On Nonlinear Geodesic Acoustic Modes in Tokamak Plasmas

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Abstract – It is shown that, in tokamak plasmas, finite drift/banana-orbit width (FOW) effects play crucial roles in the nonlinear evolution of Kinetic/ Geodesic Acoustic Modes (KGAM/GAM). In particular, it is found that, in contrast to the negligible second-harmonic generation, KGAM/GAM can generate appreciable zero-frequency zonal flow (ZFZF) due to the FOW effects. On the other hand, ZFZF is found to have negligible effects on the dynamics of GAM/KGAM. This route of generating ZFZF has important implications to the nonlinear dynamics of zonal flows and, consequently, DW turbulences.

Zonal flows (ZF) or more generally zonal structures (ZS) are toroidally and poloidally symmetric radial corrugations. ZF can be excited by drift wave turbulences [1] including drift Alfvén waves, and in turn, suppress DW turbulence by scattering DWs into stable short radial wavelength domain [2–4]. Thus, ZF are generally believed to play important roles in the nonlinear dynamics of DW turbulences [5].

There are two types of zonal flows, i.e., zero frequency zonal flow (ZFZF) [6, 7] and its finite frequency counterpart, Geodesic Acoustic Mode (GAM) [8]. It is believed that, ZFZF is more effective in suppressing DW than GAM via turbulence shearing [9]. Understanding the excitation mechanisms for both GAM and ZFZF is, thus, of fundamental importance to the understanding of turbulence transports. While gyrokinetic theory has shown that the parametric excitation rates of GAM and ZFZF are comparable with each other [10], the DW parametric processes have parameter-sensitive dependencies on the corresponding threshold conditions, which affect the relative importance of GAM and ZFZF in regulating DW turbulence. There is also the possibility of nonlinear interactions between GAM and ZFZF.

It has been shown via numerical simulations in Ref. 11 (Figs. 1c and 5a therein) that the nonlinear self interaction of finite amplitude GAM has negligible effect in generating second harmonic GAM [12] due to the cancellation between the usual perpendicular nonlinearity and the parallel nonlinearity, which is, typically, much smaller than the perpendicular nonlinearity and is generally negligible [13]. It is shown in the same work that, contrary to the previous belief [14], ZFZF can also be generated by finite-amplitude GAM and this generation is little affected by the inclusion of parallel nonlinearity [11]. This observation indicates that ZFZF can be nonlinearly generated via GAM; even though it is below the threshold value for its own spontaneous excitation. This route of generating ZFZF obviously has important implications to the nonlinear dynamics of zonal flows and, consequently, DW turbulences since ZFZF may be more efficient than GAM in suppressing micro turbulence [15], when shearing [9] prevails over the effect of scattering [3, 4, 10, 16].

This work is, thus, motivated to investigate analytically the mechanism for ZFZF generation by GAM, and naturally, the feedback modulation of ZFZF on GAM. We found that, ZFZF can indeed be driven by finite amplitude GAM via the crucial role played by finite orbit width (FOW) effects. On the other hand, ZFZF is found to have negligible effects on the dynamics of GAM/KGAM.

The nonlinear interactions between GAM and ZFZF are investigated within the theoretical framework of modulational instability [17, 18]. In this work, we assume that both GAM and ZFZF are electrostatic with a scalar potential $\delta\phi = \delta\phi_G + \delta\phi_Z$, where

$$\delta\phi_G \quad = \quad \Phi_G(r, \sigma t)\exp(-i\omega_0 t) + c.c.,$$

and

$$\delta\phi_Z \quad = \quad \Phi_Z(r, \sigma t).$$

Here, σ is a smallness parameter indicating slow temporal variations; i.e., $|d\ln\Phi_G/dt| \sim |d\ln\Phi_Z/dt| \ll \omega_0 = \omega_G$. The nonlinear equations describing the interations between GAM and ZFZF are derived from the charge quasi-neutrality condition

$$\frac{n_0 e^2}{T_i}\left(1 + \frac{T_i}{T_e}\right)\delta\phi_k = \langle eJ_k\delta H_i\rangle_k - \langle e\delta H_e\rangle_k; \tag{1}$$

with δH_s being the nonadiabatic part of the perturbed particle distribution function of specie s, which can be solved from the nonlinear gyrokinetic equation [13]

$$\left(\partial_t + v_\parallel\partial_l + v_{dr}\partial_r\right)_k \delta H_k = -\frac{q_s}{m}J_k\partial_t\delta\phi\partial_E F_0 - \sum_{\mathbf{k}}\delta\mathbf{u}_{k'}\cdot\nabla\delta H_{k''}. \tag{2}$$

A large aspect-ratio axisymmetric tokamak with equilibrium magnetic field given by $\mathbf{B}_0 = B_0(\mathbf{e}_\xi/(1+\epsilon\cos\theta) + (\epsilon/q)\mathbf{e}_\theta)$ is considered in this work; where, ξ and θ are, respectively, toroidal and poloidal angles of the torus, $\epsilon = r/R_0 \ll 1$ is the inverse aspect ratio, r and R_0 are, respectively, the minor and major radii, and (r, θ, ξ) are straight-field-line toroidal flux coordinates. Meanwhile, $v_{dr} = (v_\perp^2/2 + v_\parallel^2)/(\Omega R_0)\sin\theta \equiv \hat{v}_{dr}\sin\theta$ is the magnetic drift velocity associated with geodesic curvature, $\delta\mathbf{u} = \mathbf{b}\times\nabla J_0\delta\phi/\Omega$ is the electric field drift velocity, $\Omega = q_s B/mc$ is the gyrofrequency, q is the tokamak safety factor, $J_k = J_0(k_\perp\rho_L)$ is the Bessel function accounting for finite Larmor radius (FLR) effects, $k_\perp$ is the perpendicular wave vector, $\rho_L = mcv_\perp/q_s B$ is the Larmor radius, and $E = (v_\parallel^2 + v_\perp^2)/2$.

Note that, in equation (2), parallel nonlinearity is not included, since it is usually much smaller than the perpendicular nonlinearity and corresponds to physics on a time scale longer than that of interest here. In fact, it can be shown a posteori that the parallel nonlinearity is indeed ignorable in the nonlinear generation of ZFZF by GAM; consistent with GTC simulations [11].

Linear Theory of ZFZF. – Since our work utilizes linear properties, such as the $m \neq 0$ components of the perturbed distribution function and potential field, we first briefly review the linear theory of ZFZF. Defining $\delta H_Z^L = e^{-\widetilde{\rho_d}\partial_r}\delta H_{dZ}^L$, where the L superscript denotes the linear response, the linear gyrokinetic equation describing nonadiabatic particle response to ZFZF can be written as

$$\left(\partial_t + v_\parallel\partial_l\right)\delta H_{dZ}^L = \frac{q}{T}F_0 J_Z e^{\widetilde{\rho_d}\partial_r}\partial_t\delta\phi_Z. \tag{3}$$

Here, $\widetilde{\rho_d}$ is the drift orbit width, defined as $\partial_l\widetilde{\rho_d} = v_{dr}/v_\parallel$, and $e^{-\widetilde{\rho_d}\partial_r}$ is the operator for drift orbit center transformation. Separating the $m \neq 0$ and $m = 0$ components of δH^L_{dZ}, $\delta H^L_{dZ} = \widetilde{\delta H^L_{dZ}} + \overline{\delta H^L_{dZ}}$, where $\overline{(\cdots)}$ denotes $m = 0$ or surface-averaged component and $\widetilde{(\cdots)}$ denotes $m \neq 0$, one then has, from equation (3),

$$\begin{aligned} \partial_t\overline{\delta H^L_{dZ}} &= \frac{q}{T}F_0J_Z\overline{e^{\widetilde{\rho_d}\partial_r}\partial_t\delta\phi_Z}, \\ \text{and} \quad \left(\partial_t + v_\parallel\partial_l\right)\widetilde{\delta H^L_{dZ}} &= \frac{q}{T}F_0J_Z\left(e^{\widetilde{\rho_d}\partial_r}\partial_t\delta\phi_Z - \overline{e^{-\widetilde{\rho_d}\partial_r}\partial_t\delta\phi_Z}\right). \end{aligned}$$

For ZFZF, with $|\widetilde{\delta H^L_{dZ}}/\overline{\delta H^L_{dZ}}| \simeq |\partial_t/v_\parallel\partial_l| \ll 1$, one has $\delta H^L_{dZ} \simeq \overline{\delta H^L_{dZ}}$, and hence

$$\delta H^L_{dZ} \simeq \overline{\delta H^L_{dZ}} = \frac{q}{T}F_0J_Z\overline{e^{\widetilde{\rho_d}\partial_r}\delta\phi_Z}.$$

Thus, the nonadiabatic part of the perturbed guiding center distribution function is

$$\delta H^L_Z = e^{-\widetilde{\rho_d}\partial_r}\delta H^L_{dZ} = \frac{q}{T}F_0J_Ze^{-\widetilde{\rho_d}\partial_r}\overline{e^{\widetilde{\rho_d}\partial_r}\delta\phi_Z}, \tag{4}$$

with the $m \neq 0$ component $|\widetilde{\delta H^L_Z}/\overline{\delta H^L_Z}| \sim |k_r\rho_d| \ll 1$. Thus, the $m \neq 0$ component of electron response to ZFZF is negligible due to small electron drift orbits. We note that, from linear theory of GAM, one has $|\widetilde{\delta H^L_G}/\overline{\delta H^L_G}| \sim |k_r\rho_L| \ll |\widetilde{\delta H^L_Z}/\overline{\delta H^L_Z}|$ due to $q \gg 1$ in the tokamak edge region of interest here. This property is used in determining the ordering of nonlinear response to GAM. The dispersion relation of ZFZF, can then be derived from surface-averaged quasi-neutrality condition [7], while the $m \neq 0$ component of quasi-neutrality condition yields

$$\widetilde{\delta\phi_Z} = -\left(1 + \frac{T_i}{T_e}\right)\left\langle F_0J_Z^2\widetilde{\rho_d}\partial_r\right\rangle\overline{\delta\phi_Z} \propto \cos\theta. \tag{5}$$

We note again that $|\widetilde{\delta\phi_Z}/\overline{\delta\phi_Z}| \propto |k_r\widetilde{\rho_d}|$ is also larger than the counterpart of GAM, $\widetilde{\delta\phi_G}/\overline{\delta\phi_G}| \propto |k_r\rho_L|$ [19].

ZFZF generation by GAM. – We first derive the nonlinear equation describing ZFZF generation by GAM. Again, defining $\delta H^{NL}_Z = e^{-\widetilde{\rho_d}\partial_r}\delta H^{NL}_{dZ}$, the nonlinear gyrokinetic equation for ZFZF can be written as

$$\left(\partial_t + v_\parallel\partial_l\right)\delta H^{NL}_{dZ} = -e^{\widetilde{\rho_d}\partial_r}\sum_{\mathbf{k}}\delta\mathbf{u}\cdot\nabla\delta H.$$

For ZFZF, with $\omega_Z \sim \omega_{NL} \ll |v_\parallel\partial_l|$, one has $\delta H^{NL}_{dZ} = \overline{\delta H^{NL}_{dZ}} + \widetilde{\delta H^{NL}_{dZ}} \simeq \overline{\delta H^{NL}_{dZ}}$. Therefore,

$$\begin{aligned} \partial_t\overline{\delta H^{NL}_{dZ}} &= -\overline{e^{\widetilde{\rho_d}\partial_r}\sum_{\mathbf{k}}\delta\mathbf{u}\cdot\nabla\delta H} \\ &= -\frac{q}{T}F_0J_G\overline{e^{\widetilde{\rho_d}\partial_r}\sum_{\mathbf{k}}\delta\mathbf{u}_G\cdot\nabla\left(\delta\phi_G + \frac{\omega_d}{\omega_G}\delta\phi_G - i\frac{v_\parallel\partial_l}{\omega_G}\delta\phi_G\right)}. \end{aligned} \tag{6}$$

In deriving equation (6), we have noted [19]

$$\delta H^L_G \simeq \frac{e}{T_i}F_0J_G\left(\delta\phi_G + \frac{\omega_d}{/\omega_G}\delta\phi_G - iv_\parallel\frac{\partial}{\partial l}\widetilde{\delta\phi_G}/\omega_G + \cdots\right).$$

The first term of equation (6) vanishes since $\delta\mathbf{u}_G = c\mathbf{b}\times\nabla\delta\phi_G/B$. Noting that $\delta\mathbf{u}_G\cdot\nabla = \delta u_{G,\theta}(1/r)\partial_\theta + \delta u_{G,r}\partial_r$, with $\delta u_{G,\theta} = -(c/B)\overline{\delta E_{G,r}}$ and $\delta u_{G,r} = (c/B)\delta E_{G,\theta} \propto \cos\theta$, we then have

$$\partial_t\overline{\delta H^{NL}_{dZ}} = -\frac{e}{T_i}F_0J_G\overline{(1+\tilde\rho_d\partial_r)\left(-\frac{c}{B_0}\overline{\delta E_{G,r}}\frac{1}{r}\partial_\theta - \frac{c}{B_0}\frac{1}{r}\partial_\theta\widetilde{\delta\phi_G}\partial_r\right)\left(\frac{\omega_d}{\omega_G}\delta\phi_G - \frac{iv_\parallel\partial_l}{\omega_G}\widetilde{\delta\phi_G}\right)} \tag{7}$$

In obtaining equation (7), we have assumed that $|\widetilde{\rho_d}\partial_r| \ll 1$. Noting that $\widetilde{\rho_d} \propto \cos\theta$, $\omega_d \propto \sin\theta$, $\widetilde{\delta\phi_G} \propto \sin\theta$ [19], equation (7) can be greatly simplified to

$$\begin{aligned}\partial_t\overline{\delta H^{NL}_{dZ}} &= -\frac{e}{T_i}F_0 J_G \overline{\widetilde{\rho_d}\partial_r\left(-\frac{c}{B_0}\overline{\delta E^*_{G,r}}\frac{1}{r}\partial_\theta\right)\left(\frac{\omega_d}{\omega_G}\overline{\delta\phi_G}\right)} \times (1+O(q^{-2})) + c.c. \\ &= i\frac{e}{T_i}F_0 J_G\frac{c}{B_0}\hat{v}_{dr}\overline{\widetilde{\rho_d}\cos\theta}\left(\frac{1}{\omega_G}-\frac{1}{\omega^*_G}\right)\frac{\partial}{\partial r}\left(\frac{|\overline{\delta E_{G,r}}|^2}{r}\right)\times\left(1+O(q^{-2})\right). \end{aligned} \tag{8}$$

Noting $\omega_G = \omega_0 + i\partial_t$, we then have, after some algebra,

$$\overline{\delta H^{NL}_{dZ}} = -\frac{e}{T_i}F_0 J_G \frac{1}{\omega_0^2}\frac{c}{B_0}\hat{v}_{dr}\overline{\widetilde{\rho_{dr}}\cos\theta}\frac{\partial}{\partial r}\left(\frac{|\delta E_{G,r}|^2}{r}\right). \tag{9}$$

The main contribution comes from the second term of equation (6). The first term vanishes due to anti-symmetry in θ. The third term, meanwhile, is of order $O(q^{-2})$ smaller than the second term. It is worthy mentioning that, from equation (8), the dominant contribution comes from coupling due to finite drift-orbit width effect; that is, a neoclassical effect. Substituting the nonlinear particle response, equation (9), into the quasi-neutrality condition, we obtain the following nonlinear equation describing nonlinear excitation of ZFZF by a finite amplitude GAM

$$\chi_Z\overline{\delta\phi_Z} = -\frac{c}{B_0}\frac{1}{\omega_G^2}\frac{\partial}{\partial r}\left[\left\langle \hat{v}_{dr}\overline{\cos\theta\widetilde{\rho_d}}F_0\right\rangle\frac{|\overline{\delta E_{G,r}}|^2}{r}\right]; \tag{10}$$

where, χ_Z is the well-known neoclassical polarization of ZFZF [7]

$$\chi_Z\overline{\delta\phi_Z} \equiv \left(1-\left\langle\frac{F_0}{n_i}J_Z^2\left|\overline{e^{\widetilde{\rho_d}\partial_r}}\right|^2\right\rangle\right)\overline{\delta\phi_Z}.$$

Null modulation of GAM by ZFZF. – For GAM, with $|\omega_G| \gg |v_d\partial_r|, |v_\parallel\partial_l|$, the particle responses can be solved by asymptotic expansion with the smallness parameter $q^{-1} \sim v_\parallel/(qR_0\omega_G)$. Separating $\delta H^{NL}_G = \delta H^{NL}_{G,0} + \delta H^{NL}_{G,1}$, and noting $|\delta H^{NL}_{G,1}/\delta H^{NL}_{G,0}| \simeq O(q^{-1}) \ll 1$, the nonlinear gyrokinetic equation for GAM can be expanded order by order;

$$\partial_t\delta H^{NL}_{G,0} \quad = \quad -\nabla\cdot(\delta\mathbf{u}\delta H)_G\,, \tag{11}$$

and

$$\partial_t\delta H^{NL}_{G,1} \quad = \quad -v_\parallel\partial_l\delta H^{NL}_{G,0} - v_{dr}\partial_r\delta H^{NL}_{G,0}. \tag{12}$$

To the lowest order, the surface-averaged equation (11) yields

$$\frac{\partial}{\partial t}\left\langle\overline{\delta H^{NL}_{G,0}}\right\rangle = -\frac{c}{B_0}\frac{\partial}{\partial r}\overline{\left(\widetilde{\delta\phi}_Z\partial_\theta\langle\delta H_G\rangle + \widetilde{\delta\phi}_G\partial_\theta\langle\delta H_Z\rangle\right)}. \tag{13}$$

For $n=0$ zonal modes, with $\widetilde{\delta H}_e = 0$, the $m\neq 0$ component of quasi-neutrality condition yields:

$$\left\langle J_0\widetilde{\delta H}_i\right\rangle_k = \frac{e}{T_i}\left(1+\frac{T_i}{T_e}\right)\widetilde{\delta\phi} \Rightarrow \left\langle\widetilde{\delta H_i}\right\rangle \simeq \alpha\widetilde{\delta\phi}, \tag{14}$$

with $\alpha = (e/T_i + e/T_e)$. Substituting equation (14) into equation (13), we then have

$$\frac{\partial}{\partial t}\left\langle\overline{\delta H^{NL}_{G,0}}\right\rangle = 0; \tag{15}$$

i.e., at the leading order, there is no modulation of GAM by ZFZF.

To the next order, taking time derivative of equation (12), and then taking surface averaging, we get

$$\begin{aligned}\partial_t^2 \overline{\delta H^{NL}_{G,1}} &= -\overline{v_{dr}\partial_r\partial_t \delta H^{NL}_{G,0}} \\ &= \frac{\partial}{\partial r}\overline{\left(\widetilde{v_{dr}}\,\overline{\widetilde{\delta u_\theta}}\frac{\partial_\theta}{r}\widetilde{\delta H} + \widetilde{v_{dr}}\,\widetilde{\delta u_r}\frac{\partial}{\partial r}\overline{\delta H}\right)}.\end{aligned} \tag{16}$$

The first term of equation (16), is of order $k_r\rho_i(R_0/r)(\widetilde{\delta H}/\overline{\delta H})$ relative to the parallel nonlinearity, while the second term, is of order $k_r\rho_i(R_0/r)(\widetilde{\delta\phi}/\overline{\delta\phi})$. Note that, we have $|\widetilde{\delta\phi}_Z/\overline{\delta\phi}_Z|, |\widetilde{\delta H}_Z/\overline{\delta H}_Z| \sim k_r\rho_d \gg |\widetilde{\delta\phi}_G/\overline{\delta\phi}_G|, |\widetilde{\delta G}_G/\overline{\delta G}_G| \sim k_r\rho_i$. Thus, both terms in equation (16) are of order $k_r\rho_i k_r\rho_d(R_0/r)$ compared with parallel nonlinearity. For the ordering $1 \gg k_r\rho_d \gg k_r\rho_i \gg r/R_0 \gg k_r\rho_d k_r\rho_i$, both terms are smaller than the parallel nonlinearity. As a result, there is no modulation of GAM by ZFZF up to the order of the parallel nonlinearity.

Conclusion. – In conclusion, the nonlinear interactions between GAM and ZFZF is studied using gyrokinetic theory. It is found that, contrary to earlier belief [14], ZFZF can be generated by a finite-amplitude GAM, when the crucial FOW effect due to particle magnetic field gradient and curvature drifts are included. One implication is that ZFZF can be forced driven by a finite-amplitude GAM (e.g. driven by energetic particles [20, 21]), even if it is below the threshold condition for its own spontaneous excitation by drift wave turbulence [10]. On the other hand, we show that, ZFZF has a negligible effect on the dynamics of GAM; and that parallel nonlinearity must be taken into account if one wants to address the long time scale feedback effect on GAM of nonlinear generated ZFZF.

When extended to burning plasmas of fusion interest, the results of the present work suggest that DW turbulence, zonal flows and energetic particles will have complex interplays on long times, in the order of the transport time scale [22]. Therefore, the interesting implications of current findings to the nonlinear dynamic evolution of the coupled zonal flow and DW turbulence remain to be further studied.

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