Shielding of two slow co-moving test charges in collisional plasma

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Abstract. The three-dimensional far-field electrostatic potential of two co-moving point charges are considered. In contrast to the one-dimensional case, it is found that the shielding behavior remains qualitatively similar to that of a single moving charge.

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1. Introduction

The electrostatic potential in a sufficiently close vicinity of a free electron is the well-known single-particle Coulomb potential [1, 2]. Since in a plasma the number of moving charges contributing to the field at any point in space is extremely large, the combined potential field at the location of a given electron varies rapidly in time. An exact description of the fluctuating microfield near any given electron is therefore rather difficult. However, usually only the averaged, say over a time that is large with respect to the microfield timescale and a volume large enough to include a sufficiently large number of particles, field is of practical interest [1]. Because of the presence of many other electrons, the field of any individual electron in a plasma or other electronic medium is heavily shielded. The so-called Debye shielding [1, 2] greatly shortens, namely exponential decay instead of inversely proportional to the distance away from the charge, the range of the Coulomb interaction potential of a charged particle in plasma. It is one of the most important characteristics of plasmas [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. It is also closely related to the effect of charged grains in dusty plasmas [14, 15, 16], as well as satellites and re-entry vehicles in space and the ionosphere [17, 18, 19].

However, the expression (Coulomb potential with exponential decay) for Debye shielding of a charged particle has been obtained assuming that the latter is infinitesimally small and stationary, and the background plasma is in thermal equilibrium. Partial breakdown of Debye shielding can also occur for test charges in various situations, such as when it is moving or when the plasma is inhomogeneous, anisotropic, collisional, magnetized, etc. [3, 4, 5, 6, 7, 9, 11, 12, 13]. In earlier works [11, 12] by Lennart Stenflo and one of us on the electrostatic shielding of a slowly moving test charge in a plasma with collisions described by the Bhatnager-Gross-Krook (BGK) model [1, 20, 21], we found that in certain parameter regimes Debye shielding can break down and the charge can become completely unshielded along the direction of its motion. Such a phenomenon could affect theories of charged-particle collisions that invoke Debye cutoff of the interaction range. Recently, two of us [24, 25, 26] considered one-dimensional nonlinear disturbances excited by two finite moving identically charged pulses, and found that under conditions that are still not fully understood, almost perfect shielding can take place [24]. It is therefore of interest to see if two co-moving point charges in three dimensions can also become fully shielded. In this work we consider this problem by following the approach of [11]. For the cases considered, our results show that the far-field shielding behavior of two test charges remains qualitatively similar to that of a single charge.

2. Formulation

The kinetic equation for the electron distribution function $f(\boldsymbol{x}, \boldsymbol{v}, t)$ with the BGK collision operator [1, 21, 20] is

$$\partial_t f + \boldsymbol{v} \cdot \nabla f + (e/m)\nabla\phi \cdot \nabla_{\boldsymbol{v}} f = \nu(f - f_M), \tag{1}$$

where $-e$, m and $\boldsymbol{v}$ are the charge, mass, and velocity of the electrons, $\phi(\boldsymbol{x}, t)$ is the electrostatic potential, ν is the electron-neutral collision frequency, and f_M is the local Maxwellian distribution, where the density, mean velocity, and temperature are determined from the respective velocity moment integrals. Compared to the Fokker-Planck and other more realistic models [1, 22, 23] for charged particle collisions, the BGK model tends to overestimate the effect of collisional relaxation or transport since the participating particle becomes thermalized after a single collision event. The BGK model is nevertheless frequently invoked because of its simplicity and the observation that the expressions for the major transport coefficients such as the viscosity and thermal conductivity, in particular their dependence on the plasma parameters, derived from it are qualitatively similar to that from the more realistic models [21]. However, the full BGK model involves an integral-partial differential equation and is tedious to handle [11, 21]. A further simplification is realized in the popular reduced-BGK, or the Krook, model, in which the mean velocity and temperature in the Maxwellian distribution are fixed to the initial, or background, values. As a result, only the number, but not the momentum and energy, of the particles in a collision event is conserved. It has been shown that for problems not involving transport, in particular that of test-charge shielding, the Krook model yields results that are remarkably close to that from the full BGK model that conserve particle number, momentum, and energy in collisions [1, 11, 21].

For collisions between the electrons and neutral particles, we shall use the Krook collision model [1, 11, 21]. We assume for simplicity that the two test charges are of the same velocity, so that the distance between them does not change. The electrostatic potential $\phi(\boldsymbol{x}, t)$ of the plasma response is then given by the Poisson's equation

$$\varepsilon_0 \nabla^2 \phi(\boldsymbol{x}, t) = e n_1 - q_1 \delta(\boldsymbol{x} - \boldsymbol{v}_0 t) - q_2 \delta(\boldsymbol{x} - \boldsymbol{x}_0 - \boldsymbol{v}_0 t), \tag{2}$$

where $n_1 = n - n_0$ is the electron density perturbation, n_0 is the ion density, $-e$ is the electron charge, $q_j = Z_j e$ is the charge of the test particle $j = 1, 2$, $\boldsymbol{v}_0$ is the velocity of the test charges, and $\boldsymbol{x}_0$ the distance between them. We have assumed that $v_0 \ll c$, where c is the light speed, so that electromagnetic effects can be neglected.

The plasma ions are unperturbed by the test charges, so that n_0 is constant. The response of the plasma electrons to the two moving and non-interacting point charges is small and can be assumed to be linear [5, 7, 11, 12, 13]. The solution $\phi(\boldsymbol{x}, t)$ then follows straightforwardly from that [11] of a single test charge and can be written as

$$\phi(\boldsymbol{x}, t) = \frac{1}{8\pi^3 \varepsilon_0} \int \frac{e^{i\boldsymbol{k}\cdot(\boldsymbol{x} - \boldsymbol{v}_0 t)}[q_1 + q_2 e^{i\boldsymbol{k}\cdot\boldsymbol{x}_0}]}{k^2 D(\boldsymbol{k}, -\boldsymbol{k}\cdot\boldsymbol{v}_0)} d\boldsymbol{k}, \tag{3}$$

where the dielectric function of the plasma response is [11]

$$D(\boldsymbol{k}, \omega) = 1 + \frac{1}{k^2 \lambda_D^2} - \frac{1}{k^2 \lambda_D^2} \frac{\omega \mathcal{F}/n_0}{1 - i\nu \mathcal{F}/n_0}, \tag{4}$$

with $\mathcal{F} = \int d\boldsymbol{v} f_0/(\omega + \boldsymbol{k}\cdot\boldsymbol{v} + i\nu)$. The second term is from the adiabatic electron response that when alone gives rise to the Debye shielding potential.

We can see in (3) that except for the phase shift $\boldsymbol{k} \cdot \boldsymbol{x}_0$ due to its location, the contribution of the second test charge is similar to that of the first, which is at the origin $(0, 0)$. Such a phase shift can nevertheless lead to interference of the plasma oscillations excited by the two test charges and thus affect the behavior of the electrostatic potential. For convenience of comparison with the single test-charge case, in the following we shall use the same parameters and limits as in [11] wherever suitable.

3. Weak collisions

We first consider the weak collision limit $\nu \ll kv_t$ and $r \gg v_0/\nu$, or $\lambda_m \gg r \gg (v_0/v_t)\lambda_m$, where v_t is the thermal speed and $\lambda_m = v_t/\nu$ is the electron mean free path. The potential (3) then becomes

$$\phi \sim \sum_{j=1,2}\left[\frac{q_j}{4\pi\varepsilon_0 r_j}e^{-r_1/\lambda_D} + \frac{q_j\lambda_D^2 v_0\cos\theta_j}{\sqrt{2\pi}\pi\varepsilon_0 v_t r_j^3}\left(1-\frac{\sqrt{\pi}\nu^* r_j}{2\sqrt{2}v_t}\right)\right], \tag{5}$$

where $\nu^* = (\pi/2-1)\nu$, λ_D is the Debye length, $r_1 = |\boldsymbol{x}-\boldsymbol{v}_0 t|$ and $r_2 = |\boldsymbol{x}-\boldsymbol{x}_0-\boldsymbol{v}_0 t|$ are the distances of two test charges from the observer, and θ_j is the angle between $\boldsymbol{r_j}$ and $\boldsymbol{v}_0$. That is, the distance of the observer from the test charges is much larger than λ_D but much less than λ_m, and the effect of collisions (the second term in the parenthesis) is small.

When the observer is not too far away from the test-charge pair, the potential is dominated by the first, or the Debye shielding, term of (5). At long distances, however, the second term corresponding to the wakefield behind the test charges becomes important since it is not shielded. In fact, we see that the wake potential falls off as inverse third power of the distance from the observer. With a single test charge $q_1 = -e$, the wake potential is positive behind ($\pi/2 < \theta < 3\pi/2$) it. Comparing to the single test charge case, the angular dependence of the positive potential region in the two test charge case is more complicated since it also involves r_0 and r_1.

It is convenient to introduced the normalized quantities $\boldsymbol{r} = \boldsymbol{r}/\lambda_D$, $t = \omega_p t$ $\boldsymbol{v}_0 = \boldsymbol{v}_0/v_t$, $n = n\lambda_D^3$, and $\nu = \lambda_D\nu/v_t$, where the original dimensional quantities are on the right hand sides and v_t is the electron thermal speed. For the numerical evaluation, we assume $v_0 = 0.005$ and $\nu = 0.01$. For convenience of discussion, in the figures we shall use a frame moving at the constant velocity $\boldsymbol{v}_0 = v_0\hat{\boldsymbol{e}}_x$ of the test charges. Thus, we can assume that the first charge is at the origin and the second charge at $\boldsymbol{r_0}$.

Fig. 1 shows the far-field ($r \gg \lambda_D$) potential (a) of a single test charge, and (b) and (c) of two test charges with different $|\boldsymbol{r_0}|$ values. As expected, the potential depends strongly on $\boldsymbol{r_0}$ as well as the angle between the direction of the test charge motion and the observer. When the distance r_0 between the two test charges is small (b), the potential distribution is similar to that for the single test charge (a): the potential at the front and back of the moving test charges are antisymmetric. However, with two test charges separated at the larger distance, this symmetry is broken, as can be seen in (c). In fact, as r_0 increases, the potential in front of the test charges becomes increasingly

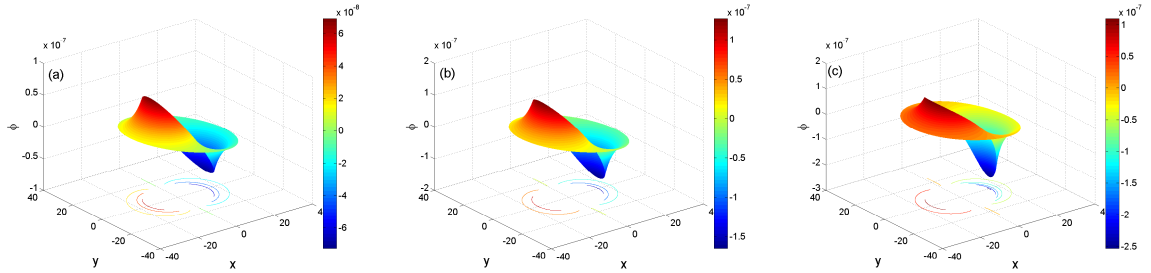

Figure 1. The far-field ($r > 15$) potential for (a) a single negative test charge at $\boldsymbol{r} = (0, 0)$, and a second negative test charge at (b) $\boldsymbol{r} = (1, 0)$ and (c) $\boldsymbol{r} = (3, 0)$. In order to show the complete space, the region around $(0, 0)$ is also included. However, the potential in this region ($r < 15$) is left blank since the solutions are not valid there. The same applies to the other figures.

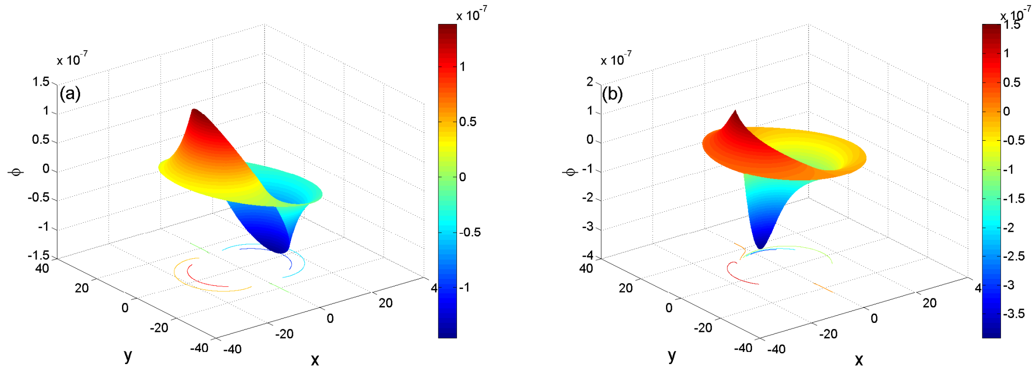

Figure 2. The potential of two parallel moving charges for the separation distances (a) $r_0 = 1$ and (b) $r_0 = 5$. That is, $\boldsymbol{r_0}$ is in the transverse (y) direction.

lower. That is, the plasma in the front becomes less perturbed, but the wake is more like that of the single test charge. This indicates that unfavorable (favorable) phase mixing of the plasma waves independently excited by the two test charges took place at the front (back) of the pair.

It may also be of interest to consider the potential distribution of two parallel co-moving test charges, i.e., $\boldsymbol{r_0}$ is along the y direction, perpendicular to the direction of propagation. Fig. 2 shows that the affected region widens as $|\boldsymbol{r_0}|$ becomes smaller. Moreover, the potential becomes smaller when the observation angle is decreased. With the charge separation distance r_0 increasing, the locations of potential peaks and valleys shift towards the $\boldsymbol{r_0}$ direction. Since the second test charge is nearer to the observer, its influence is also somewhat larger than the first test charge.

When the second test charge (i.e., a positron) is positive, the behavior of the potential distribution is somewhat more complicated. Fig. 3 shows several typical cases. As expected, the far-field potential is generally of much smaller magnitude and can even vanish locally. However, its overall profile remains qualitatively similar to that of two negative charges. This can be expected since besides modifying the charge distribution,

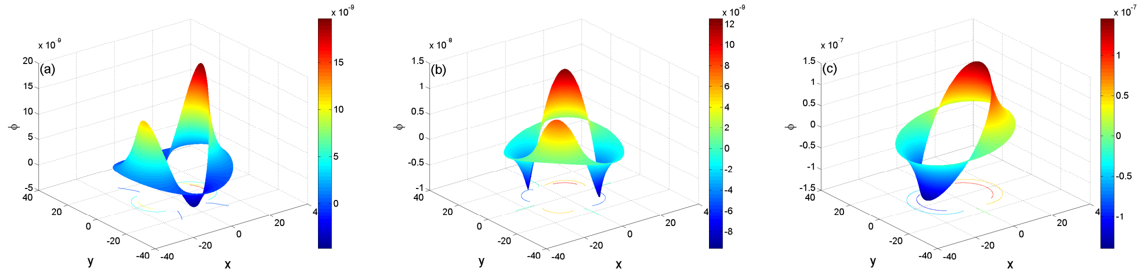

Figure 3. The potential of two co-moving charges for $r_0 = 1$. (a) The second charge is positive and $\boldsymbol{r}_0 = (1, 0)$. (b) The second charge is positive and $\boldsymbol{r}_0 = (0, 1)$. (c) Two positive test charges and $\boldsymbol{r}_0 = (0, 1)$.

the polarity of the second point charge only shifts the phases of the excited oscillations by π (i.e., exactly antisymmetric). This behavior also resembles that of appropriately changing the separation distance between the charges, such that the relative phases of the plasma waves excited independently by the latter are shifted by π.

4. Strong collisions

We now consider the high collision-frequency regime, namely $\nu \gg kv_t \gg \boldsymbol{k} \cdot \boldsymbol{v}_0$, or $r_j \gg \lambda_m \gg v_0/\nu$.

First we look at the limit of very strong collisions, namely $\nu v_0 r \gg v_t^2$. One easily obtains $\phi \sim \sum_{j=1,2} q_j/4\pi\varepsilon_0 r_j$, which we see behaves like the long-ranged Coulomb potential of two stationary charges in vacuum. This isotropic result can be expected since in this limit the thermal and test-charge velocities are to small to realize Debye shielding and test-charge motion induced anisotropy. However, this regime is relatively small and valid only when the observer is sufficiently far away.

Next, in the limit $\nu v_0 r \ll v_t^2$, we have

$$\phi \sim \sum_{j=1,2} \left(\frac{q_j}{4\pi\varepsilon_0 r_j} e^{-r_j/\lambda_D} - \frac{q_t \nu v_0}{4\pi\varepsilon_0 \omega_p^2 r_j^2} \cos\theta_j \right). \tag{6}$$

Fig. 4 shows the far-field potential of the test charges for $\nu = 1$ and different charge separation distances $\boldsymbol{r}_0$. The potential falls off as the inverse square of the distance from, and is positive in front of, the test charges. It also differs from the preceding case in that the positive potential is now in the front. However, as both cases depend on $\cos\theta$, their lateral potential profiles are similar. That is, the essential difference is that in the weak collisions limit the far-field potential falls off as the inverse third power, and in the strong collision limit as the inverse square, of the distance to the observer. Furthermore, to an observer who is sufficiently far away, the potential of two point test charges are qualitatively similar (but not identical) as that for a single test charge. However, it should be cautioned that the results here may not hold if the charges are of finite dimension [17], too fast [19], or if the background plasma response is nonlinear [24].

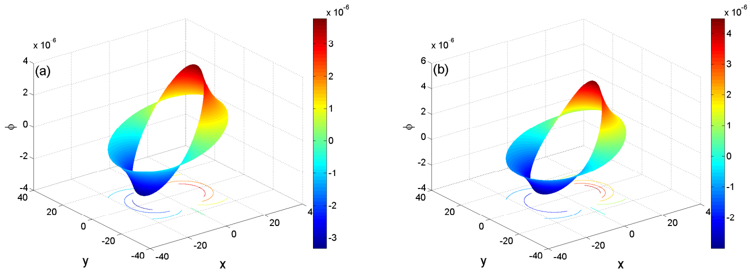

Figure 4. The potential distribution of two inline test charges in highly collisional plasma, for (a) $r_0 = 1$, (b) $r_0 = 3$, and (c) $r_0 = 5$. There is no shielding, and the decay around the axis in the wake of the charges scales as inverse square of the distance away from the pair. But there is exponential Debye shielding in the transverse direction.

5. Discussions

In this paper we have considered the potential of two or more co-moving test charges in an equilibrium plasma. For simplicity, we have considered slowly moving ($v \ll v_t$) test charges. It is found that, except for some details, the far-field potential is qualitatively similar to that of a single moving test charge. It is also found that the far-field potential of an electron-positron pair is weak but nonzero as long as their separation distance is finite. The results here can be understood in terms of the weak disturbance (namely the plasma waves) excited by the point charges in the plasma and spread by the plasma waves whose phase velocities are less than the electron thermal velocity. To an observer far away, except in the magnitude the effects of one or two or more charges cannot be very much different unless there is significant coherent phase interference among the disturbances from the different test charges. But unlike in one-dimensional systems, such coherent phase interference is highly unlikely in three-dimensional systems. Moreover, the conclusions here do not apply to the near-field (not considered here) potential distribution, as can already be observed from our results when the observer is not too far (but still within our approximations) from the test charges. There the potential distribution can be rather different for different arrangements of the test charges. It should also be pointed out that if the test charges move near the electron thermal speed, electron concentration associated with strong Landau damping of the excited plasma oscillations in the wakefield can take place and the behavior of its far-field potential can become quite different.

Besides being relevant to the physics of charged-particle shielding [11, 12, 13] and kinetic behavior of plasmas [22, 23], our results on the far-field potential of two test charges should also be of interest in practical applications such as the wake of satellites in space and reentry bodies in the upper ionosphere [17, 19], particle acceleration schemes based on the wakefield of charged particle beams [27, 28, 29, 30, 31], self-organization of dust grains in plasmas [14, 15, 16], etc. In such environments the plasma particles involved are considerably more coherent and the non-shielding effects are much more likely to accumulate.

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References

[1] Clemmow P C and Dougherty J P 1969 *Electrodynamics of Particles and Plasmas* (London: Addison-Wesley)
[2] Chen F F 1974 *Introduction to Plasma Physics* (New York: Plenum) p 249
[3] J. Neufeld and R. H. Ritchie 1955 *Phys. Rev.* **98** 1635
[4] Joyce G R and Montgomery D 1967 *Phys. Fluids* **10** 2017
[5] Montgomery D, Joyce G R and Sugihara R 1968 *Plasma Phys.* **10** 681
[6] Boris J and Roberts K V 1969 *J. Comp. Phys.* **4** 552
[7] Cooper G 1969 *Phys. Fluids* **12** 2707
[8] James C R and Vermeulen F 1970 *Can. J. Phys.* **48** 349
[9] Laing E W, Lamont A, and Fielding P J 1971 *J. Plasma Phys.* **5** 441
[10] Kan J R 1971 *J. Plasma Phys.* **6**, 223
[11] Stenflo L, Yu M Y and Shukla P K 1973 *Phys. Fluids* **16** 450
[12] Stenflo L and Yu M Y 1973 *Phys. Scr.* **8** 301
[13] Yu M Y, Tegeback R and Stenflo L 1973 *Z. Phys.* **264** 341
[14] Chen Z, Yu M Y and Luo H 2001 *Phys. Scr.* **64** 476
[15] Stoffels E, Stoffels W W, Kersten H, Swinkels G H P M, and Kroesen G M W 2001 *Phys. Scr.* **T89**, 168
[16] Denysenko I, Yu M Y, Stenflo L and Azarenkov N A 2005 *Phys. Plasmas* **12** 042102
[17] Al'pert V A L, Gurevich A V and Pitaevskii L 1963 *Sov. Phys. Uspekhi* **6** 13
[18] Liu V C 1965 *Nature* **208** 883
[19] Gurevich A V, Pitaevsky L P and Smirnova V V 1969 *Space Sci. Rev.* **9** 805
[20] Bhatnagar P L, Gross E P and Krook M 1954 *Phys. Rev.* **94** 511
[21] Huang K 1987 *Statistical Mechanics* 2nd Ed (New York: Wiley)
[22] Thompson W B 1964 *An Introduction to Plasma Physics* (Oxford: Pergamon)
[23] Perkins F 1965 *Phys. Fluids* **8** 1361
[24] Wang Y and Yu M Y 2010 *Phys. Plasmas* **17** 112116
[25] Wang Y M, Yu M Y, Chen Z Y 2011 Phys. Scr. 84 025501
[26] Wang Y M, Yu M Y, Chen Z Y and Lu G M 2013 *Laser Part. Beams* **31** 155
[27] Shchelkunov S V, Marshall T C, Hirshfield J L, Babzien M A and LaPointe M A, 2006 *Phys. Rev. ST Accel. Beams* **9** 011301
[28] Norreys, P A 2009 *Nature Photon.* **3** 423
[29] Jiang B, Jing C, Schoessow P, Power J and Gai W 2012 *Phys. Rev. ST Accel. Beams* **15** 011301
[30] Malka V 2012 *Phys. Plasmas* **19** 055501
[31] Hooker S M 2013 *Nature Photon.* **7** 775