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Effects of Zonal Fields on Energetic-Particle Excitations of Reversed Shear Alfvén Eigenmodes : Simulation and Theory†

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(I) Introduction

(II) GTC Simulations

(III) Theoretical Analyses

◎ Beat-driven zonal fields

◎ EP instability drive and phase-space zonal structures

(IV) Summary & Discussions

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(I) Introduction

⊙ EP ⇒ RSAE (AE's) ⇒ Zonal e.m. fields (ZFs) and Zonal phase-space structures (PSZS) ⇒ suppress RSAE ⇒ NL saturation

Well established (mainly via GK simulations)

?? How do ZFs/PSZS suppress RSAE?

No definitive/quantative analysis

⊙ Two possible routes:

(i) Via thermal plasmas ⇒ modification in mode structures, NL frequency shift, etc. ⇒ enhanced damping? ⇒ $\gamma_{NL} = \gamma_{NL}(ZFs)$??

* (ii) via EPs ⇒ ZFs shearing ⇒ reduces EP drive

⊙ Focus of the work: Investige route (ii) via

Combining simulations + analytical theory

{ Computer experiments to gain/improve analytical theory/physics insights }

⊙ Highlights of results (Simulation + Theory):

⇒ ZFs do NOT suppress EP's drive of RSAE !!

⇒ ZFs enhances EP's instability drive via the destabliting ZFs-induced PSZS.

⇒ ZFs suppress/saturate RSAE via NL mechanisms in thermal plasmas

: How ?? under investigation

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(II) GTC Simulations

- $$\boxed{f = \left(\frac{q}{m}\right)\frac{\partial f_0}{\partial \varepsilon}\left(1 - \underbrace{e^{-\underset{\sim}{\rho}\cdot\underset{\sim}{\nabla}} J_0}_{\text{polarization}}\right)\delta\phi + e^{-\underset{\sim}{\rho}\cdot\underset{\sim}{\nabla}} f_g}$$

- f_g : gyro-center distribution function

$$\boxed{\left(\mathcal{L}_{g0} + \underbrace{\delta\mathcal{L}_x + \delta\mathcal{L}_\varepsilon}_{\delta\mathcal{L}}\right) f_g(\varepsilon, \mu, \underset{\sim}{x}, t) = 0}$$ NLGKE

- $$\boxed{\mathcal{L}_{g0} = \partial_t + v_{\parallel}\, \underset{\sim}{b_0}\cdot\underset{\sim}{\nabla} + \underset{\sim}{v_d}\cdot\underset{\sim}{\nabla}}$$

- $\underset{\sim}{v_d}$: ∇B_0 + Curvature drift

- $$\boxed{\delta\mathcal{L}_x = \langle \delta \underset{\sim}{u_g} \rangle \cdot \underset{\sim}{\nabla}}$$

- $$\langle \delta \underset{\sim}{u_g} \rangle = \frac{c}{B_0}\, \underset{\sim}{b_0} \times \left\langle \left(\delta\phi - \frac{v_{\parallel}\delta A_{\parallel}}{c}\right)_g \right\rangle = \langle \delta \underset{-}{U_E} \rangle + v_{\parallel} \langle \delta \underset{\sim}{B_\perp} \rangle / B_0$$

- $$\boxed{\delta\mathcal{L}_\varepsilon = \delta\dot{\varepsilon}\,\frac{\partial}{\partial\varepsilon}}$$

- $$\delta\dot{\varepsilon} = \left(\frac{q}{m}\right)\left[v_{\parallel}\left(\underset{\sim}{b_0} + \frac{\langle \delta \underset{\sim}{B_\perp} \rangle}{B_0}\right)\cdot \langle \delta \underset{\sim}{E} \rangle + \underset{\sim}{v_d}\cdot \langle \delta \underset{\sim}{E} \rangle \right]$$

⊙ Keeping only a single-n_0 RSAE + the zonal components of fluctuations

$$\Rightarrow \quad \delta\mathcal{L}_x + \delta\mathcal{L}_z \equiv \boxed{\delta\mathcal{L} = \delta\mathcal{L}_0 + \delta\mathcal{L}_z}$$

$$\Rightarrow \quad \boxed{f_g = F_{g0} + \delta F_g}$$

$$\Rightarrow \quad \boxed{\left[\mathcal{L}_{g0} + \delta\mathcal{L}_0 + \delta\mathcal{L}_z\right]\delta F_g = -\left[\delta\mathcal{L}_0 + \delta\mathcal{L}_z\right] F_{g0}} \qquad (*)$$

⊙ Three cases of study

- **Case A: No ZFs** $\Rightarrow \boxed{\delta\mathcal{L}_z = 0}$ in (*)

$$\boxed{\left[\mathcal{L}_{g0} + \delta\mathcal{L}_0\right]\delta F_g = -\delta\mathcal{L}_0 F_{g0}}$$

- **Case B: Full ZFs** $\Rightarrow \boxed{\text{Eq. } (*)}$

- **Case C**: Partial ZFs $\Rightarrow \delta\mathcal{L}_z = 0$ in the g.c. Propagator (LHS of (*))

$$\boxed{\left[\mathcal{L}_{g0} + \delta\mathcal{L}_0\right]\delta F_g = -\left[\delta\mathcal{L}_0 + \delta\mathcal{L}_z\right] F_{g0}}$$

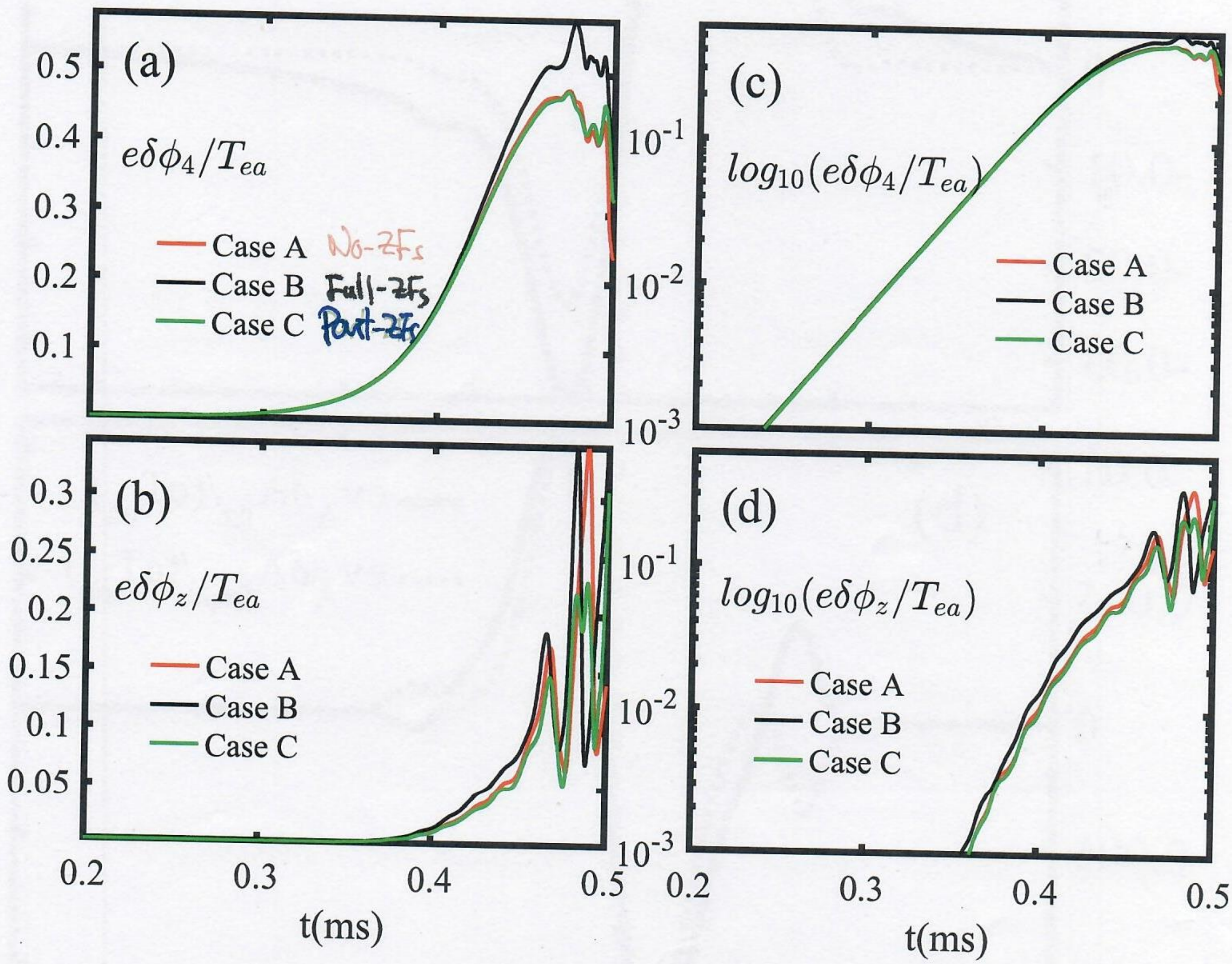
suppress shearing

(II) GTC Simulations of RSAE : ($\delta\phi_4$, $\delta\phi_z$) p.6 4h

Case A : No ZFs

Case B : Full ZFs

Case C : Partial ZFs (No zonal drift/shearing)



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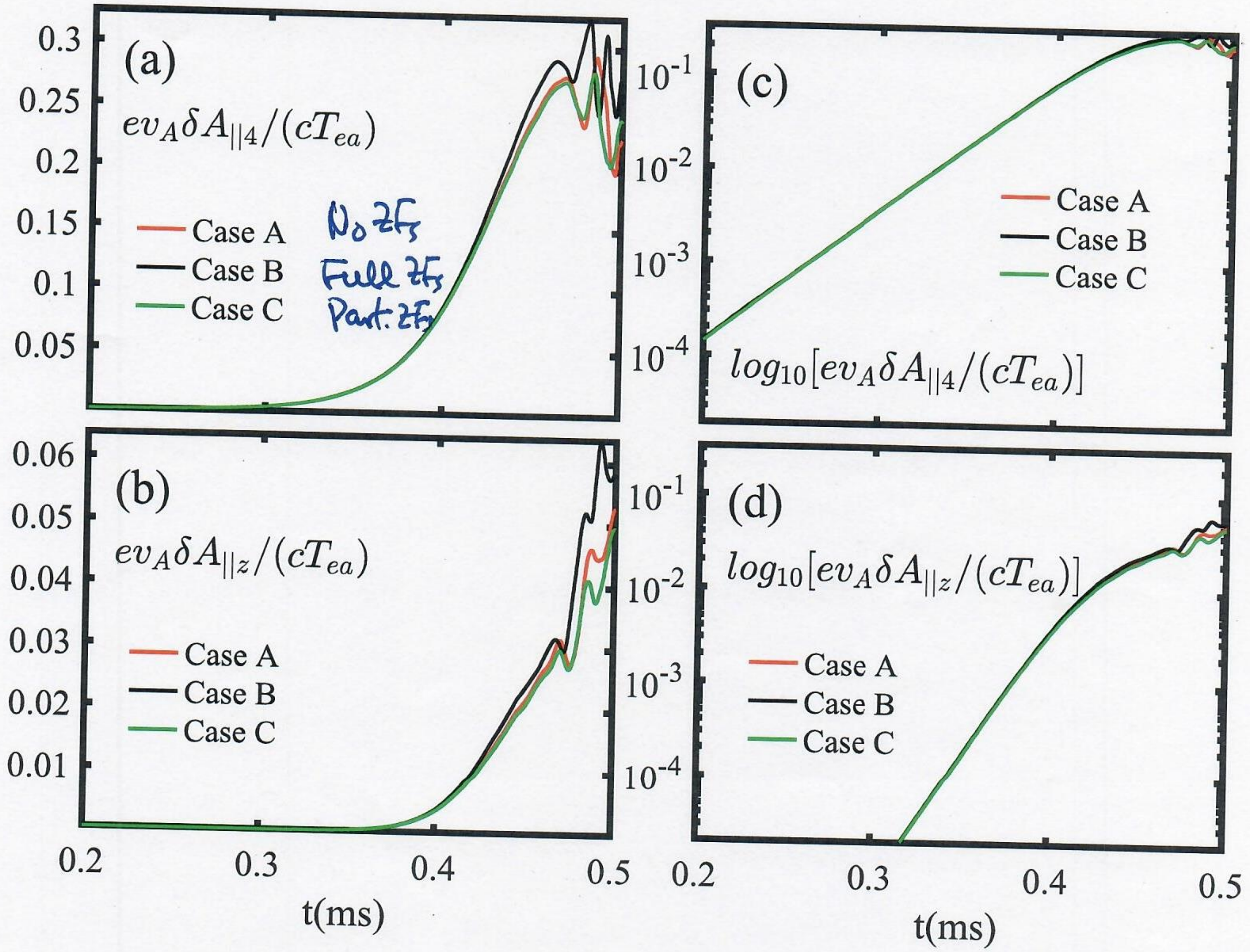
GTC Simulations of RSAE: ($\delta A_{\parallel 4}$, $\delta A_{\parallel z}$)

Case A: No-ZFs

Case B: Full-ZFs

Case C: Partial-ZFs



(III) Theoretical Analyses

(III.a) Beat-driven ZFs by RSAE

⊙ Governing eqns

- NLGKE ($\partial F_0 / \partial \mu = 0$)

⊙ $$\boxed{f = F_0 + \left(\frac{q}{m}\right)\frac{\partial F_0}{\partial \varepsilon}\delta\phi + e^{-\rho\cdot\nabla}\,\delta g}$$

⊙ $$\boxed{\begin{aligned}\mathcal{L}_g\,\delta g = (\mathcal{L}_{g0} + \delta\mathcal{L}_x)\,\delta g &= -\frac{q}{m}\frac{\partial F_0}{\partial \varepsilon}\frac{\partial}{\partial t}\left\langle \left(\delta\phi - \frac{v_\parallel \delta A_\parallel}{c}\right)\right\rangle_\alpha \\ - \delta\mathcal{L}_x F_0 &\equiv -\frac{q}{m} QF_0 \left\langle \delta\phi - \frac{v_\parallel\delta A_\parallel}{c}\right\rangle_\alpha\end{aligned}}$$

- Quasi-neutrality condition

$$\boxed{\frac{N_0 e^2}{T_e}(1+\tau)\delta\phi = \sum_{j=e,i} e_j \langle J_0 \delta g_j\rangle_v}$$

- Parallel Ampere's Law

$$\boxed{\nabla_\perp^2 \delta A_\parallel = \frac{4\pi}{c}\delta J_\parallel = \frac{4\pi}{c}\sum_j e_j \langle J_0 v_\parallel \delta g_j\rangle_v}$$

⊙ Calculate

NLGKE $\Rightarrow$

- $\delta g_{z,j} = \delta g_{z,j}^{(1)} + \delta g_{z,j}^{(2)}$
- $\delta g_{z,j}^{(1)} = \delta g_{z,j}^{(1)}[\delta\phi_z, \delta A_{\parallel z}]$
- $\delta g_{z,j}^{(2)} = \delta g_{z,j}^{(2)}[\delta\phi_0, \delta A_{\parallel 0}]$

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- $$\boxed{\begin{pmatrix} \delta\phi_0 \\ \delta A_{110} \end{pmatrix} = e^{-i\omega_{or}t + in_0\xi} \sum_m \begin{pmatrix} \Phi_m(r,t) \\ A_m(r,t) \end{pmatrix} e^{-im\theta} + c.c.}$$
    - RSAE fluctuations
    - $\underline{\delta E_{110} \simeq 0} \Rightarrow \dfrac{\omega_0 \delta A_{110}}{c\, k_{110}} \simeq \delta\phi_0$

- <u>Beat-driven ZFs</u>
    - $$\boxed{\delta\phi_z = \frac{c}{B_0\omega_{or}^2}(1 + c_0\eta_i)\frac{\partial}{\partial r}\left[\sum_m \left(\frac{n_0 q}{r}\right)\omega_{*in}|\Phi_m|^2\right]}$$
        - $\underline{c_0 \simeq 1}$ for $|k_z \rho_{bi}|^2 \ll 1$
    - $$\boxed{\frac{\delta A_z}{c} = \frac{c}{B_0\omega_{or}^2}\frac{\partial}{\partial r}\left[\sum_m \left(\frac{n_0 q}{r}\right)\frac{(n_0 q - m)}{qR}|\Phi_m|^2\right]}$$
    - <u>Valid</u> for high-frequency AE's : <u>TAE</u>, etc.

Zonal fields ($\delta\phi_z$, $\delta A_{\|z}$) beat-driven by RSAE

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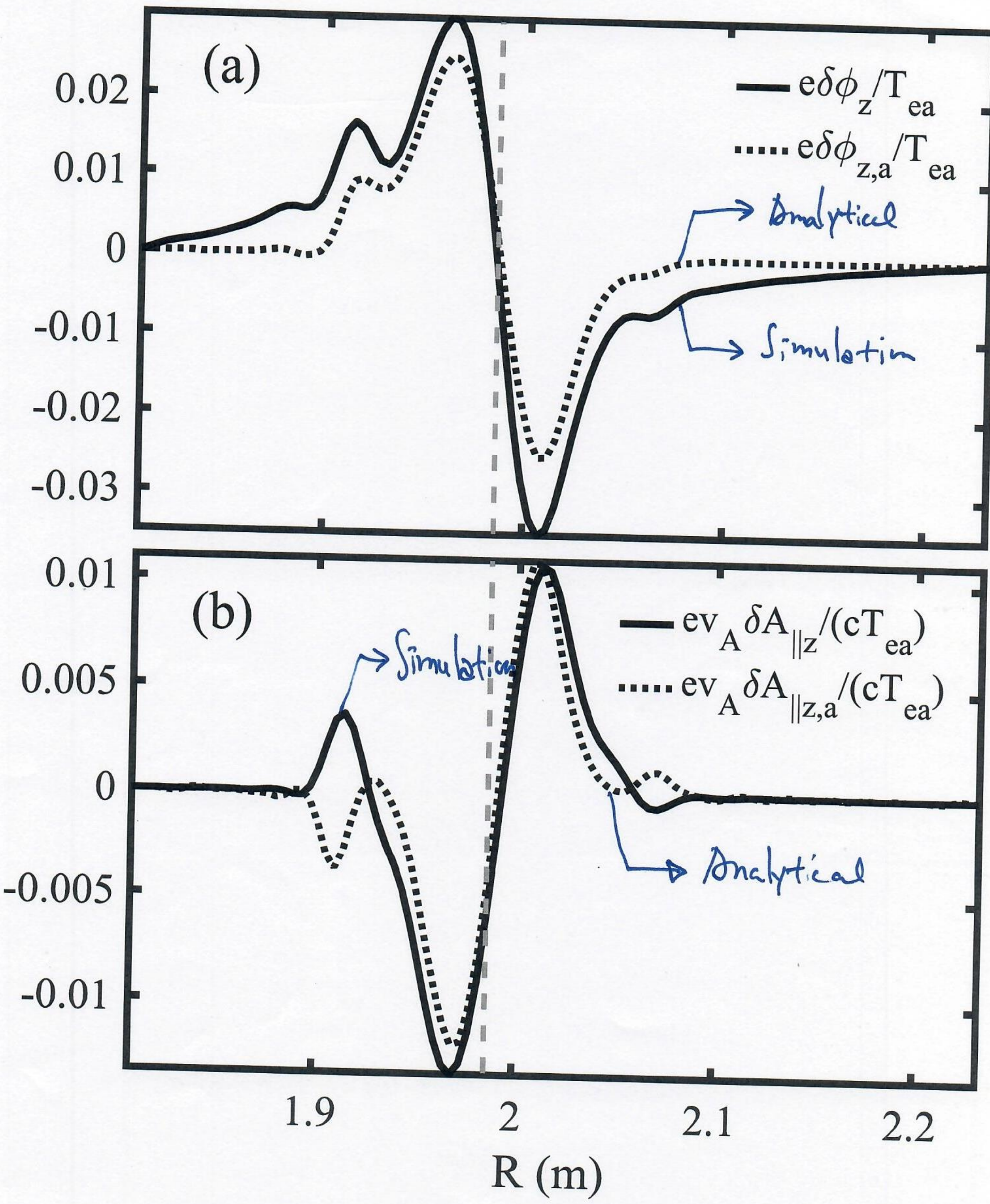


(III.b) AE stability & phase-space zonal structure

⊙ General fishbone-like dispersion relation (GFLDR)

⇒ EP contribution to the RSAE (AE) drive

$$\boxed{\mathrm{Im}\,\delta W_{k0} = e_E\, \mathrm{Im} \int d^3\underset{\sim}{x}\; \delta\phi_0^* \left\langle (J_0 \omega_z + \omega_d J_0)\,\delta K_0 \right\rangle_v}$$

- $\omega_z = -i \langle \delta \underset{\sim}{u}_g \rangle_z \cdot \underset{\sim}{\nabla}_\perp$, • $\omega_d = -i \underset{\sim}{v}_d \cdot \underset{\sim}{\nabla}_\perp$

∘ $$\boxed{(\mathcal{L}_{g0} + i\omega_z)\,\delta K_0 = i\left(\frac{e}{m}\right) J_0 \frac{(\omega_d + \omega_z)}{\omega_{0r}} \delta\phi_0\, Q\,(F_0 + \delta g_z)}$$

- $QF_0 = \left(i \frac{\partial F_0}{\partial \mathcal{E}} \frac{\partial}{\partial t} + \hat{\omega}_* F_0\right)$

- δK_0 : compressional component of δf_0 containing W-P resonance [C&H, JGR, 1991]

⇒ $\boxed{\mathrm{Im}\;\delta W_{k0} > 0 \Rightarrow \text{instability drive}}$

∘ $|\omega_{0r}| \ll |\omega_{*iE}|$

⇒ $Q \simeq \hat{\omega}_* = (\underset{\sim}{k} \times \underset{\sim}{b}_0 / \Omega) \cdot \hat{r}\, \partial/\partial r$

⇒ $QF_0 \big|_{r_m} > 0$ for $(\partial F_0/\partial r)_{r_m} < 0$: linear instability drive ($|\delta\phi_0|$ peaks at $q(r_m) = q_{min}$)

?? $(\partial \delta g_z / \partial r)_{r_m}$??

◎ Connection between simulation δF_g and analytical δg :

analytical • $$\boxed{f = F_0 + \left(\frac{e}{m}\right)\frac{\partial F_0}{\partial \mathcal{E}} + e^{-\underset{\sim}{\rho}\cdot\underset{\sim}{\nabla}}\delta g}$$

simulation • $$f = \left(\frac{e}{m}\right)\frac{\partial F_0}{\partial \mathcal{E}}\left(1 - e^{-\underset{\sim}{\rho}\cdot\underset{\sim}{\nabla}} J_0\right)\delta\phi + e^{-\underset{\sim}{\rho}\cdot\underset{\sim}{\nabla}} f_g$$

• $f_g = F_{g0} + \delta F_g$

$$\Rightarrow \boxed{\delta g = \delta F_g - \left(\frac{e}{m}\right)\frac{\partial F_{g0}}{\partial \mathcal{E}} J_0 \delta\phi}$$

(i) Case A : No ZFs in EP ($\delta\phi_z = 0 = \delta A_{\parallel z}$)

◎ $$\mathrm{Im}\,\delta W_{K0A} = \left(\frac{e^2}{m}\right)_E \left(\frac{\pi}{\omega_{0r}}\right)\int d^3\underset{\sim}{x}\, \langle J_0^2 \partial_{E0}^2 |\delta\phi_0|^2 \bar{\omega}_d^2 \times \delta(\bar{\omega}_d - \omega_0)\, Q(F_0 + \delta g_{zA})_E \rangle_v$$

• Assuming trapped EPs

◎ $$\boxed{\mathcal{L}_{g0}\delta g_{zA} = -\left[\langle \delta \underset{\sim}{U}_g \rangle_0 \cdot \underset{\sim}{\nabla}\delta g_{0A}\right]_z}$$

• PSZS due to symmetry-breaking RSAE only ! ⇒ "clump-hole" ps structures!

- $$\boxed{\delta g_{zA} \simeq \partial_{zE}^2 g_{E0}^2 \left| \frac{c}{B_0}\frac{n_0 q}{r}\frac{\overline{\omega}_d}{\omega_{0r}} \right|^2 \frac{\partial}{\partial r}\left[\frac{|\delta\overline{\Phi}_0|^2}{(\overline{\omega}_d - \omega_{0r})^2 + \gamma_L^2} \cdot \frac{\partial F_0}{\partial r} \right]}$$

$$\Rightarrow \delta g_{zA} : \begin{pmatrix} \underline{r > r_m} \Rightarrow > 0 & \underline{\text{clump}} \\ \underline{r < r_m} \Rightarrow < 0 & \underline{\text{hole}} \end{pmatrix}$$

$$\Rightarrow \boxed{\frac{\partial}{\partial r}\delta g_{zA} > 0 \quad \text{at} \quad r_m}$$

$\Rightarrow$ $\boxed{\text{stabilizing}}$

(ii) Case B : Full ZFs in EP dynamics

- $\mathrm{Im}\,\delta W_{KOB} \simeq \left(\frac{e^2}{m}\right)_E \frac{\pi}{\omega_{0r}} \int d^3\underset{\sim}{x} \langle J_0^2 \partial_{E0}^2 |\delta\Phi_0|^2 (\overline{\omega}_d + \omega_{zE})^2 \delta(\overline{\omega}_d + \omega_{zE} - \omega_0)\, Q\,(F_0 + \delta g_{zB})_E \rangle_v$

- $\boxed{\delta g_{zB} = \delta g_{zA} + \delta g_z^{(1)}}$

- $\boxed{\delta g_z^{(1)} \simeq -\left(\frac{e}{m}\frac{\partial F_0}{\partial \varepsilon}\right) J_z \partial_{Ez}^2 \delta\phi_z}$

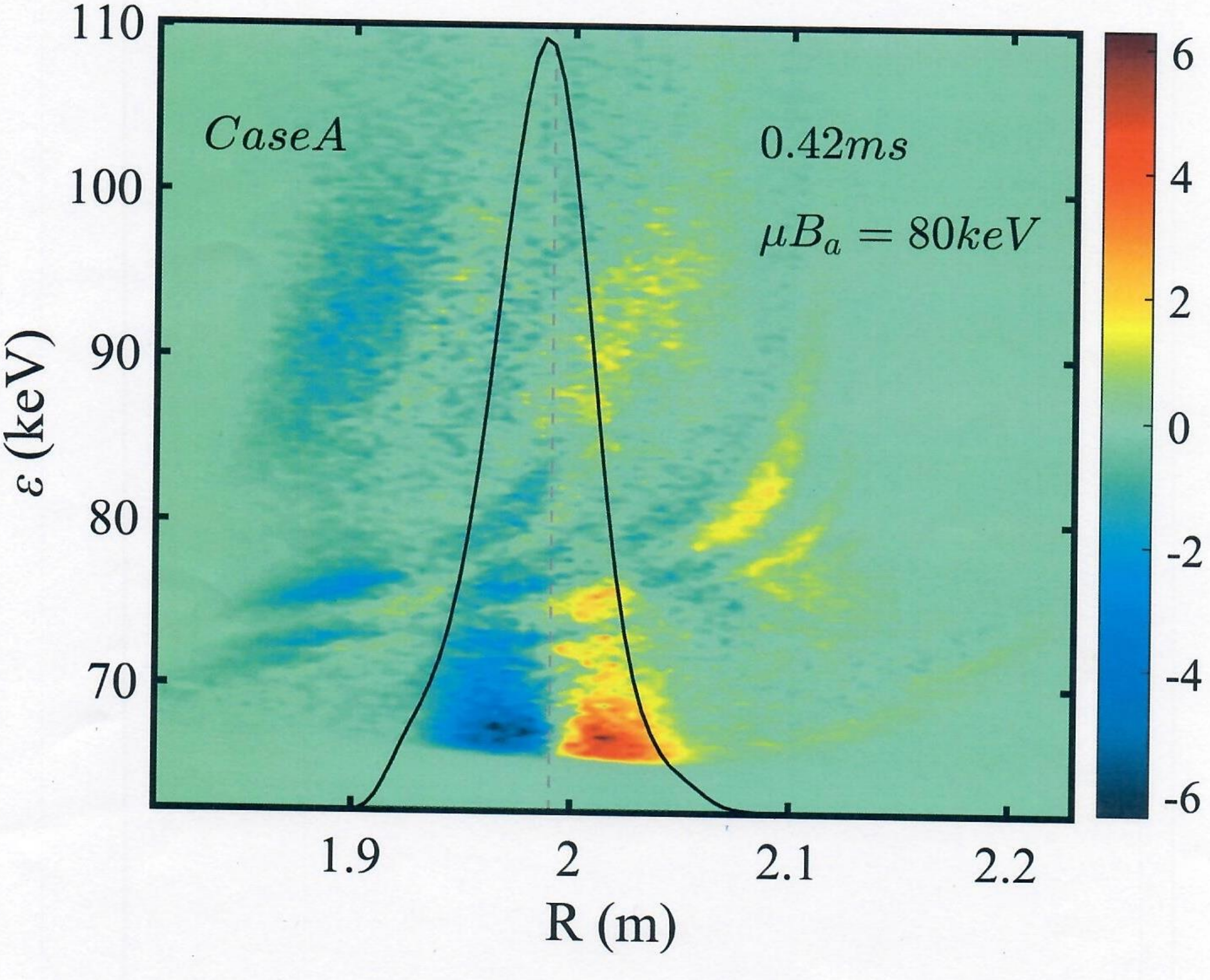
- $\omega_{zE} = \underset{\sim}{k}_0 \cdot \overline{\langle \delta \underset{\sim}{u}_{gz} \rangle} \sim O(\gamma_L) \ll |\omega_0|, |\overline{\omega}_d|$

$\Rightarrow$ Negligible shift in W-P resonance

$\delta F_z(\mu, \varepsilon, R)$

Case A : No-ZFs

hole-clump due to ($\delta\phi_x$, $\delta A_{\parallel x}$) only



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$$\Rightarrow \left(\frac{\partial}{\partial r}\delta g^{(1)}_{zE}\right)\Big|_{r_m} \doteq -\left(\frac{e}{m}\frac{\partial F_0}{\partial \varepsilon}\right)\frac{c}{B_0}\frac{(1+C_0\eta_i)}{\omega_{0r}^2} k_{\theta 0}\, \omega_{\sin}\frac{\partial^2}{\partial r^2}|\delta\phi_0|^2\Big|_{r_m} < 0 .$$

$\Rightarrow$ destablizing !

(iii) Case (c): Partial ZFs in EP dynamics

$\Rightarrow$ turning off ZF shearing/drift term

$$\odot\ \mathrm{Im}\,\delta W_{KOC} = \left(\frac{e^2}{m}\right)_E \frac{\pi}{\omega_{0r}}\int d^3\underset{\sim}{x}\, \langle J_0^2 \partial_{E0}^2 |\delta\phi_0|^2 \overline{\omega}_d^2\, \delta(\overline{\omega}_d-\omega_0)\, Q\,(F_{g0}+\delta F_{gzc})\rangle_v$$

$$\circ\ \boxed{\delta F_{gzc} = \left(\frac{e}{m}\right)\frac{\partial F_0}{\partial \varepsilon} J_0\delta\phi_z + \delta g_{zB} = \underbrace{\delta g_{zA} + \{\delta g_z^{(1)}}_{\delta g_{zB}} + \left(\frac{e}{m}\right)\frac{\partial F_0}{\partial \varepsilon}J_0\delta\phi_z\}}$$

$\Downarrow$ $\frac{\partial}{\partial r} > 0$

$$\circ\ \{\cdots\} = \left(\frac{e}{m}\right)_E \frac{\partial F_0}{\partial \varepsilon} J_z(1-\partial_{E0}^2)\delta\phi_z ,$$

$$\Rightarrow |1-\partial_{E0}^2| \ll 1,\quad |k_z\rho_{bE}| \ll 1$$

$\Rightarrow \frac{\partial}{\partial r}\{\cdots\} < 0 \Rightarrow$ weakly stablizing w.r.t. Case (A)

(IV) Summary & Discussions

⊙ NL gyrokinetic simulation & theory

⇒ ZFs beat driven by RSAE in good agreement.

⊙ ZFs ⇒ EP phase-space zonal structures ⇒ enhance the EP instability drive !!

⊙ ZFs suppress RSAE via nonlinear physics of thermal plasmas

⇒ How ?? Under investigation.

⊙ What about TAE ??