3)}\exp\left(-\frac{\langle\psi\rangle}{\Delta\psi}\right)\exp\left(\frac{-(\Lambda-\Lambda_0)^2}{\Delta\Lambda^2}\right)$$




- **Based on the HL-2A shot 30759** $(\beta_{total}=0.9\%,\ \beta_{frac}=\frac{\beta_h}{\beta_{total}}=0.50)$
- $\omega_{n_{0,HF}}=\omega_{BAE}-\omega_{TM}$**: mode coupling**

# Simulation setup and pure BAE & TM structures



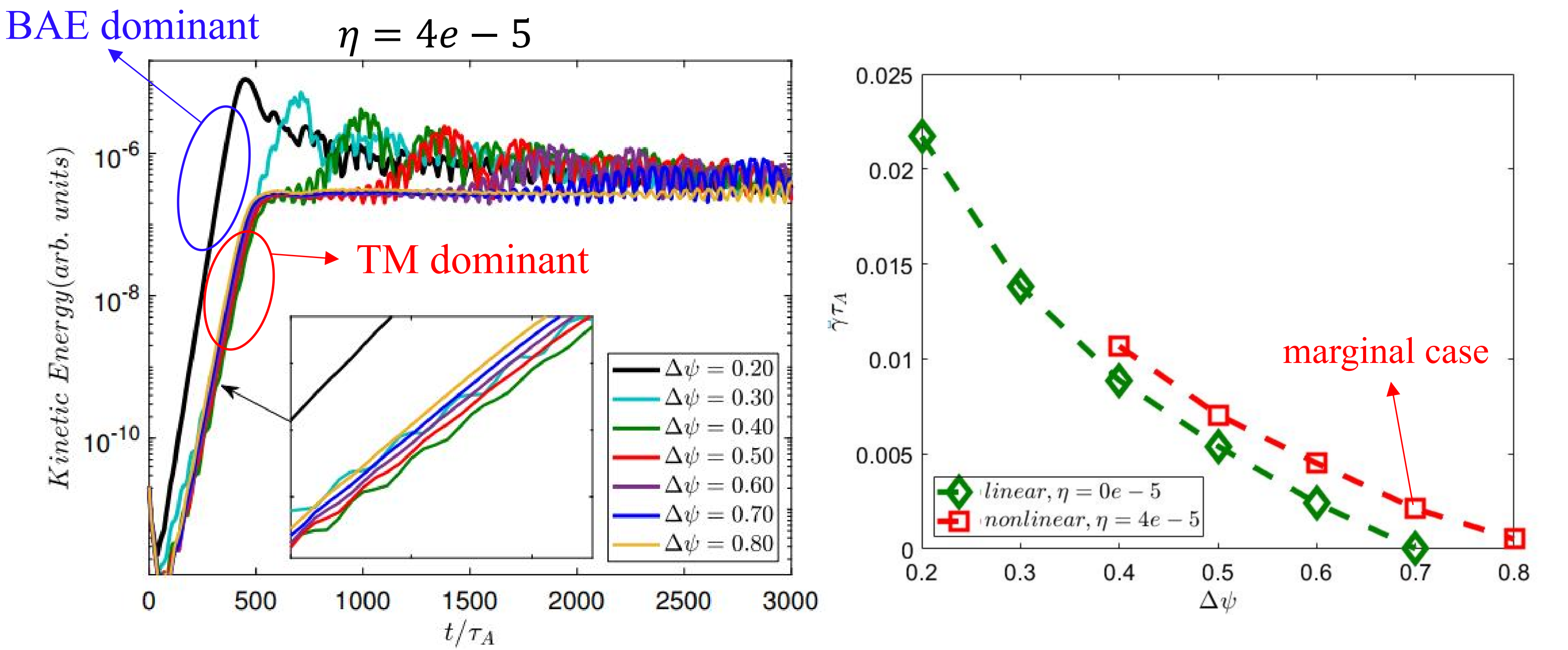

*(a) linear growth rate of TM versus resistivity η;*
*(b) growth rate of BAE versus Δψ without/with resistivity (green/red curves);*
*(c) Pure TM mode structure of stream function (U)*
*(d) Pure BAE mode structure*

**(a): TM scaling law: $\gamma \propto \eta^{3/5}$**

**(b): $\gamma_{BAE}$ dependens on EP radial gradient**

**(c)&(d): BAE structure (d) is wider than TM structure (c)**

# BAE + TM simulations

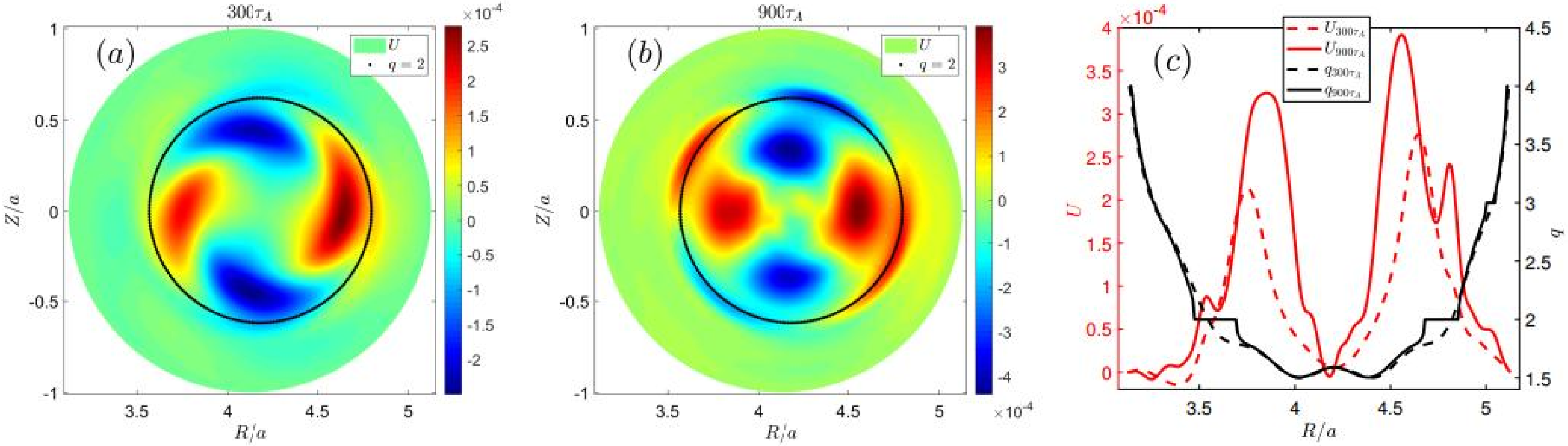



*Kinetic energy (n=1) evolution with different Δψ and growth rate of BAE versus Δψ.*

- **TM evolves to a saturation stage even BAE dominates.**
- **Linear stable BAE is excited in nonlinear stage with TM activity.**
- **The excitation threshold about $\Delta\psi$ is decreased by TM.**

# BAE linearly-dominant-unstable

$\Delta\psi = 0.20 \& \eta = 4e-5$

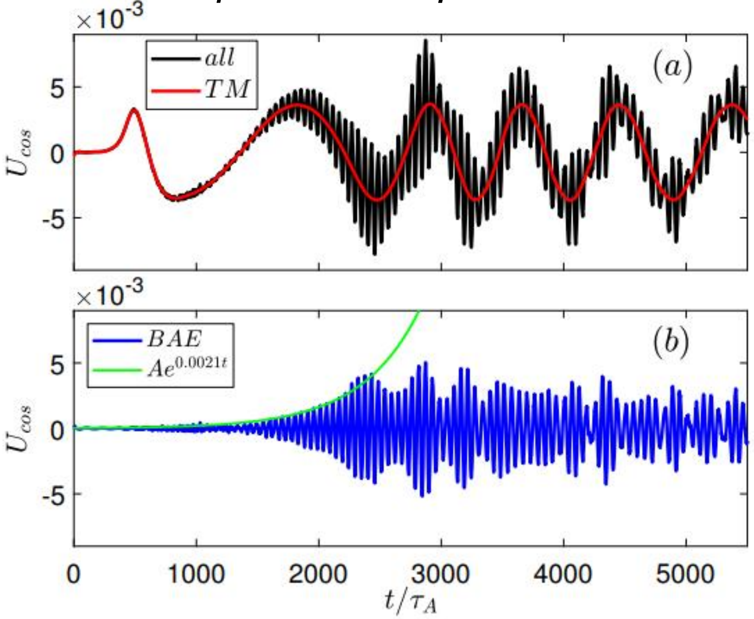


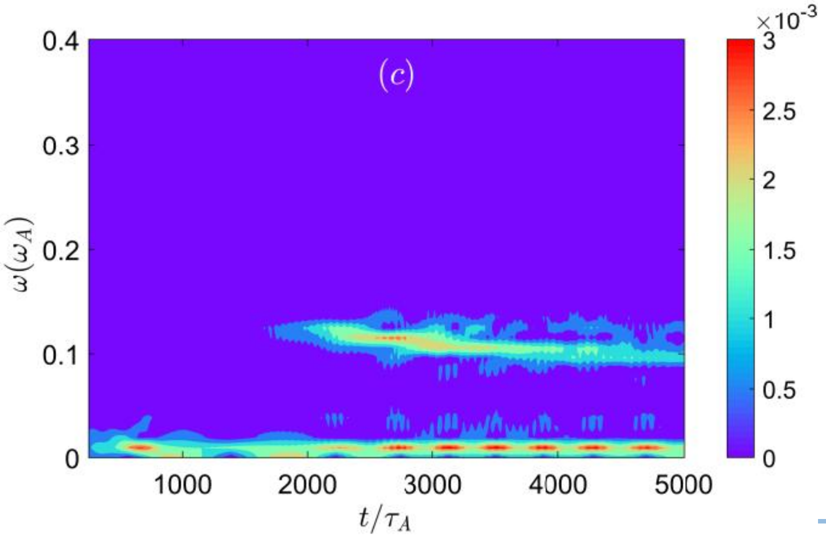

*mode structure of velocity stream function U*

- **TM coexists with BAE in the nonlinear phase.**
- **When island grows, mode structure of BAE moves inward.**
  - **q profile being flattened at rational surface by island.**
  - **EP radial gradient near island separatrix is enhanced.**

# BAE linearly-stable

$\Delta\psi = 0.70 \& \eta = 4e-5$

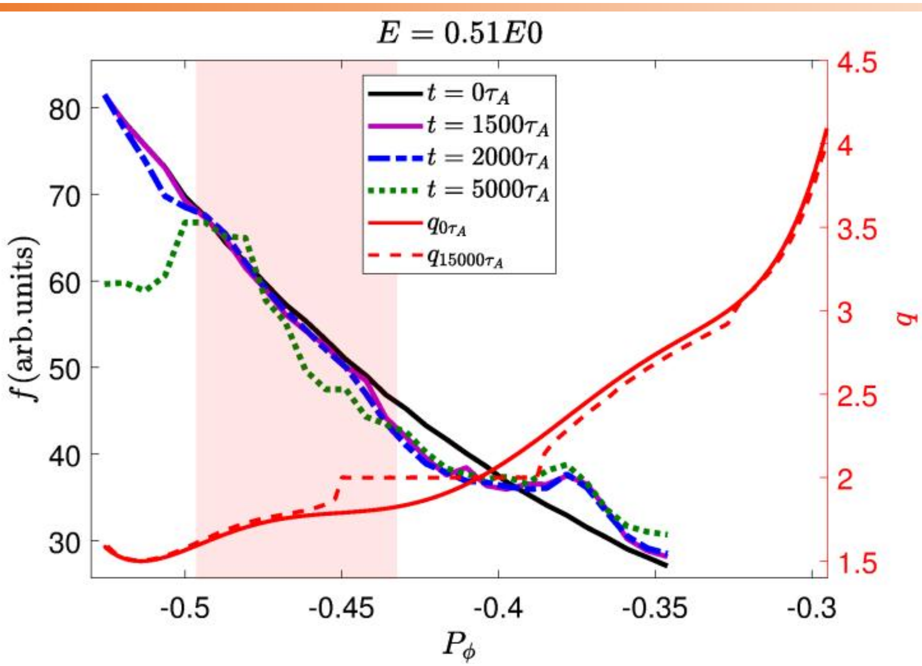



- $\boldsymbol{U_{cos,all}} \rightarrow \boldsymbol{U_{cos,BAE}}$ $(\boldsymbol{\omega > 0.05\omega_A})$ & $\boldsymbol{U_{cos,TM}}$ $(\boldsymbol{\omega \le 0.05\omega_A})$
- **BAE is triggered by TM around** $\boldsymbol{t = 1500\tau_A}$ **(with** $\boldsymbol{\gamma_{BAE} = 0.0021}$**)**
- **BAE: weakly downward chirping**

*Time evolution of stream function cos component $U_{cos}$ at r=0.60, and the corresponding frequency spectrum*

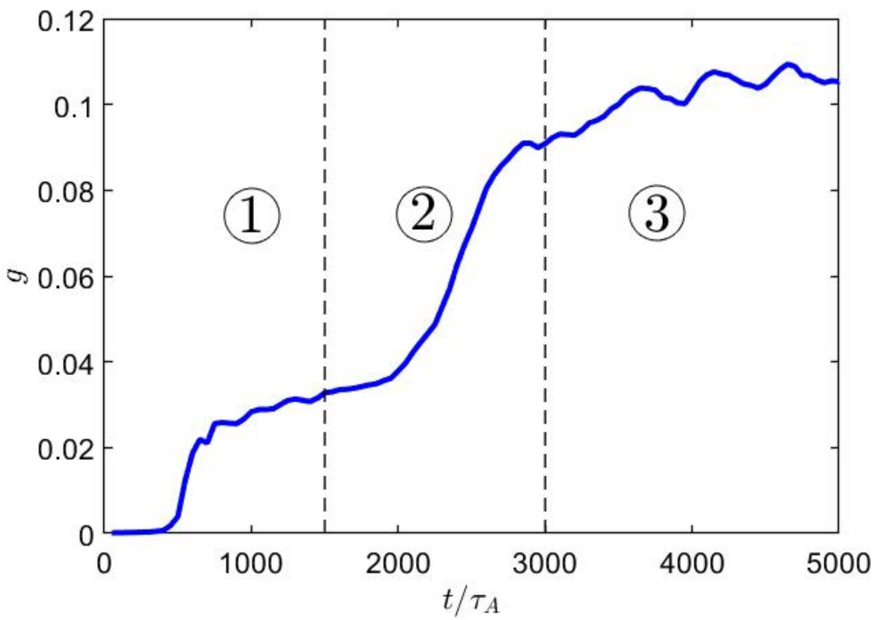
# Evolution of one-dimensional distribution $f(P_\phi)$




*$f(P_\phi)$ around $E = 0.51E_0$ and $\Lambda = 0.0$, the ions with $P_\phi > 0.346$ are lost by prompt loss. The red region denotes radial location of BAE.*

- **After TM saturation (~1500TA):**
  - **I. q profile becomes flattened at q=2 surface.**
  - **II. f is partially flattened inside the island and shifted outward to O point of the island**
  - **III. $\partial f/\partial P_\phi$ near 2/1 island separatrix is increased.**
- **Combination of I- III drive BAE being destabilized.**
- **Inner fattened region of $f(P_\phi)$ is induced by BAE.**

# EPs redistribution induced by BAE and TM




*Time evolution of redistribution level g of energetic ions with* $f(P_\phi)$ *around* $E = 0.51E_0$ *and* $\Lambda = 0.0$

$$g = \int \left| f(t = t_1) - f(t = 0) \right| \mathrm{d}P_\phi \Big/ \int f(t = t_1)\, \mathrm{d}P_\phi$$

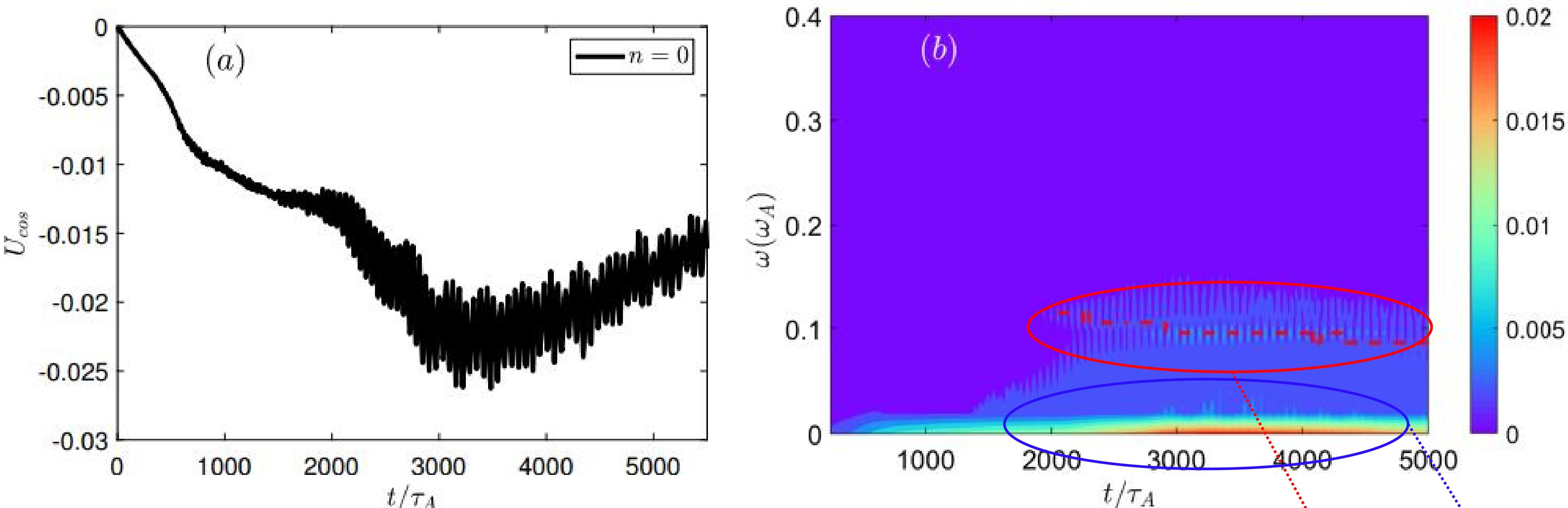
① **Firstly, EPs are redistributed by TM;**

② **Then, *g* increases significantly after the excitation of BAE;**

③ **Finally, large g value: excessive EP redistribution/losses.**

# Mode coupling between BAE and TM

$$\Delta\psi = 0.70 \& \eta = 4e-5$$




zero-frequency n=0 ($n_{0,ZF}$) component

*Time evolution of $U_{cos,n=0}$ at r=0.50, and the corresponding frequency spectrum. The red line:* $\Delta\omega = \omega_{BAE} - \omega_{TM}$.

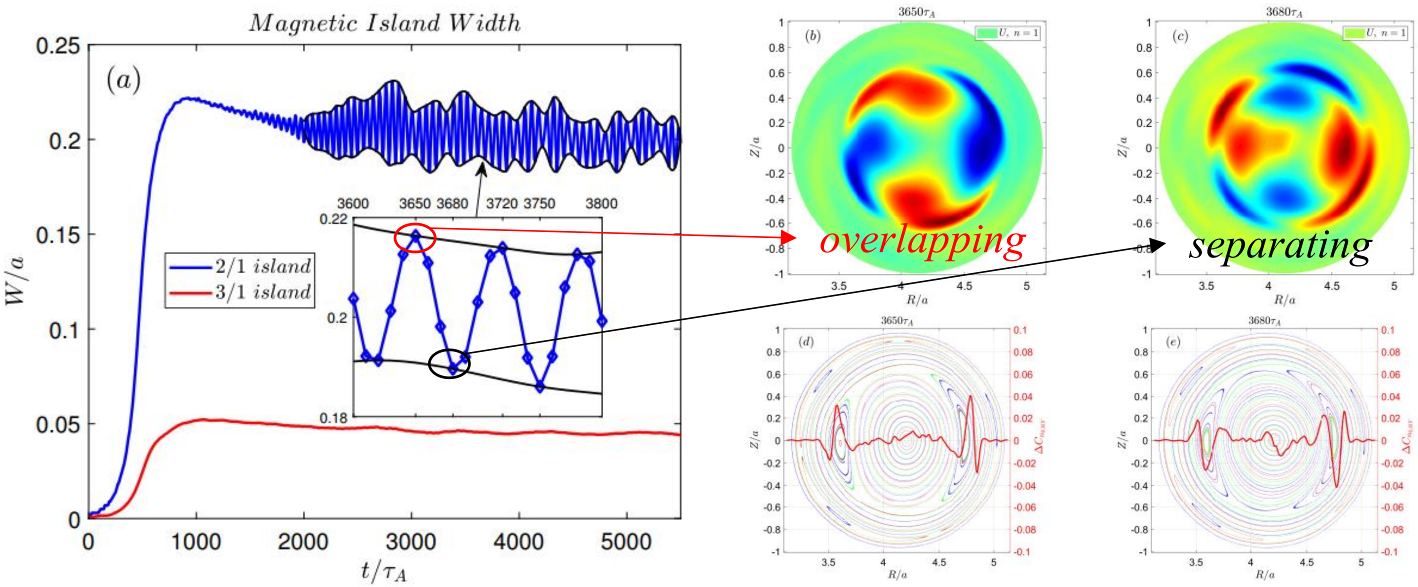
high-frequency n=0 ($n_{0,HF}$) component, called breathing mode (BM)

- $\boldsymbol{n_{BM} = n_{BAE} - n_{TM}}$.
- $\boldsymbol{\omega_{BM} = \omega_{BAE} - \omega_{TM}}$.

◆ **It is consistent with the experimental result of HL-2A.**

# Periodic oscillation of 2/1 island width

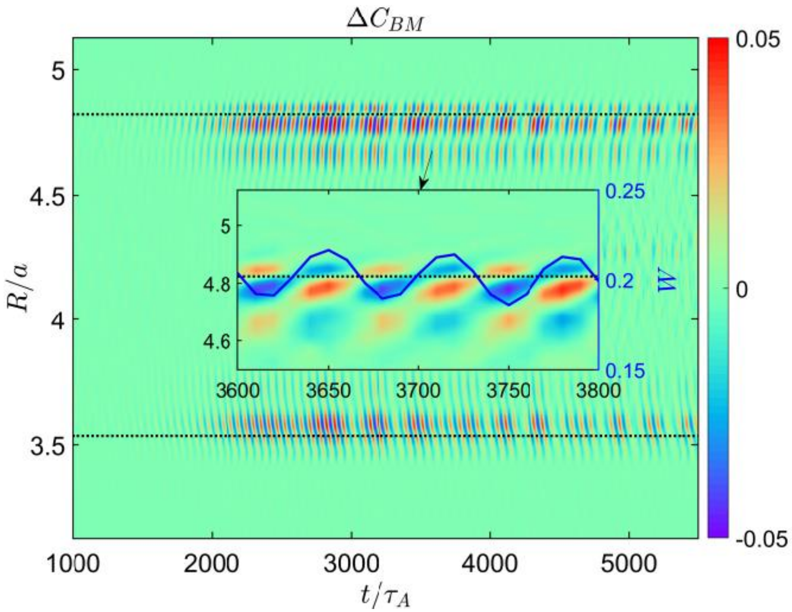
$\Delta\psi = 0.70 \& \eta = 4e-5$




*(a) magnetic island width, (b)&(c) n=1 structure of U, (d)&(e) Poincare plots of magnetic surface and* $\Delta C_{n_{0,HF}}$

- **2/1 island width exhibits periodic oscillation** **($T \sim 70\tau_A \sim 25\mu s$).**
  - **change rate is over 10%**
  - **causes inner BAE and outer TM to exhibit a periodic variation**
- **ΔC of BM component in (d)&(e):**
  - **mainly located inside 2/1 magnetic island.**
  - **changes periodically (periodic current modification).**

# $\Delta C_{BM}$ causes synchronized oscillation of $W_{2/1}$




*Time evolution of $\Delta C_{BM}$ component at Z=0 and 2/1 island width.*

- **Pearson correlation coefficient:**

$$P(x,y) = \frac{\sum_i (x(t_i)-\bar{x})(y(t_i)-\bar{y})}{(\sqrt{\sum_i (x(t_i)-\bar{x})^2})(\sqrt{\sum_i (y(t_i)-\bar{y})^2})}$$

- $\boldsymbol{P\left(W_{2/1}, \Delta C_{BM_{,R=3.557}}\right) = 0.947}$

**$\Delta C_{BM}$ component:**
**oscillates periodically around q=2 surface**
($\Delta C_{n_{0,ZF}}$ component: remain almost unchanged)

**periodic current modification**

**2/1 island width:**
**synchronized oscillation**

# Discussion and summary

- **TM activity can lead to:**
  - **significant redistribution of EPs**
  - **increase of excitation threshold of BAE.**
  - **destabilization of BAE.**
- **Excessive EPs redistribution/losses caused by BAE&TM.**
- **$n_{0,HF}$ component generated by nonlinear coupling between BAE and TM :**
  - $\omega_{BM} = \omega_{BAE} - \omega_{TM}$.
  - **mode structure ($\Delta C_{BM}$) mainly located inside 2/1 magnetic island.**

# Discussion and summary

- **Periodic oscillation of 2/1 magnetic island width**
  - **generated by high-frequency oscillation of ΔC of BM component.**
  - **small period ($T \sim 25\mu s$).**
  - **evolves in synchronization with the BM component.**
  - **ΔW is dependent on the strength of the BM component.**
  - **a larger ΔW can cause the stochasticity of magnetic field.**

# Thank you !