Parity-breaking parametric decay instability of kinetic Alfvén waves in a nonuniform plasma

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We demonstrate that, in a nonuniform plasma, the parametric decay instabilities of kinetic Alfvén waves could be quantitatively and qualitatively different from that in a uniform plasma. Specifically, for the decay via nonlinear ion Landau damping, the bare-ion Compton scattering is found to dominate over the shielded-ion scattering, and is, typically, an order of magnitude larger than that in a uniform plasma. Furthermore, the parity of the decay kinetic Alfvén waves is broken; leading to finite net wave momentum transfer and, consequently, additional convective plasma transport. Excitations of unstable eigenmodes due to a localized pump wave are also presented.

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Shear Alfvén wave (SAW) is a fundamental electromagnetic wave in magnetized plasmas existing in both nature and laboratories [1]. Due to intrinsic plasma nonuniformities, SAW frequency $\omega = k_\parallel V_A$ is spatially dependent and constitutes, thus, a continuous spectrum [2]. Here, $k_\parallel$ is the wavenumber parallel to the background magnetic field, $\mathbf{B}_0$, and $V_A = B_0/\sqrt{4\pi\rho_m}$ is the Aflvén velocity with $\rho_m \simeq N_i m_i$ being the mass density, N_i being the ion density and m_i being the ion mass. As a consequence of spatial phase mixing, SAW is shown to mode convert into short-wavelength kinetic Alfvén waves (KAWs) [3, 4]. Since there is a significant component of parallel electric field, KAWs are expected to play important dynamic roles; such as acceleration, heating and transport of charged particles [5–8]. Noting that the wave-induced phase-space dynamics depend crucially on the detailed wave spectrum, nonlinear wave-wave interactions such as the important three-wave parametric decay instability (PDI) has been investigated since the early discovery days of KAW [4, 9]. Previous theoretical studies have, however, been limited to the case of a uniform plasma [4, 9–11]. In this Letter, we employ the nonlinear gyrokinetic theory [12] and demonstrate that, in a realistic plasma with nonuniformities, the PDI of KAW could be both quantitatively and qualitatively modified from that in a uniform plasma.

Let us consider, for the present analysis, the simplified slab model with a density profile $N(x)$, constant temperatures, $\tau \equiv T_e/T_i \sim \mathcal{O}(1)$, and the thermal to magnetic pressure ratio $\beta \ll 1$. The equilibrium magnetic field $\mathbf{B}_0 \equiv B_0\hat{\mathbf{b}}$ can, thus, also be approximated as being uniform. Meanwhile, the equilibrium distribution function is taken to be a local Maxwellian F_M. With the compressional Alfvén waves being suppressed due to the frequency separation and $\beta \ll 1$, we then have $\delta(B^2/2) \simeq 0$ and the suitable field variables are $\delta\phi$ and $\delta\mathbf{A} \simeq \delta A_\parallel \hat{\mathbf{b}}$. Here, $\delta\phi$ and $\delta\mathbf{A}$ are, respectively, the scalar and vector potentials, along with the Coulomb gauge $\nabla\cdot\delta\mathbf{A} = 0$ and $\hat{\mathbf{b}} = \hat{\mathbf{z}}$. The perturbed distribution function, δf_j for species $j = e, i$, then is given by [12]

$$\delta f_j = -(q/T)_j F_{Mj}\delta\phi + \exp(-\rho_j\cdot\nabla)\delta g_j, \qquad (1)$$

with $\rho_j = \mathbf{v}\times\mathbf{b}/\Omega_j$, Ω_j being the $j_{th}-$ species cyclotron frequency, and δg satisfying the nonlinear gyrokinetic equation [12]

$$\begin{aligned}&\left(\partial_t + v_\parallel \mathbf{b}\cdot\nabla + \langle\delta u_g\rangle_\alpha\cdot\nabla\right)\delta g_j\\ &= (q/T)_j F_{Mj}\left(\partial_t + i\omega_{*j}\right)\langle\delta L_{g,j}\rangle_\alpha. \qquad (2)\end{aligned}$$

Here, $\delta L_{g,j} \equiv \exp(\rho_j\cdot\nabla)(\delta\phi - v_\parallel\delta A_\parallel/c)$ with $\langle A\rangle_\alpha$ denotes gyro-phase averaging of A, $\langle\delta\mathbf{u}_g\rangle_\alpha = (c/B_0)\hat{\mathbf{b}}\times\nabla\langle\delta L_{g,j}\rangle_\alpha$ and $\omega_{*j} = -i(cT/qB)_j\hat{\mathbf{b}}\times\nabla\ln N\cdot\nabla$. From now on, we will drop the subscripts "j" when possible in order to simplify the notations. In the present analysis of finite small-amplitude fluctuations, we let $\delta g = \delta g^{(1)} + \delta g^{(2)}$ with superscripts "(1)" and "(2)" denoting, respectively, the linear and nonlinear responses, and solve δg via successive expansions. The governing field equations, meanwhile, are the quasi-neutrality condition $\sum_j q_j\langle\delta f_j\rangle = 0$ with $\langle(\cdots)\rangle \equiv \int d^3\mathbf{v}(\cdots)$ denoting the velocity-space integration; i.e.,

$$\sum_{j=e,i}(N_0q^2/T)_j\delta\phi_k = \sum_{j=e,i} q_j\left\langle (J_0\delta g)_j\right\rangle, \qquad (3)$$

and the nonlinear gyrokinetic vorticity equation [13, 14]

$$\begin{aligned}&ik_\parallel\delta J_{\parallel,k} + (N_0e^2/T_i)(1-\Gamma_k)(\partial_t + i\omega_{*i})\delta\phi_k\\ &= \hat{\mathbf{b}}\cdot\mathbf{k}''\times\mathbf{k}'\left(\delta A_{\parallel,k'}\delta J_{\parallel,k''} - \delta A_{\parallel,k''}\delta J_{\parallel,k'}\right)/B_0\\ &- (ec/B_0)\hat{\mathbf{b}}\cdot\mathbf{k}''\times\mathbf{k}'\langle[(J_kJ_{k'} - J_{k''})\delta L_{k'}\delta g_{k'',i}\\ &\quad -(J_kJ_{k''} - J_{k'})\delta L_{k''}\delta g_{k',i}]\rangle. \qquad (4)\end{aligned}$$

In Eqs. (3) and (4), $J_0(k_\perp\rho) = \langle\exp(\rho\cdot\nabla)\rangle_\alpha$ with $k_\perp^2 = -\nabla_\perp^2$ being an operator, $\delta J_\parallel = -(c/4\pi)\nabla_\perp^2\delta A_\parallel$,

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$\Gamma_k = \langle J_0^2(k_\perp\rho_i)F_{Mi}/N_0\rangle = I_0(b_i)\exp(-b_i)$, $b_i = k_\perp^2\rho_{ti}^2$ with $\rho_{ti} = v_{ti}/\Omega_i$ being the Larmor radius defined with ion thermal velocity, I_0 being the modified Bessel function, and $J_0(k_\perp\rho_e) \simeq 1$ since $|k_\perp^2\rho_e^2| \ll 1$. The two terms on the right hand side of Eq. (4) are respectively, the Maxwell and generalized gyrokinetic ion Reynold stresses.

We now consider the nonlinear couplings between three waves, $\mathbf{\Omega}_0$, $\mathbf{\Omega}_-$ and $\mathbf{\Omega}_s$. Here, $\mathbf{\Omega}_0 \equiv [\omega_0, \mathbf{k}_0(x)]$ is the pump KAW wave, $\mathbf{\Omega}_- = [\omega_-, \mathbf{k}_-(x)]$ is the decay KAW, and $\mathbf{\Omega}_s = [\omega_s, \mathbf{k}_s(x)]$ is the electrostatic drift-sound wave (DSW). Note the frequency and wave vector matching conditions dictate $\omega_s = \omega_0 + \omega_-$ and $\mathbf{k}_s(x) = \mathbf{k}_0(x) + \mathbf{k}_-(x)$. As $\mathbf{\Omega}_0$ and $\mathbf{\Omega}_-$ are KAW normal modes, $\mathbf{k}_0(x)$ and $\mathbf{k}_-(x)$ are WKB wave vectors determined by the local linear KAW dispersion relation to be shown later. In particular, we take $\mathbf{\Omega}_0$ to be a mode-converted KAW and, hence, $\mathbf{k}_{\perp,0} \simeq k_{x,0}(x)\hat{\mathbf{x}}$. Since $\tau \sim \mathcal{O}(1)$ and our current focus is on the quasi-mode decay via nonlinear ion Landau damping, we have, as in the uniform case, $|\omega_s| \simeq |k_{\parallel,s}v_{ti}| \ll |\omega_0| \sim |\omega_-|$.

The rest of the theoretical analysis is then straightforward and follows that of Ref. 10 for the uniform case. We, therefore, will omit the details and present only the major results. For the $\mathbf{\Omega}_s$ mode, we have, from Eq. (2),

$$\delta g_{s,i}^{(1)} = -\left(\frac{q}{T}\right)_i F_{Mi}\left[\frac{\omega-\omega_{*i}}{k_\parallel v_\parallel - \omega}\right]_s J_s\delta\phi_s, \tag{5}$$

and

$$\delta g_{s,i}^{(2)} \simeq i\frac{\Lambda_0^-}{\omega_0}\left(\frac{q}{T}\right)_i F_{Mi}\left[\frac{k_\parallel v_\parallel - \omega_{*i}}{k_\parallel v_\parallel - \omega}\right]_s J_0J_-\delta\phi_0\delta\phi_-. \tag{6}$$

Here, $J_k \equiv J_0(k_\perp\rho)$, $\Lambda_{k''}^{k'} \equiv (c/B_0)\mathbf{b}\cdot\mathbf{k}''\times\mathbf{k}'$, and we have assumed, in deriving Eq. (6), $|k_\parallel v_{\parallel,i}/\omega| \ll 1$ for the $\mathbf{\Omega}_0$ and $\mathbf{\Omega}_-$ KAW modes. Meanwhile, since $|\omega/(k_\parallel v_{te})| \ll 1$ for all the three modes, one readily finds $\delta g_{s,e}^{(1)} \simeq 0$, and

$$\delta g_{s,e}^{(2)} \simeq -i\frac{\Lambda_0^s}{\omega_0}\frac{e}{T_e}F_{Me}\left[1 - \frac{k_{\parallel,0}}{k_{\parallel,s}}\left(\frac{\omega_{*e}}{\omega}\right)_-\right]\delta\psi_-\delta\psi_0. \tag{7}$$

Here, $\delta\psi_k \equiv (\omega\delta A_\parallel/(ck_\parallel))_k$ is the effective potential due to the induced parallel electric field $-\partial_t\delta A_{\parallel,k}/c$. The quasi-neutrality condition, Eq. (3), then yields, for the $\mathbf{\Omega}_s$ mode,

$$\hat{\epsilon}_{s*}\delta\phi_s = i(\Lambda_0^s/\omega_0)\hat{\beta}_1\delta\phi_0\delta\phi_-, \tag{8}$$

where $\hat{\epsilon}_{s*}$ is the linear DSW dielectric constant

$$\hat{\epsilon}_{s*} = 1 + \tau + \tau\Gamma_s(1-\omega_{*i}/\omega)_s\xi_s Z_s, \tag{9}$$

with $\xi_s \equiv \omega_s/|k_{\parallel,s}v_{ti}|$, $Z_s \equiv Z(\xi_s)$ is the plasma dispersion function,

$$\begin{aligned}\hat{\beta}_1 &= \hat{\sigma}_0\hat{\sigma}_1[1-(k_{\parallel,0}/k_{\parallel,s})(\omega_{*e}/\omega)_-] \\ &\quad + \tau F_1\left[1 + (1-\omega_{*i}/\omega)_s\xi_s Z_s\right],\end{aligned} \tag{10}$$

$F_1 \equiv \langle J_0J_sJ_-F_{Mi}/N_0\rangle$, $\hat{\sigma}_k \equiv [1+\tau-\tau\Gamma_k(1-\omega_{*i}/\omega)_k]/(1-\omega_{*e}/\omega)_k$. In deriving Eqs. (8) and (10), we have noted $\delta\psi_k \simeq \hat{\sigma}_k\delta\phi_k$ from the linear KAW wave properties.

Next we consider the decay KAW $\mathbf{\Omega}_-$ mode; including nonlinear couplings between $\mathbf{\Omega}_0$ and $\mathbf{\Omega}_s$. Noting that $\mathbf{\Omega}_s$ is a quasimode, hence, both $\delta g_s^{(1)}$ and $\delta g_s^{(2)}$ need be kept on the same footing. For electrons, one can then straightforwardly derive $\delta g_{-,e}^{(1)} \simeq (-e/T_e)F_{Me}(1-\omega_{*e}/\omega)_-\delta\psi_-$ and

$$\begin{aligned}\delta g_{-,e}^{(2)} &\simeq -\left(\frac{\Lambda_0^-}{\omega_0}\right)^2\frac{e}{T_e}F_{Me}\frac{k_{\parallel,0}}{k_{\parallel,-}} \\ &\quad\times\left[1-\frac{k_{\parallel,0}}{k_{\parallel,s}}\left(\frac{\omega_{*e}}{\omega}\right)_-\right]|\delta\psi_0|^2\delta\psi_-.\end{aligned} \tag{11}$$

Meanwhile, for ions, we have $\delta g_{-,i}^{(1)} \simeq (q/T)_iF_{Mi}(1-\omega_{*i}/\omega)_-J_-\delta\phi_-$, and

$$\begin{aligned}\delta g_{-,i}^{(2)} \simeq &-i\frac{\Lambda_0^-}{\omega_0}\left[\left(\frac{q}{T}\right)_i F_{Mi}\left(\frac{k_\parallel v_\parallel - \omega_{*i}}{k_\parallel v_\parallel - \omega}\right)_s J_0J_s\delta\phi_0^*\delta\phi_s\right. \\ &\left.-J_0\delta\phi_0^*\delta g_{s,i}^{(2)}\right].\end{aligned} \tag{12}$$

Here, $\delta g_{s,i}^{(2)}$ is given by Eq. (6). The corresponding quasi-neutrality condition, Eq. (3), then yields

$$\delta\psi_- = \left(\hat{\sigma}_- + \hat{\sigma}_-^{(2)}|\delta\phi_0|^2\right)\delta\phi_- + i(\Lambda_0^-/\omega_0)\hat{D}_1\delta\phi_s\delta\phi_0^* \tag{13}$$

here, $\hat{\sigma}_-^{(2)} = (\Lambda_0^-/\omega_0)^2\{\tau[1+(1-\omega_{*i}/\omega)_s\xi_sZ_s]F_2 - (k_{\parallel,0}/k_{\parallel,-})[1-(k_{\parallel,0}/k_{\parallel,s})(\omega_{*e}/\omega)_-]\hat{\sigma}_0^2\hat{\sigma}_-\}/(1-\omega_{*e}/\omega)_-$, $F_2 \equiv \langle J_0^2J_-^2F_{Mi}/N_0\rangle$, and $\hat{D}_1 = \tau F_1[1+(1-\omega_{*i}/\omega)_s\xi_sZ_s]/(1-\omega_{*e}/\omega)_-$.

The gyrokinetic vorticity equation, Eq. (4), meanwhile, becomes

$$\begin{aligned}&\tau b_-\left[(1-\omega_{*i}/\omega)_-(1-\Gamma_-)\delta\phi_-/b_- - (k_\parallel V_A/\omega)_-^2\delta\psi_-\right. \\ &\left.+\hat{\alpha}_-^{(2)}|\delta\phi_0|^2\delta\phi_-\right] = -i(\Lambda_0^-/\omega_0)\hat{\gamma}^{(2)}\delta\phi_s\delta\phi_s^*.\end{aligned} \tag{14}$$

Here, $\hat{\alpha}^{(2)} = (\Lambda_0^-/\omega_0)^2(F_2-F_1)[1+(1-\omega_{*i}/\omega)_s\xi_sZ_s]/b_-$, and $\hat{\gamma}_2 = \tau\{F_1[1+(1-\omega_{*i}/\omega)_s\xi_sZ_s] - \Gamma_0 - \Gamma_s(1-\omega_{*i}/\omega)_s\xi_sZ_s\}$. In deriving Eq. (14), we note that, since $\mathbf{\Omega}_s$ is an electrostatic mode, the Maxwell stress, i.e., the first term on the right hand side of Eq. (4), makes negligible contribution. Substituting Eq. (13) into (14) then yield the following equation describing $\mathbf{\Omega}_-$ generation by $\mathbf{\Omega}_0$ and $\mathbf{\Omega}_s$;

$$\tau b_-\left(\hat{\epsilon}_{A-} + \hat{\epsilon}_{A-}^{(2)}|\delta\phi_0|^2\right)\delta\phi_- = -i(\Lambda_0^-/\omega_0)\hat{\beta}_2\delta\phi_s\delta\phi_0^*; \tag{15}$$

where $\hat{\epsilon}_{A-} = (1-\omega_{*i}/\omega)_-(1-\Gamma_-)/b_- - (k_\parallel V_A/\omega)_-^2\hat{\sigma}_-$ is the linear local KAW dielectric constant, $\hat{\epsilon}_{A-}^{(2)} = \hat{\alpha}_-^{(2)} - (k_\parallel V_A/\omega)_-^2\hat{\sigma}_-^{(2)}$, and $\hat{\beta}_2 = \hat{\gamma}_2 - \tau b_-(k_\parallel V_A/\omega)_-^2\hat{D}_1$.

Equations (8) and (15) are the coupled equations between $\mathbf{\Omega}_s$ and $\mathbf{\Omega}_-$, and yield the desired WKB dispersion relation for KAW PDI;

$$\hat{\epsilon}_{s*}[\hat{\epsilon}_{A-} + \hat{\epsilon}_{s*}\hat{\chi}_{A-}|\delta\phi_0|^2] = \hat{C}_-|\delta\phi_0|^2, \quad (16)$$

where $\hat{\epsilon}_{s*}$ is given by Eq. (9),

$$\hat{\chi}_{A-} = (\Lambda_0^-/\omega_0)^2(F_2 - F_1^2/\Gamma_s)/(\tau\Gamma_s b_-\hat{\sigma}_-), \quad (17)$$

and

$$\hat{C}_- = (\Lambda_0^-/\omega_0)^2(\hat{\sigma}_0\hat{\sigma}_- - F_1\sigma_s/\Gamma_s) \times\left[\hat{\sigma}_0\hat{\sigma}_-\left(1 - \frac{k_{\parallel,0}}{k_{\parallel,s}}\frac{\omega_{*e,-}}{\omega_-}\right) - \frac{F_1\sigma_s}{\Gamma_s}\right], \quad (18)$$

with $\sigma_s \equiv 1 + \tau - \tau\Gamma_s$.

In deriving Eq. (16), we have ignored terms contributing to nonlinear frequency shift in order to concentrate on the stability properties, and noting $\mathbf{\Omega}_-$ being a normal mode, letting $(k_\parallel V_A/\omega)_-^2 \simeq (1-\omega_{*i}/\omega)_-(1-\Gamma_-)/(b_-\hat{\sigma}_-)$ in the nonlinear terms.

Focusing on the quasi-mode decay via nonlinear ion Landau damping, we then obtain, from Eq. (16), the following stability condition

$$(\gamma + \gamma_{d-})\left|\frac{\partial\hat{\epsilon}_{A-,\mathcal{R}}}{\partial\omega_-}\right| = \left[\hat{\chi}_{A-} + \frac{\hat{C}_-}{|\hat{\epsilon}_{s*}|^2}\right]|\delta\phi_0|^2\mathrm{Im}(\hat{\epsilon}_{s*}) \quad (19)$$

In Eq. (19), γ is the PDI growth rate, γ_{d-} is the linear damping rate of $\mathbf{\Omega}_-$ normal mode, and the subscript "$\mathcal{R}$" denotes the real part. Meanwhile, $\hat{\chi}_{A-}$ and $\hat{C}_-$ correspond, respectively, to the bare-ion (Compton) and shielded-ion scatterings. Noting that $\hat{\chi}_{A-}$ and $\hat{C}_-$ are generally positive, instability thus sets in when $\mathrm{Im}(\hat{\epsilon}_{s*}) > 0$, i.e., noting Eq. (9),

$$(\omega_{s,\mathcal{R}} - \omega_{*i,s})\mathrm{Im}[Z(\xi_s)] > 0. \quad (20)$$

The nonlinear ion Landau damping (or ion-induced scattering), thus, maximizes when $|\xi_s| = |\omega_s/(k_{\parallel,s}v_{ti})| \simeq 1$ for maximized $\mathrm{Im}(Z_s)$, $\omega_{s,\mathcal{R}} > 0$, and $\omega_{*i,s} < 0$. $\omega_{s,\mathcal{R}} > 0$ and $\omega_{*i,s} < 0$ correspond, respectively, to downward frequency cascading and $\mathbf{\Omega}_s$ propagating in the electron diamagnetic drift direction (or $k_{y,s} > 0$ in the present slab model). Noting that while $\omega_{s,\mathcal{R}} > 0$ is the same as in a uniform plasma considered previously [4, 9–11], $\omega_{*i,s} < 0$ ($k_{y,s} > 0$) is solely due to the plasma nonuniformity ($\nabla N \neq 0$) in the present nonuniform model. Furthermore, noting that both $\hat{\chi}_{A-}$ and $\hat{C}_-$ peak around $k_{\perp,s}\rho_i \sim \mathcal{O}(1)$, we have, typically, $|\omega_{*i,s}/\omega_s| \sim |k_{\parallel,s}L_N|^{-1} \gg 1$ with $L_N^{-1} = |\nabla N/N|$ and $|\hat{\epsilon}_{s*}| \sim |\omega_{*i,s}/\omega_s| \gg 1$. That is, in Eq. (19), the bare-ion Compton scattering dominates over the shielded-ion scattering and the PDI growth rate is $\mathcal{O}(|k_{\parallel,s}L_N|^{-1})$ larger than that in a uniform plasma.

In addition to enhancing the PDI growth rate, that the decay instability maximizes for $\omega_{*i,s} < 0$ ($k_{y,s} > 0$) also has a significant implication to the plasma transport processes. Considering the particle flux induced by the $\mathbf{\Omega}_-$ decay KAW, we have $\Gamma_x = \Gamma_{xc} + \Gamma_{xd}$ [8, 15], where Γ_{xc} and Γ_{xd} are, respectively, the convective and diffusive components in the nonuniformity x-direction. We note, furthermore, $\Gamma_{xc} \propto \sum_k k_y\omega_k|\delta\phi_k|^2$ and $\Gamma_{xd} \propto \sum_k k_y^2|\delta\phi_k|^2$, where the k_y dependence indicates that the guiding-center transport is due to the symmetry-breaking of the $P_y = mv_y + eA_y/c$ generalized momentum. In a uniform plasma, with $\mathbf{k}_{\perp,0} = k_{x,0}\hat{\mathbf{x}}$, the decay process possesses parity in k_y; i.e., the PDI growth rates are the same for $\pm k_y$; which implies $|\delta\phi_-(k_y)|^2 = |\delta\phi_-(-k_y)|^2$. This k_y-parity then dictates that there is no net k_y wave momentum transfer, and from total momentum conservation, $\Gamma_{xc} \simeq 0$; i.e., the transport is mainly due to Γ_{xd}. On the other hand, in a nonuniform plasma, $\mathbf{\Omega}_-$ decay KAW maximizes for $k_{y,-} \simeq k_{y,s} > 0$; i.e., $\mathbf{\Omega}_-$ propagates in the ion diamagnetic drift direction, and the k_y-parity is broken. Γ_{xc} is, therefore, finite and generally comparable to Γ_{xd}.

Finally, we remark that while the WKB analysis suggests that the $\mathbf{\Omega}_-$ decay KAW is convectively unstable, the nature of nonlinear coupling and the localization of $\mathbf{\Omega}_0$ pump intensity, as will be shown below, could render $\mathbf{\Omega}_-$ as an absolutely unstable eigenmode. Let $|\delta\phi_0|^2$ peak at $x = 0$ with a localization width Δ_0. For a mode-converted pump KAW $\mathbf{\Omega}_0$, we have $\Delta_0 \simeq \mathcal{O}(\rho_i^2L_N)^{1/3}$; i.e., the Airy scale length [4]. With, typically, $\rho_i/L_N \sim \mathcal{O}(10^{-3})$, then $|\Delta_0/L_N| \sim 10^{-2} \ll 1$. Furthermore, since the nonlinear coupling maximizes for $\mathbf{k}_{\perp,-} \perp \mathbf{k}_{\perp,0}$ via $|\Lambda_0^-| \propto |\mathbf{k}_{\perp,-} \times \mathbf{k}_{\perp,0}|$ and $|\hat{\epsilon}_{s*}| \sim \mathcal{O}(|k_{\parallel,s}L_N|^{-1}) \gg 1$, Eq. (16) then yields the following wave equation for $\mathbf{\Omega}_-$

$$\left[\rho_i^2\left|\frac{\partial\hat{\epsilon}_{A-,\mathcal{R}}}{\partial b_-}\right|\frac{\partial^2}{\partial x^2} - (\delta\omega + i\gamma_{d-})\left|\frac{\partial\hat{\epsilon}_{A-,\mathcal{R}}}{\partial\omega_{-0}}\right| + \hat{\epsilon}_{s*}(0)\hat{\chi}_{A-}(0)|A_0|^2\left(1 - \frac{x^2}{\Delta_0^2}\right)\right]A_-(x) = 0; \quad (21)$$

where we have noted $\delta\phi_- = A_-(x)\exp[ik_{y,-}y + ik_{\parallel,-}z]$, $k_{y,-}^2 \gg |\partial^2/\partial x^2|$, $\omega_- = \omega_{-0} + \delta\omega$, $\hat{\epsilon}_{A-,\mathcal{R}}(\omega_{-0}, k_{y,-}, k_{\parallel,-}, x = 0) = 0$, and approximated $|\delta\phi_0|^2(x) = |A_0|^2(1 - x^2/\Delta_0^2)$. Also, the equilibrium parameters can be approximated by the values at $x = 0$. Eq. (21) then readily gives the condition for the absolute unstable eigenmode as

$$(\gamma + \gamma_{d-})\left|\frac{\partial\hat{\epsilon}_{A-,\mathcal{R}}}{\partial\omega_{-0}}\right| = \frac{\gamma_p}{\omega_0} - \left(\frac{\gamma_p}{\omega_0}\right)_{th}, \quad (22)$$

where $\gamma_p/\omega_0 = |A_0|^2\hat{\chi}_{A-}(0)\mathrm{Im}[\hat{\epsilon}_{s*}(0)]$, and $(\gamma_p/\omega_0)_{th} = |A_0|(\rho_i/\Delta_0)\sqrt{|\partial\hat{\epsilon}_{A-,\mathcal{R}}/\partial b_-|\hat{\chi}_{A-}(0)}\mathrm{Im}[\sqrt{\hat{\epsilon}_{s*}(0)}]$. Physically, that instability sets in when $\gamma_p > \gamma_{p,th}$ means that $\mathbf{\Omega}_-$ wave packet gets sufficiently amplified when it reaches the turning point x_T located at $x_T^2 = (\Delta_0\rho_i)[|\partial\hat{\epsilon}_{A-,\mathcal{R}}/\partial b_-|/|\hat{\epsilon}_{s*}\hat{\chi}_{A-}|]^{1/2}/|A_0|$. Taking, as typical tokamak parameters, $|\Omega_i/\omega_0| \sim \mathcal{O}(10^2)$, $\beta \sim$

$\mathcal{O}(10^{-2})$, $|\omega_{*i,s}/k_{\parallel,s}v_{ti}| \sim \mathcal{O}(10)$ for $|k_{y,-}\rho_i| \simeq k_{y,s}\rho_i \sim \mathcal{O}(1)$, $|\gamma_{d-}/\omega_0| \sim \mathcal{O}(10^{-2})$ and $\rho_i/\Delta_0 \sim \mathcal{O}(10^{-1})$, the threshold amplitude in terms of $|\delta B_{\perp,0}/B_0|$ is $|\delta B_{\perp,0}/B_0|_{th} \simeq \mathcal{O}(10^{-4})$; which is compatible with fluctuation amplitudes observed in tokamak experiments [16].

In summary, it is found that, in a nonuniform plasma, the parametric decay instability of kinetic Alfvén waves via nonlinear ion Landau damping could be quantitatively and qualitatively different from that in a uniform plasma. Not only the ion Compton scattering rate is enhanced by, typically, an order of magnitude, but also the parity of the decay waves is broken; leading to, in addition to the usual diffusive component, a finite convective component of the transport flux. We, furthermore, demonstrate that, due to the radial localization of the pump, the decay KAW forms an absolute unstable eigenmode and the required pump threshold amplitude is consistent with experimental observed fluctuations. Noting that, while the present analysis adopts a slab model with only density nonuniformity in order to simplify the analysis and illuminate the underlying physics processes, the results obtained here can be expected to be also applicable in realistic plasmas; such as tokamaks. It will be interesting and, in fact, desirable to explore how these physics effects affect the nonlinear evolution and eventual saturation of Alfvén eigenmodes; e.g., toroidal Alfvén eigenmode [17–19] excited by energetic particles in tokamak fusion plasmas.

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