On Energetic-Particle Excitations of Low-Frequency Alfvén Eigenmodes in Toroidal Plasma[a)]

Liu Chen[1,2] and Fulvio Zonca[3,1]

[1)] *Institute for Fusion Theory and Simulation and Department of Physics, Zhejiang University, Hangzhou 310027, China*

[2)] *Department of Physics and Astronomy, University of California, Irvine CA 92697-4575, U.S.A.*

[3)] *ENEA C.R. Frascati, Via Enrico Fermi 45, C.P. 65-00044 Frascati, Italy.*

It is well-known that, within the ideal magnetohydrodynamic (MHD) description, there exists two low-frequency Alfvén continuous spectra in toroidal plasma devices; such as tokamaks. The corresponding three accumulation frequencies are the beta-induced Alfvén eigenmode (BAE) frequency, the ion-sound wave (ISW) frequency, and the zero frequency accumulation point at vanishing parallel wave number, $k_\parallel$=0. To form localized discrete eigenmodes, the plasma must be ideal MHD unstable. The zero-frequency branch then corresponds to the ideal MHD unstable discrete mode; while the BAE and ISW discrete eigenmode frequencies could be significantly shifted away from the respective accumulation frequencies. Energetic-particle (EP) effects can be analyzed and understood based on the generalized fishbone linear dispersion relation (GFLDR). In particular, for an ideal MHD stable plasma, EP could play the roles of both localization and destabilization. It can then be shown that EP preferentially excite the BAE branch over the ISW branch. The zero-frequency branch, meanwhile, becomes the well-known fishbone dispersion relation; giving rise to energetic-particle modes (EPM). Extensions to the case of reversed magnetic shear and the kinetic effects will also be discussed.

PACS numbers: 52.35.Bj; 52.35.Lv; 52.55.Fa; 52.55.Tn

[a)] Supported by US DoE, NSF, ITER-CN, NSFC and EUROfusion grants.

I. INTRODUCTION

There exists a vast literature concerning the effects of energetic particles (EPs) on low frequency magnetohydrodynamic (MHD) fluctuations. A recent overview of these physics in the International Thermonuclear Experimental Reactor (ITER) is given in Ref. 1. Other overviews cover, on the one hand, the interpretation and modeling of experimental measurements by innovative diagnostic techniques developed recently[2,3] and, on the other hand, the recent efforts in comparing numerical simulation results with observations[4,5].

A comprehensive analysis of these physics within a unified and self-contained theoretical framework[6] is provided via the general fishbone like dispersion relation (GFLDR)[7,8]. Various applications of the GFLDR theoretical framework to numerical simulation results and experimental observations can be found in Refs. 6,8. Pertinent, in particular, to the present work are analyses of the low-frequency fluctuation spectrum; that is of frequencies, ω, much less than the toroidal Alfvén frequency, $\omega_A = v_A/(qR_0)$, with v_A the Alfvén speed, q the safety factor and R_0 the major radius of the considered toroidal plasma equilibrium. Some of such analyses are the study of fishbone excitation by supra-thermal electrons in the Frascati Tokamak Upgrade (FTU)[9], the interpretation of beta induced Alfvén eigenmode (BAE) excited by finite size magnetic islands in FTU[10,11], TEXTOR[12] and HL-2A[13], the observation of BAE in Tore Supra[14–16], and the excitation of BAE by supra-thermal electrons in HL-2A[13,17].

In the present work, we apply the GFLDR to low-frequency ($|\omega| \ll \omega_A$) Alfvén eigenmodes (AEs) excited by EPs in toroidal plasmas. Our scope is to provide the insights and delineate the properties of Alfvénic and Alfvén-acoustic fluctuations (beta induced Alfvén-acoustic eigenmode, or BAAE)[18–20] that are known to exist in the low-frequency domain and their possible excitations by EPs[20]. Recently, the physics of BAE/BAAE fluctuations has been investigated by gyrokinetic numerical simulations in realistic conditions and the results are found to be in good agreement when compared with Doublet III-D (DIII-D) experimental observations[21]. It is quite clear that, consistent with theoretical predictions[22–25], a truly kinetic analysis is necessary for a proper description of the low frequency fluctuation spectrum. Nonetheless, the detailed nature of the observed fluctuations and the underlying physics remains unclear and is still being debated[26]. This is where the GFLDR theoretical framework demonstrates its usefulness, since "it elevates the interpretative capability for both experiments and numerical simulations"[6,8]. More precisely, the GFLDR allows identifying relevant processes and corresponding spatiotemporal scales and, thereby, illuminates the crucial physics that are responsible of the observed behaviors, either experimentally or by numerical simulations.

This paper is organized as follows. Mode structures and dispersion relation are introduced and discussed in Sec. II. In Sec. III, we derive the relevant expression for the generalized inertia in toroidal plasmas using MHD. In Sec. IV, we then illustrate physical insights on the low frequency fluctuation spectrum excited by EPs, by straightforward manipulations of the GFLDR and the use of the underlying concepts and physical interpretations. Section V is devoted to additional remarks on equilibrium non-uniformity, the case of reversed magnetic shear, and kinetic physics. Finally, Sec. VI gives concluding remarks and discussions.

II. MODE STRUCTURES AND DISPERSION RELATION

Following Refs. 6,7,27, we adopt straight magnetic field line toroidal flux coordinates (r, θ, ζ), with r the radial "magnetic flux" variable; and θ and ζ periodic angular coordinates along poloidal and toroidal directions, respectively. The equilibrium magnetic field can be represented in the Clebsch form as

$$\boldsymbol{B}_0 = \boldsymbol{\nabla}(\zeta - q\theta) \times \boldsymbol{\nabla}\psi \ ,$$

with $\psi = \psi(r)$ the poloidal flux function and the pitch of $\boldsymbol{B}_0$ field lines described by the safety factor $q(r)$

$$q(r) = \boldsymbol{B}_0 \cdot \boldsymbol{\nabla}\zeta / \boldsymbol{B}_0 \cdot \boldsymbol{\nabla}\theta \ . \tag{1}$$

Mode structures in strongly magnetized toroidal plasmas, including low frequency fluctuations considered here, are generally elongated along $\boldsymbol{B}_0$ field lines[27] and, given a generic fluctuation field $\delta\phi(r, \theta, \zeta) = \sum_m \exp(in\zeta - im\theta)\delta\phi_m(r)$, it can be decomposed as[28]

$$\begin{aligned} \delta\phi(r, \theta, \zeta) &= 2\pi \sum_{\ell \in \mathbb{Z}} e^{in\zeta - inq(\theta - 2\pi\ell)} \delta\hat{\phi}(r, \theta - 2\pi\ell) \\ &= \sum_{m \in \mathbb{Z}} e^{in\zeta - im\theta} \int e^{i(m-nq)\vartheta} \delta\hat{\phi}(r, \vartheta) d\vartheta \ . \end{aligned} \tag{2}$$

Equation (2) is valid for arbitrary wavelength and it reduces to the well-known "ballooning mode representation" in the high mode number limit[28]. The parallel mode structure is in the ϑ dependence of $\delta\hat{\phi}(r,\vartheta)$, while its radial variation generally exhibits two length scales: (a long) one reflecting equilibrium non-uniformities and another (short) one accounting for inertia and kinetic plasma response. As an example, these "fine radial scales" are those related to the so-called inertial layer in ideal MHD. A natural scale length for separating equilibrium non-uniformity length scale from fine radial scales is (a few times) the thermal ion Larmor radius[7].

The GFLDR theoretical framework provides the self-consistent solution of wave equations for mode structures and frequencies of discrete spectra, based on the asymptotic matching of fine radial scales behaviors with equilibrium non-uniformity length scale dependences. Thus, the GFLDR is a global theory that provides more than the dispersion relation, and it implies the knowledge of mode structures. In its general form, assuming fine radial structures at one singular layer, it is written as[7,8]

$$i|s|\Lambda(\omega) = \delta\hat{W}_f + \delta\hat{W}_k(\omega) \quad . \tag{3}$$

Here, $s \equiv rq'/q$ is the magnetic shear, $\Lambda(\omega)$ accounts for the generalized inertia response; while $\delta\hat{W}_f$ and $\delta\hat{W}_k$ correspond, respectively, to the generalized potential energy due to fluid-like plasma response and the kinetic plasma behavior due to, *e.g.*, EPs. Note that, consistent with the low frequency ordering $|\omega/\omega_A| \ll 1$ adopted here, $\delta\hat{W}_f$ is independent of ω, while both $\Lambda(\omega)$ and $\delta\hat{W}_k(\omega)$ generally depend on mode frequency. In terms of physics, $\Lambda(\omega)$ is due to the plasma response in the singular layer. Thus, recalling that continuous spectra are made of singular radially local fluctuations[6], $\Lambda(\omega)$ determines the structure of continuous spectra, including frequency gaps. In fact, the low frequency continuous spectrum is generally given by

$$\Lambda^2 = k_\parallel^2 q^2 R_0^2 \quad , \tag{4}$$

with R_0 the major radius of the toroidal plasma. While Eq. (3) is global and describes the discrete spectrum of collective eigenmode structures, Eq. (4) is local and reflects the continuous spectrum radial dependence on plasma equilibrium and $k_\parallel(r)$. Meanwhile, the causality constraints imposed upon the discrete bound modes, requires that[7]

$$|s|\mathbb{R}\mathrm{e}\left(i\Lambda(\omega)\right) = \delta\hat{W}_f + \mathbb{R}\mathrm{e}\delta\hat{W}_k(\omega) < 0 \quad . \tag{5}$$

Global mode structures and plasma response are accounted for by $\delta\hat{W}_f$ and $\delta\hat{W}_k(\omega)$, which physically represent the effect of the radial potential well on the considered bound state. Thus, Eqs. (3) and (5) illuminate the respective role of fluid and kinetic plasma response in establishing the radial potential well and in determining the long radial scale mode structures. Fine radial structures, meanwhile, are controlled by $\Lambda(\omega)$. The self-consistent plasma response to these processes, with their distinctive spatiotemporal scales, determines how much the discrete eigenmode frequency is shifted away from the accumulation point(s) $\Lambda(\omega) = 0$ and the corresponding mode structure.

A practical advantage of the GFLDR theoretical framework is that, in the linear limit analyzed here, it is variational and, thus, "various models and computation techniques with different levels of approximation can also be employed"[6], including trial function methods, analytical approaches and/or numerical techniques.

III. LOW FREQUENCY FLUCTUATION SPECTRUM IN TOROIDAL FLUID PLASMAS

For simplicity, we assume a large aspect ratio toroidal plasma equilibrium; thus, $\epsilon = a/R_0 \ll 1$, with a being the plasma minor radius. Meanwhile, by MHD stability, we take $\beta \equiv 8\pi P_0/B_0^2 \sim \mathcal{O}(\epsilon) \ll 1$ and the equilibrium plasma pressure, $P_0(r)$, satisfies the radial force balance

$$\boldsymbol{\nabla}_\perp \ln B_0 + \frac{4\pi}{B_0^2}\boldsymbol{\nabla}_\perp P_0 = \boldsymbol{\kappa} \ , \tag{6}$$

with $\boldsymbol{\nabla}_\perp \equiv \boldsymbol{\nabla} - \boldsymbol{b}\boldsymbol{\nabla}_\parallel$, $\boldsymbol{b} \equiv \boldsymbol{B}_0/B_0$ and $\boldsymbol{\kappa} \equiv \boldsymbol{b}\cdot\boldsymbol{\nabla}\boldsymbol{b}$ the magnetic curvature vector.

Noting that parallel and perpendicular scales are well separated (cf. Sec. II), *i.e.*$|k_\parallel/k| \sim \mathcal{O}(\epsilon)$, one can show that, accounting for fluctuations, Eq. (6) extends to

$$B_0\delta B_\parallel + 4\pi\delta P \simeq \mathcal{O}(\epsilon^2)\left|4\pi\delta P\right| \ ; \tag{7}$$

i.e., the compressional Alfvén wave is suppressed here. Thus, we can focus on the linear dynamics of shear Alfvén (SAW) and ion sound wave (ISW). Furthermore,

$$\delta P = -\delta\boldsymbol{\xi}\cdot\boldsymbol{\nabla}P_0 + \delta P_{\rm comp} \ , \tag{8}$$

where $\delta\boldsymbol{\xi}$ is the standard notation for the plasma displacement and $\delta P_{\rm comp}$ denotes the compressional component of δP. In the ideal MHD fluid model, denoting by Γ the ratio of specific heats,

$$\delta P_{\rm comp} = -\Gamma P_0\left(\boldsymbol{\nabla}\cdot\delta\boldsymbol{\xi}\right) \ , \tag{9}$$

which, in high-temperature collisionless plasmas, must be properly modified to retain crucial kinetic effects such as finite Larmor radii and wave-particle interactions[6]. The fluid thermal plasma description is, however, sufficient for the present purpose to delineate EP effects on the low frequency fluctuation spectrum.

Noting Eqs. (7) – (9), and following the standard approach in ideal MHD linear analysis, one can show

$$\boldsymbol{\nabla}\cdot\delta\boldsymbol{\xi}_\perp\left[1+\mathcal{O}(\epsilon^2)\right]=-2\boldsymbol{\kappa}\cdot\delta\boldsymbol{\xi}_\perp+\frac{4\pi\delta P_{\rm comp}}{B_0^2}\;. \tag{10}$$

We then have, to $\mathcal{O}(\epsilon^2)$

$$\boldsymbol{\nabla}\cdot\delta\boldsymbol{\xi}_\perp=-2\boldsymbol{\kappa}\cdot\delta\boldsymbol{\xi}_\perp\;; \tag{11}$$

and representing $\delta\boldsymbol{\xi}_\perp=\delta\boldsymbol{\xi}_{\perp 0}+\delta\boldsymbol{\xi}_{\perp 1}+...$ as asymptotic series with ϵ the expansion parameter, we can express $\delta\boldsymbol{\xi}_{\perp 0}$ as

$$\delta\boldsymbol{\xi}_{\perp 0}=\frac{c}{B_0}\boldsymbol{b}\times\boldsymbol{\nabla}\Phi_s\;, \tag{12}$$

with Φ_s being the perturbed stream function. Correspondingly, we have the leading order expression for the perturbed perpendicular magnetic field

$$\delta\boldsymbol{B}_{\perp 0}=c\boldsymbol{b}\times\boldsymbol{\nabla}\left(\nabla_\parallel\Phi_s\right)\;. \tag{13}$$

Substituting the above results into the quasi-neutrality condition, $\boldsymbol{\nabla}\cdot\delta\boldsymbol{J}=0$, we then derive the desired SAW vorticity equation[29–34]:

$$\begin{aligned}\mathcal{L}_A(\Phi_s)\equiv\;& B_0\nabla_\parallel\left(\frac{1}{B_0}\nabla_\perp^2\nabla_\parallel\Phi_s\right)-\boldsymbol{\nabla}_\perp\cdot\left(\frac{\partial_t^2}{v_A^2}\boldsymbol{\nabla}_\perp\Phi_s\right)\\ &+\frac{8\pi}{B_0}\left(\boldsymbol{b}\times\boldsymbol{\kappa}\right)\cdot\boldsymbol{\nabla}_\perp\left[\frac{\boldsymbol{b}}{B_0}\cdot\left(\boldsymbol{\nabla}P_0\times\boldsymbol{\nabla}_\perp\Phi_s\right)\right]\\ &+\frac{4\pi}{cB_0}\left[\boldsymbol{b}\times\boldsymbol{\nabla}_\perp\left(\nabla_\parallel\Phi_s\right)\right]\cdot\boldsymbol{\nabla}J_{0\parallel}\\ =&-\frac{8\pi}{cB_0}\left(\boldsymbol{b}\times\boldsymbol{\kappa}\right)\cdot\boldsymbol{\nabla}_\perp\delta P_{\rm comp}\;.\end{aligned} \tag{14}$$

Equation (14) is coupled with the perturbed parallel force balance equation, which, along with the adiabatic equation of state, can be rewritten as

$$\begin{aligned}\mathcal{L}_S(\delta P_{\rm comp})\equiv\;&\partial_t^2\delta P_{\rm comp}-c_S^2B_0\nabla_\parallel\left(\frac{1}{B_0}\nabla_\parallel\delta P_{\rm comp}\right)\\ =&-\frac{2\Gamma P_0 c}{B_0}\left(\boldsymbol{b}\times\boldsymbol{\kappa}\right)\cdot\boldsymbol{\nabla}_\perp\partial_t^2\Phi_s\;,\end{aligned} \tag{15}$$

where $c_S^2 = \Gamma P_0/\rho_{m0}$ is the sound speed and ρ_{m0} is the equilibrium plasma mass density. Equations (14) and (15) form the complete set of equations describing SAW, with the field variable Φ_s, and ISW, with the field variable $\delta P_{\rm comp}$, coupled by the finite magnetic curvature $\boldsymbol{\kappa}$. By asymptotic matching of fine radial scales behaviors with equilibrium non-uniformity length scale dependences, thus, their solution yields the appropriate form of GFLDR, Eq. (3). Meanwhile, Eqs. (14) and (15) also describe the SAW and ISW continuous spectra for radial singular perturbations by taking the $|k_r| \gg |k_\theta|$ limit; yielding the proper form of $\Lambda(\omega)$ that applies in the considered ideal MHD limit. This calculation is most easily performed applying the fluctuation representation of Eq. (2), which gives $\nabla_\parallel \mapsto (qR_0)^{-1}\partial_\vartheta$, $k_\theta \simeq -(nq/r)$ and $k_r \simeq k_\theta(s\vartheta)$ for fine radial scale structures. Thus, taking the $|s\vartheta| \gg 1$ limit and noting

$$\hat{\Phi}_s \simeq |s\vartheta|^{-1} e^{i\Lambda|\vartheta|} \;, \tag{16}$$

consistent with Eqs. (4) and (14), Eq. (15) yields

$$\begin{aligned}\delta\hat{P}_{\rm comp} \simeq \frac{2\Gamma P_0 c}{B_0 R_0}(ik_\theta s\vartheta)\bigg[&\frac{\epsilon_S}{\epsilon_S^2 - 4\Lambda^2\omega_S^4/\omega^4}\sin\vartheta \\ &-i\frac{2\Lambda(\vartheta/|\vartheta|)\omega_S^2/\omega^2}{\epsilon_S^2 - 4\Lambda^2\omega_S^4/\omega^4}\cos\vartheta\bigg]\hat{\Phi}_s \;,\end{aligned} \tag{17}$$

where

$$\epsilon_S = 1 - \frac{\omega_S^2}{\omega^2}\left(1+\Lambda^2\right) \;, \tag{18}$$

and $\omega_S^2 = c_S^2/(q^2R_0^2)$. Substituting Eq. (17) back into Eq. (14), one obtains, for the present case, the following expression of $\Lambda(\omega)$, which may be regarded as a generalized inertia including enhancement by plasma compression,

$$\Lambda^2 = \frac{\omega^2}{\omega_A^2} - \frac{\Gamma\beta q^2\epsilon_S}{\epsilon_S^2 - 4\Lambda^2\omega_S^4/\omega^4} \;. \tag{19}$$

Note that Eq. (19) has been derived self-consistently; that is, it assumes and incorporates the mode structures of Eqs. (16) and (17). Note, again, for a discrete eigenmode, one must impose $\mathbb{R}{\rm e}(i\Lambda) < 0$ as in Eq. (5) to form a bound state. Furthermore, it is consistent with the low frequency continuous spectrum, Eq. (4). Thus, Eq. (19) not only accounts for the low frequency continuum, but can consistently be adopted in the GFLDR, Eq. (3), to describe EP effect on the discrete spectrum. Note also that Eq. (19) can be readily obtained from the more general kinetic expression[6–8] (see, *e.g.*, Eq. (13) of Ref. 24), taking $T_i/T_e \to 0$, $\Gamma = 1$, and the large argument expansion of plasma dispersion functions. Meanwhile, mode structures of coupled SAW and ISW fluctuations are consistent with those derived from kinetic theory (see, *e.g.*, Eqs. (8) and (12) of Ref. 24).

IV. LOW FREQUENCY DISCRETE FLUCTUATION SPECTRUM EXCITED BY ENERGETIC PARTICLES

In this section, we adopt the generalized inertia expression derived above in Eq. (19) and use it in the GFLDR, Eq. (3), to investigate excitation of the low frequency fluctuation spectrum by EPs. To illustrate the underlying physics with the simplest cases, we assume that the frequency of BAE and BAAE accumulation points are well separated, and are much larger than the frequencies of MHD fluctuations.

As anticipated above, this will illuminate physical insights on the low frequency fluctuation spectrum excited by EPs, obtained by straightforward manipulations of Eq. (3). Once again, the use of the generalized inertia expression in Eq. (19) allows us to self-consistently take into account the coupled SAW and ISW continuous spectra, as well as the mode structures of fluctuations belonging to the discrete spectrum. The causality constraint, $\delta\hat{W}_f + \mathbb{R}\mathrm{e}\delta\hat{W}_k(\omega) < 0$, Eq. (5), indicates, in particular, that no discrete mode can exist in an ideal MHD stable ($\delta\hat{W}_f > 0$) plasma without the non-resonant EPs providing an effective potential well. Meanwhile, the resonant kinetic response, $\mathbb{I}\mathrm{m}\delta\hat{W}_k(\omega)$, due to, *e.g.*, EPs, may provide instability drive, depending on the wave-particle resonance condition.

A. Beta induced Alfvén Eigenmodes

Assuming $|\omega^2| \gg \omega_S^2$ (consistent with $q^2 \gg 1$), Eq. (18) yields $\epsilon_S \simeq 1$. Thus, Eq. (19) can be cast as

$$\omega^2 = \omega_{BAE}^2 \left(1 + \frac{\Lambda^2}{\Gamma\beta q^2}\right) , \tag{20}$$

where $\omega_{BAE}^2 \equiv \Gamma\beta q^2 \omega_A^2 = 2q^2\omega_S^2$ is the BAE accumulation point. Invoking the GFLDR, Eq. (3), Eq. (20) becomes

$$\omega^2 = \omega_{BAE}^2 \left(1 - \frac{\delta\hat{W}^2}{\Gamma\beta q^2 s^2}\right) , \tag{21}$$

with $\delta\hat{W} \equiv \delta\hat{W}_f + \delta\hat{W}_k$. Letting $\omega = \omega_r + i\gamma$ with $|\gamma/\omega_r| \ll 1$, *i.e.*, assuming weakly unstable modes, we obtain

$$\omega_r^2 = \omega_{BAE}^2 \left[1 - \frac{\left(\delta\hat{W}_f + \mathbb{R}\mathrm{e}\delta\hat{W}_k(\omega_r) \right)^2}{\Gamma\beta q^2 s^2} \right] . \tag{22}$$

Thus, the effect of EP is non-perturbative, as anticipated above. In fact, in the absence of EP, the discrete BAE could not exist in ideal stable MHD plasma, which would violate the causality constraint, Eq. (5). In addition, the real mode frequency could be significantly modified by the non-resonant EP response. Furthermore,

$$\begin{aligned} \gamma = & -\mathbb{I}\mathrm{m}\delta\hat{W}_k(\omega_r) \frac{\omega_{BAE}^2}{\omega_r} \frac{\left(\delta\hat{W}_f + \mathbb{R}\mathrm{e}\delta\hat{W}_k(\omega_r) \right)}{\Gamma\beta q^2 s^2} \\ & \times \left[1 + \frac{\omega_{BAE}^2}{\omega_r} \frac{\partial \mathbb{R}\mathrm{e}\delta\hat{W}_k}{\partial\omega_r} \frac{\left(\delta\hat{W}_f + \mathbb{R}\mathrm{e}\delta\hat{W}_k(\omega_r) \right)}{\Gamma\beta q^2 s^2} \right]^{-1} \\ \simeq & -\mathbb{I}\mathrm{m}\delta\hat{W}_k(\omega_r) \frac{\omega_{BAE}^2}{\omega_r} \frac{\left(\delta\hat{W}_f + \mathbb{R}\mathrm{e}\delta\hat{W}_k(\omega_r) \right)}{\Gamma\beta q^2 s^2} . \end{aligned} \tag{23}$$

Assuming, for simplicity, modes with positive frequency, Eqs. (5) and (23) impose $\mathbb{I}\mathrm{m}\delta\hat{W}_k(\omega_r) > 0$ for BAE excitation by EPs via resonant wave-particle interaction. Equation (23) is a further proof of the non-perturbative EP effects in ideal stable MHD plasmas.

B. Beta induced Alfvén acoustic Eigenmodes

Consider now the limit $|\epsilon_S^2| \ll |\Lambda^2| \ll 1$. This is the slow sound wave (SSW) approximation and implies $\omega^2 \simeq \omega_S^2$. Thus, Eq. (19) can be cast as

$$\omega^2 = \omega_S^2 \left[1 + \Lambda^2 \left(1 - \frac{2}{q^2} + \frac{4\Lambda^2}{\Gamma\beta q^2} \right) \right] . \tag{24}$$

Proceeding as for Eq. (20) and invoking the GFLDR, Eq. (3), Eq. (24) becomes

$$\omega^2 = \omega_S^2 \left[1 - \frac{\delta\hat{W}^2}{s^2} \left(1 - \frac{2}{q^2} - \frac{4\delta\hat{W}^2}{\Gamma\beta q^2 s^2} \right) \right] . \tag{25}$$

By direct inspection of Eqs. (21) and (25), it is readily recognized that the EP effect on BAAEs is much weaker than on BAEs, namely the induced relative frequency shift,

$$\left(\frac{\Delta\omega}{\omega} \right)_{BAAE} \sim \mathcal{O}(\beta q^2) \left(\frac{\Delta\omega}{\omega} \right)_{BAE} \sim \mathcal{O}\left(\frac{\delta\hat{W}^2}{s^2} \right) , \tag{26}$$

is $\sim \mathcal{O}(\beta q^2)$ smaller in the BAAE case. Assuming weakly unstable modes, Eq. (25) yields

$$\omega_r^2 \simeq \omega_S^2 \left[1 - \frac{\left(\delta\hat{W}_f + \mathbb{R}\mathrm{e}\delta\hat{W}_k(\omega_r) \right)^2}{s^2} \left(1 - \frac{2}{q^2} \right) \right] . \tag{27}$$

As in the discrete BAE case, Eq. (22), the effect of EP should in general be considered as non-perturbative, since discrete BAAE could not exist in ideal stable MHD plasma. As to the linear growth rate, one has

$$\gamma \simeq - \left(1 - \frac{2}{q^2} \right) \mathbb{I}\mathrm{m}\delta\hat{W}_k(\omega_r) \frac{\omega_S^2}{\omega_r} \frac{\left(\delta\hat{W}_f + \mathbb{R}\mathrm{e}\delta\hat{W}_k(\omega_r) \right)}{s^2} . \tag{28}$$

A direct comparison of Eqs. (23) and (28) confirms that $(\gamma/\omega_r)_{BAAE} \sim \mathcal{O}(\beta q^2)(\gamma/\omega_r)_{BAE}$, consistent with Eq. (26). In the BAAE case, however, causality constraint, Eq. (5), and wave particle resonance condition with $\mathbb{I}\mathrm{m}\delta\hat{W}_k(\omega_r) > 0$ yield mode growth only if $q^2 > 2$.

C. Low frequency MHD fluctuations

Finally, let us consider the low frequency MHD limit, $|\omega^2| \ll \omega_S^2$. Thus, $\epsilon_S \simeq -\omega_S^2/\omega^2$ and Eq. (3) can be cast as

$$\Lambda^2 \simeq \frac{\omega^2}{\omega_A^2} - \frac{\Gamma\beta q^2}{\epsilon_S} \simeq \frac{\omega^2}{\omega_A^2}(1 + 2q^2) . \tag{29}$$

This is the well-known inertia enhancement as originally derived by Glasser, Greene and Johnson[35]. It is also connected to the ion flow within the considered magnetic flux surface under the incompressibility condition, as pointed out in the classic work by Pfirsch and Schlüter[36]. Invoking, again, the GFLDR, Eq. (3), Eq. (29) yields

$$\omega = -i \frac{\omega_A}{(1 + 2q^2)^{1/2}} \frac{\delta\hat{W}}{|s|} . \tag{30}$$

The MHD (gap) mode is then given by

$$\begin{aligned} \omega_r &= \frac{\omega_A/|s|}{(1 + 2q^2)^{1/2}} \mathbb{I}\mathrm{m}\delta\hat{W}_k(\omega_r) , \\ \gamma &= -\frac{\omega_A/|s|}{(1 + 2q^2)^{1/2}} \left(\delta\hat{W}_f + \mathbb{R}\mathrm{e}\delta\hat{W}_k(\omega_r) \right) . \end{aligned} \tag{31}$$

Similar to BAE and BAAE, EP effects should be considered non-perturbative on the MHD (gap) mode, since no discrete mode could be excited in an ideal MHD stable plasma in the absence of EPs; due to the violation of the causality constraint, Eq. (5).

In addition to this MHD mode, Chen, White and Rosenbluth[37] have demonstrated that another "fishbone" mode exists, also known as energetic particle mode (EPM)[38], whose frequency and mode structure are entirely determined by maximization of wave particle power transfer. In fact, for such a mode both mode drive and radial mode structure (potential well) are set by the resonant wave-particle power transfer[6–8]. Near excitation threshold, this mode dispersion relation is given by

$$\delta\hat{W}_f + \mathbb{R}\mathrm{e}\delta\hat{W}_k(\omega_r) \simeq 0 \ ,$$
$$\gamma = \left[-\frac{\partial \mathbb{R}\mathrm{e}\delta\hat{W}_k}{\partial\omega_r} \right]^{-1} \left[\mathbb{I}\mathrm{m}\delta\hat{W}_k(\omega_r) - |s|(1+2q^2)^{1/2}\frac{\omega_r}{\omega_A} \right] \ . \qquad (32)$$

On the right hand side of the growth rate expression, Eq. (32), the competition between resonant EP drive, $\propto \mathbb{I}\mathrm{m}\delta\hat{W}_k(\omega_r)$, and the SAW continuum damping, $\propto |s|(1+2q^2)^{1/2}(\omega_r/\omega_A)$, is clearly recognizable and sets the threshold condition for EPM/fishbone excitation. We remark that the EPM/fishbone branch can be shown to exist also for the BAE/BAAE branches discussed in Secs. IV A and IV B[6–8].

V. ADDITIONAL REMARKS ON EQUILIBRIUM NON-UNIFORMITY AND KINETIC PHYSICS

Equation (3) applies when fine radial structures of considered fluctuations exist at one singular layer. More generally, multiple singular layers could exists; *i.e.*, different localized regions where fluctuations may exhibit fine radial structures[7,8]. The contribution of each region is additive, by definition, and Eq. (3), for the case of one dominant (central) singular layer and two nearest neighbors, could be written as

$$i\left[|s_0|\Lambda_0(\omega) + \mathcal{C}_+^2|s_+|\Lambda_+(\omega) + \mathcal{C}_-^2|s_-|\Lambda_-(\omega)\right] = \delta\hat{W}_f + \delta\hat{W}_k(\omega) \ . \qquad (33)$$

Here, s_0 and $\Lambda_0(\omega)$ are, respectively, magnetic shear and the generalized inertia at the dominant (central) singular layer, while $s_\pm$ and $\Lambda_\pm(\omega)$ are the same quantities at the two (left/right) nearest neighbors singular layers. Meanwhile, the $\mathcal{C}_\pm^2$ constants account for the radial decay of the mode amplitude away from the dominant localization region. Equation (33) can be recursively extended to multiple nearest neighbors. In the continuum limit, where a large number of coupled singular layers are considered for short wavelength modes, Eq. (33) becomes the radial envelope formulation of the GFLDR[6–8]

$$\left[i\Lambda - \left(\delta\bar{W}_f + \delta\bar{W}_k\right)\right] A(r) = D(r, \theta_k, \omega) A(r) = 0 \ . \tag{34}$$

Here, $D(r, \theta_k, \omega)$ corresponds to a local WKB dispersion function and the eikonal representation of the radial envelope of fluctuation amplitudes is given by $A(r) \sim \exp i \int nq'\theta_k(r)dr$. In Eq. (34), the generalized inertia is the same as in Eqs. (3) and (33); and is intended to be computed at the local radial position r by means of, *e.g.*, Eq. (19). The potential energies $\delta\bar{W}_f$ and $\delta\bar{W}_k$ are also functions of r. $\delta\hat{W}_f$ and $\delta\hat{W}_k(\omega)$ in Eq. (33), meanwhile, are radial integrals of $\delta\bar{W}_f$ and $\delta\bar{W}_k$, respectively, weighted over the fluctuation intensity and other geometric factors[6–8].

While discussing the solution of Eq. (34) is beyond the intended scope of the present work, the structure of Eq. (33) allows further illuminating the results one may expect from numerical simulations and experimental observations. For the low frequency modes, considered here, a small number of poloidal harmonics is effectively coupled to form the fluctuation structures. Meanwhile, for sufficiently weak EP drive, we can reasonably assume that one dominant mode should be observed experimentally for a given toroidal mode number, corresponding to the fastest growing instability predicted by Eq. (33). This typically occurs for imaginary $\Lambda_0(\omega)$, *i.e.* an AE[7,8], while nearly real $\Lambda_\pm(\omega)$ (either one or both) account for coupling to the SAW continuum, which can be sufficiently small if the $\mathcal{C}_\pm^2$ coupling constants are also small. In the short wavelength (high mode number) limit, the non-local coupling to the SAW continuum can be reduced to the expression derived in Refs. 39,40 (cf. Refs. 6–8 for a recent review of this problem).

A. Reversed shear

Consistent with the GFLDR, Eqs. (3) and (33), and the analysis of Sec. IV, the EP effect on AEs is enhanced at low magnetic shear. This is also consistent with numerical simulation results[21] and experimental observations[21,41]. It is expected, thus, that favorable experimental conditions for low frequency AE excitation by EP are characterized by weak and/or reversed magnetic shear. In this case, when $|s|\Lambda(\omega) \to 0$ in Eq. (3) or $|s_0|\Lambda_0(\omega) \to 0$ in Eq. (33), the GFLDR needs to be properly extended to deal with reversed magnetic shear configurations[7,8,42]. In particular, one needs to formally replace $|s|\Lambda(\omega)$ (or $|s_0|\Lambda_0(\omega)$) by

$$|s|\Lambda(\omega) \to S\left(\Lambda^2 - k_{\|0}^2 q_0^2 R_0^2\right)^{1/2} (1/n)^{1/2} \times \left[k_{\|0} q_0 R_0 - i\left(\Lambda^2 - k_{\|0}^2 q_0^2 R_0^2\right)^{1/2}\right]^{1/2} \quad , \tag{35}$$

with $S \equiv (r/q)[q'']^{1/2}\big|_{r=r_0}$, to be computed at the minimum-q surface $r = r_0$, where $q = q_0$ and $k_\| = k_{\|0}$; and $\Lambda(\omega)$ still provided by Eq. (19). More precisely, near the accumulation points $\Lambda^2 = k_{\|0}^2 q_0^2 R_0^2$, Eq. (35) is further simplified and gives[6,8,43]

$$\begin{aligned} |s|\Lambda(\omega) &\to S\left(\Lambda^2 - k_{\|0}^2 q_0^2 R_0^2\right)^{1/2} (1/n)^{1/2} \\ &\times \left[k_{\|0} q_0 R_0 - i\left(\Lambda^2 - k_{\|0}^2 q_0^2 R_0^2\right)^{1/2}\right]^{1/2} \\ &\simeq i\left(\frac{2}{n}\right)^{1/2} S\left(\frac{k_{\|0}^2 q_0^2 R_0^2 - \Lambda^2}{k_{\|0} q_0 R_0}\right)^{1/2} k_{\|0} q_0 R_0 \quad . \end{aligned} \tag{36}$$

Considering $n > 0$ without loss of generality, for a given toroidal mode number there must exist two adjacent poloidal mode numbers m and $m-1$, for which $k_{\|0}\big|_{m,n} < 0$ and $k_{\|0}\big|_{m-1,n} > 0$[43]. Thus, Eq. (36) shows that a natural frequency gap appears in the SAW continuum at the minimum-q surface for $-q_0 R_0\, k_{\|0}\big|_{m,n} < \Lambda < q_0 R_0\, k_{\|0}\big|_{m-1,n}$, where reversed-shear AEs (RSAEs)[44–46] can exist under proper conditions. Taking $k_{\|0}\big|_{m,n} = -|k_{\|0}|$; *i.e.*, considering the most typical condition for RSAE excitation by EPs with the mode frequency upshifted from the accumulation point $\Lambda = -k_{\|0} q_0 R_0 > 0$, one obtains, by substitution of Eq. (36) into Eq. (3),

$$\left(\frac{\Lambda^2 - k_{\|0}^2 q_0^2 R_0^2}{|k_{\|0}| q_0 R_0}\right)^{1/2} = \left(\frac{n}{2}\right)^{1/2} \frac{\delta\hat{W}_f + \delta\hat{W}_k(\omega)}{S|k_{\|0}| q_0 R_0} \,. \tag{37}$$

This equation illuminates why mode existence requires $\delta\hat{W}_f + \mathbb{R}\mathrm{e}\,\delta\hat{W}_k(\omega) > 0$[6–8]; *i.e.*, Mercier-Suydam ideal stability in the absence of EPs, as noted by Fu and Berk[47]. It is worthwhile noting that a direct comparison with ideal MHD stability is subtle, since it involves not only the sign of $\delta\hat{W}_f$ but, perhaps more importantly, the underlying mode structure that is used for $\delta\hat{W}_f$ minimization in deriving the stability criterions. That is, AEs mode structures are, in general, not the same as the mode structures computed for MHD stability. Thus, the theoretical framework to properly assess AE stability properties taking into account the corresponding mode structure is the GFLDR, Eq. (3) or Eq. (37).

By direct inspection of Eqs. (3) and Eq. (37), it is possible to conclude that normal- and reversed-shear AE dispersion relations are essentially the same, provided one formally substitutes

$$\frac{\delta\hat{W}^2}{s^2} \to \frac{n}{2k_{\parallel 0}q_0R_0}\frac{\delta\hat{W}^2}{S^2} - k_{\parallel 0}^2 q_0^2 R_0^2 \, . \tag{38}$$

In other words, one generally finds the same three branches excited by EPs discussed in Sec. IV. That is, the reversed-shear BAE branch is obtained by the formal substitution of Eq. (38) into Eq. (21). Similarly, the reversed-shear BAAE branch is obtained from Eq. (25) by the same formal substitution; and, finally, the low-frequency MHD branch is readily obtained from Eq. (30). In general, it is possible to conclude that the relative EP induced frequency shift remains $\mathcal{O}(\beta q^2)$ weaker in the BAAE than in the BAE case also for reversed-shear plasma equilibria. Furthermore, the EP effect on the mode structures can be non-perturbative, as discussed in Sec. IV.

B. Kinetic effects

The description adopted so far for illustrating the GFLDR is based on a fluid treatment of the singular layer(s), while ideal regular region(s) account for the fluid response of thermal plasma particles as well as the kinetic response of EPs. In nearly collisionless fusion plasmas, however, the kinetic response of thermal plasma ions in the singular layer at low frequency becomes crucial[6]. The SAW continuum accumulation points, in fact, are generally shifted into the complex plane because of ion Landau damping in non-uniform toroidal plasma equilibria[48–50], with important consequences on the overall stability and mode structures of low frequency electromagnetic modes[50–52]. In particular, the importance of kinetic analyses for mode stability was recognized in the mid nineties, with the discovery of Alfvénic ion temperature gradient (AITG) driven modes[52], which can be excited even in the absence of EP drive and are the short wavelength counterpart of EP driven BAEs[6–8].

The GFLDR theoretical framework allows computing the general kinetic expression of $\Lambda(\omega)$[6–8], extending Eq. (19) and including proper generalizations accounting for finite Larmor radius and finite orbit width effects via gyrokinetic descriptions (numerical and/or analytical). The structure of coupled SAW and ISW discrete and continuous fluctuation spectra are significantly modified by kinetic effects to the point that the concept of a low frequency kinetic thermal ion (KTI) gap has been proposed[22] for self consistently accounting for the relevant kinetic interactions where needed. Nonetheless, accumulations points of the SAW/ISW spectra can still be dubbed as BAE, BAAE and low-frequency MHD, or kinetic ballooning mode (KBM), by continuously tracking their frequency as a function of the relevant kinetic plasma parameter; *e.g.*, the diamagnetic frequency[25].

In general, numerical approach is necessary for the calculation of the generalized inertia $\Lambda(\omega)$[6–8]. However, simplified analytic expressions have been provided including circulating as well as trapped particle thermal plasma responses[23–25]. Mostly due to the difference in mode polarization and frequency, the general finding is that the BAAE accumulation point is heavily damped by ion Landau damping unless $T_e \gg T_i$, while the least damped response is either BAE or KBM depending on the strength of plasma non-uniformity and mode number[6,25]. In other words, BAAE are effectively suppressed with respect to BAE for $T_e \lesssim T_i$, typical of burning fusion plasmas, for two major reasons: (i) the strong ion Landau damping; and (ii) the weaker effect of EP drive (cf. Sec. IV).

Numerical simulations often allow to probe the plasma behavior by a radially localized "antenna", whose response in frequency identifies damping of plasma resonant cavity modes (plasma eigenmodes) as well as continuous spectra. Kinetic effects can, thus, be easily measured by this type of analysis, including the frequency shift of SAW/ISW accumulation points into the complex plane due to Landau damping. This is shown, *e.g.*, in Ref. 21, which confirms the strong damping of the BAAE accumulation point expected theoretically[23–25]. In the same work, however, it is also shown that eigenmode excitation by EP due to wave particle resonances is viable in the same frequency range due to weak coupling of the mode structure with the ISW continuum. This can be readily understood by the present analysis and, in particular, of Eq. (33), possibly modified by means of Eq. (36) for reversed-shear equilibria. In fact, due to the non-perturbative EP effects on the mode structure, the $\mathcal{C}_\pm^2$ coupling constants to the SAW/ISW continuum are small and mode excitation is thereby allowed. Furthermore, as shown in the present work, EP effects on the BAE are stronger than on BAAE. Thus, the mode structure excited by non-perturbative EPs is expected to be that of a BAE or low-frequency RSAE. At sufficiently low frequencies, the resonant excitation of the well-known MHD fluctuations by EPs is also possible.

VI. CONCLUSIONS AND DISCUSSIONS

In this work, we have analyzed EP excitations of low-frequency, long wave-length, discrete fluctuations adopting a two component plasma description, where thermal plasma is considered as an ideal MHD fluid, while resonant wave-EP interactions are treated kinetically. We have shown that the discrete fluctuation spectrum consists of modes originating from the accumulation points of the SAW/ISW continuous spectra; *i.e.*, in decreasing frequency oder, the BAE, BAAE and MHD ($k_{\parallel} = 0$) zero frequency accumulation points.

In particular, we have shown that, for an ideal MHD stable plasma, EP play important roles not only in the mode destabilization but also in the definition of mode structures and mode localization. This crucial feature is due to the fact that modes of the discrete fluctuation spectrum do not exist in ideal MHD stable plasmas and, thus, the effect of EPs should be considered as non-perturbative.

The GFLDR theoretical framework is adopted here to take into account the self-consistent solution of wave equations for mode structures and frequencies of discrete spectra. In this way, it can then be shown that EP preferentially excite the BAE over the BAAE branch due to the stronger wave-EP interaction. This conclusion applies for both standard (positive) as well as reversed magnetic shear. Meanwhile, properly accounting for kinetic effects at low frequencies further suppresses the BAAE with respect to the BAE branch due to the typically stronger ion Landau damping. The dispersion relation of the zero-frequency branch, meanwhile, consistently describes the EPM/fishbone excitations by EPs.

The findings of the present work are consistent with and illuminate both observations and numerical simulation results of discrete low-frequency fluctuations excited by EPs. Furthermore, they also demonstrate the usefulness of the GFLDR theoretical framework as well as its capability of providing insights and interpretations for both experiments and numerical simulations.

ACKNOWLEDGMENTS

This work was supported by US DoE, NMCFERP-CN, NSFC grants and by EUROfusion Consortium under grant agreement No. 633053.

REFERENCES

[1]S. D. Pinches, I. T. Chapman, Ph. W. Lauber, H. J. C. Oliver, S. E. Sharapov, K. Shinohara, and K. Tani, Phys. Plasmas **22**, 021807 (2015).

[2]S. E. Sharapov, B. Alper, H. L. Berk, D. N. Borba, B. N. Breizman, C. D. Challis, I. G. J. Classen, E. M. Edlund, J. Eriksson, A. Fasoli, E. D. Fredrickson, G. Y. Fu, M. García-Muñoz, T. Gassner, K. Ghantous, V. Goloborodko, N. Gorelenkov, M. Gryaznevich, S. Hacquin, W. W. Heidbrink, C. Hellesen, V. G. Kiptily, G. J. Kramer, Ph. Lauber, M. K. Lilley, M. Lisak, F. Nabais, R. Nazikian, R. Nyqvist, M. Osakabe, C. Perez von Thun, S. D. Pinches, M. Podesta, M. Porkolab, K. Shinohara, K. Schoepf, Y. Todo, K. Toi, M. A. Van Zeeland, I. Voitsekhovich, R. B. White, V. Yavorskij, ITPA EP TG, and JET-EFDA Contributors, Nucl. Fusion **53**, 104022 (2013).

[3]N. N. Gorelenkov, S. D. Pinches, and K. Toi, Nucl. Fusion **54**, 125001 (2014).

[4]Ph. Lauber, Phys. Rep. **533**, 33 (2013).

[5]A. Fasoli, S. Brunner, W. A. Cooper, J. P. Graves, P. Ricci, O. Sauter, and L. Villard, Nat. Phys. **12**, 411 (2016).

[6]L. Chen and F. Zonca, Rev. Mod. Phys. **88**, 015008 (2016).

[7]F. Zonca and L. Chen, Phys. Plasmas **21**, 072120 (2014).

[8]F. Zonca and L. Chen, Phys. Plasmas **21**, 072121 (2014).

[9]F. Zonca, P. Buratti, A. Cardinali, L. Chen, J. Dong, Y. Long, A. V. Milovanov, F. Romanelli, P. Smeulders, L. Wang, Z. Wang, C. Castaldo, R. Cesario, E. Giovannozzi, M. Marinucci, and V. Pericoli Ridolfini, Nucl. Fusion **47**, 1588 (2007).

[10]P. Buratti, P. Smeulders, F. Zonca, S. Annibaldi, M. De Benedetti, H. Kroegler, G. Regnoli, O. Tudisco, and the FTU-team, Nucl. Fusion **45**, 1446 (2005).

[11]S. V. Annibaldi, F. Zonca, and P. Buratti, Plasma Phys. Control. Fusion **49**, 475 (2007).

[12]O. Zimmermann, H. R. Koslowski, A. Krämer-Flecken, Y. Liang, R. Wolf, and TEC-team, in *Proceedings of the 32.nd EPS Conference on Plasma Physics, Tarragona, Spain, 27 June - 1 July, (2005), ECA*, Vol. 29C (EPS, 2005) pp. CD–ROM file P4.059.

[13]W. Chen, X. T. Ding, Yi. Liu, Q. W. Yang, X. Q. Ji, G. L. Yuan, Y. P. Zhang, M. Isobe, Y. B. Dong, Y. Huang, J. Zhou, Y. Zhou, W. Li, B. B. Feng, X. M. Song, J. Q. Dong, Z. B. Shi, X. R. Duan, and HL-2A Team, Nucl. Fusion **51**, 063010 (2011).

[14]R. Sabot, F. Clairet, G. D. Conway, L. Cupido, X. Garbet, G. Falchetto, T. Gerbaud, S. Hacquin, P. Hennequin, S. Heuraux, C. Honoré, G. Leclert, L. Meneses, A. Sirinelli, L. Vermare, and A. Truc, Plasma Phys. Control. Fusion **48**, B421 (2006).

[15]V. S. Udintsev, M. Goniche, G. Giruzzi, G. T. A. Huysmans, F. Imbeaux, P. Maget, X. Garbet, R. Sabot, J. L. Ségui, F. Turco, T. P. Goodman, D. Molina, H. Weisen, and

the Tore Supra team, Plasma Phys. Control. Fusion **48**, L33 (2006).

[16]C. Nguyen, X. Garbet, R. Sabot, L.-G. Eriksson, M. Goniche, P. Maget, V. Basiuk, J. Decker, D. Elbèze, G. T. A. Huysmans, A. Macor, J.-L. Ségui, and M. Schneider, Plasma Phys. Control. Fusion **51**, 095002 (2009).

[17]W. Chen, X. T. Ding, Q. W. Yang, Yi Liu, X. Q. Ji, Y. P. Zhang, J. Zhou, G. L. Yuan, H. J. Sun, W. Li, Y. Zhou, Y. Huang, J. Q. Dong, B. B. Feng, X. M. Song, Z. B. Shi, Z. T. Liu, X. Y. Song, L. C. Li, X. R. Duan, Y. Liu, and HL-2A team, Phys. Rev. Lett. **105**, 185004 (2010).

[18]N. N. Gorelenkov, H. L. Berk, E. Fredrickson, S. E. Sharapov, and JET EFDA Contributors, Phys. Lett. A **370**, 70 (2007).

[19]N. N. Gorelenkov, H. L. Berk, N. A. Crocker, E. D. Fredrickson, S. Kaye, S. Kubota, H. Park, W. Peebles, S. A. Sabbagh, S. E. Sharapov, D. Stutmat, K. Tritz, F. M. Levinton, H. Yuh, the NSTX Team, and JET EFDA Contributors, Plasma Phys. Control. Fusion **49**, B371 (2007).

[20]N. N. Gorelenkov, M. A. Van Zeeland, H. L. Berk, N. A. Crocker, D. Darrow, E. Fredrickson, G. Fu, W. W. Heidbrink, J. Menard, and R. Nazikian, Phys. Plasmas **16**, 056107 (2009).

[21]H. S. Zhang, Y. Q. Liu, Z. Lin, and W. L. Zhang, Phys. Plasmas **23**, 042510 (2016).

[22]L. Chen and F. Zonca, Nucl. Fusion **47**, S727 (2007).

[23]I. Chavdarovski and F. Zonca, Plasma Phys. Control. Fusion **51**, 115001 (2009).

[24]F. Zonca, A. Biancalani, I. Chavdarovski, L. Chen, C. Di Troia, and X. Wang, J. Phys.: Conf. Ser. **260**, 012022 (2010).

[25]I. Chavdarovski and F. Zonca, Phys. Plasmas **21**, 052506 (2014).

[26]Y. Q. Liu and Z. Lin *et al.*, Phys. Plasmas, submitted.

[27]F. Zonca, L. Chen, S. Briguglio, G. Fogaccia, A. V. Milovanov, Z. Qiu, G. Vlad, and X. Wang, Plasma Phys. Control. Fusion **57**, 014024 (2015).

[28]Z. X. Lu, F. Zonca, and A. Cardinali, Phys. Plasmas **19**, 042104 (2012).

[29]S. M. Mahajan, W. Ross, and G. L. Chen, Phys. Fluids **26**, 2195 (1983).

[30]J. P. Goedbloed, Physica **12D**, 107 (1984).

[31]C. Z. Cheng, L. Chen, and M. S. Chance, Ann. Phys. (N.Y.) **161**, 21 (1985).

[32]C. Z. Cheng and M. S. Chance, Phys. Fluids **29**, 3695 (1986).

[33] L. Chen, *Waves and Instabilities in Plasmas* (World Scientific Publication Co., Singapore, 1987).

[34] L. Chen, in *Theory of Fusion Plasmas*, edited by J. Vaclavik, F. Troyon, and E. Sindoni (Association EURATOM, Bologna, 1988) p. 327.

[35] A. H. Glasser, J. M. Greene, and J. L. Johnson, Phys. Fluids **18**, 875 (1975).

[36] D. Pfirsch and A. Schlüter, MPI/PA/7/62 Max-Planck Institut für Physik und Astrophysik, München (1962).

[37] L. Chen, R. B. White, and M. N. Rosenbluth, Phys. Rev. Lett. **52**, 1122 (1984).

[38] L. Chen, Phys. Plasmas **1**, 1519 (1994).

[39] F. Zonca and L. Chen, Phys. Rev. Lett. **68**, 592 (1992).

[40] F. Zonca and L. Chen, Phys. Fluids B **5**, 3668 (1993).

[41] W. Chen, R. R. Ma, Y. Y. Li, Z. B. Shi, H. R. Du, M. Jiang, L. M. Yu, B. S. Yuan, Y. G. Li, Z. C.Yang, P. W. Shi, X. T. Ding, J. Q. Dong, Yi. Liu, M. Xu, Y. H. Xu, Q. W. Yang, and X. R. Duan, arXiv:1611.05538 [physics.plasm-ph] (2016).

[42] R. J. Hastie, T. C. Hender, B. A. Carreras, L. A. Charlton, and J. A. Holmes, Phys. Fluids **30**, 1756 (1987).

[43] F. Zonca, S. Briguglio, L. Chen, S. Dettrick, G. Fogaccia, D. Testa, and G. Vlad, Phys. Plasmas **9**, 4939 (2002).

[44] H. Kimura, Y. Kusama, M. Saigusa, G. J. Kramer, K. Tobita, M. Nemoto, T. Kondoh, T. Nishitani, O. Da Costa, T. Ozeki, T. Oikawa, S. Moriyama, A. Morioka, G. Y. Fu, C. Z. Cheng, and V. I. Afanas'ev, Nucl. Fusion **38**, 1303 (1998).

[45] S. E. Sharapov, D. Testa, B. Alper, D. N. Borba, A. Fasoli, N. C. Hawkes, R. F. Heeter, M. Mantsinen, M. G. Von Hellermann, and contributors to the EFDA-JET workprogramme, Phys. Lett. A **289**, 127 (2001).

[46] M. Takechi, A. Fukuyama, K. Shinohara, M. Ishikawa, Y. Kusama, S. Takeji, T. Fujita, T. Oikawa, T. Suzuki, N. Oyama, T. Ozeki, A. Morioka, C. Z. Cheng, N. N. Gorelenkov, G. J. Kramer, R. Nazikian, and JT-60 Team, in *Proceedings of the 19th International Conference on Fusion Energy 2002* (International Atomic Energy Agency, Vienna, 2002) pp. CD–ROM file EX/W–6 and http://www.iaea.org/programmes/ripc/physics/fec2002/html/fec2002.htm.

[47] G. Y. Fu and H. L. Berk, Phys. Plasmas **13**, 052502 (2006).

[48] A. B. Mikhailowskii, Nucl. Fusion **13**, 259 (1973).

[49]M. Kotschenreuther, Phys. Fluids **29**, 2898 (1986).

[50]F. Zonca, L. Chen, and R. A. Santoro, Plasma Phys. Control. Fusion **38**, 2011 (1996).

[51]F. Zonca, L. Chen, R. A. Santoro, and J. Q. Dong, Plasma Phys. Control. Fusion **40**, 2009 (1998).

[52]F. Zonca, L. Chen, J. Q. Dong, and R. A. Santoro, Phys. Plasmas **6**, 1917 (1999).