A neoclassically optimized compact stellarator with four planar coils

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A neoclassically optimized compact stellarator with simple coils has been designed. The magnetic field of the new stellarator is generated by only four planar coils including two interlocking coils of elliptical shape and two circular poloidal field coils. The interlocking coil topology is the same as that of the Columbia Non-neutral Torus (CNT)[1]. The new configuration was obtained by minimizing the effective helical ripple [2] directly via the shape of the two interlocking coils. The optimized compact stellarator has very low effective ripple in the plasma core implying excellent neoclassical confinement. This is confirmed by the results of the drift-kinetic code SFINCS[3] showing that the particle diffusion coefficient of the new configuration is one order of magnitude lower than CNTs.

I. INTRODUCTION

Stellarator is one of main approaches to magnetic fusion energy. Compared to the main stream tokamaks, stellarators have advantages of naturally stead state operation without disruptions. The magnetic field of stellarators are mainly provided by external coils. Therefore the physics properties of stellarators can be largely controlled by external coils and can thus be optimized by varying coil geometry. However 3D stellarator coils usually have complex 3D geometry and they are difficult and costly to build. It is important to explore the possibility of optimized stellarators with simple coils.

In this work a neoclassically optimized compact stellarator with only four simple coils has been designed. The new stellarator is of CNT type with two InterLocking (IL) coils and two poloidal field coils. The new configuration is obtained by direct optimization from the shape of the two interlocking coils. This direct method is different from the conventional two-stage optimization where the first stage is optimization of physics properties of stellarators from the shape of the last closed flux surface. The second stage is design of 3D coil set which is optimized in such a way that the shape of plasma boundary it generates closely matches the plasma boundary obtained in the first stage. The two-stage method usually works well and it was successful in design of advanced stellarators such as HSX[4], W7-X[5], and NCSX[6] etc. However it suffers from the fact that the coils found in the second stage cannot perfectly recover the optimized plasma boundary obtained in the first stage. Thus usually some iterations between the first stage and second stage are needed in order to obtain the desired physics and engineering properties. In view of this, we adopted the direct optimization method from coils. Specifically we carry out optimization by varying the shape of stellarator coils to directly control the physics properties of vacuum magnetic field of stellarators. Our primary optimization target is the so called $1/\nu$ neoclassical transport[2] due to helical ripple. This transport is due to finite magnetic drift of trapped particles in helical wells. The neoclassical transport is a serious issue for stellarators. The $1/\nu$ scaling is very unfavorable for fusion reactors where plasma temperature is necessary high and collision frequency is very low.

The primary goal of our design is to find a candidate for a toroidal magnetic confinement device to be built at Zhejiang University. We chose compact stellarator topology of CNT type in our design for two reasons. First, CNT is arguably the world simplest stellarator with only four circular coils including two interlocking coils and two poloidal field coils. Therefore it is relatively easy to build. Second, our direct optimization method is suitable for stellarator design of CNT type because the shape of only one coil needs to be considered in the optimization and thus the degree of freedom is modest. It should be pointed out that the original goal of CNT was non-neutral plasma experiment and thus the neoclassical transport due to helical ripple was not emphasized. In contrast, our design goal is experimental study of fully ionized neutral plasmas. Therefore the neoclassical confinement is our primary focus in the configuration optimization. We will show that excellent neoclassical confinement in the plasma core can be achieved with two interlocking coils of elliptical planar shape. The optimized configuration is to be called Zhejiang university Compact Stellarator (ZCS).

The paper is organized as following. Section II describes the detailed optimization process. In section III, an optimized configuration with good neoclassical confinement is described including coil geometry and magnetic flux surfaces as well as rotational transform profile. Section IV shows the calculated $1/\nu$ neoclassical transport coefficient $\epsilon_{eff}^{3/2}$ of ZCS. In section V, the simulation results of single particle confinement and neoclassical transport are presented and discussed. In section VI, the effects of finite plasma beta on equilibrium and the effective ripple are studied. In section VII, conclusions of

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this work are given.

II. NUMERICAL METHODS

For the purpose of carrying out stellarator optimization directly from coils, a code suite has been developed for calculating vacuum magnetic field from coils, magnetic flux surfaces as well as particle motions in the magnetic field. The magnetic field is calculated from current-carrying coils straight forwardly using the Biot-Sarvant law. A line current is assumed for simplicity. The vacuum magnetic flux surfaces and corresponding rotational transform profile are calculated by following the magnetic field lines. The $1/\nu$ neoclassical coefficient is calculated by integrating along magnetic field lines[2]. The neoclassical transport is also evaluated by the drift kinetic code SFINCS[3].

III. OPTIMIZATION METHOD

Here we describe the method used for optimizing stellarators directly from coils. In this work we mainly aim to minimize the effective ripple coefficient $\epsilon_{eff}^{3/2}$ of neoclassical transport in the $1/\nu$ regime. In addition we also consider rotational transform profile in optimization targets. Therefore we aim to minimize the following combined target function.

$$\chi^2 = \sum_i w_i |f_i^{actual} - f_i^{target}|^2. \tag{1}$$

where f_i^{target} is the desired value of ith target f_i^{target} and w_i is the optimization weight of the ith target. The goal of the optimization is to minimize the total deviation of the targets from the desired target values by varying the shape of each coil. The size of each weight is chosen based on the relative importance of the corresponding target.

Assuming every coil is a closed smooth curve, we use Fourier series[7] to define the shape of each coil in Cartesian coordinate as

$$\begin{cases} x = x_{c,0} + \sum_{n=1,n_f} [x_{c,n}\cos(n\theta) + x_{s,n}\sin(n\theta)], & (2) \\ y = y_{c,0} + \sum_{n=1,n_f} [y_{c,n}\cos(n\theta) + y_{s,n}\sin(n\theta)], & (3) \\ z = z_{c,0} + \sum_{n=1,n_f} [z_{c,n}\cos(n\theta) + z_{s,n}\sin(n\theta)], & (4) \end{cases}$$

where the angle parameter θ ranges $[0, 2\pi]$ so that the coil curve is closed. From above formula we see that the shape of each coil is determined by $3 \times (2n_f + 1)$ Fourier harmonics, with n_f being the cutoff harmonic number. The coil current I is also a free parameter, so the total degree of freedom for each coil is $3 \times (2n_f + 1) + 1$. It should be noted that, almost any closed smooth curve without straight sections or sharp corners can be represented well by Fourier harmonics.

As mentioned above, we choose CNT as starting point of our optimization. The main reason is that CNT is the simplest stellarator with only four circular coils. Our approach is optimizing neoclassical confinement by varying the shapes of the two interlocking coils. Furthermore, because it is a two period stellarator, the shapes of the two interlocking coils are necessarily the same. Therefore the degree of freedom is minimized and is much smaller than that of conventional stellarators. Regarding the two other coils, their circular shapes and the distance between them can be fixed because they only provide a vertical field for the CNT-like stellarator. The current of the two vertical field coils can also be fixed because only the ratio of their current and that of the interlocking coils is important.

FIG.1 plots the coil configuration of CNT. The two IL coils can be represented by only four Fourier harmonics including $x_{c,0} = \pm 0.313, x_{c,1} = \pm 0.405, y_{s,1} = \mp 0.255, z_{s,1} = 0.315$, in which $x_{c,0}$ and $|x_{c,1}| = \sqrt{y_{s,1}^2 + z_{s,1}^2}$ represent the half distance between the two coil centers and the radius of the coils, respectively. The shapes of coils are elliptic when $|x_{c,1}| \neq \sqrt{y_{s,1}^2 + z_{s,1}^2}$. The angel between the two coil planes is controlled by $\theta = 2 * \arctan(z_{s,1}/y_{s,1})$. If other higher order Fourier harmonics $(n > 1)$ are kept, the shape of interlocking coils changes from planar coil to three dimensional coil. For poloidal field coils, Fourier harmonics $x_{s,1} = x_{c,1} = 1.08$ and $z_{c,1} = \pm 0.405$ represent the coil radius and the half distance between the center of two coils, respectively.

A combination of global optimization and Levenberg-Marquardt[8] algorithm is adopted in our optimization process. For the global optimization method, we chose an appropriate parameter range and associated n_i discrete grid points for each harmonic. Thus, the total mesh points in the multi-dimensional phase space of all Fourier harmonics is $n_i^{N_F}$ with $N_F = 3 \times (2n_f + 1) + 2$ being the total degree of freedom. Here N_F includes the current of the two IL coils and the radius of the two poloidal field coils. Considering the stellarator symmetry, the degree of freedom is reduced to $N_F = 3 \times (n_f + 1)$. Each mesh point represents one unique stellarator configuration and the corresponding combined target function is evaluated. In this way, a global minimum can be found as long as the total number of mesh points are limited and the required computational resource and time is reasonable. For the case of only n=0 and n=1 harmonics are included, the total degree of freedom is only $N_F = 6$ and the total number of mesh points is 10^6 for $n_i = 10$. Once a global minimum is found, we can then do a refined local search near the neighborhood of this global minimum using Levenberg-Marquardt algorithm. In this second phase of optimization, higher harmonics can be included

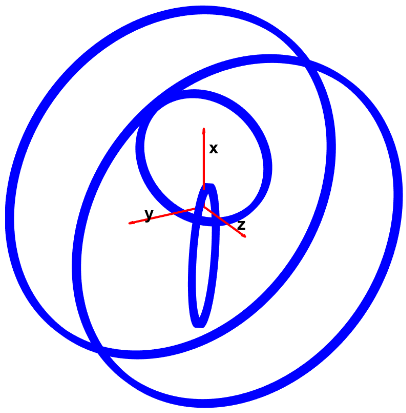


FIG. 1: CNT coils

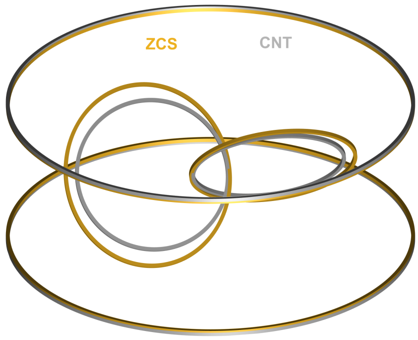
	Parameter	Value	Range
Interlocking coil	$x_{c,0}$	0.313	0.3∼0.5
	$x_{c,1}$	0.405	0.2∼0.6
	$y_{s,1}$	0.255	0.2∼0.4
	$z_{s,1}$	0.315	0.2∼0.6
Poloidal field coil	$x_{s,1} = x_{c,1}$	1.08	0.5∼1.2
	$z_{c,1}$	0.405	0.405
Current ratio	I_{IL}/I_{PF}	2.25	1∼5

TABLE I: The specific parameters variation in optimization

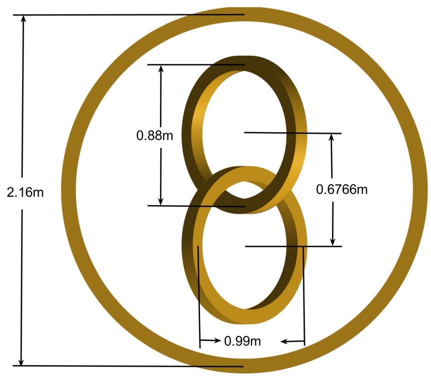
for three dimensional coils. In the actual optimization, up to $n_f = 3$ was included. It turns out that, as will be shown later, inclusion of $n > 1$ harmonics only leads to a slight improvement in the target function. Therefore in this work we focus on the globally optimized configuration with Fourier harmonics up to $n_f = 1$ and the specific parameters variation are showed in Table I. In this case the shape of IL coils is simply planar.

IV. BASIC PARAMETERS OF NEW CONFIGURATION ZCS

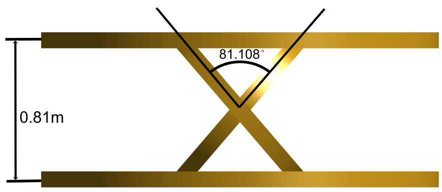
FIG.2 shows the coil system of the new configuration ZCS (orange color) obtained using global optimization with only $n = 0$ and $n = 1$ Fourier harmonics. The IL coils of CNT are also shown in FIG.2(a) for comparison. Table II lists coil parameters of ZCS and C-NT including Fourier coefficients of the interlocking coils and coil current ratio between IL coils and vertical field coils. The shape of IL coils of the new configuration is now elliptical instead of circular shape of CNT's. The long and short diameter is $0.99m$ and $0.88m$ respectively (FIG.2(b)). The angle and center distance between the two IL coils are $81.108°$ and $0.6766m$ (FIG.2(c)). The radius of the circular poloidal field (PF) coils and the center distance is $1.08m$ and $0.81m$ respectively. These two parameters are the same as CNT's. The current ratio between IL coils and PF coils is $1.6 : 1.0$. The main difference between ZCS's coils and CNT's is the shape of the interlocking coils. As a result, the neoclassical confinement of the new configuration is much better than CNT's.



(a)3D schematic



(b)Top view



(c)Side view

FIG. 2: View of the new configuration

FIG.3 plots the 3D magnetic flux surfaces relative to the two IL coils of ZCS. FIG.4 plots the cross sections of last closed flux surfaces of ZCS (solid lines) and CNT (dashed lines) respectively for three toroidal angles, $\phi = 0°, 45°$ and $90°$. We observed that the shapes of flux surfaces of the new configuration are similar to those of CNTs. A notable difference is that the last closed surface

	Parameter	CNT	ZCS
Interlocking coil	$x_{c,0}$	0.313	0.3383
	$x_{c,1}$	0.405	0.44
	$y_{s,1}$	0.255	0.322
	$z_{s,1}$	0.315	0.376
Poloidal field coil	$x_{s,1} = x_{c,1}$	1.08	1.08
	$z_{c,1}$	0.405	0.405
Current ratio	I_{IL}/I_{PF}	2.25	1.6

TABLE II: The Fourier Harmonics of ZCS in comparison with CNT

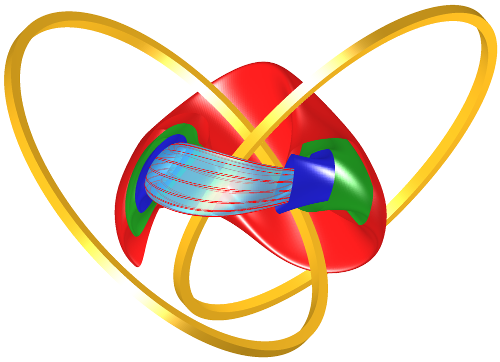


FIG. 3: The 3-D magnetic surface construction

at $\phi = \pi/2$ shifts inward considerably as compared to that of CNT.

The rotation transform profile of the new configuration is plotted in FIG.5. We observed that the new profile is close to that of CNT. This is not surprising since The rotational transform profile of CNT was chosen as a target.

V. THE NEOCLASSICAL CONFINEMENT OF ZCS

Here we show that the neoclassical confinement of the optimized compact stellarator is much better than that of CNT. The neoclassical transport is evaluated by calculating the effective ripple parameter and by using the

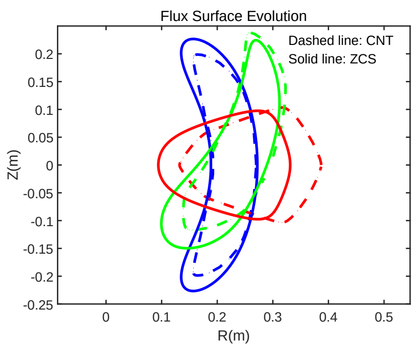


FIG. 4: Cross-sections of the boundary magnetic surface

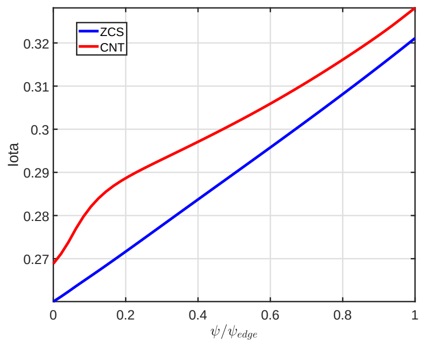


FIG. 5: Rotation transform evolution

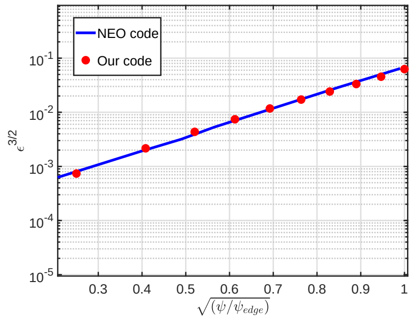


FIG. 6: $\epsilon_{eff}^{3/2}$ comparison between the NEO code and our code

drift-kinetic code SFINCS.

A. The effective ripple

For stellarators, the neoclassical transport due to helical ripple is a serious problem since this transport scales as $1/\nu$ for small collision frequency ν. This scaling is very unfavorable for fusion reactors where plasma temperatures are necessary high and collision frequencies are small. Thus this neoclassical transport must be minimized to achieve high plasma confinement. This $1/\nu$ transport is proportional to the effective helical ripple parameter $\epsilon_{eff}^{3/2}$. Thus this ripple parameter is chosen as the main target for our optimization. It was shown that $\epsilon_{eff}^{3/2}$ is only a function of magnetic field geometry and can be calculated straight forwardly by integrating along a magnetic field line[2]. A module has been developed in our code suite for calculating $\epsilon_{eff}^{3/2}$ and has been benchmarked against the NEO code[9]. FIG.6 compares the calculated effective ripple of a stellarator using our module with that of the NEO code. The agreement between the results of two codes is excellent.

FIG.7 shows the calculated $\epsilon_{eff}^{3/2}$ of ZCS and CNT as functions of the normalized radial variable $\sqrt{\psi/\psi_{edge}}$, in which ψ_{edge} is the boundary poloidal flux. We observe that the effective ripple of the optimized configu-

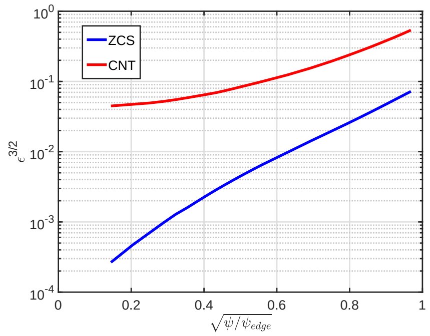


FIG. 7: Parameter $\epsilon_{eff}^{3/2}$ for ZCS and CNT

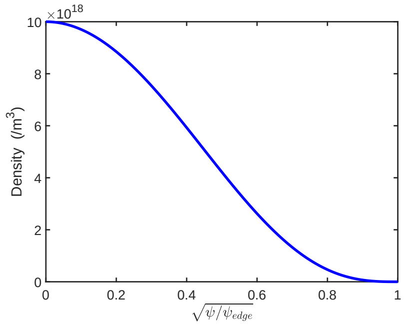
ration is much smaller than that of CNT especially in the core where it is two order of magnitude smaller. It is remarkable that this huge improvement in neoclassical confinement is achieved by simple planar coils of elliptical shape.

B. Evaluation of neoclassical transport using SFINCS

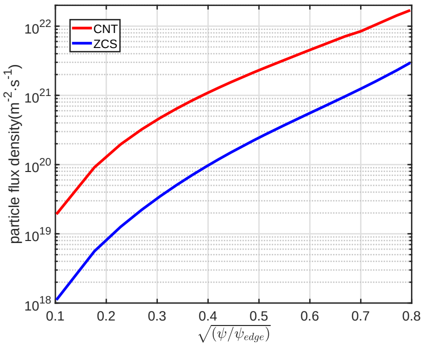
SFINCS is a kinetic code for calculation of neoclassical transport in stellarators by solving the steady-state drift-kinetic equation for multiple species. We use SFINCS code to calculate neoclassical particle fluxes of both electrons and ions with effects of ambipolar radial electric field. The density profiles of ions and electrons considered are shown in FIG.8(a). The temperature profiles are chosen to be uniform at $T_e = 2T_i = 200eV$ for simplicity. FIG.8(b) shows the electron particle fluxes of both CNT and ZCS. The results indicate that the neoclassical transport of ZCS is much lower than that of CNT especially in the plasma core where the particle flux of ZCS is one order of magnitude lower. Based on these findings, we conclude that the new optimized configuration has very good neoclassical confinement.

C. quasi-symmetry and quasi-omnigeneity

We now consider the degree of quasi-symmetry and quasi-omnigeneity to understand the reason of good neoclassical confinement of the optimized configuration ZCS. Quasi-symmetry is an effective concept for improving neoclassical transport in stellarators. Boozer showed that the particle drift orbits in stellarator are equivalent to those of axi-symmetric tokamaks if the magnetic field strength is axis-symmetric in Boozer coordinates, even though the structure of magnetic field is three dimensional[10]. The magnetic field distribution on a flux surface can be expressed by $B = \sum_{m,n} B_{m,n} \cos(m\theta - n\zeta)$, where θ and ζ are boozer coordinates. For quasi helical-symmetry configuration, such as Helical Symmetric Experiment(HSX)[4], the dominant Fourier components $B_{m,n}$ have a single helicity and other components are very small. For quasi axi-symmetry configurations, the magnetic field spectrum is nearly axi-symmetric with all the non-axi-symmetry components being very small. Quasi-axisymmetry has been used to design compact stellarators with excellent neoclassical confinement. Examples of quasi-axisymmetric stellarators include the National Compact Stellarator Experiment (NCSX)[6] etc. Another approach of optimizing neoclassical confinement is quasi-omnigeneity. This approach was used to design Wendelstein 7-X (W7-X)[5] by minimizing the averaged particle drift.



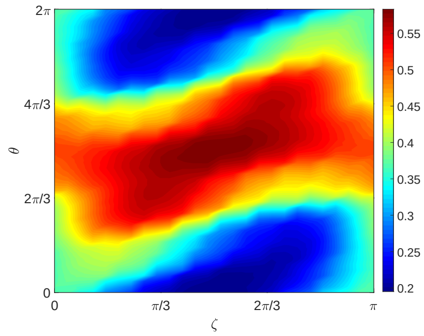
(a)Initial density profile in SFINCS



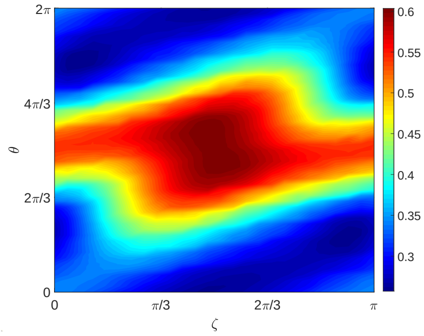
(b)Electron transport flux density

FIG. 8: SFINCS results for initial density profiles

FIG.9 plots the distribution of magnetic field strength on the last closed flux surface for both CNT (a) and the optimized configuration (b). We observe that the new configuration is closer to quasi-axisymmetry than CNT. This is confirmed by Fourier spectrum of magnetic field strength shown in FIG.10(a) for CNT and Fig.10(b) for the optimized configuration. Furthermore we evaluate quasi-omnigeneity by looking at the minimums of magnetic field strength along a field line. It is known that the degree of quasi-omnigeneity can be largely measured by how close the minimums of magnetic field strength being a constant. In Fig.11, we choose 12 local minimums at $B<0.35T$ and calculate the standard deviations of these local minimums. The result shows that the standard deviation $\delta_{ZCS} = 0.05$ of ZCS is about 50% of

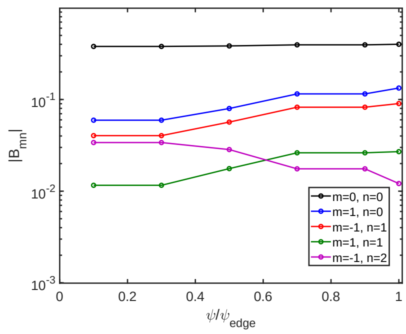


(a)CNT

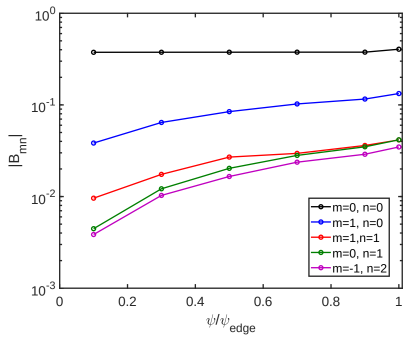


(b)ZCS

FIG. 9: $|B|$ on the boundary flux surface



(a)CNT



(b)ZCS

FIG. 10: Dominant $|B_{mn}|$ modes

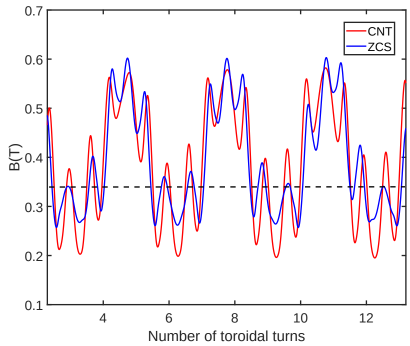


FIG. 11: Magnetic field strength distribution on field line

CNT's ($\delta_{CNT} = 0.10$). This indicates that ZCS is much closer to quasi-omnigeneity than CNT.

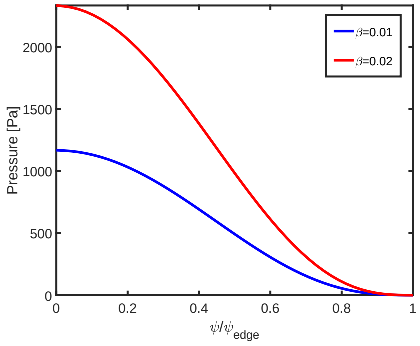
VI. THE FINITE BETA EFFECTS ON THE EFFECTIVE RIPPLE

So far we have only consider stellarators with vacuum magnetic field and effects of finite plasma pressure have been neglected. Here we consider the effects of finite plasma beta on the helical ripple. The equilibria of ZCS at finite pressures are calculated using the VMEC code. The fixed boundary condition is used for simplicity. The bootstrap current is calculated using SFINCS and is included in the finite beta equilibria. We use the NEO code to calculate the effective ripple parameter $\epsilon_{eff}^{3/2}$ at finite plasma beta.

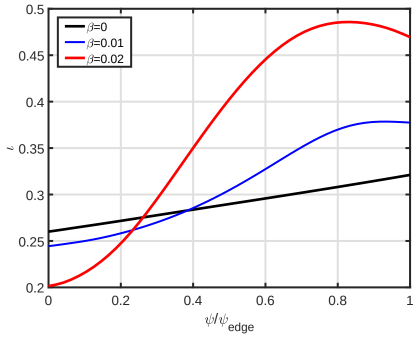
The pressure profile is chosen to be $p = p_0(1 - r^2)^3$ as shown in Fig.12(a) for two values of the volume-averaged plasma beta β, where p_0 is a constant used to control the equilibrium beta and r is the square root of the normalized poloidal flux. From Fig.12(b) we observe that, as β increases from zero to 2%, the central iota decreases slightly while the edge iota increases substantially due to bootstrap current. Fig.12(c) shows that, as β increases to 2%, the effective ripple parameter $\epsilon_{eff}^{3/2}$ changes little in the core but decreases by about a factor of two near the edge. Optimization of neoclassical confinement at finite beta will be considered in future work.

VII. CONCLUSIONS

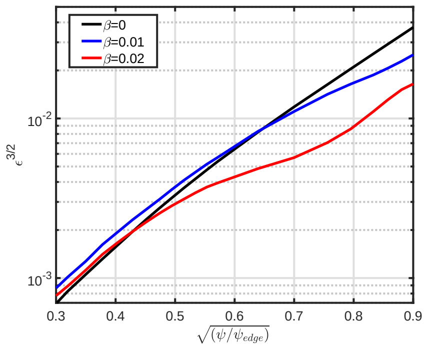
In conclusion, a new compact stellarator with simple coils and good neoclassical confinement has been designed. The magnetic field of the new stellarator is generated by only four planar coils including two interlocking coils of elliptical shape and two circular poloidal field coils. The neoclassical optimized configuration was obtained by a global minimization of the effective helical ripple directly from the shape of the two interlocking coils. The optimized compact stellarator has very low level



(a)Pressure profile for different plasma β



(b)Rotation transform profile for different plasma β



(c)Effective ripple $\epsilon_{eff}^{3/2}$ profile for different plasma β

FIG. 12: Pressure profile,rotation transform and effective ripple for different plasma β

of effective ripple in the plasma core implying excellent neoclassical confinement. The results of the drift-kinetic code SFINCS show that the particle flux of the new configuration is one order of magnitude lower than CNT's in the core. Future work will explore the possibility of a compact stellarator with additional desired physics properties including robust MHD stability and low turbulent transport.

Acknowledgement

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